

Analysis of Algorithms Part 1: Binary Search

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Binary Search : Problem Description

Problem: Does the given **element** belong to the **sorted list of numbers**?

Input:

sorted list of numbers: [1, 6, 7, 19, 22, 25, 31, 55]
element: 18

Expected Answer:

False (element is not in the list).

Input:

sorted list of numbers: [1, 6, 7, 14, 17, 21, 25]
element: 6

Expected Answer:

True, Index : 1

Binary Search: Basic Idea

Search for element: 6

[1, 6, 7, 14, 17, 21, 25]

Binary Search: Basic Idea # 2

Search for element: 18

[1, 6, 7, 14, 17, 21, 25]

Binary Search Implementation

```
def binarySearchHelper( lst, elt, left, right)
    # Requirements:
    # 0 <= left <= right < size(lst)
    # Invariant:
    # If elt is found in lst, it must be found in the
    # sub-list [ lst[left],...,lst[right] ]
```

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    # 0 <= left <= right < size(lst)
    # Invariant:
    # If elt is found in lst, it must be found in the
    #   sub-list [ lst[left],...,lst[right] ]
    if (left > right):
        return None # Search region is empty -- let us bail.
    else:
        mid = (left + right)//2 # Note that // is integer division
        if lst[mid] == elt:
            return mid # BINGO -- we found it. Return its index and that we found it
        elif lst[mid] < elt:
            return binarySearchHelper(lst, elt, mid+1, right)
        else: # lst[mid] > elt
            return binarySearchHelper(lst, elt, left, mid-1)
```

Binary Search

```
def binarySearch(lst, elt)
    binarySearchHelper(lst, elt, 0, size(lst) - 1)
```

Correctness of Binary Search

`binarySearchHelper(lst, elt, left, right)` has the following behavior for `lst` sorted in ascending order:

- If `elt` belongs to the sub-list `lst[left]...lst[right]`, it returns the index of `elt` in `lst`.
- If `elt` does not belong to the sub-list, it returns the special value `None`.

How to prove correctness?

Running Time Analysis of Binary Search

Running Time Analysis of Binary Search (Cont)