A

$\sum_{i=1}^{n}\sum_{j=1}^{m}\gcd(i,j)=\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{d=1}^{n}[\gcd(i,j)=d]d$

$=\sum_{d=1}^{n}d\sum_{i=1}^{\lfloor\frac{n}{d}\rfloor}\sum_{j=1}^{\lfloor\frac{m}{d}\rfloor}[\gcd(i,j)=1]$

$=\sum_{d=1}^{n}d\sum_{i=1}^{\lfloor\frac{n}{d}\rfloor}\sum_{j=1}^{\lfloor\frac{m}{d}\rfloor}\sum_{k|i,k|j}\mu(k)$
$=\sum_{d=1}^{n}d\sum_{k=1}^{\lfloor\frac{n}{d}\rfloor}\mu(k)\sum_{k|i}^{\lfloor\frac{n}{d}\rfloor}\sum_{k|j}^{\lfloor\frac{m}{d}\rfloor}1$
$=\sum_{d=1}^{n}d\sum_{k=1}^{\lfloor\frac{n}{d}\rfloor}\mu(k)\lfloor\frac{n}{kd}\rfloor\lfloor\frac{m}{kd}\rfloor$

B

有个神奇的柿子$D(ab)=\sum_{i|a}\sum_{j|b}[\gcd(i,j)=1]$

$Ans=\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{a|i}\sum_{b|j}[\gcd(a,b)=1]$
$=\sum_{a=1}^{n}\sum_{b=1}^{m}\lfloor\frac{n}{a}\rfloor\lfloor\frac{m}{b}\rfloor[\gcd(a,b)=1]$
$=\sum_{a=1}^{n}\sum_{b=1}^{m}\lfloor\frac{n}{a}\rfloor\lfloor\frac{m}{b}\rfloor\sum_{k|a,k|b}\mu(k)$
$=\sum_{k=1}^{n}\mu(k)\sum_{a=1}^{\lfloor\frac{n}{k}\rfloor}\sum_{b=1}^{\lfloor\frac{m}{k}\rfloor}\lfloor\frac{n}{ak}\rfloor\lfloor\frac{m}{bk}\rfloor$

可以发现右边那两个东西实际上只和n/k和m/k有关

所以说再整除分块预处理出来$\sum_{i=1}^{a}\lfloor\frac{n}{i}\rfloor$

对每个询问都整除分块，发现再处理一下μ的前缀和即可

C

$f(k)=\sum_{d=1}^{a}\mu(d)\lfloor\frac{a}{dk}\rfloor\lfloor\frac{b}{dk}\rfloor$
显然可以整除分块

D

C加点容斥即可

E

$\sum_{i=1}^{n}\sum_{j=1}^{m}\gcd(i,j)^k$
$=\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{d=1}^{n}[\gcd(i,j)=d]d^k$
$=\sum_{d=1}^{n}d^k\sum_{i=1}^{\lfloor\frac{n}{d}\rfloor}\sum_{j=1}^{\lfloor\frac{m}{d}\rfloor}[\gcd(i,j)=1]$
$=\sum_{d=1}^{n}d^k\sum_{i=1}^{\lfloor\frac{n}{d}\rfloor}\sum_{j=1}^{\lfloor\frac{m}{d}\rfloor}\sum_{l|i,l|j}\mu(l)$
$=\sum_{d=1}^{n}d^k\sum_{l=1}^{\lfloor\frac{n}{d}\rfloor}\mu(l)\lfloor\frac{n}{dl}\rfloor\lfloor\frac{m}{dl}\rfloor$

设$D=dl$

$=\sum_{d=1}^{n}\lfloor\frac{n}{D}\rfloor\lfloor\frac{m}{D}\rfloor\sum_{d|D}d^k\mu(\frac{D}{d})$

发现右边$f(D)=\sum_{d|D}d^k\mu(\frac{D}{d})$,是积性函数（$d^k$,$\mu(\frac{D}{d})$均为积性函数，他们的狄利克雷卷积也为积性函数）

$f(p)=p^k-1$

$f(p^{x_p})=p^{x_pk}-p^{(x_p-1)k}=p^{(x_p-1)k}*(p-1)$

令p为a的最小质因数

$$f(a)=\begin{cases}f(a/(p^{x_p}))*f(p_p^x)=f(a/p)\times p^k & (x_p>1)\\ f(a/p)\times(p^k-1) & (x_p=1)\end{cases}$$

然后就可以线性递推了

左边的$\sum_{d=1}^{n}\lfloor\frac{n}{D}\rfloor\lfloor\frac{m}{D}\rfloor$可以整除分块

线性递推代码

```
void check()
{
    sum[1]=1;
    for (int i=2;i<=5000000;i++)
    {
        if (!is_prime[i])
        {
            prime[++cnt]=i;
            sum[i]=ksm(i,k)-1;
            if (sum[i]<0) sum[i]+=mod;
        }
        for (int j=1;j<=cnt;j++)
        {
            if (prime[j]*i>5000000) break;
            is_prime[i*prime[j]]=1;
            if (i%prime[j]==0)
            {
                sum[i*prime[j]]=sum[i]*(sum[prime[j]]+1)%mod;
                break;
            }
            else
            {
                sum[i*prime[j]]=sum[i]*sum[prime[j]]%mod;
            }
        }
    }
    for (int i=2;i<=5000000;i++)
        sum[i]=(sum[i-1]+sum[i])%mod;
}
```