Stochastic Simulation Techniques and Structural Reliability SoSe 2025
Assignment 4
Random Fields

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1 Generate 1D Random Field using Cholesky decomposition

1. Generate a Gaussian random field using Cholesky decomposition and the Markov correlation model with mean 5 and standard deviation 0.7 on a one dimensional domain $x \in [0, 5]$ using 50 discretization points. Start with a sample number $N = 1000$, and a correlation length $\theta = 1.0$. Kindly, do the following:
   - Plot mean and standard deviation along the specified domain.
   - Calculate the empirical correlation matrix and compare it to the predefined one.
2. Study the effect of the correlation length θ. For example, take three values from the range $\theta = [0.5 - 4]$. Compare the stochastic properties, i.e., the mean, the standard deviation, and first three generated random fields. Plot the results. Give your interpretations.

2 Random Fields using Kahunen-Loève Expansion

1. Find the minimum sample number N needed to generate a 1D random field using Kahunen-Loève Expansion and the Markov correlation model and the followings parameters:
   - Generate a Gaussian random field with mean 0 and standard deviation 1 on a one dimensional domain $x \in [0, 1]$ using 30 discretization points.
   - The correlation length $\theta = 0.75$. Change the sample number within the range $N = [1000 - 50000]$.
2. Change the correlation length to $\theta = 0.02$. Repeat the previous task. How does this affect the minimum sample number N needed to generate the random field.

Hints: calculate the error between the generated sample covariance matrix and the original covariance matrix [C] for each sample number and then draw the calculated errors against the different sample numbers.