A

$$(\sum_{x=1}^{n}\sum_{y=1}^{m}2\gcd(x,y)-1) \tag{1}$$

$$2(\sum_{x=1}^{n}\sum_{y=1}^{m}\gcd(x,y))-m*n \tag{2}$$

$$2(\sum_{x=1}^{n}\sum_{y=1}^{m}\sum_{p=1}^{n}p*[\gcd(x,y)=p])-m*n \tag{3}$$

$$2(\sum_{p=1}^{n}\sum_{x=1}^{\lfloor\frac{n}{p}\rfloor}\sum_{y=1}^{\lfloor\frac{m}{p}\rfloor}p*[\gcd(x,y)=1])-m*n \tag{4}$$

$$2(\sum_{p=1}^{n}p*\sum_{x=1}^{\lfloor\frac{n}{p}\rfloor}\sum_{y=1}^{\lfloor\frac{m}{p}\rfloor}\sum_{d|\ \gcd(x,y)}\mu(d))-m*n \tag{5}$$

$$2(\sum_{p=1}^{n}p*\sum_{d=1}^{\lfloor\frac{n}{p}\rfloor}\mu(d)\sum_{d|x}^{\lfloor\frac{n}{p}\rfloor}\sum_{d|y}^{\lfloor\frac{m}{p}\rfloor}1)-m*n \tag{6}$$

$$2(\sum_{p=1}^{n}p*\sum_{d=1}^{\lfloor\frac{n}{p}\rfloor}\mu(d)\lfloor\frac{m}{dp}\rfloor\lfloor\frac{n}{dp}\rfloor)-n*m \tag{7}$$

$$2\sum_{T=1}^{n}\sum_{d|T}d*\mu(\frac{T}{d})\lfloor\frac{m}{T}\rfloor\lfloor\frac{n}{T}\rfloor-m*n \tag{36}$$

$$2\sum_{T=1}^{n}\lfloor\frac{m}{T}\rfloor\lfloor\frac{n}{T}\rfloor\sum_{d|T}d*\mu(\frac{T}{d})-m*n \tag{9}$$

$$2\sum_{T=1}^{n}\lfloor\frac{m}{T}\rfloor\lfloor\frac{n}{T}\rfloor\phi(T)-m*n \tag{10}$$

B

$$\sum_{i=1}^{n}\sum_{j=1}^{m}d(ij) \tag{11}$$

$$=\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{x|i}\sum_{y|j}[\gcd(x,y)=1] \tag{12}$$

$$=\sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{x|i}\sum_{y|j}\sum_{d|\ \gcd(x,y)}\mu(d) \tag{13}$$

$$= \sum_{x=1}^{n}\sum_{y=1}^{m}\sum_{d|\gcd(x,y)}\mu(d)\sum_{x|i}^{n}\sum_{y|i}^{m}1 \tag{14}$$

$$= \sum_{x=1}^{n}\sum_{y=1}^{m}\sum_{d|\gcd(x,y)}\mu(d)\lfloor\frac{n}{x}\rfloor\lfloor\frac{m}{y}\rfloor \tag{15}$$

$$= \sum_{d=1}^{n}\mu(d)\sum_{x=1}^{\lfloor\frac{n}{d}\rfloor}\sum_{y=1}^{\lfloor\frac{m}{d}\rfloor}\sum_{i=1}^{\lfloor\frac{n}{dx}\rfloor}\sum_{j=1}^{\lfloor\frac{m}{dy}\rfloor}1 \tag{16}$$

$$= \sum_{d=1}^{n}\mu(d)\sum_{x=1}^{\lfloor\frac{n}{d}\rfloor}\sum_{y=1}^{\lfloor\frac{m}{d}\rfloor}\lfloor\frac{n}{dx}\rfloor\lfloor\frac{m}{dy}\rfloor \tag{17}$$

令$\lfloor\frac{n}{d}\rfloor = p$ $\lfloor\frac{m}{d}\rfloor = q$

$$= \sum_{d=1}^{n}\mu(d)\sum_{x=1}^{p}\sum_{y=1}^{q}\lfloor\frac{p}{x}\rfloor\lfloor\frac{q}{y}\rfloor \tag{18}$$

$$= \sum_{d=1}^{n}\mu(d)\sum_{x=1}^{p}\lfloor\frac{p}{x}\rfloor\sum_{y=1}^{q}\lfloor\frac{q}{y}\rfloor \tag{19}$$

令$f(p) = \sum_{x=1}^{p}\lfloor\frac{p}{x}\rfloor$

$$= \sum_{d=1}^{n}\mu(d)f(p)f(q) \tag{20}$$

$$= \sum_{d=1}^{n}\mu(d)f(\lfloor\frac{n}{d}\rfloor)f(\lfloor\frac{m}{d}\rfloor) \tag{21}$$

C

$$\sum_{i=1}^{n}\sum_{j=1}^{m}[\gcd(i,j)=p] \tag{22}$$

$$\sum_{i=1}^{\lfloor\frac{m}{p}\rfloor}\sum_{j=1}^{\lfloor\frac{m}{p}\rfloor}[\gcd(i,j)=1] \tag{23}$$

$$\sum_{i=1}^{\lfloor\frac{n}{p}\rfloor}\sum_{j=1}^{\lfloor\frac{m}{p}\rfloor}\sum_{d|\gcd(i,j)}\mu(d) \tag{24}$$

$$\sum_{d=1}^{n}\mu(d)\sum_{d|i}^{\lfloor\frac{n}{p}\rfloor}\sum_{d|j}^{\lfloor\frac{m}{p}\rfloor}1 \tag{25}$$

$$\sum_{d=1}^{n}\mu(d)\lfloor\frac{m}{dp}\rfloor\lfloor\frac{n}{dp}\rfloor \tag{26}$$

$$\sum_{T=1}^{n}\mu(\frac{T}{p})\lfloor\frac{m}{T}\rfloor\lfloor\frac{m}{T}\rfloor \tag{27}$$

D

容斥 同C

E

$$\sum_{i=1}^{n}\sum_{j=1}^{m}\gcd(i,j)^k \tag{28}$$

$$=\sum_{p=1}^{n}\sum_{i=1}^{n}\sum_{j=1}^{m}[\gcd(i,j)=p]*p^k \tag{29}$$

$$=\sum_{p=1}^{n}p^k\sum_{i=1}^{\lfloor\frac{n}{p}\rfloor}\sum_{j=1}^{\lfloor\frac{m}{p}\rfloor}[\gcd(i,j)=1] \tag{30}$$

$$=\sum_{p=1}^{n}p^k\sum_{i=1}^{\lfloor\frac{n}{p}\rfloor}\sum_{j=1}^{\lfloor\frac{m}{p}\rfloor}\sum_{d|\ \gcd(i,j)}\mu(d) \tag{31}$$

$$=\sum_{p=1}^{n}p^k\sum_{d=1}^{n}\mu(d)\sum_{d|i}^{\lfloor\frac{n}{p}\rfloor}\sum_{d|j}^{\lfloor\frac{m}{p}\rfloor}1 \tag{32}$$

$$=\sum_{p=1}^{n}p^k\sum_{d=1}^{n}\mu(d)\lfloor\frac{n}{dp}\rfloor\lfloor\frac{m}{dp}\rfloor \tag{33}$$

$$=\sum_{T=1}^{n}\lfloor\frac{n}{T}\rfloor\lfloor\frac{m}{T}\rfloor\sum_{p|T}p^k*\mu(\frac{T}{p}) \tag{34}$$

$g(T)=\sum_{p|T}p^k\mu(\frac{T}{p})$

$$=\sum_{T=1}^{n}\lfloor\frac{n}{T}\rfloor\lfloor\frac{m}{T}\rfloor g(T) \tag{35}$$

考虑如何预处理$g(T)$

$g(T)=\sum_{p|T}p^k\mu(\frac{T}{p})=\sum_{p|T}(\frac{T}{p})^k\mu(p)$

因为$A(x)=x^p$和$B(x)=\mu(x)$ 均为积性函数，所以$(A*B)$ 也为积性函数。

那么当T为质数的时候，那么$g(T)=p^k*\mu(1)+1*\mu(p)=p^k-1$

当T为x^{p+1}次时候，$g(T)=\sum_{p|T}(\frac{T}{p})^k\mu(p)$，$\mu(p)$在$p|T$时，当且仅当$p=x$或$p=1$不为0，所以$g(x^{p+1})=x^{(p+1)*k}-x^{p*k}$，又因为$g(x^p)=x^{p*k}-x^{(p-1)*k}$，那么$g(x^{p+1})=g(x^p)*x^k$

用线性筛来做这个式子。

当$i \bmod p_j=0$时，$g(i*p_j)=g(i)*p_j^k$

否则，$g(i*p_j)=g(i)*g(p_j)$

```
inline void init() {
    g[1] = 1;
    for (int i = 2; i <= M; i++) {
        if (!flag[i]) {
            prime[++tot] = i;
            h[i] = qpow(i, k);
            g[i] = (h[i] - 1 + mod) % mod;
        }
        for (int j = 1; j <= tot && prime[j] * i <= M; j++) {
            flag[i * prime[j]] = 1;
            if (i % prime[j] == 0) {
                g[i * prime[j]] = 1ll * g[i] * h[prime[j]] % mod;
            } else {
                g[i * prime[j]] = 1ll * g[i] * g[prime[j]] % mod;
            }
        }
    }
    for (int i = 1; i <= M; i++) s[i] = (s[i - 1] + g[i]) % mod;
}
```