\sum_{i=1}^n\sum_{j=1}^mgcd(i,j)^k \\=\sum_{d=1}^{min(n,m)}d^k\sum_{i=1}^{\frac{n}{d}}\sum_{j=1}^{\frac{m}{d}}1 \\=\sum_{d=1}^{\min(n,m)}d^k\frac{n}{d}\\frac{m}{d} \\预处理d^k,O(n+T\sqrt n)

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$$egin{aligned} \sum_{i=1}^n \sum_{j=1}^m gcd(i,j)^k \ &= \sum_{d=1}^{min(n,m)} d^k \sum_{i=1}^{rac{n}{d}} \sum_{j=1}^{rac{m}{d}} 1 \ &= \sum_{d=1}^{min(n,m)} d^k rac{n}{d} rac{m}{d} \ &rac{n}{d} \sum_{i=1}^n \sum_{j=1}^m gcd(i,j)^k \ &= \sum_{d=1}^{min(n,m)} d^k \sum_{i=1}^{rac{n}{d}} \sum_{j=1}^{rac{m}{d}} 1 \ &= \sum_{d=1}^{min(n,m)} d^k rac{n}{d} rac{m}{d} \ &rac{m}{d} rac{m}{d} \ &rac{m}{d} rac{m}{d} \ &rac{m}{d} rac{m}{d} \ &rac{m}{d} rac{m}{d} \ &rac{m}{d$$