Efficient Methods for Solving Power System Operation Scheduling Challenges: The Thermal Unit Commitment Problem with Staircase Cost and the Very Short-term Load Forecasting Problem

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Motivation

Determines	the optimal schedule for generating electricity.
Short time	available for obtaining an optimal or near-optimal solution.
Family	of NP-Hard combinatorial problems.
Heart	of scheduling power generators worldwide.
Daily	task for electric system planners.

Background

Two avenues for addressing the complexities of the UCP

Reformulation with tight and compact models

- A tighter model reduce the solver's search space.
- A compact model uses fewer constraints and variables.
- Morales-España et al. (2013) were pioneers.
- Knueven et al. (2021) studied the trade-off between T&C models.
- **Use only off-the-shelf solvers.**

Matheuristics methods

- Fazur et al. (2014) used the Local Branching (LB) original version.
- Dupin and Talbi (2018) used Relax and Fix (R&F), and Relaxation-Induced Neighborhood Search (RINS).
- Santos et al. (2021) incorporate a variant of the Feasibility Pump (FP) and LB.
- Harjunkoski et al. (2022) introduced an efficient constructive method based on R&F.

Related work

Research	UCP feature	Objective Function	Start-up Cost	Generation Limits	Ramp up / Ramp down	Startup / Shutdown Rams	Matheuristic method used
Fazur et al. (2014)	Thermal MIQP 1 day ahead	Quadratic	Cold and hot start-up	yes	yes	no	Local branching´s original version and cost linearized.
Saboia and Diniz (2016)	Thermal Stochastic 1 day ahead	Linear	Exponential cost	yes	yes	no	A variant of LB that incorporates violated flow limits in each node of the LB tree. Tested in one instance.
Dupin and Talbi (2016)	Thermal Discrete Real-time	Linear	FIxed cost	discretized	no	no	Uses RINS and LB, among other strategies, to define neighborhoods.
Dupin and Talbi (2018)	Thermal Discrete T&C model Real-time	Linear	Fixed cost	discretized	min-stop ramping	no	Several constructives methods running in parallel. Kernel Search is only used as a concept in two constructives.
Santos et al. (2021)	Hydro-thermal 7 days ahead	Linear	Fixed cost	yes	yes	yes	Use the LB version from Saboia and Diniz and a variant of Feasibility Pump matheuristic.
Harjunkoski et al. (2021)	Thermal	Linear	Fixed cost	yes	yes	no	A 'Relax and Fit' method that selects the variables to fix with a selection rule and introduces them into the solver as a warm start.
Our Research	Thermal T&C model 7 days ahead	Staircase cost with valid inequalities	Variable start-up cost	yes	yes	yes	A new constructive method HARDUC. Four variants of LB (applying RCL and soft-fixing concepts) A variant of Kernel Search. A new T&C with staircase production cost.

Objective

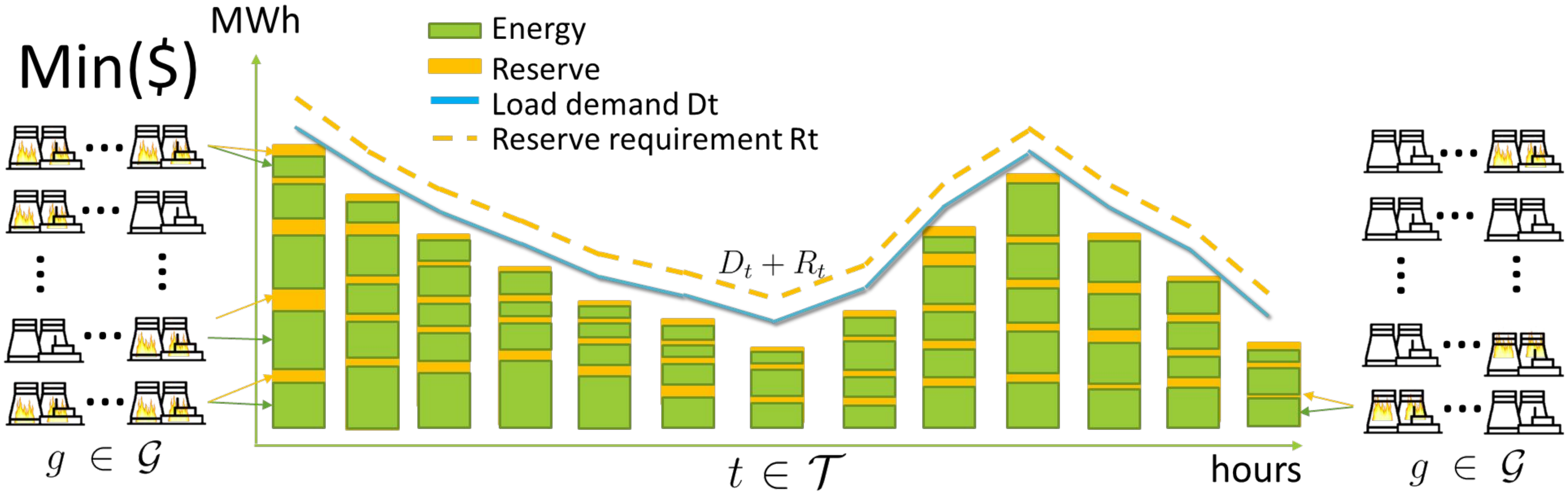
To improve the efficiency of operation schedules in power systems by **reducing computational runtime and enhancing solution accuracy**, specifically focusing on the thermal unit commitment problem with a staircase cost function.

Hypothesis

By employing custom matheuristic methods, including **Relax&Fix**, **Local Branching**, and **Kernel Search** approaches that **leverage the inherent mathematical structure** of our problem, it is possible to enhance the efficiency and accuracy of energy system operation scheduling.

Problem statement

The Unit Commitment Problem (UCP) is a family of problems that optimizes power generation schedules to meet electricity demand at the lowest cost while considering technical and operational constraints.



Mathematical model

Parameters

Generation costs

C_g^{R} Fixed cost

C_g^{l} Staircase energy cost

C_{gs}^{S} Variable startup cost

C_g^{SD} Shutdown cost

System requirements

De_t Load demand

R_t Reserve requirement

Generator technical constraints

$\overline{P}_g$, $\underline{P}_g$ Generation Limits

RD_g, RU_g Ramp up Ramp down

SD_g, SU_g Startup ramp Shut down ramp

UT_g, DT_g Minimum uptime Minimum downtime

Mathematical model

Decision variables

- **Binary** decision variables: commitment, start-up, and shut-down.

We have identified this binary variable as dominant over the other variables

$$\boxed{u_{gt}} \qquad v_{gt} \qquad w_{gt}$$

- **Continuous** decision variable: output of each generator (MW).

$$\boxed{p_{gt}}$$

Mathematical model

Mixed-Integer Linear Programming model

Fixed cost | Staircase energy cost | Variable startup cost | Shutdown cost

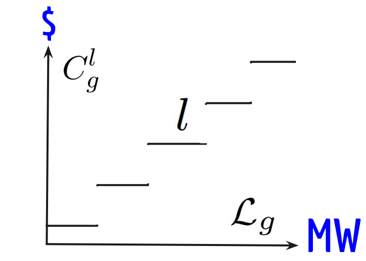
Objective function

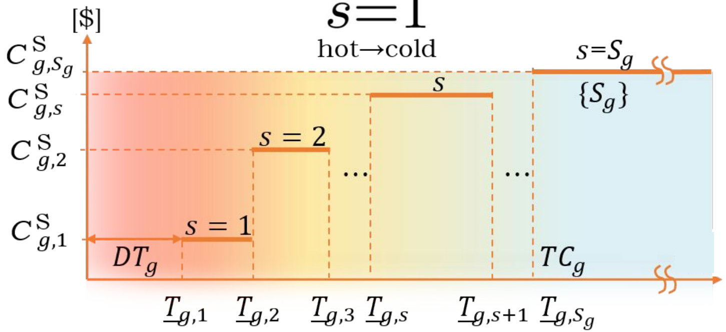
$$\min \sum_{t\in\mathcal{T}} \sum_{g\in\mathcal{G}} C^{\mathrm{R}}_{g} u_{gt} + c^{\mathrm{p}}_{gt} + c^{\mathrm{SU}}_{gt} + c^{\mathrm{SD}}_{gt}$$

$$\sum_{l\in\mathcal{L}_g} C^{l}_{g} p^{l}_{gt} = c^{\mathrm{p}}_{gt}$$

$$c^{\mathrm{SU}}_{gt} = \sum_{s=1}^{|\mathcal{S}_g|} C^{\mathrm{S}}_{g} \delta_{gts}$$

$$c^{\mathrm{SD}}_{gt} = C^{\mathrm{SD}}_{g} w_{gt}$$





time

Morales-España et al. (2013)

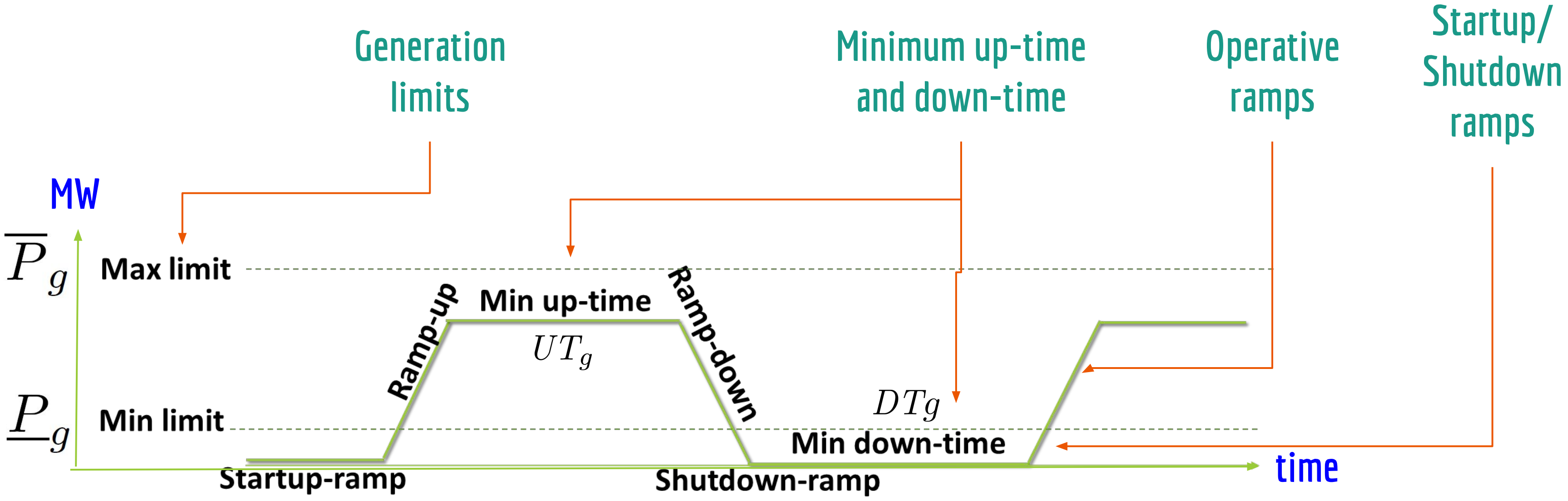
Mathematical model

Mixed-Integer Linear Programming model

Generations constraints

Logical constraints

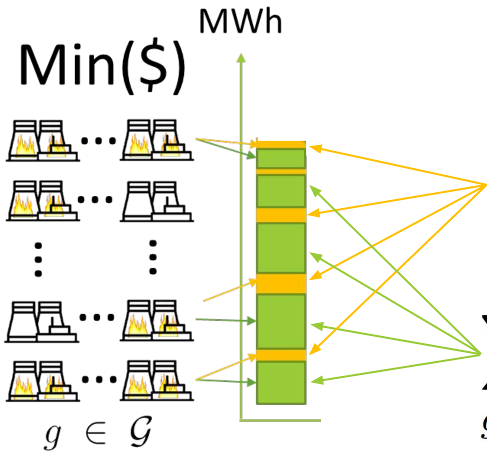
$$u_{g,t} - u_{g,t-1} = v_{g,t} - w_{g,t}$$



Mathematical model

Mixed-Integer Linear Programming model

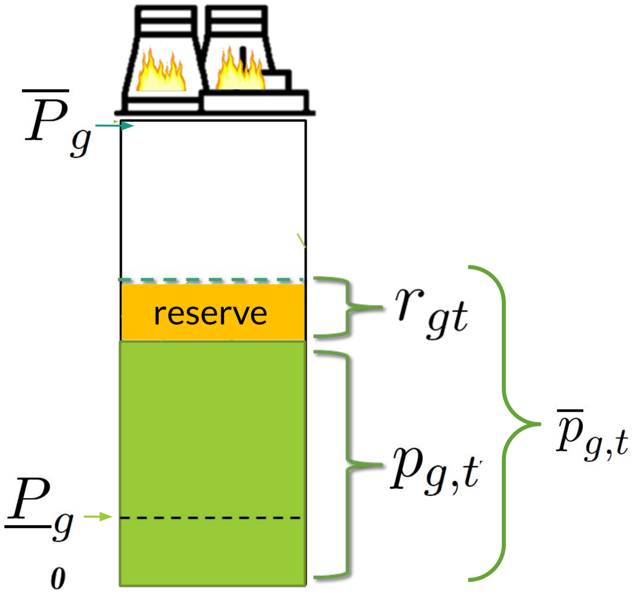
Load and reserve demand
System constraints



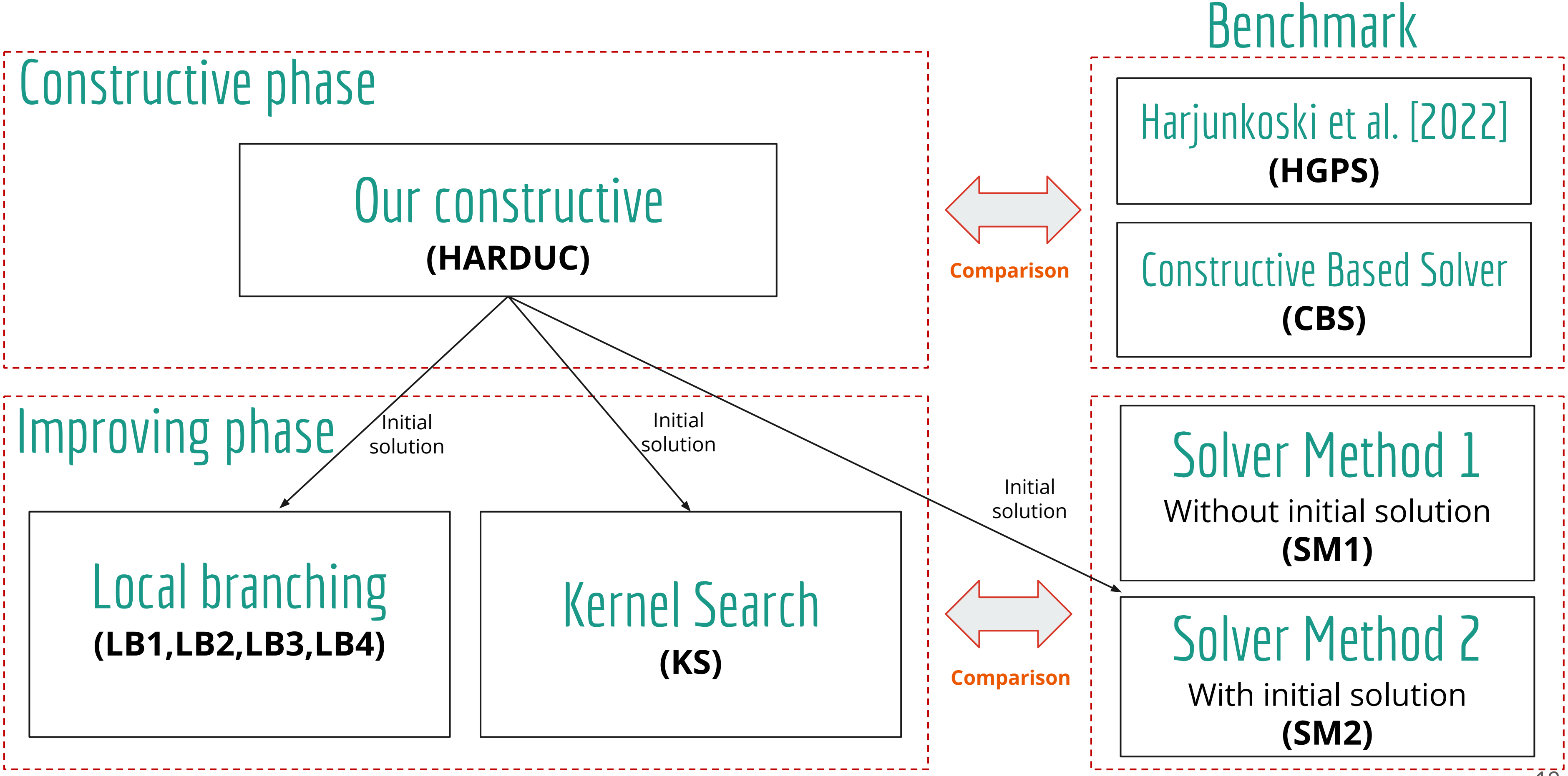
$$\sum_{g \in \mathcal{G}} r_{g,t} \geq R_t$$

$$\sum_{g \in \mathcal{G}} p_{g,t} = De_t$$

$$\sum_{g \in \mathcal{G}} \overline{p}_{gt} \geq De_t + R_t$$



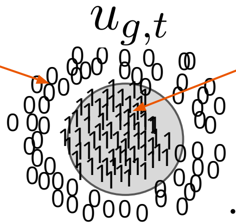
Research methodology



Phase constructive

Our **HARDUC** constructive method was compared to the **HGPS** introduced by Harjunkoski et al. (2022)

fixing variables HARDUC



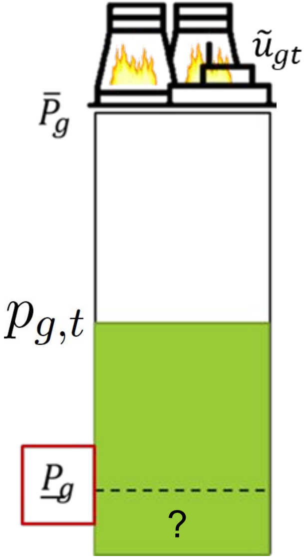
fixing variables HGPS

... solve sub-MILP

Constructive algorithm

(Relax&Fix)

1. Solving the UCP problem as a Linear Relaxation (LR).
2. Identifying the commitment variables to be fixed using the **HARDUC** or **HGPS**.
3. Fixing the commitment variables identifying and solving the sub-MILP.



Our **HARDUC** Fixing Rule

$$u_{gt} = \mathbf{0}:\ \tilde{u}_{gt} \cdot p_{gt} = 0$$

generators probably unassigned

The Fixing Rule of HGPS

$$u_{gt} = \mathbf{1}:\ \tilde{u}_{gt} \cdot p_{gt} \geq \underline{P}_g$$

generators probably assigned

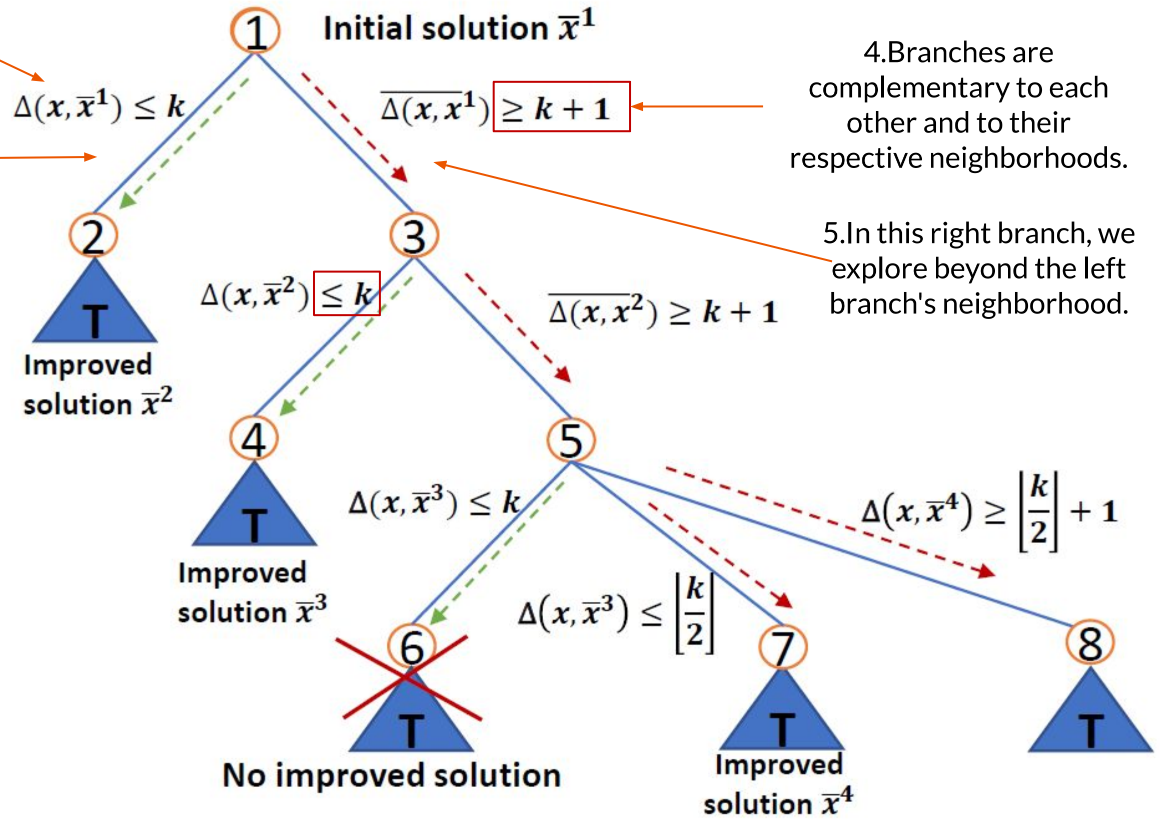
Local Branching: Improving phase

2. A local branching constraint is added to MILP.

3. In this branch, an complete neighborhood is explored.

4.Branches are complementary to each other and to their respective neighborhoods.

5.In this right branch, we explore beyond the left branch's neighborhood.



Representation of branching in the LB method, based on Fischetti and Lodi (2006).

- External local branching tree node
- Solver invocation
- Emulating local search
- Adding LBC to MILP
- Reducing search space
- Simplifying the problem

Local Branching: Improving phase

Restricted Candidate List (RCL)

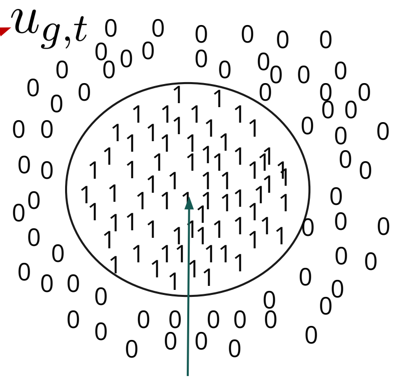
$$u_{gt}\colon \tilde{u}_{gt} \cdot \tilde{p}_{gt} < \underline{P}_g$$
$$\cap\ \tilde{u}_{gt} \cdot \tilde{p}_{gt} \neq 0$$

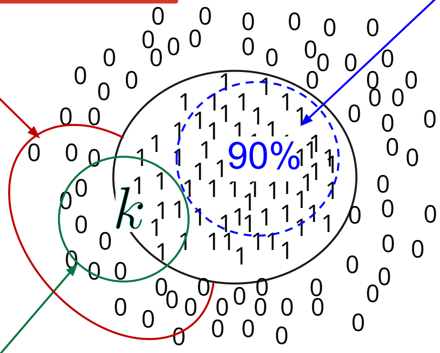
Soft-fixing

$$\sum_{j \in BS} \bar{u}_j u_j \geq \lceil 0.9 \sum_{j \in BS} \bar{u}_j \rceil$$

First solution

We use the commitment binary variable in method.





Binary support

$$BS = \{u_{g,t}\} : u_{g,t} = 1$$

Local Branching Constraint

Fischetti and Lodi (2006)

$$\Delta(x, \bar{x}) = \sum_{j \in BS} (1 - u_j) + \sum_{j \in \overline{BS}} u_j \leq k$$

(1 ⇨ 0) (0 ⇨ 1)

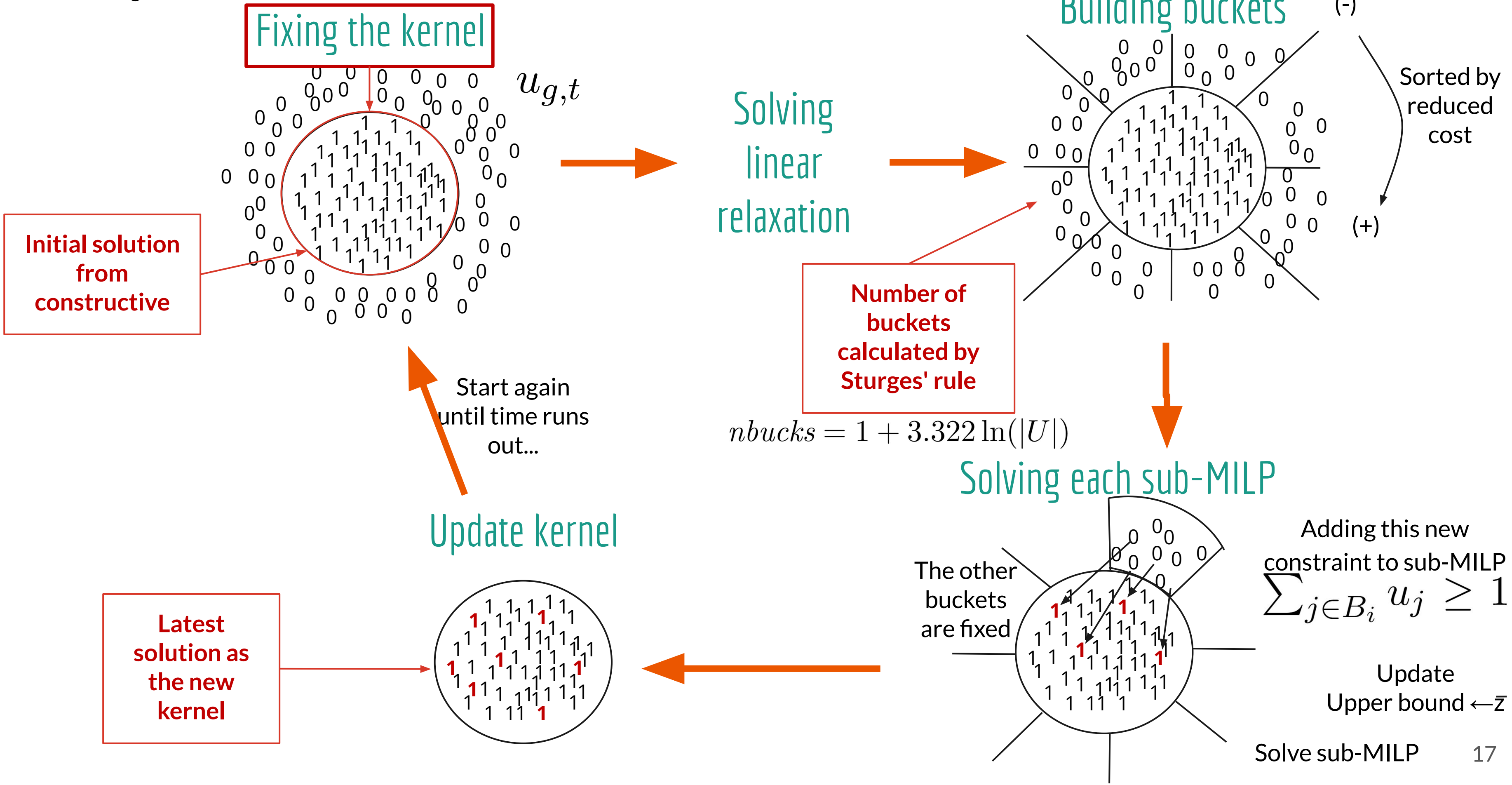
Local Branching: Improving phase

Local Branching variations

Local branching version	Soft-Fixing	RCL from Harjunkoski et al. (2022) rule	RCL from negative reduced cost	Use commitment variable dominant
LB1	✓	✓		✓
LB2		✓		✓
LB3				✓
LB4	✓		✓	✓

Kernel Search: Phase of improving

Base on Angelelli et al. 2010

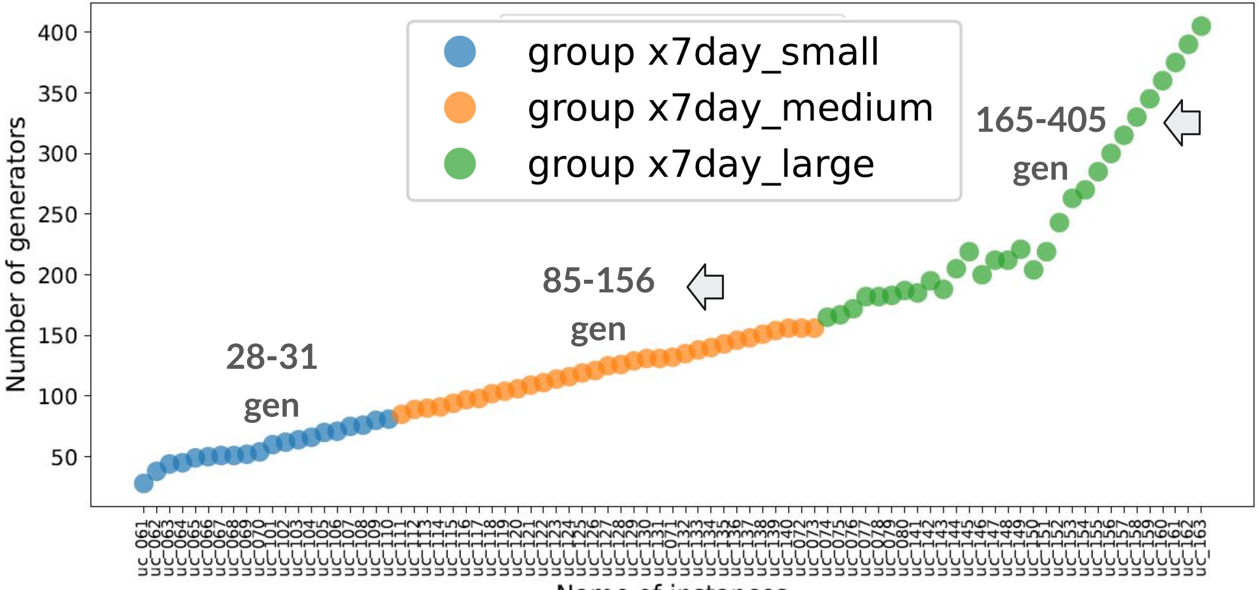


Research questions

HARDUC	Does having information about the unassigned units in a solution help in obtaining feasible solutions more quickly in our UCP?
LB1-2,4	Does the combination of RCL (Restricted Candidate List) and Soft Fixing concepts enhance the performance of the LB algorithm in the search for optimal solutions in our UCP?
KS	Does applying the Kernel Search method to UCP lead to an improvement in the initial solution to achieve high-quality solutions in our UCP?
SOLVER	Can these matheuristic methods obtain better and faster solutions in our UCP than using the solver alone?

Experimental work

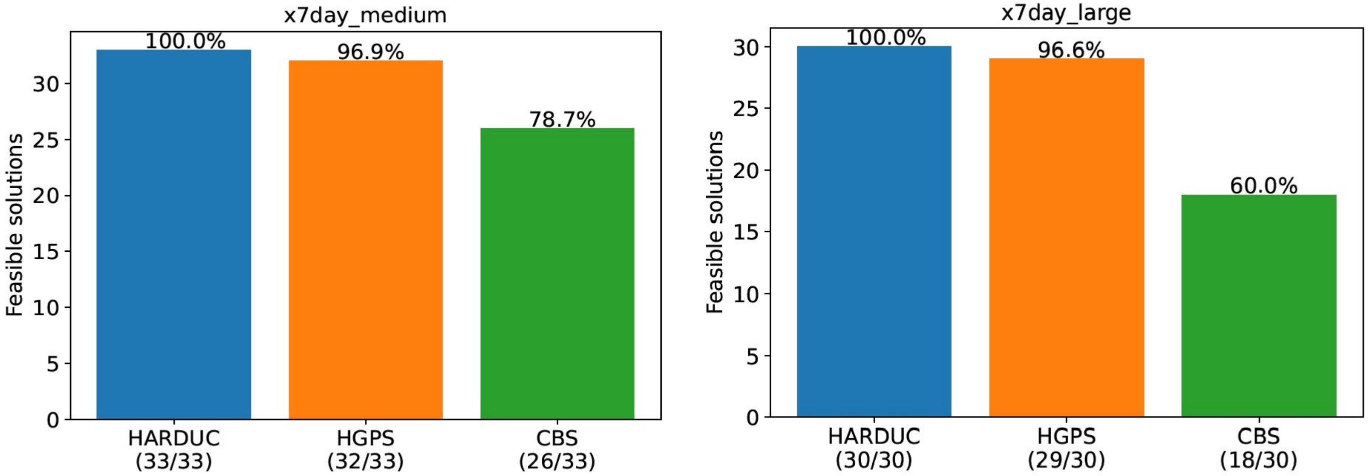
- 20 instances from Morales-España et al. [2013]
- 63 instances generated by random parameter combinations.
- All instances running at time limit: 4000 and 7200 seconds
- Periods of planning horizon: t=160
- CPLEX parameters: time_limit=7200, symmetry_detection=0, emphasis_mip=feasibility, strategy_heuristic_freq=50, tolerances_mipgap=1E-5, mip_strategy_file=disk and compress.
- PC 64-bit , 64GB of RAM , 2.50 GHz Intel(R) i7(R) 11700 CPU, 65W.



Results

Number of feasible solutions

Obtained by constructive methods



HARDUC
Our method

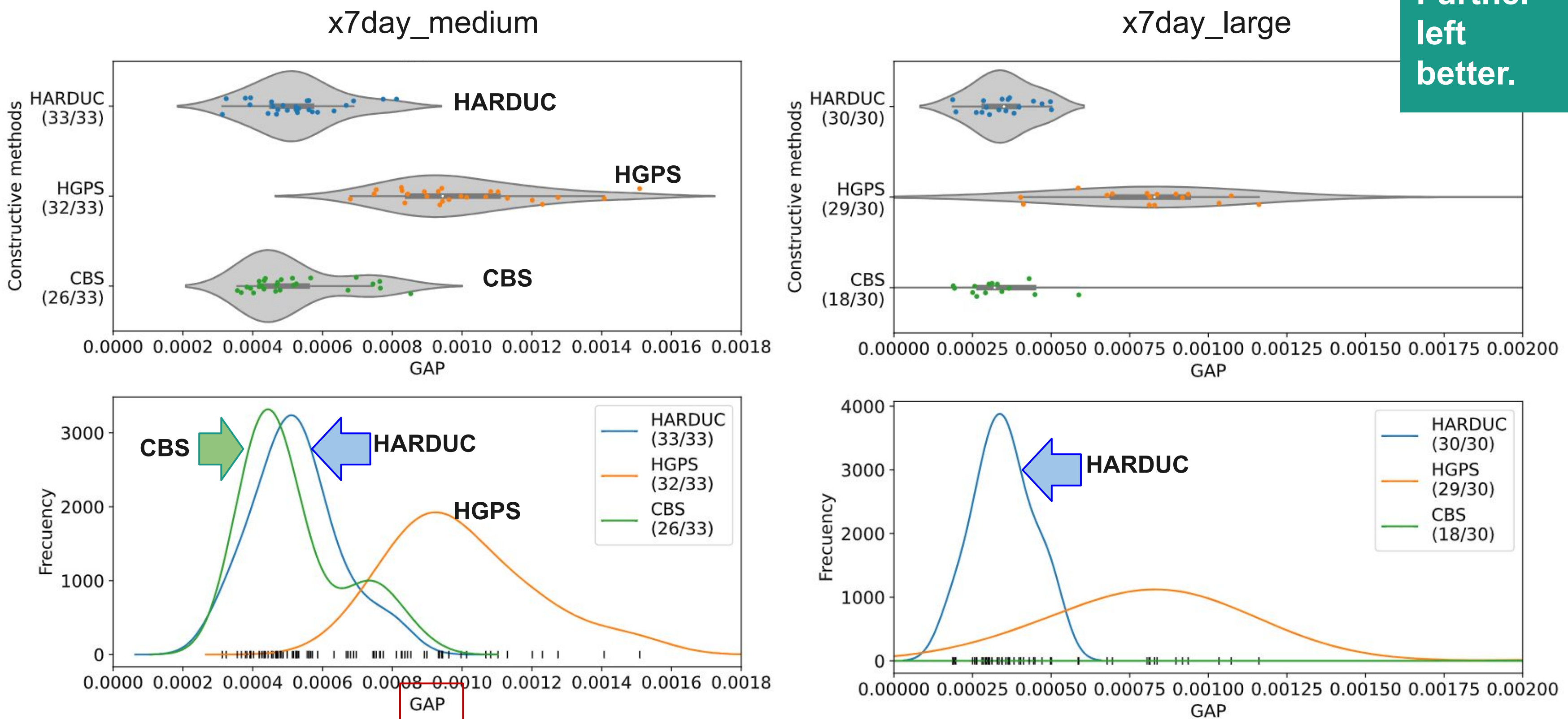
HGPS
Harjunkoski et al. (2022)

CBS
Solver

Results: HARDUC constructive

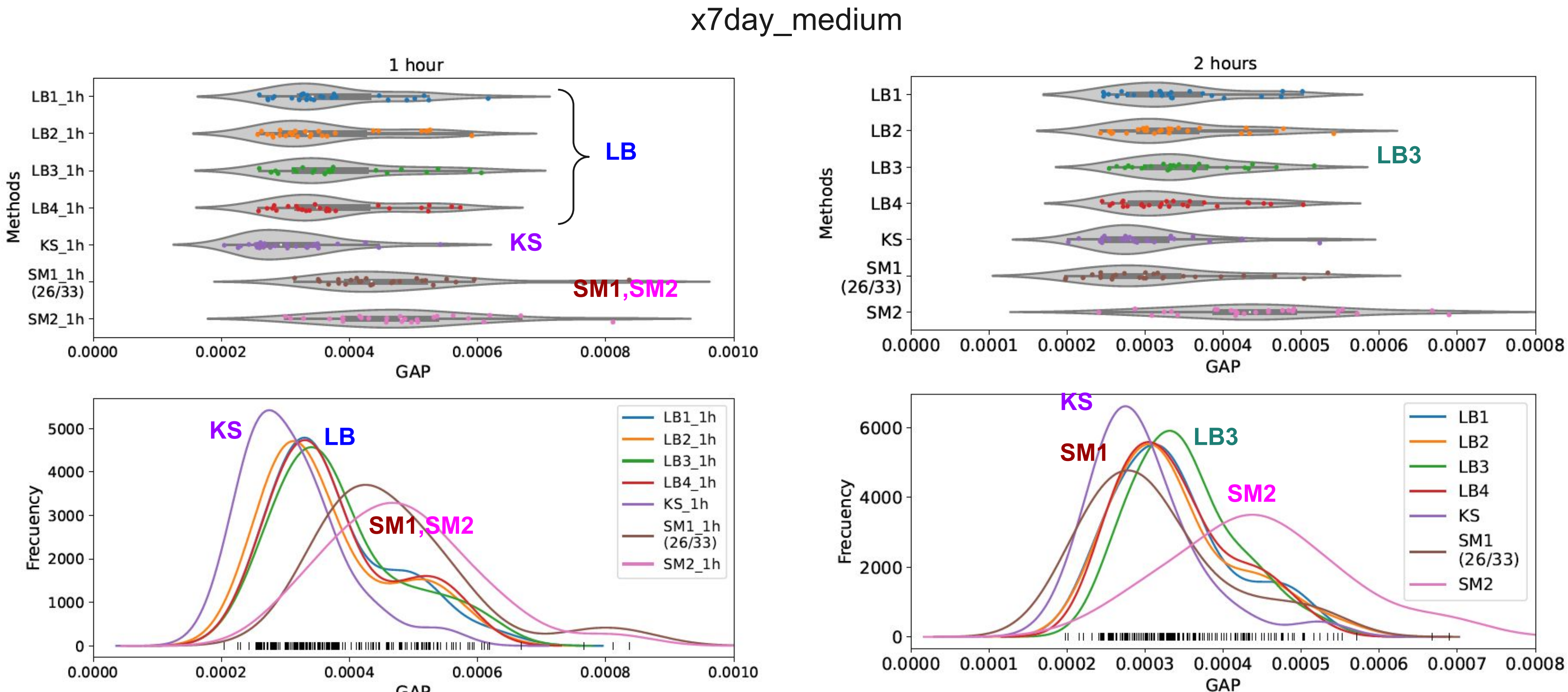
Relative optimality gap distributions of constructive methods in the instances medium and large.

Further left better.



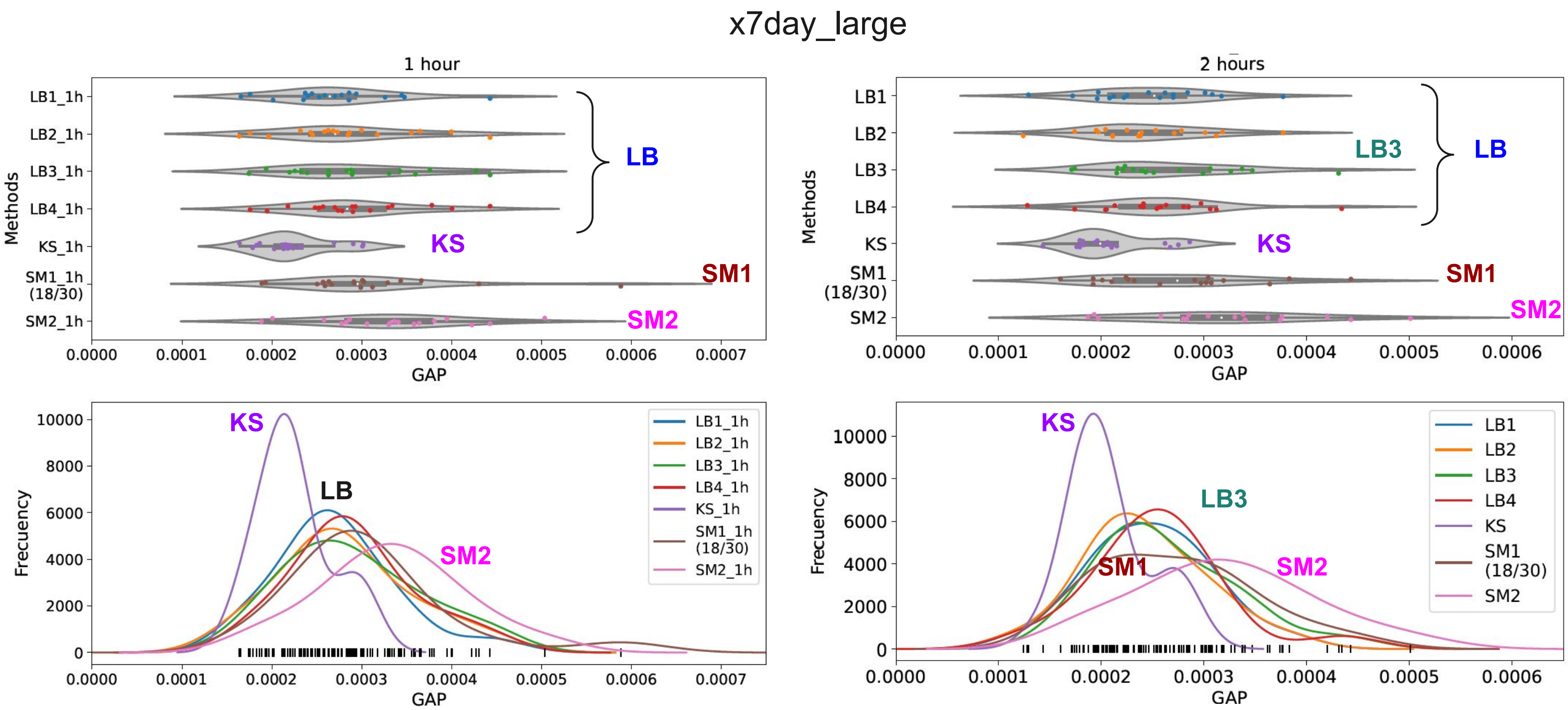
Results: Improving methods

Distribution of relative gap improvement methods for medium instances with allowed time limits of 4000 and 7000 seconds.



Results: Improving methods

Distribution of relative gap improvement methods for large instances with allowed time limits of 4000 and 7000 seconds.

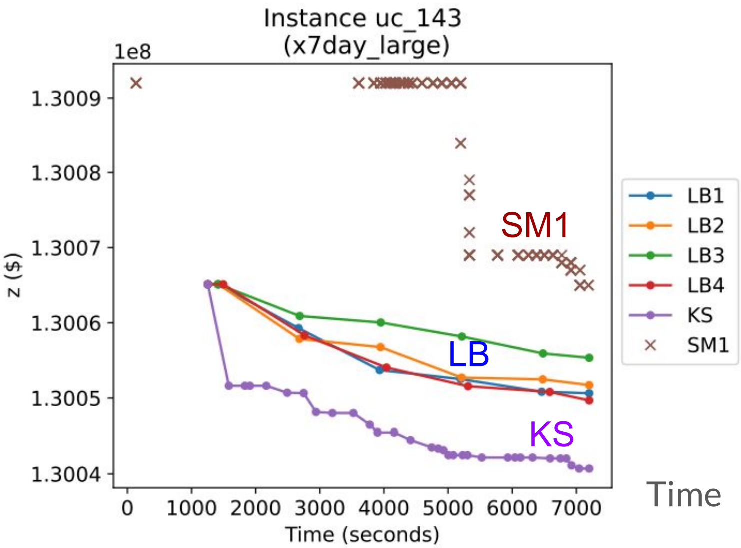


Results: Highlights

Method comparison for instances where solver (SM1) found a solution but had slower solution-finding speed compared to matheuristics.

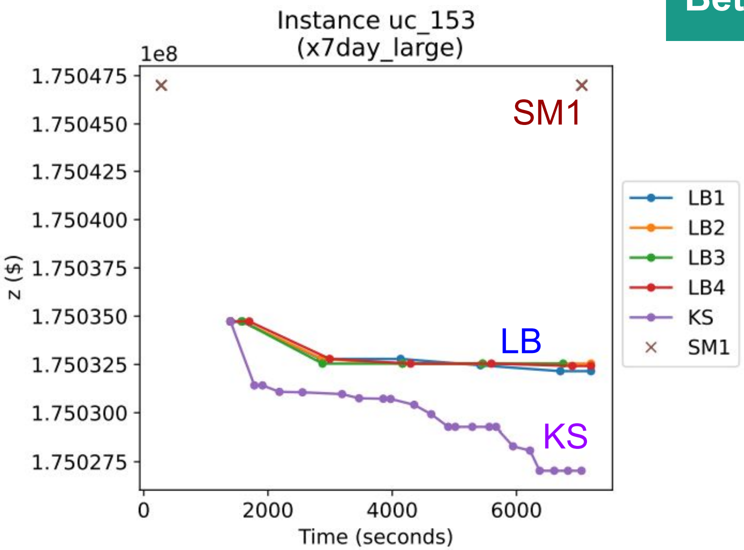
Lower is Better.

Objective function



Time

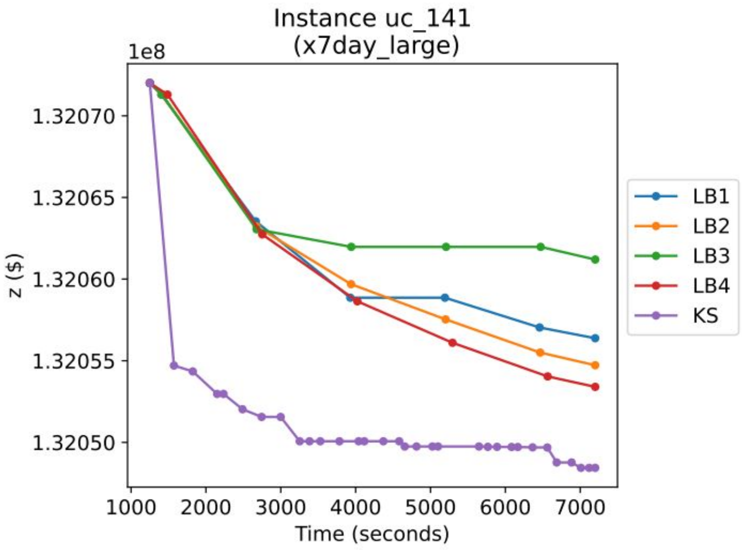
(a) Instance 143.



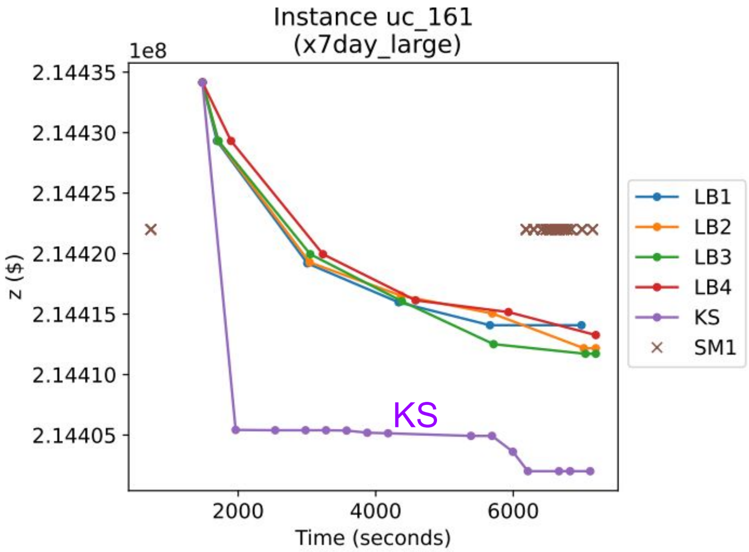
(b) Instance 153.

Results: Highlights

Comparison of methods on instances where the KS exhibits a stagnation phenomenon.



(a) Instance 141.



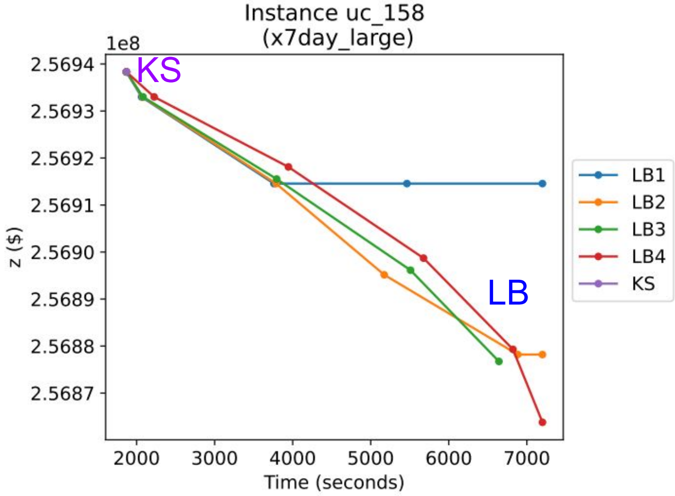
(b) Instance 161.

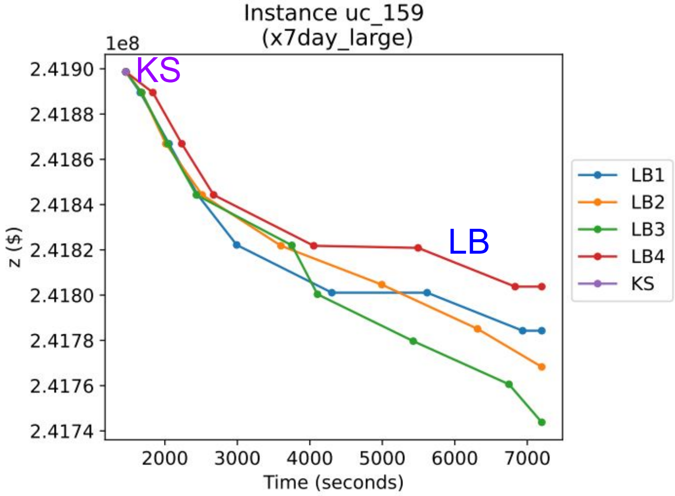
Results: Highlights

Comparison of methods on instances where the KS remained in the initial solution without improvement.

(a) Instance 155.

(b) Instance 156.





(c) Instance 158.

(d) Instance 159.

Results: Summary of Statistical test

We performed statistical tests, including ANOVA and paired T-Tests for datasets with parametric distributions, as well as Kruskal-Wallis and paired Mann-Whitney tests for non-parametric cases.

KS	consistently achieved a lower average optimality gap, making it the preferred choice for medium and large instances with both 4000 and 7200-second time limits.
KS	generally excelled, however it couldn't surpass the initial solution provided by HARDUC in around 4% of instances.
Any LB	outperformed SM2 in medium and large instances with a 4000-second time limit. Also, consistently improved the initial solution.
LB1-2,4	showed a higher performance than LB3, which is more similar to the original method of Fischetti and Lodi.
HARDUC	outperformed HGPS and CBS giving an initial solution.

Conclusions: Research contributions

- A new thermal T&C UCP model with staircase production cost has been developed.
- We have developed five methods for solving thermal Unit Commitment Problems (UCP) within a matheuristic framework.
- Four methods are based on local branching (LB1, LB2, LB3, LB4), and another is based on the kernel search (KS) method.
- A constructive method (HARDUC) was developed to provide initial solutions to the matheuristic methods.

Conclusions: Research contributions

- All methods used the dominant variable commitment.
- The methods were tested on small, medium, and large instances, and the results were compared to the solver's performance with and without an initial solution.
- For large instances, all methods outperformed the solver, with KS showing the best performance.
- Our matheuristics can significantly improve thermal UCP problem solving, and the KS method can be valuable in addressing other UCPs.

Future work

Hybridizing KS and LB

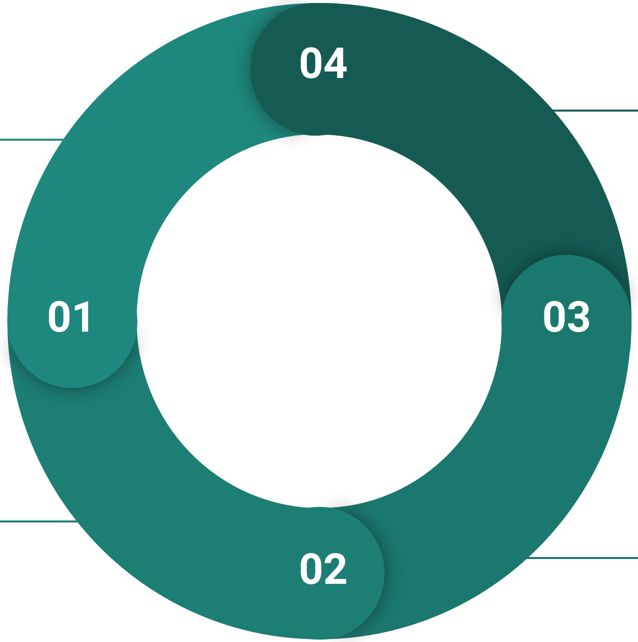
Fast KS convergence.

LB for escaping local optima.

Test KS with UCP extensions

Hydrothermal-UCP.

Network constraint-UCP.



Integrating Local Branching into Branch and Cut

Callback functions to introduce local branching cuts into the solver's to enhancing efficiency.

Apply our methods to a Stochastic UCP

Uncertainty in renewable generation and energy demand.

These methods may reduce computation time.

Scientific Production

Paper: Uriel Iram Lezama Lope, Alberto Benavides-Vázquez, Guillermo Santamaría-Bonfil, and Roger Z. Ríos-Mercado. Fast and Efficient Very Short-Term Load Forecasting Using Analogue and Moving Average Tools. *IEEE Latin America Transactions: Special Issue on Sustainable Energy Sources for an Energy Transition*, 21(9):1015, 2023

Paper: Uriel Iram Lezama Lope, Roger Z. Ríos-Mercado, José Luis Ceciliano Meza, and Miguel Ceceñas Falcón. A Network-Constrained Hydrothermal Unit Commitment Model in the Day-Ahead Market *IEEE Latin America Transactions*. **Submitted** 2023.

Paper: Uriel Iram Lezama Lope, Roger Z. Ríos-Mercado, and Diana Lucia Huerta Muñoz., Surveying Matheuristic Techniques for the Unit Commitment Problem: Emphasizing the Kernel Search Approach and Constructive Methods. **In process.**

Conference: Uriel Iram Lezama Lope, Roger Z. Ríos-Mercado, and José Luis Ceciliano Meza. An hydro-thermal unit commitment problem in the wholesale electricity market in Mexico. IX Congreso de la Sociedad Mexicana de Investigación de Operaciones. Aguascalientes, México 2021.

Conference: Uriel Iram Lezama Lope, Roger Z. Ríos-Mercado, and José Luis Ceciliano Meza. A Matheuristic based on Local Branching to solve the Unit Commitment problem. X Congreso de la Sociedad Mexicana de Investigación de Operaciones. Mérida 2022.

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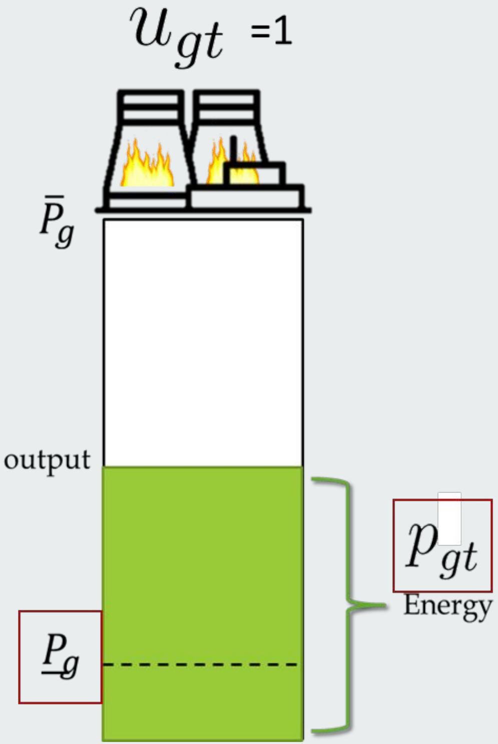
Boschetti, M.A., Maniezzo, V. Matheuristics: using mathematics for heuristic design. 4OR-Q J Oper Res (2022).

Método Constructivo

* [Harjunkoski et al. 2021]



Solution of the linear relaxation

Regla para fijar*

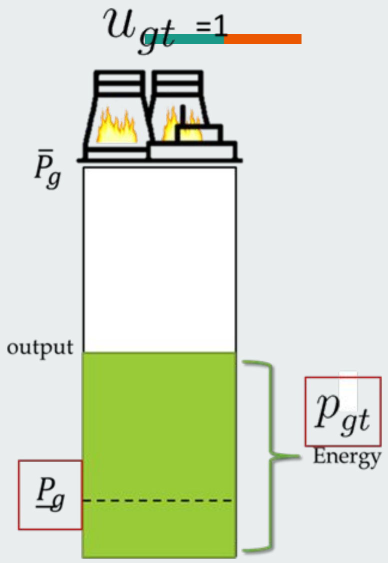
$$\tilde{u}_{gt} \cdot p_{gt} \geq \underline{P}_g$$

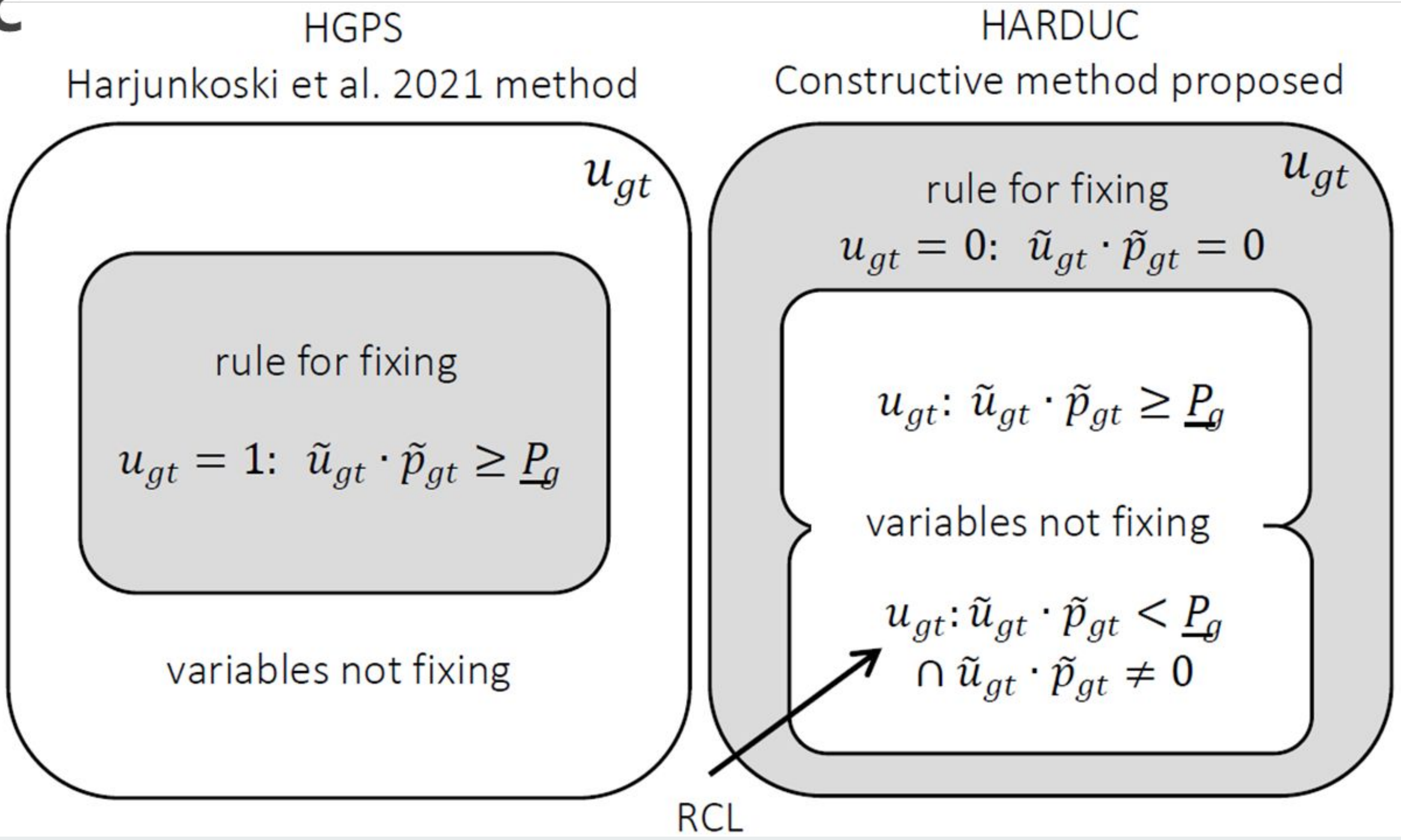
Redondea y fija la variable

u_{gt} = 1

u_{gt} ∈ [0,1]

Opción HARDUC





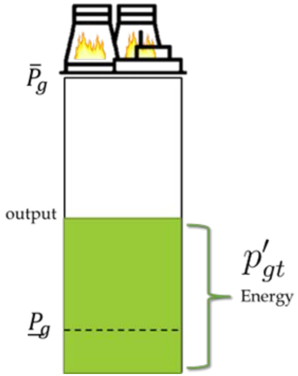
Mathematical model

The commitment variable u_{gt} is established as the dominant variable over other binary variables.

$$\boxed{u_{gt}}, v_{gt}, w_{gt}, \delta_g^s \in \{0,1\}, g \in \mathcal{G}, t \in \mathcal{T}$$

$$p'_{gt}, \bar{p}'_{gt}, c^p_{gt}, c^{SU}_{gt}, r_{gt} \geq 0, g \in \mathcal{G}, t \in \mathcal{T}$$

u_{gt} =1



Objective function

$$\min \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} c^p_{gt} + C^R_g \boxed{u_{gt}} + c^{SU}_{gt}$$

Constraints

Logical constraints.

$$\boxed{u_{gt}} - \boxed{u_{g,t-1}} = v_{gt} - w_{gt} \qquad g \in \mathcal{G}, t \in \mathcal{T} \qquad (2)$$

Generation limits.

$$p'_{gt} \leq \bar{p}'_{gt} \qquad g \in \mathcal{G}, t \in \mathcal{T} \qquad (3)$$

$$p'_{gt} + r_{gt} \leq (\bar{P}_g - \underline{P}_g)\boxed{u_{gt}} - (\bar{P}_g - SU_g)v_{gt} - (\bar{P}_g - SD_g)w_{g,t+1} \qquad g \in \mathcal{G}^{>1}, t \in \mathcal{T} \qquad (4)$$

$$p'_{gt} + r_{gt} \leq (\bar{P}_g - \underline{P}_g)\boxed{u_{gt}} - (\bar{P}_g - SU_g)v_{gt} \qquad g \in \mathcal{G}^1, t \in \mathcal{T} \qquad (5)$$

$$p'_{gt} + r_{gt} \leq (\bar{P}_g - \underline{P}_g)\boxed{u_{gt}} - (\bar{P}_g - SD_g)w_{g,t+1} \qquad g \in \mathcal{G}^1, t \in \mathcal{T} \qquad (6)$$

Minimum uptime and downtime.

$$\sum_{i=1}^{\min\{U_g,T\}} \boxed{u_{gi}} = \min\{U_g, T\} \qquad g \in \mathcal{G}, t \in \mathcal{T} \qquad (7)$$

$$\sum_{i=1}^{\min\{D_g,T\}} \boxed{u_{gi}} = 0 \qquad g \in \mathcal{G}, t \in \mathcal{T} \qquad (8)$$

$$\sum_{i=t-UT_g+1}^{t} v_{gi} \leq \boxed{u_{gt}} \qquad g \in \mathcal{G}, t \in \{UT_g, ..., T\} \qquad (9)$$

$$\sum_{i=t-DT_g+1}^{t} w_{gi} \leq 1 - \boxed{u_{gt}} \qquad g \in \mathcal{G}, t \in \{DT_g, ..., T\} \qquad (10)$$

Startup and operative ramps.

$$p'_{gt} - p'_{g,t-1} \leq (SU_g - \underline{P}_g - RU_g)v_{gt} + RU_g\boxed{u_{gt}} \qquad g \in \mathcal{G}, t \in \mathcal{T} \qquad (11)$$

$$p'_{g,t-1} - p'_{g,t} \leq (SD_g - \underline{P}_g - RD_g)w_{gt} + RD_g\boxed{u_{g,t-1}} \qquad g \in \mathcal{G}, t \in \mathcal{T} \qquad (12)$$

Piecewise cost.

$$c^p_{gt} \geq C^l_g p'_{gt} + (\bar{C}^{l-1}_g - C^l_g(\bar{P}^{l-1}_g - \underline{P}_g))\boxed{u_{gt}} \qquad g \in \mathcal{G}, t \in \mathcal{T}, l \in \mathcal{L}_g \qquad (13)$$

Variable start-up cost.

$$\delta_{gts} \leq \sum_{i=\underline{T}^s_g}^{\underline{T}^{s+1}_g - 1} w_{g,t-i} \qquad g \in \mathcal{G}, t \in \mathcal{T} s \in \mathcal{S}_g \setminus \{S_g\} \qquad (14)$$

$$v_{gt} = \sum_{s=1}^{S_g} \delta_{gts} \qquad g \in \mathcal{G}, t \in \mathcal{T} \qquad (15)$$

$$c^{SU_g}_{gt} = \sum^{S_g} C^s_g \delta_{gts} \qquad g \in \mathcal{G}, t \in \mathcal{T} \qquad (16)$$

load and reserve.

$$\sum_{g \in \mathcal{G}} (p'_{gt} + \underline{P}_g\boxed{u_{gt}}) = \sum_{n \in \mathcal{N}} d_{nt} \qquad t \in \mathcal{T} \qquad (17)$$

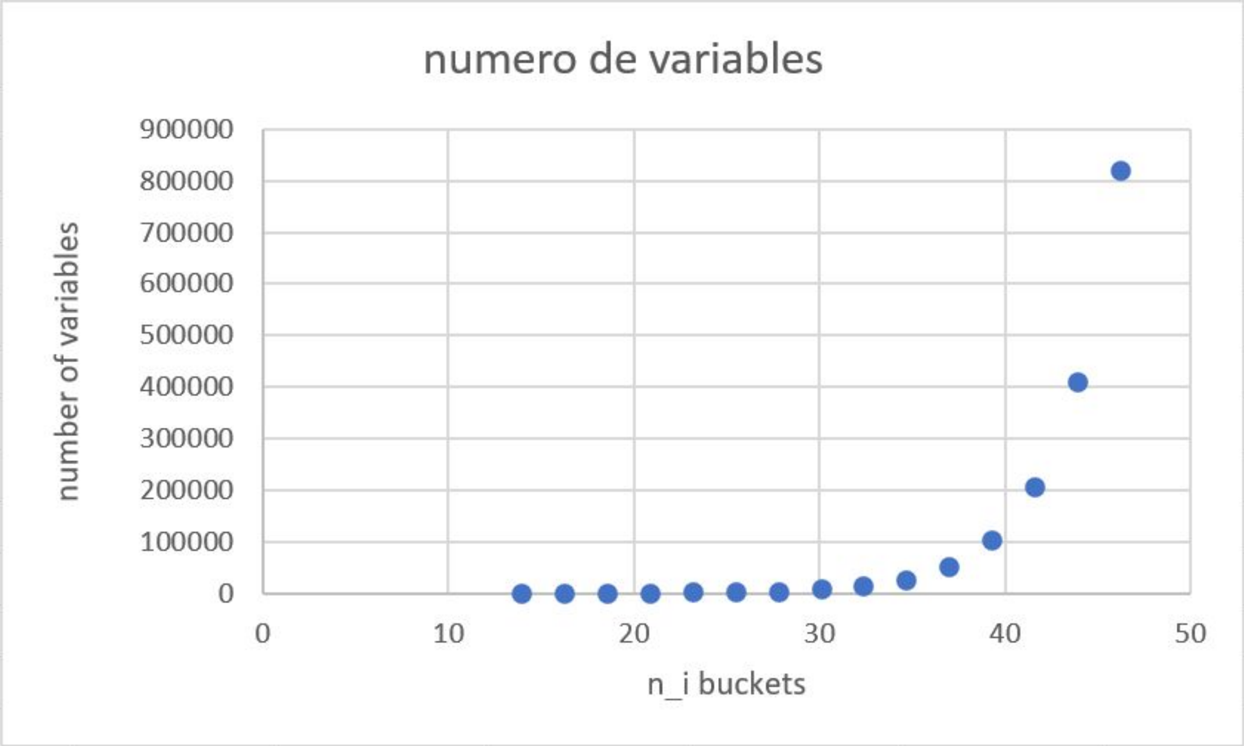
$$\sum_{g \in \mathcal{G}} (p'_{gt} + \underline{P}_g\boxed{u_{gt}} + r_{gt}) \geq \sum_{n \in \mathcal{N}} d_{nt} + R_t, t \in \mathcal{T} \qquad (18)$$

Summary of differences in improvement methods

Comparison between our Kernel Search version versus KS developed by Angelelli et al. [2010], and also the distinctions among the four Local Branching implementations compared to Fischetti and Lodi [2016].

LB1	LBC built using $\boxed{\text{dominant variable } u_{g,t}.}$ $\boxed{\text{RCL}}$ from Harjukovski's rule [41]. $\boxed{\text{Soft-fixing to 90\%}}$ of binary support.
LB2	LBC built using dominant variable $u_{g,t}$. RCL from Harjukovski's rule [41].
LB3	LBC built using dominant variable $u_{g,t}$.
LB4	LBC built using dominant variable $u_{g,t}$. RCL from reduced costs. Soft-fixing to 90% of binary support.
KS	$\boxed{\text{The first solution}}$ is given by the HARDUC constructive method. Kernel and buckets are built only with $\boxed{\text{dominant variables } u_{g,t}}$. The number of buckets is calculated by $\boxed{\text{Sturge's rule [81]}}$. The buckets are built using the reduced costs from linear relaxation $\boxed{\text{(fixing kernel)}}$.

$$nbucks = 1 + 3.322\ln(|U|)$$



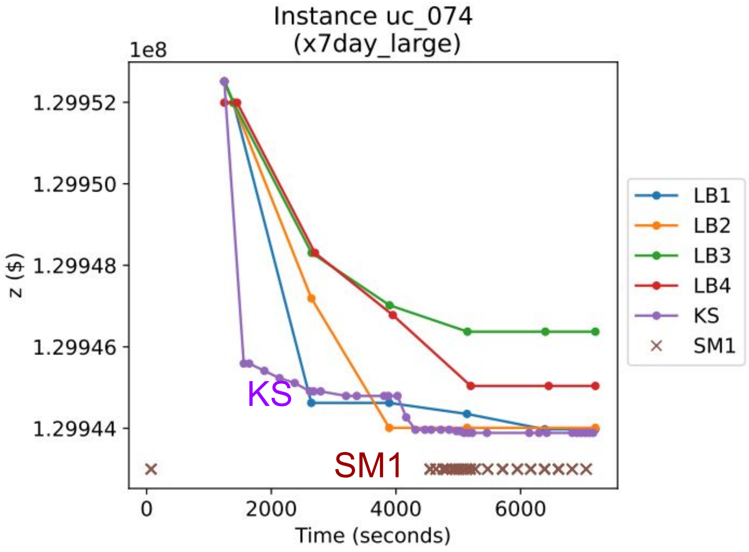
- LB2: This version is similar to LB1 except that it does not have soft-fixing; therefore, constraints (6.5) are not applied.

- LB3: This version is the closest to the original one proposed by Fischetti and Lodi [32]. This version does not do a local search with any RCL and is not narrowed down by soft-fixing. The local search is defined only between the BS and the variables that do not form the binary support $\overline{BS}$.

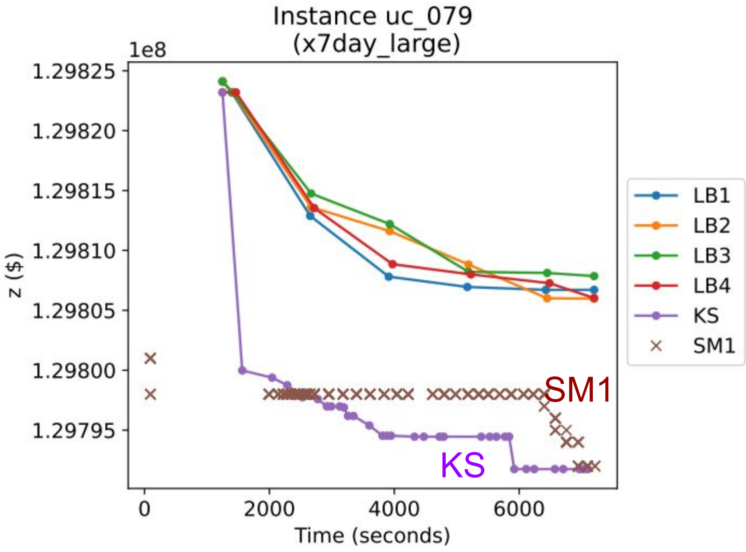
- LB4: This version is similar to LB1 except that the RCL is formed by the variables with negative values in the reduced costs. The reduced costs are calculated by fixing to 1 the variables that form the BS of the initial solution $\bar{x}$ and solving the linear relaxation of the problem.

Results: Highlights

Comparison of methods on instances where the solver (SM1) produced results that were on par with or better than those obtained through the matheuristics.



(a) Instance 074.



(b) Instance 079.