**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Given: Mean (μ) = 45 minutes Standard deviation (σ) = 8 minutes

The service manager plans to start the work 10 minutes after the car is dropped off, which means there's 60 minutes - 10 minutes = 50 minutes available for servicing the transmission within the 1-hour commitment.

Now, we want to find the probability that the service time exceeds 50 minutes (i.e., the car won't be ready within the committed 1 hour).

To find this probability, we'll standardize the value of 50 minutes using the z-score formula:

�=�−��*Z*=*σX*−*μ*​

Where: �*X* = 50 minutes (time available for servicing) �*μ* = 45 minutes (mean) �*σ* = 8 minutes (standard deviation)

�=50−458=58=0.625*Z*=850−45​=85​=0.625

Now, we need to find the probability associated with this z-score.

Looking up the z-score of 0.625 in a standard normal distribution table or using a calculator that provides probabilities from the standard normal distribution, the probability is approximately 0.2676.

Therefore, the correct answer is B. 0.2676.

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Given: Mean (μ) = 38 years Standard deviation (σ) = 6 years

A. More employees at the processing center are older than 44 than between 38 and 44.

To determine the number of employees older than 44 and between 38 and 44, we'll use the z-score formula and the properties of the normal distribution.

For employees older than 44: �44=44−386=66=1*Z*44​=644−38​=66​=1 From the standard normal distribution table, the probability corresponding to �=1*Z*=1 is about 0.8413. This represents the proportion of employees older than 44.

For employees between 38 and 44: To find the proportion of employees between 38 and 44, we can subtract the cumulative probability up to 38 from the cumulative probability up to 44. �38=38−386=0*Z*38​=638−38​=0 �44=44−386=1*Z*44​=644−38​=1 The probability corresponding to �=0*Z*=0 is 0.5 (as it's the mean in a standard normal distribution). Subtracting the probability up to �=38*Z*=38 from the probability up to �=44*Z*=44 gives 0.8413−0.5=0.34130.8413−0.5=0.3413.

Therefore, the statement A is False. More employees fall between the ages of 38 and 44 (about 34.13%) than are older than 44 (about 84.13%).

B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

To find the proportion of employees under the age of 30, we'll calculate the z-score for 30 and then find the corresponding probability.

�30=30−386=−86=−1.33*Z*30​=630−38​=6−8​=−1.33 From the standard normal distribution table or calculator, the probability corresponding to �=−1.33*Z*=−1.33 is about 0.0912.

Multiplying this probability by the total number of employees (400) gives the expected number of employees under 30: 0.0912×400≈36.480.0912×400≈36.48

Therefore, statement B is True. The training program for employees under the age of 30 would be expected to attract around 36 employees.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Given:

* �1∼�(�,�2)*X*1​∼*N*(*μ*,*σ*2)
* �2∼�(�,�2)*X*2​∼*N*(*μ*,*σ*2)
* �1*X*1​ and �2*X*2​ are independent

**Distribution and Parameters of 2�12*X*1​:**

The random variable 2�12*X*1​ represents a transformation of �1*X*1​ where each value of �1*X*1​ is doubled.

Distribution:

Since �1*X*1​ is normally distributed, 2�12*X*1​ will also be normally distributed.

Parameters:

For 2�12*X*1​:

* Mean: �[2�1]=2⋅�[�1]=2⋅�*E*[2*X*1​]=2⋅*E*[*X*1​]=2⋅*μ*
* Variance: ���(2�1)=4⋅���(�1)=4⋅�2*Var*(2*X*1​)=4⋅*Var*(*X*1​)=4⋅*σ*2

**Distribution and Parameters of �1+�2*X*1​+*X*2​:**

The random variable �1+�2*X*1​+*X*2​ represents the sum of two independent normal random variables.

Distribution:

The sum of independent normal random variables is also a normal random variable.

Parameters:

For �1+�2*X*1​+*X*2​:

* Mean: �[�1+�2]=�[�1]+�[�2]=�+�=2⋅�*E*[*X*1​+*X*2​]=*E*[*X*1​]+*E*[*X*2​]=*μ*+*μ*=2⋅*μ*
* Variance: ���(�1+�2)=���(�1)+���(�2)=�2+�2=2⋅�2*Var*(*X*1​+*X*2​)=*Var*(*X*1​)+*Var*(*X*2​)=*σ*2+*σ*2=2⋅*σ*2

**Difference between 2�12*X*1​ and �1+�2*X*1​+*X*2​:**

* Both 2�12*X*1​ and �1+�2*X*1​+*X*2​ follow normal distributions.
* Their means are the same, 2⋅�2⋅*μ*, but their variances differ: 4⋅�24⋅*σ*2 for 2�12*X*1​ and 2⋅�22⋅*σ*2 for �1+�2*X*1​+*X*2​.

In summary, while both random variables have the same mean, their variances differ due to their different operations (scaling for 2�12*X*1​ and summation for �1+�2*X*1​+*X*2​).

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1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

To find two values, �*a* and �*b*, symmetric about the mean of a normal distribution such that the probability of the random variable taking a value between them is 0.99, we need to consider the properties of the standard normal distribution and the z-score.

For a normal distribution �∼�(�,�2)*X*∼*N*(*μ*,*σ*2), the values �*a* and �*b* can be determined using z-scores:

�=�−��*Z*=*σX*−*μ*​

Given: Mean (�*μ*) = 100 Variance (�2*σ*2) = 20² = 400

First, let's find the z-scores corresponding to the probabilities that will enclose the middle 99% of the distribution. For a 99% probability, we need to find the z-scores that correspond to the tails of 0.0050.005 and 0.9950.995 (as the distribution is symmetric).

Using a standard normal distribution table or calculator:

For 0.0050.005 tail probability (on the left): �0.005≈−2.576*Z*0.005​≈−2.576

For 0.9950.995 tail probability (on the right): �0.995≈2.576*Z*0.995​≈2.576

Now, we'll use these z-scores to find the corresponding values of �*a* and �*b* in the original distribution:

�=�+�0.005×�*a*=*μ*+*Z*0.005​×*σ* �=100+(−2.576)×20=100−51.52=48.48*a*=100+(−2.576)×20=100−51.52=48.48

�=�+�0.995×�*b*=*μ*+*Z*0.995​×*σ* �=100+2.576×20=100+51.52=151.52*b*=100+2.576×20=100+51.52=151.52

The values of �*a* and �*b* are approximately 48.48 and 151.52, respectively.

Comparing these values to the given choices:

* D. 48.5, 151.5 closely matches the calculated values of �*a* and �*b*, making it the most appropriate answer.

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45

Given:

* Profit1 follows a normal distribution: ������1∼�(5,32)*Profit*1∼*N*(5,32)
* Profit2 follows a normal distribution: ������2∼�(7,42)*Profit*2∼*N*(7,42)
* The exchange rate is $1 = Rs. 45

1. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company

To find a range centered on the mean that contains 95% probability, we'll sum the means of Profit1 and Profit2 and then find the range that covers 95% of the total probability.

Total mean profit in $: Total mean=Mean of Profit1+Mean of Profit2=5+7=12 million dollarsTotal mean=Mean of Profit1+Mean of Profit2=5+7=12 million dollars

Total standard deviation: Total variance=Variance of Profit1+Variance of Profit2=32+42=9+16=25 million dollars2Total variance=Variance of Profit1+Variance of Profit2=32+42=9+16=25 million dollars2 Total standard deviation=25=5 million dollarsTotal standard deviation=25​=5 million dollars

Now, to find the range containing 95% probability: Using the properties of the normal distribution and the fact that 95% of the probability falls within approximately 1.96 standard deviations from the mean:

Lower bound=Total mean−1.96×Total standard deviation=12−1.96×5Lower bound=Total mean−1.96×Total standard deviation=12−1.96×5 Upper bound=Total mean+1.96×Total standard deviation=12+1.96×5Upper bound=Total mean+1.96×Total standard deviation=12+1.96×5

Calculating these values in million dollars and converting to Rupees using the exchange rate ($1 = Rs. 45):

Lower bound in Rupees: (12−1.96×5)×45(12−1.96×5)×45 Upper bound in Rupees: (12+1.96×5)×45(12+1.96×5)×45

### B. 5th percentile of profit (in Rupees) for the company:

The 5th percentile corresponds to a value below which 5% of the data lies.

Using the properties of the standard normal distribution, the 5th percentile is approximately -1.645 standard deviations from the mean.

5th percentile in Rupees: (Total mean−1.645×Total standard deviation)×45(Total mean−1.645×Total standard deviation)×45

### C. Which division has a larger probability of making a loss in a given year?

For a normal distribution, a loss occurs when the profit is less than zero.

Let's find the probability of each division making a loss using their respective means and standard deviations:

For Profit1: Probability of loss for Profit1 �(Profit1<0)*P*(Profit1<0) can be calculated using the cumulative distribution function (CDF) of a normal distribution with mean 5 and standard deviation 3.

For Profit2: Probability of loss for Profit2 �(Profit2<0)*P*(Profit2<0) can be calculated using the CDF of a normal distribution with mean 7 and standard deviation 4.

The division with a larger probability of making a loss in a given year will be the one with a higher probability obtained from these calculations.

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