Task 1:

Distribution type: Binomial, X with parameters $n=12$, $P=0.15$

Probability mass function of x is

$$f(x) = P(X=x) = \binom{n}{x} p^{x} (1-p)^{n-x}$$

$$= P(X=x) \binom{12}{x} (0.15)^{x} (0.85)^{12-x}$$

* Mean: $\mu = np = 12 \times 0.15 = \underline{\underline{1.8}}$.

Variance, $\sigma^2 = V(x) = np(1-p)$

$$= 12 \times 0.15 (1-0.15)$$

$$= \underline{\underline{1.53}}.$$

* Probability that David recieves exactly 3 invitations

$$P(x=3) = \binom{12}{3} (0.15)^3 \cdot (0.85)^9$$

$$= \frac{12!}{3! \cdot 9!} \times (0.15)^3 \cdot (0.85)^9$$

$$= 220 \times (0.15)^3 \cdot (0.85)^9 = \underline{\underline{0.1719}}.$$

* Probability that david receives fewer than 3 invitations $= P(x<3) = P(x=0) + P(x=1) + P(x=2)$

$$P(x=0) = \binom{12}{0} (0.15)^0 (0.85)^{12} = 1 \times 1 \times 0.142$$
$$= \underline{\underline{0.142}}$$

$$P(x=1) = \binom{12}{1} (0.15)^1 (0.85)^{11} = 12 \times 0.15 \times 0.167$$
$$= \underline{\underline{0.301}}$$

$$P(x=2) = \binom{12}{2} (0.15)^2 (0.85)^{10} = 66 \times (0.0225) \times (0.196)$$
$$= \underline{\underline{0.291}}$$

$$P(x<3) = 0.142 + 0.301 + 0.291 = \underline{\underline{0.734}}$$

* Probability that David receives atleast 10 invitations $= P(x \geq 10) = P(x=10) + P(x=11) + P(x=12)$

$$P(x=10) = \binom{12}{10} (0.15)^{10} (0.85)^{2}$$
$$= 66 \cdot (5.77 \times 10^{-9}) \times 0.7225$$
$$= \underline{\underline{2.75 \times 10^{-7}}}$$

$$P(x=11) = \binom{12}{11} (0.15)^{11} (0.85)^{1}$$
$$= \underline{\underline{8.82 \times 10^{-9}}}$$

$$P(x=12) = \binom{12}{2} \times (0.15)^{12} \times (0.85)^{0}$$
$$= 1.29 \times 10^{-10}$$

$$P(x \geq 10) = 2.75 \times 10^{-7} + 8.82 \times 10^{-9} + 1.29 \times 10^{-10}$$
$$= 2.84 \times 10^{-7}$$

Task 2:

Distribution type : Geometric, Y with $P = 0.15$

Probability mass function, $f(y) = P \cdot (1-P)^{y-1}$
$$= 0.15\,(1-0.15)^{y-1}$$
$$= 0.15\,(0.85)^{y-1}$$

* Mean, $E(x) = 1/p = 1/0.15 = 6.66$

Variance, $\sigma^2 = (1-P)/P^2 = \frac{0.85}{0.15^{2}} = 37.78$

* Probability that michael receives his first invitation exactly on 5^{th} letter
$$= P(Y=5) = 0.15 \times (0.85)^4$$
$$= 0.0783$$

* Probability that michael receives his first invitation not later than 4 letters $= P(Y \leq 4)$

$$= P(Y=1) + P(Y=2) + P(Y=3) + P(Y=4)$$

$$= 0.15(0.85)^0 + 0.15(0.85)^1 + 0.15(0.85)^2 + 0.15(0.85)^3$$

$$= \underline{\underline{0.4779}}$$

* Probability that michael sends more than 3 letters before getting an invitation $= P(Y>3) = 1 - P(Y \leq 3)$

$$= 1 - (P(Y=1) + P(Y=2) + P(Y=3))$$

$$= 1 - (0.15(1 + 0.85 + 0.7225)$$

$$= \underline{\underline{0.6141}}$$

Task 3:

Distribution type: Negative binomial, Z with parameters $p = 0.15$, $r = 6$

Probability mass function, $f(z) = \binom{z-1}{r-1}(1-p)^{z-r} p^r$

$$= \binom{z-1}{6-1}(1-0.15)^{z-6} \cdot (0.15)^6$$

* Mean $= E(x) = \frac{r}{p} = 6/_{0.15} = \underline{40}$

Variance, $V(x) = \frac{r(1-p)}{p^2} = \frac{6(1-0.15)}{0.15^2}$

$= \underline{\underline{226.66}}$.

* Probability that emma sends exactly 30 letters to collect 6 invitations

$P(Z=30) = \binom{29}{5} (0.85)^{24} \cdot (0.15)^6$

$= \underline{\underline{0.0273}}$.

* Probability that Emma needs atmost 9 letters to collect 6 invitations $= P(Z \leq 9)$

$= P(Z=6) + P(Z=7) + P(Z=8) + P(Z=9)$

$P(Z=6) = \binom{5}{5} (0.85)^0 (0.15)^6 = \underline{\underline{1.14 \times 10^{-5}}}$.

$P(Z=7) = \binom{6}{5} (0.85)^1 (0.15)^6 = 6 \times 0.85 \times 1.14 \times 10^{-5}$

$= \underline{\underline{5.81 \times 10^{-5}}}$

$$P(Z=8) = \binom{7}{5}(0.85)^2(0.15)^6$$

$$= 21 \times 0.7225 \times 1.14 \times 10^{-5}$$

$$= 1.72 \times 10^{-4}$$

$$P(Z=9) = \binom{8}{5}(0.85)^3(0.15)^6 = 56 \times 0.614 \times 1.14 \times 10^{-5}$$

$$= 3.92 \times 10^{-4}$$

$$P(Z \leq 9) = 6.33 \times 10^{-4}$$

* Probability that Emma needs atleast 10 letters to collect 6 invitations $= P(Z \geq 10) = 1 - P(Z < 10)$

$$= 1 - P(Z \leq 9)$$

$$= 1 - 0.000633$$

$$= 0.9993$$

Task 4 :

Distribution type : Hypergeometry with Parameters $N = 15$, $n = 4$, $k = 6$

Probability mass function, $f(m) = \dfrac{\binom{k}{m}\binom{N-k}{n-m}}{\binom{N}{n}}$

$$= \frac{\binom{6}{m}\binom{15-6}{4-m}}{\binom{15}{4}}$$

* Mean, $E(x) = \dfrac{nk}{N} = \dfrac{4 \times 6}{15} = \underline{\underline{1.6}}$

Variance, $V(x) = \dfrac{nk}{N}\left(1 - \dfrac{k}{N}\right)\left(\dfrac{N-n}{N-1}\right)$

$$= \frac{4 \times 6}{15}\left(1 - \frac{6}{15}\right)\left(\frac{15-4}{15-1}\right)$$

$$= 1.6 \times 0.6 \times 0.7857$$

$$= \underline{\underline{0.7542}}$$

* Probability that exactly 2 of destroyed letters were invitations $= P(M=2)$

$$= \frac{\binom{6}{2}\binom{9}{2}}{\binom{15}{4}} = \frac{\frac{6!}{2!\,4!} \cdot \frac{9!}{2!\,7!}}{\frac{15!}{4!\cdot 11!}}$$

$$= \underline{\underline{0.3956}}.$$

* Probability that atleast 3 of the destroyed letters were invitations $= P(M \geq 3) = P(M=3) + P(M=4)$

$$= \frac{\binom{6}{3}\binom{9}{1}}{\binom{15}{4}} + \frac{\binom{6}{4}\binom{9}{0}}{\binom{15}{4}}$$

$$= \frac{\frac{6!}{3!\,3!} \cdot \frac{9!}{1!\,8!}}{\frac{15!}{4!\,11!}} + \frac{\frac{6!}{4!\,2!} \cdot \frac{9!}{0!\,9!}}{\frac{15!}{4!\,11!}}$$

$$= \frac{20 \cdot 9}{1365} + \frac{15 \cdot 1}{1365} = \frac{180+15}{1365} = \underline{\underline{0.1428}}$$

* Probability that none of destroyed letters were invitations = $P(M=0) = \dfrac{\binom{6}{0}\binom{9}{4}}{\binom{15}{4}}$

$$= \frac{\frac{6!}{0!\,6!} \cdot \frac{9!}{4!\,5!}}{\frac{15!}{4!\cdot 11!}}$$

$$= \frac{1 \cdot 126}{1365} = \underline{\underline{0.0923}}.$$