SVF equations

Juan Uriel Legaria Peña

August 10, 2025

Here I will justify the equations applied to obtain a fisheye image, which is an operation used in the SVF calculation. I am sure this is irrelevant, as most things I calculated or experimented with, but I have to do these sort of scribbles; else I cannot understand problems very well.

Let's consider a panoramic where sky has been segmented. It would look something like this:

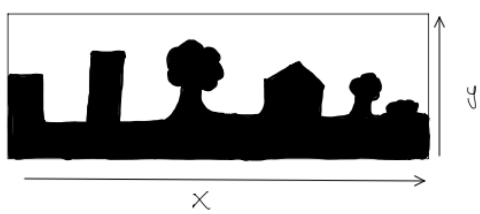

Figure 1: Segmented panoramic drawing.

This panoramic image has width W and height H.

What we essentially want to do now is fold this image over a halfsphere (a planetarium type of projection). The halfsphere over which the image will be folded is shown in Figure 2.

The x axis in the panoramic will go into the azimuth angle θ, while the y axis will cover the polar angle ϕ of the half sphere. Thus, we have the following linear transformations:

$$\theta = \frac{2\pi}{W}x \tag{1}$$

$$\phi = \frac{\pi/2}{H}y \tag{2}$$

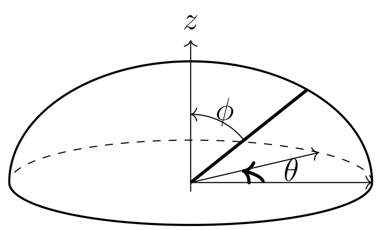

Figure 2: Hemisphere with polar angle ϕ (from z) and azimuth θ on the base.

Of course, even if we now have mapped the panoramic into the surface, this is still a 3D representation. We now need to “flatten” the halfsphere into a 2D image. This flattening transformation will project lines at constant θ to lines in the image that radiate from the center. I show the idea behind these mapping operation in Figure 3.

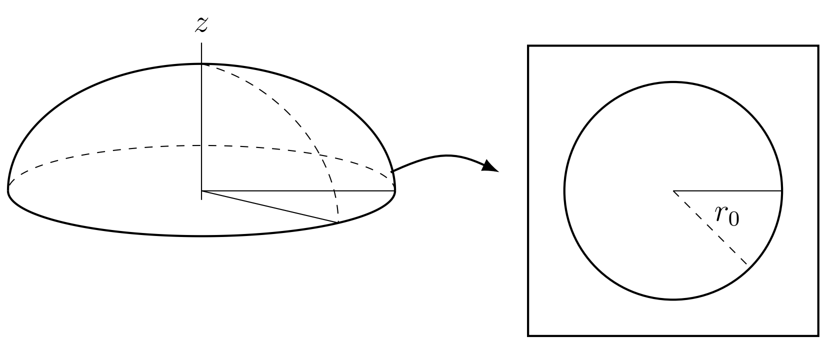

Figure 3: Lines along the hemishphere will be mapped to lines radiating from the center in the image in the 2D representation.

So we are mapping ϕ to points in the 2D image $\mathbf{P_f} = (x_f, y_f)$ of the following form:

$$\mathbf{p}_f = \mathbf{c_f} + \phi \times \alpha \times \hat{\mathbf{r}} \tag{3}$$

where $\mathbf{c_f} = (x_{cf}, y_{cf})$ is the center of the fisheye image, $\hat{\mathbf{r}}$ is a unit vector in the radial direction, and α is a scaling factor allowing to go from values of ϕ to pixels. In this case, since we have defined the circle in the projected image to have a radius of r_0 pixels, we have:

$$\alpha = \frac{r_0}{\frac{\pi}{2}} \tag{4}$$

The points are schematized in figure 4

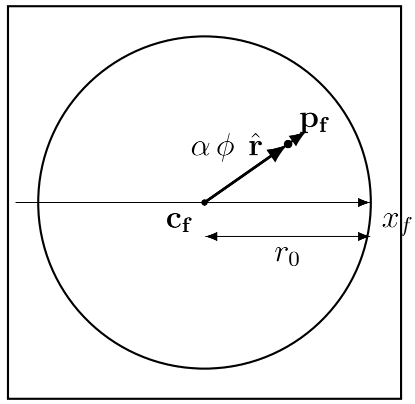

Figure 4: Polar projection points

If we expand Equation 3 we get the following:

$$(x_f, y_f) = (x_{cf}, y_{cf}) + \phi(\cos(\theta), \sin(\theta))\frac{2r_0}{\pi} \tag{5}$$

Now we substitute θ and ϕ in terms of the panoramic coordinates using Equations 1 and 2.

$$\begin{aligned}(x_f, y_f) &= (x_{cf}, y_{cf}) + \frac{\pi}{2H}y\left(\cos\left(\frac{2\pi}{W}x\right), \sin\left(\frac{2\pi}{W}x\right)\right)\frac{2r_0}{\pi} && (6)\\ &= (x_{cf}, y_{cf}) + \frac{r_0}{H}y\left(\cos\left(\frac{2\pi}{W}x\right), \sin\left(\frac{2\pi}{W}x\right)\right) && (7)\end{aligned}$$

Let's separate this by components:

$$\begin{aligned}x_f &= x_{cf} + \frac{r_0}{H}y\cos\left(\frac{2\pi}{W}x\right) && (8)\\ y_f &= y_{cf} + \frac{r_0}{H}y\sin\left(\frac{2\pi}{W}x\right) && (9)\end{aligned}$$

This maps every pixel (x, y) to its corresponding location on the fisheye image (x_f, y_f). However, this is not actually how you transform images in practice. In practice, you start with a "mold" of the output image and see what position in the initial image corresponds to each pixel of the mold; you are effectively applying the inverse transform of what I obtained. Sometimes you just assign the inverted position (nearest interpolation); you can also consider values over a small neighborhood around that inverse coordinate (bilinear or bicubic interpolation).

Let's get the inverse of Equations 8 and 9, which is what will be used in practice.

Rearrange Equations 8 and 9:

$$x_f - x_{cf} = \frac{r_0}{H}\, y \cos\left(\frac{2\pi}{W}x\right), \tag{10}$$

$$y_f - y_{cf} = \frac{r_0}{H}\, y \sin\left(\frac{2\pi}{W}x\right). \tag{11}$$

Square both equations and add them. Use $\cos^2 t + \sin^2 t = 1$:

$$(x_f - x_{cf})^2 + (y_f - y_{cf})^2 = \left(\frac{r_0}{H}y\right)^2 \left[\cos^2\left(\tfrac{2\pi}{W}x\right) + \sin^2\left(\tfrac{2\pi}{W}x\right)\right] \tag{12}$$

$$= \left(\frac{r_0}{H}y\right)^2 \cdot 1 \tag{13}$$

$$\implies \boxed{y = \frac{H}{r_0}\sqrt{(x_f - x_{cf})^2 + (y_f - y_{cf})^2}}. \tag{14}$$

Divide Equation 10 by Equation 11

$$\frac{y_f - y_{cf}}{x_f - x_{cf}} = \tan\left(\frac{2\pi}{W}x\right) \tag{15}$$

Obtain x from that:

$$\boxed{x = \frac{2\pi}{W}\arctan\left(\frac{y_f - y_{cf}}{x_f - x_{cf}}\right)} \tag{16}$$