The two-K method predicts h

This is a new technique that requires only two constants, plus the Reynolds number and fitting diameter, to predict the head loss in an elbow, valve or tee. It is accurate even for large-diameter and alloy fittings, and at low Reynolds numbers.

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☐ Forcing a fluid through a pipe fitting consumes energy, which is provided by a drop in pressure across the fitting. This pressure drop—or head loss—is caused by friction between the fluid and the fitting wall and by creation of turbulence in the body of the fluid.

The loss due to wall friction is best handled by treating the fitting as a piece of straight pipe, of the same physical length as the fitting. All common prediction methods, and the two-K method, do this. But each method predicts the remaining "excess" head loss a different way.

Equivalent length

The equivalent-length method adds some hypothetical length of pipe to the actual length of the fitting, yielding an "equivalent length" of pipe (L_e) that has the same total loss as the fitting. The unfortunate drawback to this simple approach is that the equivalent length for a given fitting is not constant, but depends on Reynolds number and roughness, as well as size and geometry. Therefore, use of the equivalent-length method requires consideration of all these factors.

The *excess* head loss in a fitting is due mostly to turbulence caused by abrupt changes in the direction and speed of flow. Thus it is best to predict this loss by using a velocity-head approach.

Velocity head

The amount of kinetic energy contained in a stream is the velocity head. An equivalent statement is that the velocity head is the amount of potential energy (head) necessary to accelerate a fluid to its flowing velocity.

For example: Pressure gages on both sides of a gradual, friction-free pipe entrance would show that the pressure in the flowing fluid is lower than the pressure in the feed tank by one velocity head. (This is why an eductor works.) The potential (pressure) energy of the fluid in the tank is not lost; it has been converted to kinetic energy. The number of velocity heads (H_d) in a flowing stream is calculated directly from the velocity of the stream (v):

$$H_d = v^2/2g$$

With this background, consider a square elbow. The entering fluid experiences a pipelike frictional head loss as it moves down the inlet leg. At the turn, the flow stops abruptly and starts in a new direction. Since the inlet velocity vector has no component in the outlet direction, all of the inlet kinetic energy is lost. Thus, this part of the loss in a square elbow is close to one velocity head. The remaining losses are the frictional losses in the turn and the outlet leg.

The total head loss in the elbow is the sum of the frictional and directional losses. The excess head loss (ΔH) is less than the total by the amount of frictional loss that would be experienced by straight pipe of the same physical length. (Of course, the *actual* frictional loss in the fitting will be different than the loss in a pipe.) The excess loss in a fitting is normally expressed by a dimensionless "K factor":

$$\Delta H = K\,H_d$$

The two-K method

K is a dimensionless factor defined as the *excess* head loss in a pipe fitting, expressed in velocity heads. In general, it does not depend on the roughness of the fitting (or the attached pipe) or the size of the system, but it is a function of Reynolds number and of the exact geometry of the fitting. The two-K method takes these dependencies into account in the following equation:

$$K = K_1/N_{Re} + K_\infty(1 + 1/ID)$$

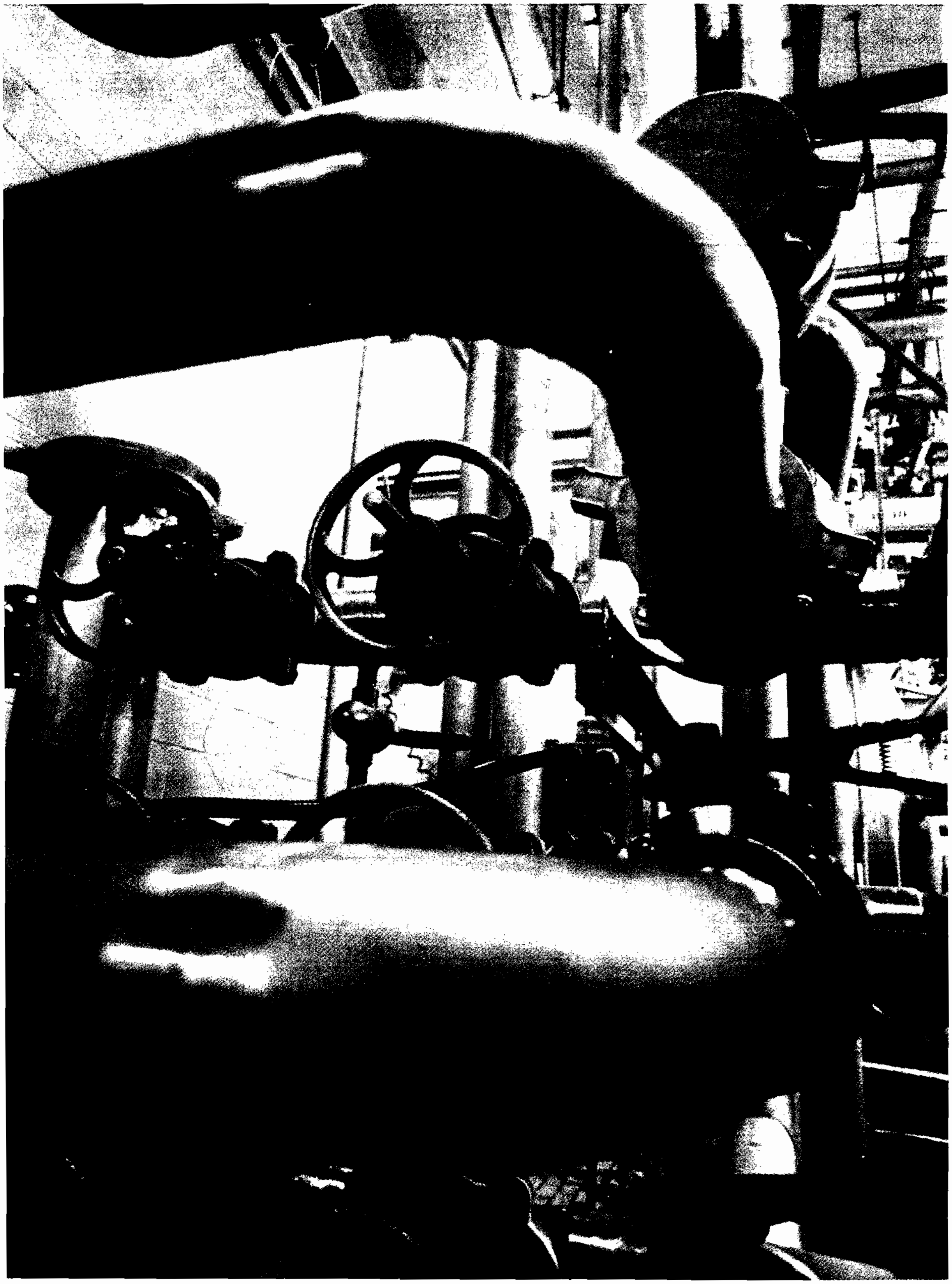
where:

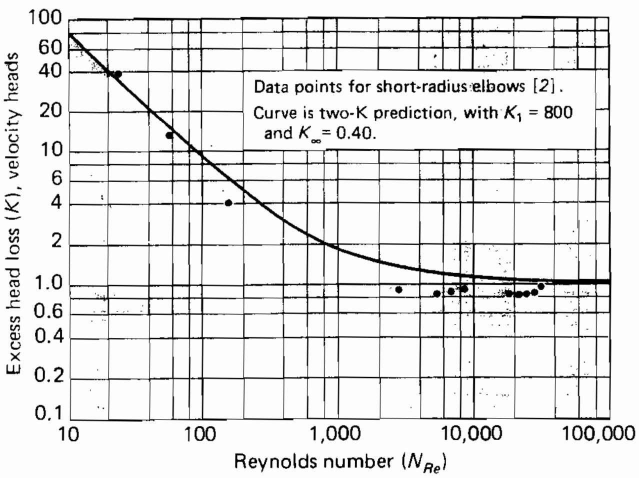
$K_1 = K$ for the fitting at $N_{Re} = 1$
$K_\infty = K$ for a large fitting at $N_{Re} = \infty$
ID = Internal dia. of attached pipe, in.

How N_{Re} and fitting size affect K

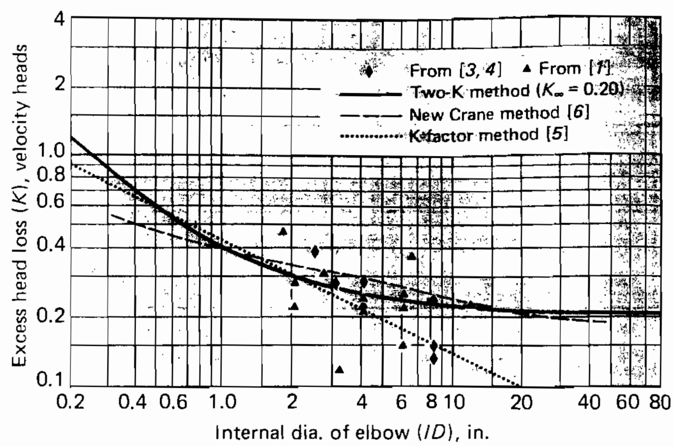
Why two Ks, when the literature usually reports a single K value? Most published K values apply to fully-developed turbulent flow. This is convenient because K is independent of N_{Re} when N_{Re} is sufficiently high. However, K starts to rise as N_{Re} decreases toward 1,000, and becomes inversely proportional to N_{Re} when N_{Re} is below 100.

head losses in pipe fittings





The two-K method fits head-loss data for laminar, transitional and turbulent flow **Fig. 1**



Size of elbow affects K **Fig. 2**

Constants for two-K method

		Fitting type	K_1	K_∞
Elbows	90°	Standard (R/D = 1), screwed	800	0.40
		Standard (R/D = 1), flanged/welded	800	0.25
		Long-radius (R/D = 1.5), all types	800	0.20
		Mitered elbows (R/D=1.5) 1 Weld (90° angle)	1,000	1.15
		Mitered elbows (R/D=1.5) 2 Weld (45° angles)	800	0.35
		Mitered elbows (R/D=1.5) 3 Weld (30° angles)	800	0.30
		Mitered elbows (R/D=1.5) 4 Weld (22½° angles)	800	0.27
		Mitered elbows (R/D=1.5) 5 Weld (18° angles)	800	0.25
	45°	Standard (R/D = 1), all types	500	0.20
		Long-radius (R/D = 1.5), all types	500	0.15
		Mitered, 1 weld, 45° angle	500	0.25
		Mitered, 2 weld, 22½° angles	500	0.15
	180°	Standard (R/D = 1), screwed	1,000	0.60
		Standard (R/D = 1), flanged/welded	1,000	0.35
		Long radius (R/D = 1.5), all types	1,000	0.30
Tees	Used as elbow	Standard, screwed	500	0.70
		Long-radius, screwed	800	0.40
		Standard, flanged or welded	800	0.80
		Stub-in-type branch	1,000	1.00
	Run-through tee	Screwed	200	0.10
		Flanged or welded	150	0.50
		Stub-in-type branch	100	0.00
Valves	Gate, ball, plug	Full line size, β = 1.0	300	0.10
		Reduced trim, β = 0.9	500	0.15
		Reduced trim, β = 0.8	1,000	0.25
	Globe, standard		1,500	4.00
	Globe, angle or Y-type		1,000	2.00
	Diaphragm, dam type		1,000	2.00
	Butterfly		800	0.25
	Check	Lift	2,000	10.00
		Swing	1,500	1.50
		Tilting-disk	1,000	0.50

Note: Use R/D = 1.5 values for R/D = 5 pipe bends, 45° to 180°. Use appropriate tee values for flow through crosses.

Fig. 1 is a plot of K vs. N_{Re} for short-radius elbows [2]. Note that the two-K expression, with 800 for K_1 and 0.40 for K_∞, fits the points accurately in all flow regimes. In this case, K_1 has no effect on the predicted K at N_{Re} above 10,000; K_∞ is negligible below an N_{Re} of 50.

Theoretically, K should be the same for all fittings that are geometrically similar. In fact, smaller fittings are more sensitive to surface roughness and have more abrupt changes in cross-section. Thus K is greater for smaller fittings of a given type.

The $1/ID$ correction in the two-K expression accounts for the size differences: K is higher for small sizes, but nearly constant for large sizes. Fig. 2 is a plot of K vs. pipe size data for long-radius ($R/D = 1.5$) elbows [1,3,4]. The solid line shows how the two-K correlation fits these points; the other lines are correlations that will be discussed later.

Recommended values

The table lists values of K_1 and K_∞ derived from plots of K vs. N_{Re} and size (similar to Fig. 1, 2). The reader is encouraged to keep this and use it, because it is the heart of the two-K method.

Three special cases are not listed in the table because the size correction of the two-K equation does not apply to them. The following equation applies to pipe entrances, exits and orifices:

$$K = K_1/N_{Re} + K_\infty$$

The constants are (K_∞ is the "classic" K):

1. **Pipe entrances** (Fig. 3): $K_1 = 160$; $K_\infty = 0.50$ for "normal" entrance, and 1.0 for "Borda" entrance.
2. **Pipe exit:** $K_1 = 0$; $K_\infty = 1.0$.
3. **Orifice:** K_1 is variable; $K_\infty = 2.91\ (1 - \beta^2)\ ((1/\beta^4) - 1)$, where β is the ratio of orifice dia. to pipe inside dia.

Two-K vs. equivalent length

Why use the two-K method when the equivalent-length method is more familiar? and easier to use? This

Nomenclature

D	Inside pipe dia., ft
f	Moody friction factor ($f = 64/N_{Re}$ for laminar flow)
f_T	"Standard" friction factor for head loss in fitting
g	Acceleration due to gravity, 32.17 ft/s^2
H_d	Velocity head, ft of fluid
ΔH	Head loss, ft of fluid
ID	Inside pipe dia., in.
K	Excess head loss for a fitting, velocity heads
K_1	K for fitting at $N_{Re} = 1$, velocity heads
K_∞	K for very large fitting at $N_{Re} = \infty$, velocity heads
L	Length of pipe, including physical length of fittings, ft
L_e	Equivalent length of a fitting ($L_e = K\,D/f$), ft
N_{Re}	Reynolds number for flow ($N_{Re} = \rho D\,v/\mu$)
n	Number of fittings of a given type
ΔP	Pressure drop ($\Delta P = \rho\Delta H/144$), psi
R/D	Bend radius of an elbow divided by inside dia. of pipe
v	Fluid velocity, ft/s
β	Ratio of orifice dia. to pipe inside dia.
ϵ	Roughness of pipe wall, ft
μ	Viscosity of fluid, lb/ft-s
ρ	Density of fluid, lb/ft^3

Two-K method

Form: $\Delta H = K\,H_d$; $K = K_1/N_{Re} + K_\infty(1 + 1/ID)$

Find K for fittings:

Fittings	n	K_1	nK_1	K_∞	nK_∞
90° elbows	6	800	4,800	0.20	1.20
Tees (side outlet)	2	800	1,600	0.80	1.60
Gate valves	2	500	1,000	0.15	0.30
Totals			7,400		3.10

$K = 7{,}400/1{,}210{,}000 + 3.10(1 + 1/15.624) = 3.305$

Find K for exit and straight pipe:

$K = 1.0$ for normal exit; $K = f\,L/D = 0.937$ for pipe

Find head loss:

$$\Delta H = K\,H_d$$
$$= (3.305 + 1.0 + 0.937)(1.554)$$
$$= 8.15 \text{ ft}$$

K-factor method [5]

Form: $\Delta H = ((fL/D) + K)\,H_d$

Find K for fittings and exit:

Fittings	n	K	nK
90° elbows	6	0.22	1.32
Tees (side outlet)	2	0.44	0.88
Gate valves	2	0.03	0.06
Exit	1	1.0	1.00
Total			3.26

Find K for straight pipe:

$$K = f\,L/D = 0.937 \text{ (given)}$$

Find head loss:

$$\Delta H = K\,H_d$$
$$= (3.26 + 0.937)(1.554)$$
$$= 6.52 \text{ ft}$$

Old equivalent-length method [1]

Form: $\Delta H = (f\,L_e/D)\,H_d$

Find equivalent lengths:

Fittings	n	L_e	nL_e
90° elbows	6	42	252
Tees (side outlet)	2	89	178
Gate valves	2	9	18
Exit	1	89	89
Straight pipe			100
Total L_e			637 ft

Find head loss:

$$\Delta H = (f\,L_e/D)H_d$$
$$= (0.0122 \times (637/1.302))(1.554)$$
$$= 9.28 \text{ ft}$$

New Crane method [6]

Form: $\Delta H = ((fL/D) + K)\,H_d$

f_T for this system is 0.013 (p. A-26)

Find K for fittings and exit:

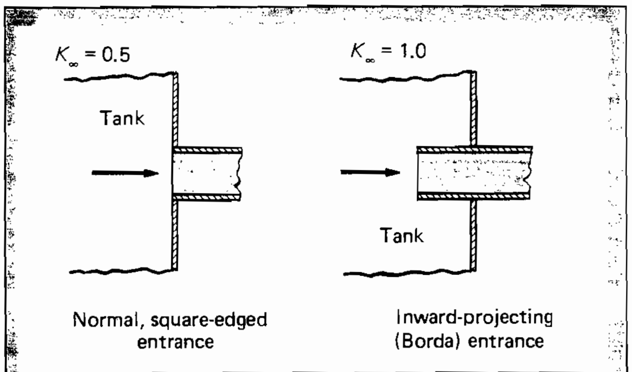
Fittings		K	n	nK
90° elbows	$K = 20\,f_T$	0.260	6	1.560
Tees (side outlet)	$K = 60\,f_T$	0.780	2	1.560
Gate valves	$K = 8\,f_T$	0.104	2	0.208
Exit		1.00	1	1.00
Total				4.328

Find K for straight pipe:

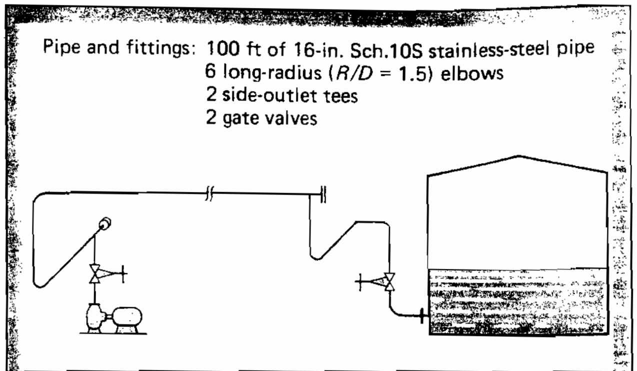
$$K = fL/D = 0.937 \text{ (given)}$$

Find head loss:

$$\Delta H = ((fL/D) + K)\,H_d$$
$$= (0.937 + 4.328)(1.554)$$
$$= 8.18 \text{ ft}$$



Pipe-entrance K values do not depend on diameter Fig. 3



Pipe layout for examples Fig. 4

classic method, in which each type of fitting has one "equivalent length," is reliable for 1–6 in. carbon-steel piping in normal runs (see the dashed line in Fig. 2). In large, complex alloy systems, the method could predict head losses 1.5–3 times too high. That means oversized pumps and a large waste of energy and capital. In laminar flow, on the other hand, it could predict head losses a whole order of magnitude too low.

The equivalent-length concept also contains a booby trap for the unwary. Every equivalent length has a specific friction factor (f) associated with it, because the equivalent lengths were originally developed from K factors by the formula $L_e = KD/f$. This is why the latest version of the equivalent-length method (the 1976 edition of Crane Technical Paper 410 [6]) properly requires the use of two friction factors. The first is the actual friction factor for flow in the straight pipe (f), and the second is a "standard" friction factor for the particular fitting (f_T). Thus the two-K method is as easy to use as the updated equivalent-length method. And the results are as accurate.

What about the widely-used K-factor graphs published by the Hydraulic Institute? (See [5] for a good presentation of these graphs.) The graphs are good for 1–8 in. pipe in fully turbulent flow (see dotted line in Fig. 2), but extrapolation to larger sizes can cause errors. For example, the K-factor line in Fig. 2 shows a K of 0.075 for a 36-in. elbow, but the actual K is about 0.200. Of course, these charts greatly underestimate laminar head losses, and should not be used for N_{Re} below 10,000.

Example

Consider a 16-in. Sch 10S stainless-steel system as shown in Fig. 4. The system contains 100 actual ft of pipe; 6 long-radius (normal for most systems) elbows; 2 side-outlet tees; 2 gate valves and an exit into a tank. The fluid has a viscosity of 1 cP, a specific gravity of 1, and is flowing at 10 ft/s. What is the head loss through this system?

Let us first calculate and convert the given data to get the needed information:

$$\rho = 1 \times 62.43 = 62.43 \text{ lb/ft}^3$$
$$\mu = 1 \times 6.72 \times 10^{-4} = 6.72 \times 10^{-4} \text{ lb/ft-s}$$
$$ID = 15.624 \text{ in. for Sch 10S pipe}$$
$$D = 15.624/12 = 1.302 \text{ ft}$$
$$N_{Re} = (10)(1.302)(62.43)/(6.72 \times 10^{-4}) = 1{,}210{,}000$$
$$H_d = v^2/2g = 10^2/64.34 = 1.554 \text{ ft of fluid}$$

Given $\epsilon = 0.00005$ ft for stainless pipe, we can find f from the Colebrook equation: $f = 0.0122$. Thus, $fL/D = (0.0122)(100)/(1.302) = 0.937$ (this is the K value for the pipe itself).

The four boxes (on p. 99) show how to calculate the total head loss by the two-K method and three other methods. The results:

1. Two-K method: $\Delta H = 8.15$ ft.
2. Old equivalent-length method: $\Delta H = 9.28$ ft (14% high).
3. K-factor method: $\Delta H = 6.52$ ft (20% low).
4. Revised Crane method: $\Delta H = 8.18$ ft.

Note that flow was fully turbulent in this example. For laminar flow, the equivalent-length and K-factor methods would have been off considerably more.

Mark Lipowicz, Editor

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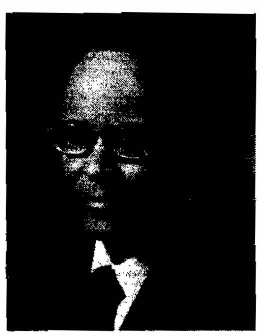

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