Boom-Bust Housing Cycles and Structural Heterogeneity in Currency Areas

Ulrich Roschitsch, Hannes Twieling*

April 1, 2023

Abstract

The intensity of boom-and-bust cycles in house prices differs markedly between the member countries of the Euro Area. We provide empirical evidence that links the heterogeneity in country-wise house price responses to monetary policy shocks to a proxy for housing supply elasticities. We then construct a model of a currency area where (i) structural heterogeneity in countries' ability to quickly produce many new housing units and (ii) subjective private sector house price expectations in the form of capital gain extrapolation interact to endogenously generate inefficient boom-bust cycles in house prices that differ in their intensity between countries. To solve the model, we propose a new method for linearizing models with asset price learning. Specifically, we explicitly solve for the subjectively optimal plan of households to first order, given that they hold subjective expectations over house prices and rational expectations elsewhere. Our model qualitatively matches empirical patterns and uncovers new challenges for the monetary authority in a currency area.

JEL Codes: E31, E32, E52, F45

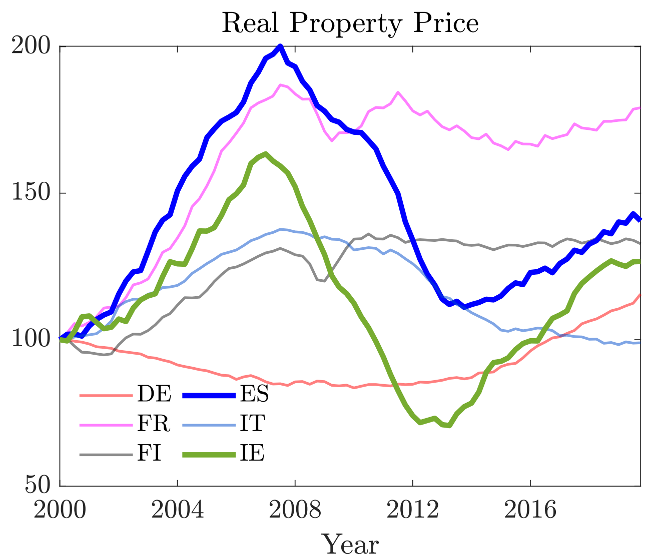
Keywords: Monetary Policy, Currency Area, Structural Heterogeneity, Subjective House Price Expectations, Housing Booms, Asset Price Learning

*University of Mannheim, Department of Economics, ulrich.roschitsch@uni-mannheim.de, hannes.twieling@uni-mannheim.de. The authors gratefully acknowledge financial support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) through CRC TR 224 (Project C02) and by Stiftung Geld und Währung.

1 Introduction

House prices in many advanced economies exhibit large and sustained booms and busts. Such episodes are often thought to reflect subjective private sector expectations and their shifts between over-optimism and over-pessimism (Kaplan et al., 2020). Not all advanced economies experience equally large housing cycles, though. While the timing of the house price episode of the early 2000s has been quite synchronous between most countries that did experience a cycle, in particular across the Euro Area, its intensity has been very different. In the Euro Area, Spain and Ireland experienced a large boom and bust cycle in house prices (Figure 1), while Italy experienced a more tempered cycle, France experienced a secular increase and German real house prices followed a convex, smooth and moderate path.

Figure 1: The 2000s House Price Cycle in the Euro Area



Notes: GDP-deflated prices for housing, for selected Euro Area countries (2000 = 100); *Source:* Bank for International Settlements (BIS).

Investigating the causes and consequences of boom-bust episodes in house prices is important: To the extent that house price episodes may reflect over-optimism and -pessimism, they not only amplify macroeconomic and financial risk, but directly distort consumption, investment, and labor choices. The notion of belief-driven and thus inefficient cycles, and the observation of the different intensities of the recent housing cycle across Euro Area countries jointly point to an important issue for the design of monetary policy in a currency area: The differing intensities of country-specific house price cycles may reflect a structural heterogeneity that, like heterogeneity in price-setting frictions, or in frictions on the labor market, renders both policy transmission and the intensity of frictions heterogeneous across member countries.

In this paper, we propose a model of a currency union with endogenously arising house price cycles that differ in their intensity across countries, to examine the implications of this novel type of heterogeneity.

Using Euro Area macro-data, we first document that house prices in Euro Area countries react heterogeneously to monetary policy shocks and that the heterogeneous responses can be linked to differences in a proxy for the elasticity of housing construction: the time it takes to obtain a building permit.[1] We regress the logarithm of a real property price index, and the logarithm of the price-to-rent ratio for selected Euro Area countries on a high-frequency-identified monetary policy shock and its interaction with the time it takes to obtain a building permit. In countries where it takes comparatively long to start the construction process, house prices react much more strongly to a positive monetary policy shock: Increasing the time it takes to obtain a building permit by one standard deviation (ca. 50 days) increases the peak house price reaction to a one standard deviation monetary policy shock (ca. 13 basis points) by almost 50%.

Informed by this stylized fact, we then construct a model of a two-country currency area with cross-country asymmetry in the elasticity of housing construction. Embedding subjective house price expectations in the form of capital gain extrapolation into this model gives rise to endogenous boom-bust cycles in house prices that differ in their intensity across countries: cycles are strong where the elasticity of housing supply is low. The same model under rational expectations (RE) is unable to produce cycles in house prices absent further modifications such as, e.g., habit formation. Subjective expectations in the form of capital gain extrapolation, in turn, represent a parsimonious deviation from the standard currency union model and have been shown to provide a good fit to the available empirical evidence on private sector expectations over stock prices (Adam et al., 2017) and house prices (Adam et al., 2022). That is, households in the currency area optimize given their beliefs and hold rational expectations on all variables except house prices: As in Adam et al. (2012; 2022), households filter a perceived long-run house price growth rate from past house price observations.[2] Our model provides a theoretical framework for explaining cross-country heterogeneity in the intensity of housing cycles. Specifically, the extrapolative nature of private sector expectations creates a positive feedback loop between house price growth and housing demand. This leads to booms that are most pronounced in countries where market clearing with an inelastic housing supply requires

[1] Aastveit et al. (2020) and Aastveit and Anundsen (2022) show how the differential house price reactions between US metropolitan areas to monetary policy shocks relate negatively to differences in the elasticity of housing supply.

[2] (i) Agents are *internally rational* (see Adam and Marcet (2011): agents are endowed with a set of time-consistent beliefs and act optimally given these beliefs); and (ii) agents hold rational expectations with respect to each variable that they take as given, except for house prices. (That means that for each such variable, they know the true, equilibrium-implied distribution.)

large house price increases after an initial shock. As a result of housing cycles being driven by subjective expectations, decisions are distorted: during the boom phase households work too much, consume too little, and invest too much into housing, relative to the RE-allocation. The difference in intensity of the cycle manifests in differences between the countries' allocations of consumption, housing, housing investment, and hours worked. This is relevant for policy, as there are welfare gains associated with monetary policy leaning against high house prices.

Our paper relates to the literature on structural asymmetries in currency areas (Benigno, 2004; Calza et al., 2013; Farhi and Werning, 2017; Bletzinger and von Thadden, 2021; Pica, 2021; Corsetti et al., 2022; Kekre, 2022). Most of this literature has focused on asymmetries that regard either price-setting frictions for goods (cf. e.g. Benigno, 2004), or on labor market frictions (cf. e.g. Kekre, 2022). To the best of our knowledge, we are the first to examine the ramifications of structural asymmetries that lead to cross-country differences in the intensity of asset price cycles. Pica (2021) does investigate the role of heterogeneities in housing market institutions across Euro Area countries for shaping monetary policy transmission. He focuses on the effects of monetary policy that operate through mortgage rates and household balance sheets, while we abstract from household heterogeneity and focus on the role of house price cycles and their distortionary influence on the economy.

We also relate to the literature on subjective asset price expectations. Adam et al. (2022), Winkler (2020), and Caines and Winkler (2021), among others, examine the implications of capital gain extrapolation in closed economies. Relative to this literature we make the methodological contribution of providing a method to solve models with capital gain extrapolation to first order. Previous research has either solved such models globally (Adam and Merkel, 2019), linearized part of the model while keeping the subjective expectations law of motion nonlinear (Adam et al., 2022), or assumed that household beliefs are 'conditionally model consistent' (Winkler, 2020; Caines and Winkler, 2021), an assumption that implies that beliefs over all equilibrium variables are distorted relative to the variables' equilibrium-implied distributions. In contrast, we solve directly for the linearized subjectively optimal plan of households. This carries two main advantages. First, despite retaining the ability of subjective beliefs to generate boom-bust patterns in equilibrium house prices, linearization provides a fast solution method that readily scales to large models and yields a model representation that is amenable to the analysis of optimal policy using linear-quadratic approximations. Second, by explicitly solving for the subjectively optimal plans, our method of linearizing the model preserves the ability to solve for the equilibrium concept where belief distortions are confined to asset prices. This is attractive not because the rational expectations hypothesis is the best model of expectations regarding inflation or wages, say, but because it is still widely regarded as the natural default for capturing private sector expectations in macroeconomic models. Our linearization method allows to deviate from this default at exactly those points where survey data allow to discipline the modeling of the expectations formation process.

The rest of the paper is structured as follows. In Section 2 we present the stylized facts on the responses of house prices to monetary policy shocks across Euro Area countries that motivate our modeling choices. In Section 3, we present the structure of our model economy, explain how agents form expectations and which equilibrium concept we use. Section 4 elaborates how we solve for the subjectively optimal plans to first order, presents analytical insights into households' housing plans and the role of expected capital gains, and presents our parameterization. Finally, in Section 5 we examine the model-implied impulse responses to a monetary policy shock, and Section 6 concludes.

2 House prices, monetary policy shocks and heterogeneity across Euro Area countries

In this section, we investigate how house prices react to monetary policy shocks and document heterogeneous responses across Euro Area countries. We find that lower supply elasticities in the housing market are associated with stronger house price responses. In our empirical analysis, we rely on country panel local projections. Our panel covers the following Euro Area countries: Austria, Germany, Spain, Finland, France, Ireland, Italy, the Netherlands, and Portugal. These countries roughly cover the founding members of the Euro Area.[3] In our exercise we are interested in the reaction of a certain variable of interest to a monetary policy shock. To investigate what drives cross-country differences we further add an interaction term to the monetary policy shock. Our empirical specification reads as follows:

$$y_{n,t+h} = \alpha_n^h + t + \beta^h \epsilon_t^{MP} + \gamma^h \epsilon_t^{MP} \times \text{Inter}_n + \text{controls}_{n,t} + u_{n,t+h} \qquad h = 0, 1, \ldots, H \tag{1}$$

As left-hand-side, y_n, we consider the log of the real property price index and of the price-to-rent ratio. ϵ_t^{MP} denotes the monetary policy shock which is a high-frequency identified shock at a one-year horizon and taken from Altavilla et al. (2019). As an interaction term, we use the days it takes to obtain a building permit in a given country. We interpret this variable as a proxy for supply-side elasticities. Finally, we include country-fixed effects, a time trend, and a vector of controls.[4] The sample runs from 2000 to 2019 and is in quarterly frequency. Figure 2 plots the responses of our variables of interest to an expansionary monetary policy shock.

[3] We exclude Belgium and Luxembourg due to insufficient data availability.

[4] The vector of controls consists of 6 lags of the following variables: The left-hand-side variable, log GDP, log HICP, the EONIA, the shock, and the shock interacted with the interaction term.

Figure 2: Empirical evidence for heterogeneous housing responses

Notes: Responses to expansionary MP shock (1 std = 13 bpt); Interaction term: Time to obtain a building permit in days (World Bank; 1 std = 49 days); MP-shock: high-frequency-identified based on OIS at one year horizon from Altavilla et al. (2019), applied poor man's approach & aggregated to quarterly frequency; Controls: 6 lags of LHS variable, log GDP, log HICP, EONIA, MP shock, MP shock × interaction; Sample: all quarters from 2000Q1 to 2019Q4; Countries: AT, DE, ES, FI, FR, IR, IT, NL, PT; Confidence Intervals: 68% and 95% (Driscoll and Kraay, 1998).

The left column shows the mean responses, the β^h in equation (1), and the right column the interaction term, the γ^h in equation (1). We find that the expansionary shock leads to an increase in house prices and price-to-rent ratios. All responses are significant. With respect to the interaction terms, the interpretation is as follows: a one-standard-deviation higher interaction term leads to a roughly 50% stronger response in house prices. The interpretation is equivalent for the price-to-rent ratio. We can therefore conclude that a longer time to obtain a building permit is associated with a stronger response in house prices and price-to-rent ratios. These results suggest that housing supply elasticities play an important role in the reaction of house prices to monetary policy. As part of a robustness analysis, we considered several additional interaction terms in our empirical specifications. However, none of these terms were found to be statistically significant. Details and results of this exercise are presented in Appendix A.

3 Model Outline

In this section, we describe our two-country currency union model. First-order closed-form solutions to the household program are in Section 4. Foreign variables are denoted by an asterisk (*). For expositional brevity, we describe only the domestic economy wherever possible; the corresponding details for the foreign economy are then analogous.

3.1 Households

A representative domestic household derives utility from consuming domestic and foreign varieties, leisure, and housing. The preferences are as follows:

$$\mathbb{E}_0^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t u(c_t, h_t, n_t), \quad u(c_t, h_t, n_t) = \frac{\xi_{c,t} c_t^{1-\sigma}}{1-\sigma} + \frac{\xi_{h,t} h_t^{1-\nu}}{1-\nu} - \chi \frac{n_t^{1+\varphi}}{1+\varphi}$$

$$c_t = \left[\lambda^{\varsigma} c_{H,t}^{1-\varsigma} + (1-\lambda)^{\varsigma} c_{F,t}^{1-\varsigma} \right]^{\frac{1}{1-\varsigma}}$$

$$c_{H,t} = \gamma \left[\int_0^1 c_{H,t}(j)^{\frac{\epsilon-1}{\epsilon}} dj \right]^{\frac{\epsilon}{\epsilon-1}}, \quad c_{F,t} = (1-\gamma) \left[\int_0^1 c_{F,t}(j^*)^{\frac{\epsilon-1}{\epsilon}} dj^* \right]^{\frac{\epsilon}{\epsilon-1}}$$

where $\mathbb{E}_0^{\mathcal{P}}$ denotes the subjective expectations operator discussed in Section 3.2. $\xi_j, j \in \{c, h, a, x, i\}$, denote model-exogenous shock terms, h_t and n_t denote housing and hours worked respectively, and γ is the measure of households in the home economy. Following Benigno (2004), γ is simultaneously the economic size of the home region, i.e. the mass of variety-producing firms. c_t denotes consumption of the domestic basket that is assembled from the home-good and the foreign-good which in turn are CES-aggregates of two groups of varieties. Consumers in F also consume the home- and foreign-good, albeit with different weights: $c_t^* = [(1-\lambda^*)^{\varsigma} c_{H,t}^{1-\varsigma} + (\lambda^*)^{\varsigma} c_{F,t}^{1-\varsigma}]^{\frac{1}{1-\varsigma}}$. A preference bias for goods produced in the respective country of residence ("home bias") arises if $\lambda, 1-\lambda^* \neq \gamma$ and throughout the paper we maintain the assumptions that (i) the degree of home bias is symmetric: $\gamma(1-\lambda) = (1-\gamma)(1-\lambda^*)$, and (ii) the bias is such that households favor domestically produced products, $\lambda \geq \gamma$. Standard algebra on CES-aggregation allows to aggregate the market prices for consumption goods into price indices

$$\gamma \int_0^1 P_{H,t}(j) c_{H,t}(j) dj = c_{H,t} P_{H,t}, \quad P_{H,t} = \left[\int_0^1 P_{H,t}(j)^{1-\epsilon} dj \right]^{\frac{1}{1-\epsilon}},$$

$$c_{H,t} P_{H,t} + c_{F,t} P_{F,t} = P_t c_t, \quad P_t = \left[\lambda P_{H,t}^{1-\frac{1}{\varsigma}} + (1-\lambda) P_{F,t}^{1-\frac{1}{\varsigma}} \right]^{\frac{1}{1-\frac{1}{\varsigma}}},$$

and the household budget constraint is then given by:

$$c_t + q_t(h_t - (1-\delta)h_{t-1}) + b_{t+1} + x_t = w_t n_t + (1+r_t) b_t + q_t \cdot H(x_t, \xi_{x,t}) - T_t + \Sigma_t + \mathscr{b}_t.$$

The budget constraint is expressed in units of the country-H final consumption basket, c. Σ_t are profits from all domestic firms, which are owned evenly by all domestic households, w_t is the real wage, T_t are government lump-sum taxes, and b_t is a one-period nominal zero-coupon bond that is traded union-wide. q_t denotes the real house price and x_t is the number of consumption units dedicated to the production of new housing units, using the technology $H(x_t, \xi_{x,t}) = \xi_{x,t} \frac{x_t^\eta}{\eta}$, $\eta \in (0,1)$. Housing units can be retained to enjoy housing services, or sold on the housing market. It is convenient to express the bond holdings in units of country H's final basket. The real interest rate r_t is taken as given by households and is determined in equilibrium by the following Fisher-type equation: The value of bond holdings in units of numéraire is $B_t = P_t \cdot b_t$ and the nominal bond pays $i_{t-1} - \psi b_t$ units of currency as interest.[5] The real interest rate is thus given by

$$1 + r_t = \frac{1 + i_{t-1} - \psi b_t}{1 + \pi_t}$$

where $\pi_t := P_t / P_{t-1} - 1$. Finally, $\mathscr{b}_t := (\beta^{-1} - 1)\left(\gamma + (1-\gamma)\frac{P_{t-1}^*}{P_{t-1}}\right)(1+\pi_t)^{-1}\bar{b}$, taken as exogenous by the household, captures payment streams between H and F that guarantee that households are content with holding no bonds in the non-stochastic steady state with zero inflation and real exchange rate parity.[6]

[5]The nominal interest rate is elastic in the aggregate holdings of bonds by domestic households. We follow Schmitt-Grohé and Uribe (2003) to ensure stationarity of the first-order dynamics. In Appendix B we provide a simple micro-foundation for debt-elastic interest rates.

[6]Given that bond holding entails a real cost in equilibrium, see footnote 5, introducing the payments $\mathscr{b}_t$ is a way to ensure that there are no bond holding costs in the non-stochastic steady state with zero inflation and real exchange rate parity (i.e. $1 + \pi_t = 1 + \pi_t^* = 1 + \pi_{H,t} = 1 + \pi_{F,t} = \frac{P_{H,t}}{P_{F,t}} = 1$). This ensures that this steady state is efficient, given that fiscal policy undoes the monopolistic competition distortion. $\mathscr{b}_t$ may be interpreted as the real interest rate paid by a non-marketable nominal consol, that perpetually pays the nominal rate $(\beta^{-1} - 1)(\gamma + (1-\gamma)P_{t-1}^*/P_{t-1})$ and of which the household is endowed with $\bar{b}$ units. The endowments of these consols ensure that nominal payments balance, i.e. $\gamma\bar{b} + (1-\gamma)\bar{b}^* = 0$, see Appendix B.

3.2 Subjective House Price Expectations

As is standard in the literature on capital gain extrapolation (e.g. Adam and Marcet, 2011; Adam et al., 2017), households are endowed with a set of beliefs in the form of a probability measure over the full sequence of variables that they take as given ("external" variables): $(\xi_t, r_t, w_t, \Sigma_t, T_t, \pi_t, (P_t/P_t^*), q_t)_{t\geq 0}$. This measure we denote as $\mathcal{P}$; Rational expectations are a special case of this setup in the form that households' beliefs agree with the objective (or sometimes "true" or "equilibrium-implied") distribution of external variables, $\mathcal{P} = \mathbb{P}$. Although households may hold expectations that are generally inconsistent with the equilibrium-implied (conditional) distribution of external variables, it is worth emphasizing that (i) they have a time-consistent set of beliefs, and (ii) they behave optimally given their beliefs. That is, households are *internally rational* in the sense of Adam and Marcet (2011). Moreover, the fact that all households are identical in beliefs and preferences is not common knowledge among agents so there is no possibility for households to discover the misspecification of their beliefs, $\mathcal{P} \neq \mathbb{P}$, by eductively reasoning through the structure of the economy. Given the observed path of external variables up to period t, households then use this information and $\mathcal{P}$ to form a conditional expectation over the continuation sequence of external variables, which we denote as $\mathbb{E}_t^{\mathcal{P}}$. We denote the conditional rational expectations operator as usual by $\mathbb{E}_t$.

We assume that agents have rational expectations with respect to all external variables, except for house prices, q_{t+s}.[7] Households entertain the idea that house prices follow a simple state-space model:

$$\begin{aligned}
\ln \frac{q_{t+1}}{q_t} &= \ln m_{t+1} + \ln e_{t+1} \\
\ln m_{t+1} &= \varrho \ln m_t + \ln v_{t+1}, \qquad \varrho \in (0, 1) \\
\begin{pmatrix} \ln e_t & \ln v_t \end{pmatrix}' &\sim \mathcal{N}\left(\begin{pmatrix} -\frac{\sigma_e^2}{2} & -\frac{\sigma_v^2}{2} \end{pmatrix}, \begin{pmatrix} \sigma_e^2 & 0 \\ 0 & \sigma_v^2 \end{pmatrix} \right)
\end{aligned} \tag{2}$$

where ϱ is assumed to be close to unity. Hence, agents perceive house price growth rates as the sum of a transitory and a persistent component. Crucially, $\ln e_t$ and $\ln v_t$ are not observable to the agents, rendering $\ln m_t$ unobservable. Agents apply the optimal Bayesian filter, i.e. the

[7]Formally, $\mathcal{P} := \mathbb{P}_{-q} \otimes \mathcal{P}_q$, where $\mathbb{P}_{-q}$ is the objective measure over sequences of external variables without house prices, $\mathcal{P}_q$ is the measure over sequences of house prices implied by the described perceived model of house prices, and $\otimes$ is the product measure. Since we are interested in a first-order solution to the model, it does not matter what households perceive to be the dependence structure between house prices and the other external variables.

Kalman filter, to arrive at the observable system:[8]

$$\begin{aligned}\ln \frac{q_{t+1}}{q_t} &= \varrho \ln \overline{m}_t + \ln \widehat{e}_{t+1} \\ \ln \overline{m}_t &= \varrho \ln \overline{m}_{t-1} - \frac{\sigma_v^2}{2} + g \cdot \left(\ln \widehat{e}_t + \frac{\sigma_e^2 + \sigma_v^2}{2} \right)\end{aligned}$$

where $\ln \overline{m}_t := \mathbb{E}_t^{\mathcal{P}}(\ln m_t)$ is the posterior mean, $g = \frac{\sigma^2+\sigma_v^2}{\sigma^2+\sigma_v^2+\sigma_e^2}$ is the steady-state Kalman filter, $\sigma^2 = \frac{1}{2}[-\sigma_v^2+\sqrt{\sigma_v^4 + 4\sigma_v^2\sigma_e^2}]$ is the steady-state Kalman filter uncertainty, and $\ln \widehat{e}_t$ is perceived to be a white noise process.

To avoid simultaneity in the house price we modify the belief setup following Adam et al. (2017).[9] We obtain the same observable system but with lagged information being used in the posterior mean updating equation:

$$\ln \overline{m}_t = (1-g) \left(\varrho \ln \overline{m}_{t-1} - \frac{\sigma_v^2}{2} \right) + g \left(\ln \frac{q_{t-1}}{q_{t-2}} + \frac{\sigma_e^2}{2} \right) \tag{3}$$

Under this formulation the posterior mean is pre-determined. We may now derive the posterior mean on the $s > 0$ periods ahead of price:

$$\mathbb{E}_t^{\mathcal{P}} q_{t+s} = q_t \cdot \exp\left(\ln \overline{m}_t \cdot \varrho \tfrac{1-\varrho^s}{1-\varrho} + \frac{1}{2}\sigma^2 \left(\varrho \tfrac{1-\varrho^s}{1-\varrho}\right)^2 \right) \cdot \exp(V), \quad V \propto \sigma_v^2 \tag{4}$$

Explicit derivations can be found in Appendix C. This arrangement of subjective beliefs over house prices follows closely Winkler (2020) and Caines and Winkler (2021), where the interested reader may find a detailed explanation of and intuition for the dynamics of beliefs in response to price movements.

3.3 Firms and price setting

We assume a continuum of monopolistically competitive firms that produce intermediate good varieties and have the same beliefs as households. Firm beliefs, however, concern only variables over which households have rational expectations, so firms are rational. Firm j buys labor $n_t(j)$ from the representative labor packer and produces the variety $y_t(j)$ with a linear technology where labor is the only production factor. The variety is bought by households from both

[8] We assume agents' prior variance equals the steady-state Kalman variance.

[9] q_t appears twice: in the forecast equation, and in the Kalman-updating equation through $\ln \widehat{e}_t$. Since q_t depends on $\overline{m}_t$, but the latter also depends on the former, it is not assured that at any point an equilibrium asset price exists and whether it is unique. See Adam et al. (2017) for the details. The idea of the modification is to alter agents' perceived information setup in that they observe each period one component of the lagged transitory price growth.

regions. The firm sets its retail price $P_{H,t}(j)$ and maximizes the expected discounted stream of profits, subject to Rotemberg-type adjustment costs. Formally the firm solves:

$$\max_{P_{H,t}(j)} \mathbb{E}_0^{\mathcal{P}} \sum_{t=0}^{\infty} \beta^t \frac{\Lambda_t}{P_t} \left[P_{H,t}(j) y_{H,t}(j) - (1-\tau^\ell) W_t n_t(j) - P_{H,t} \frac{\kappa}{2} \left(\frac{P_{H,t}(j)}{P_{H,t-1}(j)} - 1 \right)^2 y_{H,t} \right]$$

$$s.t. \quad y_{H,t}(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} y_{H,t}$$

with $y_{H,t}(j) = \xi_{a,t} n_t(j)$. $\Lambda_t = u'_{c,t}/u'_{c,0}$ denotes the stochastic discount factor and τ^ℓ is a wage subsidy paid by the government. It is selected such that the monopolistic competition distortion is offset in the non-stochastic steady state. The subsidy is financed through a lump-sum tax on the firm. In symmetric equilibrium, all firms choose the same price, $P_{H,t}(j) = P_{H,t} \forall j$ and we receive the New Keynesian Phillips curve:

$$(\Pi_{H,t}-1)\Pi_{H,t} = \beta \mathbb{E}_t^{\mathcal{P}} \left[\frac{\Lambda_{t+1} y_{H,t+1}}{\Lambda_t y_{H,t} \Pi_{t+1}} (\Pi_{H,t+1} - 1) \Pi_{H,t+1}^2 \right] + \frac{1}{\kappa} \left((1-\epsilon) + \epsilon (1-\tau^\ell) \frac{w_t P_t}{\xi_{a,t} P_{H,t}} \right) \quad (5)$$

The real wage and gross producer price inflation are defined as $w_t = \frac{W_t}{P_t}$ and $\Pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}}$ respectively.

3.4 Monetary authority

The monetary authority sets the nominal interest rates according to a standard Taylor rule targeting currency union consumer price inflation:

$$i_t = \frac{1}{\beta} \left(\Pi_t^{cu} \right)^{\phi} e^{\xi_{i,t}} \quad (6)$$

Currency union inflation is the average of country-level consumer price inflation, weighted by the country size: $\Pi_t^{cu} = (\Pi_t)^{\gamma} \left(\Pi_t^* \right)^{1-\gamma}$.

3.5 Market clearing

To achieve goods market clearing, each goods market for a variety j must clear. For notational convenience, we define $y_{H,t}(j) := c_{H,t}(j) + x_{H,t}(j) + \Psi_{t,H}(j)$, as the total demand for good (H, j) coming from one typical H-consumer. $\Psi_t := (1+\pi_t)^{-1} \frac{\psi}{2} (b_t)^2$ is the real cost of intermediating the position of an H-citizen in the union-wide bond. This cost, just like consumption and housing investment, gets passed along down to the varieties: $\Psi_{t,H} := (\frac{P_{H,t}}{P_t})^{-\frac{1}{\varsigma}} \lambda \Psi_t$. Goods market clearing across all goods markets requires:

$$y_{H,t} := \int_0^1 y_{H,t}(j)dj = \gamma \int_0^1 y_{H,t}(j)dj + (1-\gamma) \int_0^1 y^*_{H,t}(j)dj + \int_0^1 \Phi_t(j)dj$$

where $\Phi_t(j) = \left(\frac{P_{H,t}(j)}{P_{H,t}}\right)^{-\epsilon} \Phi_t$ and $\Phi_t = \frac{\kappa}{2}\left(\Pi_{H,t}-1\right)^2 y_{H,t}$ account for the price adjustment costs from the firm side. Aggregation and successive substitution eventually yields the domestic and foreign aggregate good market clearing conditions:

$$\left(1-\frac{\kappa}{2}(\Pi_{H,t}-1)^2\right) y_{H,t}\gamma = \gamma y_{H,t} + (1-\gamma) y^*_{H,t}$$
$$\left(1-\frac{\kappa}{2}(\Pi_{F,t}-1)^2\right) y^*_{F,t}(1-\gamma) = \gamma y_{F,t} + (1-\gamma) y^*_{F,t}$$

Further, the bond market clearing condition is given by:

$$\gamma P_t b_{t+1} + (1-\gamma) P^*_t b^*_{t+1} = 0.$$

Market clearing in the housing sectors is given by:

$$H(x_t, \xi_t) = (h_t - (1-\delta)h_{t-1}),$$
$$H(x^*_t, \xi^*_{x,t}) = (h^*_t - (1-\delta)h^*_{t-1}).$$

Finally, the balance-of-payments equation ensures that the household budget constraints hold:

$$\gamma y_{F,t} P_{F,t} - P_{H,t}(1-\gamma) y^*_{H,t} + \gamma \left(P_t b_{t+1} - (1+i_{t-1})P_{t-1}b_t - \ell_t\right) = 0.$$

3.6 Equilibrium

We adopt the equilibrium concept of Internally Rational Expectations Equilibrium as defined in Adam and Marcet (2011):

Definition 1 (Internally Rational Expectations Equilibrium (IREE)). *An IREE consists of three bounded stochastic processes, shocks* $(\xi_t)_{t\geq 0}$*, allocations* $([c_{H,t}(j)]_{j\in[0,1]}, [c^*_{F,t}(j^*)]_{j^*\in[0,1]},$ $b_t, b^*_t, h_t, h^*_t, x_t, x^*_t, n_t, n^*_t)_{t\geq 0}$ *and prices* $(w_t, w^*_t, q_t, q^*_t, i_t, [P_{H,t}(j)]_{j\in[0,1]}, [P_{F,t}(j^*)]_{j^*\in[0,1]})_{t\geq 0}$*, such that in all t*

1. *households choose* $[c_{H,t}(j)]_{j\in[0,1]}, [c^*_{F,t}(j^*)]_{j^*\in[0,1]}, b_t, b^*_t, h_t, h^*_t, x_t, x^*_t, n_t, n^*_t$ *optimally, given their beliefs* $\mathcal{P}$*,*

2. *firms choose* $([P_{H,t}(j)]_{j\in[0,1]}, [P_{F,t}(j^*)]_{j^*\in[0,1]})_{t\geq 0}$ *optimally, given their beliefs* $\mathcal{P}$*,*

3. *the monetary authority acts according to the Taylor rule,*

4. *markets for consumption good varieties, hours and housing clear given the prices, and the balance-of-payments equation holds.*

4 Solution method and parameters

We solve our model to first order around a non-stochastic and efficient steady state. This preserves analytic tractability at many points in the model and allows us to derive results on household behavior under capital gain extrapolation in closed form. It furthermore carries the advantage of providing a much better starting point for characterizing Ramsey-optimal monetary policy, an analysis of which would be an interesting and natural extension of our present work.

Linearizing models with capital gain extrapolation is not straightforward. In fact, to the best of our knowledge, we are the first to provide a first-order approximation to a model with capital gain extrapolation under the assumption that agents hold rational expectations outside of the asset pricing block.[10] In the following, we first describe how we solve for the linearized household decision rules, then we present analytical insights into the subjectively optimal housing choices of households, and finally we discuss the parameterization of our model.

4.1 Solution method

Notation. For any variable $\text{var}_t \notin \{b_t, b_t^*, \Sigma_t, \Sigma_t^*\}$ define $\widehat{\text{var}_t} := \frac{\text{var}_t - \text{var}_{ss}}{\text{var}_{ss}} \simeq \ln \text{var}_t - \ln \text{var}_{ss}$ to first order. For b_t, Σ_t (analogously for b_t^*, Σ_t^*) define $\widehat{\text{var}_t} := \frac{\text{var}_t - \text{var}_{ss}}{y_{ss}}$, which allows for the case that $\text{var}_{ss} = 0$. (That is, we scale deviations in bond holdings and profits by GDP.) Note furthermore that $\widehat{1 + r_{t+1}} \simeq \ln(1 + r_{t+1}) - \ln(1 + r_{ss}) \simeq r_{t+1} + \ln \beta$. We abuse notation slightly and write $\hat{r}$ instead of $\widehat{1 + r}$.

Standard first-order solution techniques for models with rational expectations rely on a recursive representation of the equilibrium conditions, e.g. the inter-temporal consumption decision is captured by the forward recursion referred to as Euler equation:

$$\widehat{c_t} = \mathbb{E}_t \widehat{c}_{t+1} - \frac{1}{\sigma} \mathbb{E}_t \hat{r}_{t+1}.$$

[10] Winkler (2020) proposes the "conditionally model-consistent expectations" (CMCE) concept as a starting point for linearizing models with capital gain extrapolation. Under CMCE, however, beliefs over all external variables are distorted relative to rational expectations. In our approach, linearized decision rules may be obtained under the assumption that belief distortions apply only to asset prices, allowing to confine the deviations from rational expectations to exactly those variables where survey data allow to discipline the expectations-modeling choices.

Nothing, in principle, prohibits writing such a representation for a model with subjective expectations:

$$\widehat{c}_t = \mathbb{E}_t^{\mathcal{P}} \widehat{c}_{t+1} - \frac{1}{\sigma} \mathbb{E}_t^{\mathcal{P}} \hat{r}_{t+1}.$$

The general difficulty that arises in this class of models is the following. We know (i) how to characterize subjective expectations for external variables over which households have distorted expectations (here $\mathbb{E}_t^{\mathcal{P}} \widehat{q}_{t+s}$): we simply use the perceived model; we also know (ii) how to capture expectations for external variables over which households have rational expectations (e.g. $\mathbb{E}\hat{r}_{t+s}$): we formulate the relevant equilibrium conditions recursively. What is less obvious is how to characterize, or capture in the equilibrium representation, households' subjective expectations over their *own choices*, e.g. $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_{t+s}$. These expectations cannot be rational, i.e. consistent with the distribution of $\widehat{c}_{t+s}$ under $\mathbb{P}$.[11] This is not an insurmountable obstacle, though: $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_{t+s}$ is a well-defined object and can be computed – by explicitly solving for the *subjectively optimal plan* of a household.

In solving for the subjectively optimal plan we exploit two key insights into how households behave to first order that are valid irrespective of which (time-consistent) set of beliefs they hold. (In particular, our solution method can be used to solve RE models.) First, since there is only one budget constraint, there is only one inter-temporal trade-off, namely in consumption, c. Given a path for consumption, the first order conditions for housing, hours, and housing investment uniquely pin down a mapping from external sequences to decisions for these variables. This insight allows us to concentrate on finding the optimal path for consumption. The second insight is that to first-order, the permanent income hypothesis holds and consumption depends only on the path of real interest rates (an external variable) and the subjectively expected lifetime income. In other words, by iterating forward the Euler equation we receive

$$\widehat{c}_t = -\frac{1}{\sigma} \sum_{s \geq 0} \mathbb{E}_t \hat{r}_{t+s+1} + \mathbb{E}_t^{\mathcal{P}} \lim_{s \to \infty} \hat{c}_{t+s},$$

which goes to show that the only subjective expectation of a choice variable left to characterize is that of $\mathbb{E}_t^{\mathcal{P}} \lim_{s \to \infty} \hat{c}_{t+s}$, which we refer to as "terminal consumption".

Once the household first-order conditions have been linearized we, therefore, proceed in a three-step approach:

1. Iterate over the linearized household budget constraint to find a closed-form expression for terminal consumption in terms of the sequence of future external variables.

[11] The reason is that households have distorted expectations over at least one price sequence and therefore will make distorted choices; in particular they plan to make choices in the future that are inconsistent with what these choices will be in equilibrium. If we ignored this, i.e. exchanged $\mathbb{E}_t^{\mathcal{P}} \widehat{c}_{t+s}$ for $\mathbb{E}_t \widehat{c}_{t+s}$ in the forward iteration above, the computed equilibrium would be different from the IREE in Definition 1.

2. Use the subjective house price model to characterize the expected future house price values in the expression of terminal consumption in terms of current house prices, $\widehat{q}_t$, and expectations about the future evolution of house prices, $\widehat{\overline{m}}_t$.

3. Bring the remaining future values of external variables, all of which are under the rational expectations operator, back into a recursive form.

After applying steps 1–3, the model representation can be solved with standard methods in a fraction of a second. This method of explicitly characterizing choices in terms of lifetime income is general in the sense that it allows solving for household decisions under any time-consistent set of beliefs. The linearized first-order conditions of our subjective expectations model and the equivalent model formulation in rational expectations, read as follows:

$$\begin{array}{cc} \text{Rational Expectations} & \text{Subjective Expectations} \\ \varphi\widehat{n}_t + \sigma\widehat{c}_t = \widehat{w}_t & \varphi\widehat{n}_t + \sigma\widehat{c}_t = \widehat{w}_t \\ \widehat{q}_t = (1-\eta)\hat{x}_t & \widehat{q}_t = (1-\eta)\hat{x}_t \\ \widehat{c}_t = \mathbb{E}_t\widehat{c}_{t+1} - \frac{1}{\sigma}\mathbb{E}_t\widehat{r}_{t+1} & \widehat{c}_t = -\frac{1}{\sigma}\sum_{s\geq 0}\mathbb{E}_t\hat{r}_{t+s+1} + \mathbb{E}_t^{\mathcal{P}}\lim_{s\to\infty}\hat{c}_{t+s} \\ \widehat{h}_t = -\frac{\sigma}{\nu(1-\overline{\beta})}(\widehat{c}_t - \overline{\beta}\mathbb{E}_t\widehat{c}_{t+1}) - \frac{1}{\nu}\frac{\hat{q}_t - \overline{\beta}\mathbb{E}_t\hat{q}_{t+1}}{1-\overline{\beta}} & \widehat{h}_t = -\frac{\sigma}{\nu(1-\overline{\beta})}(\widehat{c}_t - \overline{\beta}\mathbb{E}_t^{\mathcal{P}}\widehat{c}_{t+1}) - \frac{1}{\nu}\frac{\hat{q}_t - \overline{\beta}\mathbb{E}_t^{\mathcal{P}}\hat{q}_{t+1}}{1-\overline{\beta}} \end{array} \tag{7}$$

Where we have defined $\overline{\beta} = \beta(1-\delta)$. The first two lines show the household labor supply choices and investment choices. These are equivalent across rational and subjective expectations. Current consumption and housing choices differ across models, as they depend on expected consumption choices and expected house prices. From the subjective house price model (3) and (4), we receive the following linearized expressions for the expected evolution of house prices:

$$\begin{aligned} \mathbb{E}_t^{\mathcal{P}}\hat{q}_{t+s} &= \hat{q}_t + (1-\varrho^s)\frac{\varrho}{1-\varrho}\widehat{\overline{m}}_t, \\ \widehat{\overline{m}}_t &= \varrho\widehat{\overline{m}}_{t-1} + g(\hat{q}_{t-1} - \hat{q}_{t-2} - \varrho\widehat{\overline{m}}_{t-1}) \end{aligned} \tag{8}$$

Hence, we can simply use the subjective house price model to express subjective house price expectations. The closed-form expression for terminal consumption, which equals the annuity value of subjectively perceived net lifetime income, is given by:

$$
\begin{aligned}
\mathbb{E}_t^{\mathcal{P}} \lim_{s\to\infty} \widehat{c}_{t+s} = & \underbrace{\frac{\delta q_{ss} h_{ss}/\nu}{c_{ss} + \sigma/\varphi \cdot n_{ss} w_{ss} + \delta q_{ss} h_{ss} \sigma/\nu}}_{=:\ \sigma^{-1}\cdot a,\ a\in(0,1)} \cdot \left[\widehat{q}_t + \widehat{\overline{m}}_t \cdot \frac{\varrho}{1-\varrho} \underbrace{\left(1 + \frac{1-\overline{\beta}\,\varrho}{1-\overline{\beta}} \frac{1-\varrho-\delta}{1-\beta\,\varrho} \frac{1-\beta}{\delta}\right)}_{=:\ b>0\ \forall \beta,\delta,\varrho\in(0,1)} \right] \qquad (9) \\
& + \frac{y_{ss}}{c_{ss} + \sigma/\varphi \cdot n_{ss} w_{ss} + \delta q_{ss} h_{ss} \sigma/\nu} \frac{1-\beta}{\beta} \cdot \sum_{n=1}^{\infty} \beta^n \mathbb{E}_t \Bigg\{ (1 + 1/\varphi) \frac{n_{ss} w_{ss}}{y_{ss}} \widehat{w}_{t+n} + \widehat{\Sigma}_{t+n} \\
& - \frac{c_{ss} + \frac{\sigma}{\varphi} n_{ss} w_{ss}}{y_{ss}} \sum_{s\geq n} \widehat{r}_{t+s+1} \\
& + \frac{q_{ss} h_{ss}}{y_{ss} \nu} \left[\frac{\sum_{s\geq n} \widehat{r}_{t+s+1} - \beta(1-\delta) \sum_{s\geq n+1} \widehat{r}_{t+s+1}}{1-\beta(1-\delta)} - (1-\delta) \frac{\sum_{s\geq n-1} \widehat{r}_{t+s+1} - \beta(1-\delta) \sum_{s\geq n} \widehat{r}_{t+s+1}}{1-\beta(1-\delta)} \right] \Bigg\}
\end{aligned}
$$

The first line in this expression captures the net income resulting from adjusting housing choices in response to variations in current house prices, $\widehat{q}_t$, and beliefs over future house price growth, $\widehat{\overline{m}}_t$. It consists of capital gains/losses from selling/buying housing units, and housing depreciation costs. The remaining expressions capture changes in net lifetime income due to expected changes in wages, profits, and interest rates. After putting all infinite sums in a recursive formulation, the linearized household decisions as given in equations (7), (8), and (9), can be combined with the other equilibrium conditions (which are unchanged relative to the model under rational expectations) and solved using standard techniques.

Discussion. Our method has two important advantages over previous approaches to solving asset price learning models. First, we solve the model using a first-order approximation which makes it fast to solve, easily scalable, and amenable to the analysis of Ramsey-optimal policies. The literature has previously relied on non-linear solution techniques (Adam et al., 2017), or hybrid[12] techniques (Adam et al., 2022) to solve these models. Hence, solution procedures are much more involved and limits in computational capability constrain the solution of larger-scale models. Second, our solution method confines subjective expectations to house prices. A previously developed method by Winkler (2020) and Caines and Winkler (2021), which also rely on perturbation, assumes household expectations to conform with the concept of conditionally model-consistent expectations. Under this concept, subjective expectations about one variable lead to spillovers to expectations about other variables. Thus, households will form subjective beliefs across all model variables. In our approach, households only hold subjective expectations with respect to one variable, while they remain rational with respect to all other variables.

[12] Adam et al. (2022) linearize all model equations except the households' demand equation for housing.

4.2 Equilibrium household behavior

Now that we have characterized how household choices are determined under subjective expectations, we can analyze households' equilibrium behavior. In particular, we are interested in the housing demand response to changes in the current house price and beliefs about future house price growth. We can make the following

Proposition 1 (Subjectively optimal housing plans, house prices and price growth beliefs).
To first order around the non-stochastic steady state,

1. *the partial effect of house prices on subjectively perceived future housing choices is negative and constant:*
$$\frac{\partial \mathbb{E}_t^{\mathcal{P}} \widehat{h}_{t+s}}{\partial \hat{q}_t} = -\frac{1-a}{\nu} \in (-1/\nu, 0) \ \forall s \geq 0$$

2. *the partial effect of house price growth expectations on subjectively perceived future housing choices*
$$\frac{\partial \mathbb{E}_t^{\mathcal{P}} \widehat{h}_{t+s}}{\partial \widehat{\bar{m}}_t} = \frac{1}{\nu}\left[ab - \frac{\varrho}{1-\varrho}\frac{1-\varrho^s - \overline{\beta}(1-\varrho^{s+1})}{1-\overline{\beta}}\right], \ \ a \in (0,1), b > 0$$

   (a) *is positive at $s = 0$,*

   (b) *decreases monotonically and convexly as s increases,*

   (c) *and becomes negative as $s \to \infty$;*

the constants a, b are defined in equation (9).

Proposition 1 first states that if current house prices increase and are expected to stay high, households decrease their housing demand now and expect to continue doing so in the future. This is intuitive: housing suddenly becomes a more expensive good and households substitute away from it. The substitution is less than one for one (adjusted by the elasticity ν) because lowering their housing consumption means households have to maintain a smaller stock of housing and save on the replacement cost. This increase in net lifetime income is used to increase consumption of the final basket, of leisure – and of housing. Second, when households expect house prices to increase in the future, they scale up their demand for housing immediately and plan to subsequently reduce their housing stock in the future, first by selling many housing units in $s = 1$, then by selling fewer and fewer units, until in the limit $s \to \infty$ their planned housing stock stabilizes at a level strictly lower than $\widehat{h}_{t-1}$. The intuition is the following: as house prices are unchanged today but are expected to rise in the future, households buy housing units today – not primarily for consuming it but to sell it over time to realize capital gains.

We can further characterize how the cash flows accruing to subjectively perceived net lifetime income evolve over time $s > 0$, in response to an increase in expected house price growth and the ensuing changes in the planned demand for housing.

Proposition 2 (Cash flows associated with subjectively optimal housing plans).
The cash flows associated with the time-varying component of housing demand $\partial \mathbb{E}_t^{\mathcal{P}} \widehat{h}_{t+s} \,/\, \partial \widehat{\overline{m}}_t - ab/\nu$, behave as follows. To first order around the non-stochastic steady state, the household plans to adjust housing in response to an upward belief revision as shown in Proposition 1. This influences future net income (= income − expenses) via

- *Housing replacement cost, $-\delta \mathbb{E}_t^{\mathcal{P}} \widehat{h}_{t+s-1}$:*
  $\hookrightarrow$ this cost is positive initially, reducing lifetime income, but decreases geometrically to a strictly negative level.
- *Capital gains from selling housing units, $-\mathbb{E}_t^{\mathcal{P}} (\widehat{h}_{t+s} - \widehat{h}_{t+s-1})$:*
  $\hookrightarrow$ this cash flow is positive $\forall s > 0$ but geometrically decreasing.

The overall flow $-\mathbb{E}_t^{\mathcal{P}} (\widehat{h}_{t+s} - (1-\delta)\widehat{h}_{t+s-1})$ is positive $\forall s > 0$ and decreasing iff capital gains decrease faster than replacement costs do, which is the case iff $1 - \varrho - \delta > 0$.

As expected, the time-varying component of belief-driven housing demand

$$\frac{\partial \mathbb{E}_t^{\mathcal{P}} \widehat{h}_{t+s}}{\partial \widehat{\overline{m}}_t} - \frac{ab}{\nu} = -\frac{\varrho}{1-\varrho} \frac{1 - \varrho^s - \overline{\beta}(1 - \varrho^{s+1})}{1 - \overline{\beta}},$$

which also satisfies the claims in Proposition 1, point 2a–c, generates positive expected income flows in all future periods. These flows are then discounted, summed up, put into annuity value and – through $\mathbb{E}_t^{\mathcal{P}} \lim_{s\to\infty} \widehat{c}_{t+s}$ – reinserted into the first order condition for housing in the form $+\frac{ab}{\nu}$. Propositions 1 and 2 make clear the precise way in which subjective expectations on house prices, subjectively optimal choices, and expected capital gains interact to create a feedback-loop between house price growth and housing demand.

4.3 Parameterization

We choose parameters such that countries are symmetric in every aspect but the elasticity of housing production, η, η^*. For the household side, we choose parameters that are standard in the literature. Additionally, we assume that countries are symmetric in size. We choose parameters for the production side and trade following Bletzinger and von Thadden (2021). For the subjective house price model, we choose parameters similar to Winkler (2020). Finally, the Taylor coefficient on inflation is standard.

Table 1: Model parameters (symmetric parameters)

Parameter		Value	Description	Source
Households	χ	1.000	labor disutility shifter	standard
	φ	1.000	inverse Frisch elasticity	standard
	σ	2.000	inverse of intertemporal EOS	standard
	ν	1.000	housing utility elasticity	standard
	δ	0.008	housing depreciation	3% annual depreciation
	β	0.995	discount factor	standard for quarterly frequency
	γ	0.500	relative region size	symmetric regions
Goods aggregation & production	λ	0.800	home bias	Bletzinger and von Thadden (2021)
	ς	1.000	EOS across regions	
	ϵ	6.000	EOS across varieties	
	κ	28.650	price adjustment costs	
House price beliefs	ϱ	0.950	Autocorrelation of perceived long-run house price growth	tbd
	g	0.011	Kalman gain	tbd
Policy	ϕ	1.500	Taylor coefficient	standard

Notes: All parameters depicted above are equal across countries. One period in the model is one quarter.

Table 2 shows the parameter values for the non-symmetric parameters, as well as the allocation and prices in the non-stochastic steady state. We choose a higher degree of production elasticity in the foreign country. The steady-state values for the housing productivity shifter and housing preference shifter are chosen such that we attain a symmetric steady-state in the allocation variables. We make this modeling choice so that we may concentrate on the dynamic implications of structural heterogeneity. The steady-state value for the house price is the only variable that differs across countries. Symmetric steady-state values for bond levels, which are zero for both countries, also imply that there is no net-borrower or net-saver country in the steady-state. Changes in monetary policy will therefore not lead to Fisherian debt revaluation effects.

Table 2: Asymmetric model parameters and steady-state values

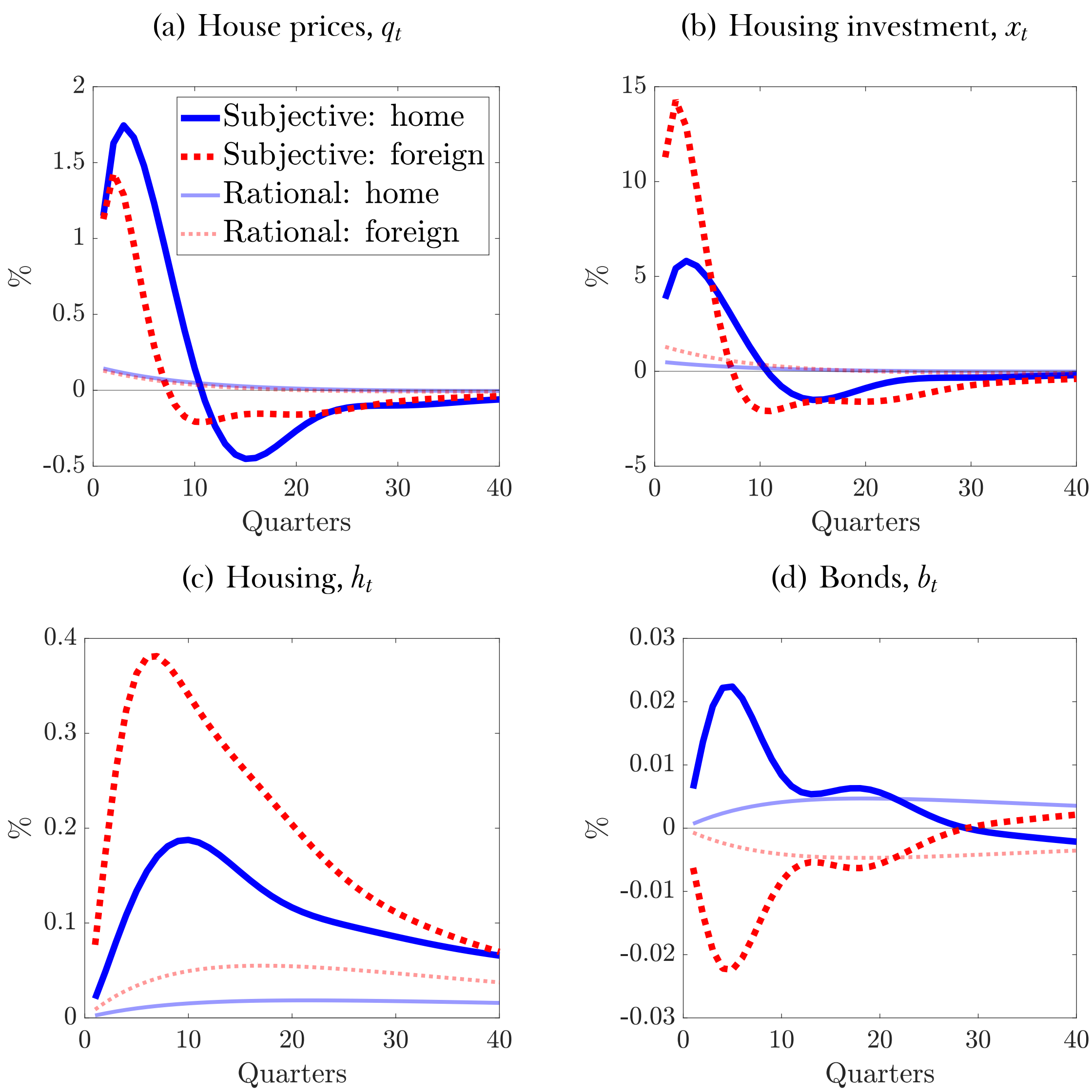
Domestic	Value	Foreign	Value	Description
η	0.700	η^*	0.900	elasticity of housing production
$\xi_{x,ss}$	1.400	$\xi^*_{x,ss}$	5.373	housing productivity shifter
$\xi_{h,ss}$	0.010	$\xi^*_{h,ss}$	0.008	housing preference shifter
c_{ss}	0.999	c^*_{ss}	0.999	consumption
x_{ss}	0.004	x^*_{ss}	0.004	housing investment
h_{ss}	5.738	h^*_{ss}	5.738	housing
y_{ss}	1.003	y^*_{ss}	1.003	output
b_{ss}	0.000	b^*_{ss}	0.000	bond holdings
q_{ss}	0.139	q^*_{ss}	0.108	house price
w_{ss}	1.000	w^*_{ss}	1.000	wage

Notes: All steady-states except the house price are symmetric across countries. The elasticity of housing production, steady-state values for the housing productivity shifter, and housing preference shifter are not symmetric across countries.

5 Results

In the following, we study the response of model variables to a 25 bp expansionary monetary policy shock. We plot the responses for the two countries of the subjective beliefs model and additionally report responses of the same model under rational expectations. Figure 3 shows the responses of house prices, housing investment, housing, and bonds. With respect to the subjective beliefs model, we find that house prices respond stronger in the domestic country, which has a less elastic housing supply sector and the boom-bust cycle is more pronounced in this country. For housing investment and housing, the responses flip. Here the foreign country responds stronger to the monetary policy shock. As the foreign country produces houses more elastically, investment and housing will react more strongly than in the home economy to any given price change. Finally, we see that in order to finance housing investment, the foreign economy will borrow from the domestic economy, as bond holdings decrease in the foreign economy. Comparing the subjective belief model to the rational expectations version, we find that the subjective beliefs responses with respect to house prices, housing investment, and housing are an order of magnitude larger.

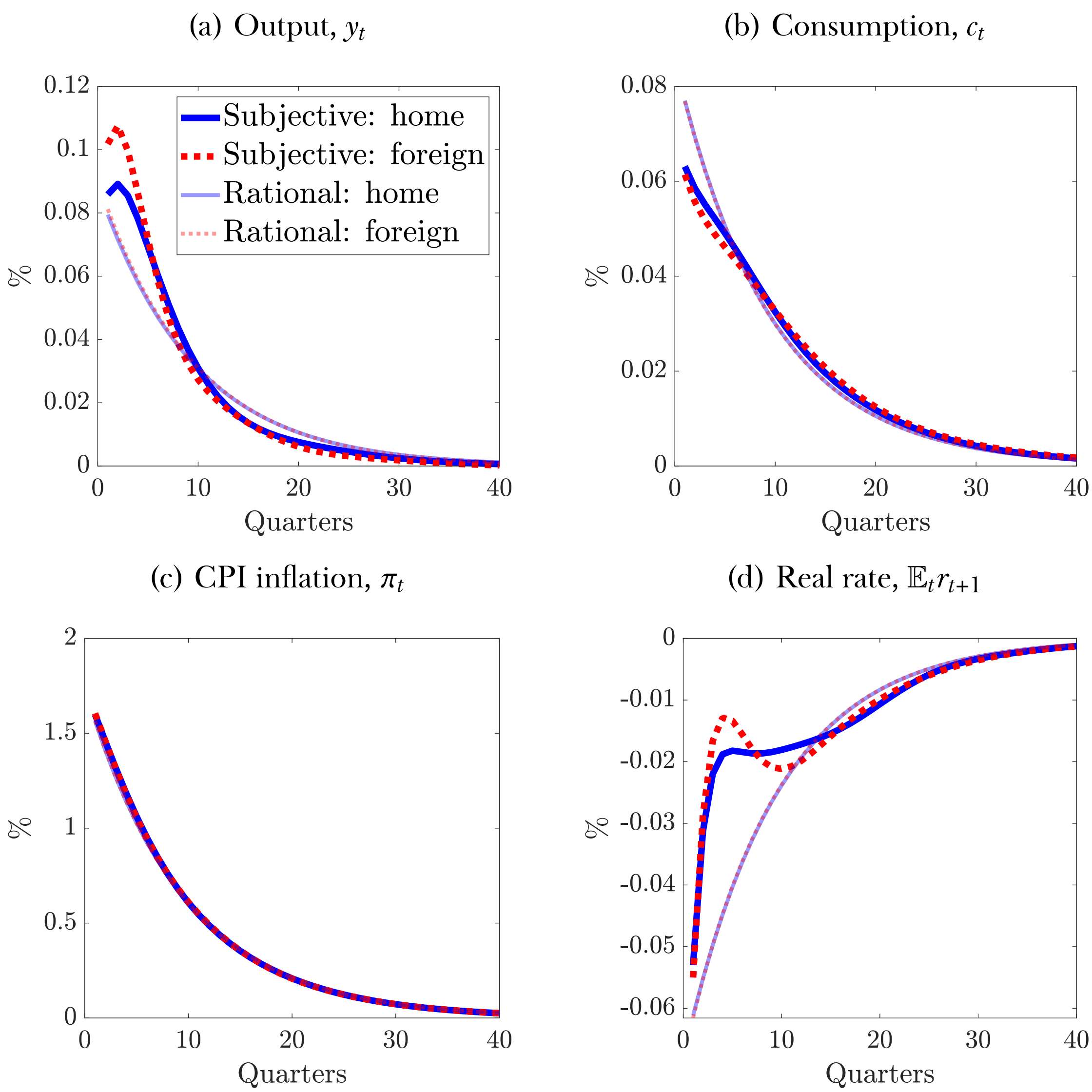
Figure 3: Model responses, monetary policy shock



Notes: The home country (blue) has the lower elasticity of housing supply. Figure depicts the model-implied impulse responses to a monetary policy shock of 25 basis points with persistency 0.9. Inflation and interest rates are annualized, variables are expressed in % deviation from their steady state values, except for bond holdings which are expressed in % of steady state output. Model parameters may be found in Section 4.3.

It has been shown in the literature that subjective expectations models can match asset price volatility fairly well (Adam et al., 2017; Winkler, 2020), while rational expectations models have a hard time doing so. Our model clearly shares this feature. If we focus on house prices we see that we can generate boom-bust cycles in the subjective beliefs model. These dynamics are absent in the rational expectations model unless enriched otherwise. Finally, the subjective beliefs model is able to capture the heterogeneity in house price responses across countries from Section 2, while the rational expectations model is not.

Figure 4: Model responses, monetary policy shock



Notes: The home country (blue) has the lower elasticity of housing supply. Figure depicts the model-implied impulse responses to a monetary policy shock of 25 basis points with persistency 0.9. Inflation and interest rates are annualized, variables are expressed in % deviation from their steady state values, except for bond holdings which are expressed in % of steady state output. Model parameters may be found in Section 4.3.

In Figure 4 we plot the impulse response functions for output, consumption, inflation, and real interest rates. Compared to the responses in the housing sector, the cross-country differences are relatively small in the subjective beliefs model. In the rational expectations model, they are even less pronounced. We see that in the subjective expectations model, relative to the rational expectations model, that output is higher, while consumption is lower. This holds for both countries. In Figure 3 we already noted that housing investment is much larger compared to the RE model. These dynamics reflect the new inefficiency in the subjective expectations model: Households work too much, consume too little and invest too much into housing, all in pursuit of capital gains that never materialize. These allocations are inefficient because they purely arise through subjective beliefs about house prices. Therefore, there is scope for policy to lean against these dynamics.

6 Conclusion

In this paper, we construct a model of a currency area where structural heterogeneity in countries' ability to produce new housing units and subjective house price expectations in the form of capital gain extrapolation interact to generate boom-bust cycles in house prices that differ in their intensity between countries. Households in our model economy do not know the true, equilibrium-implied distribution of house prices but they perceive a simple statistical model of house price growth rates. This model leads them to extrapolate capital gains observed in the past into the future. When positive exogenous shocks push house prices upwards, households become optimistic and the ensuing capital gains expectations distort households' choices: they work too much, consume too little, and invest too much into housing, all in order to realize capital gains that ultimately do not materialize in equilibrium.

While this sounds like a deceivingly simple mistake on the part of households, and indeed these distortions induce an inefficiency on top of what is present in the economy under rational expectations, models of capital gain extrapolation provide a good fit to the available survey evidence on private sector house price expectations. Built into structural models, this form of a subjective expectations arrangement generates boom-bust cycles in asset prices that match the movements in the data much better than the results from corresponding models under rational expectations. Our model, despite being linearized, shares the important qualitative features for this to occur: house price movements in the model are much larger under subjective beliefs than under rational expectations, and they exhibit momentum and subsequent mean-reversion. In linearizing the model by solving for the subjectively optimal plans explicitly, we manage to retain the assumption that households are rational to variables that they take as given other than house prices. Expectational distortions are thus confined to house prices.

In future research, we plan to exploit the linearity of our model to pursue the analysis of optimal monetary policy in a currency area where structural heterogeneities in housing markets and a realistic house price expectation formation process jointly generate house price cycles that differ in size between member countries.

References

Aastveit, K. A., B. Albuquerque, and A. K. Anundsen (2020): "Changing Supply Elasticities and Regional Housing Booms," **Bank of England Working Paper No. 844**.

Aastveit, K. A. and A. K. Anundsen (2022): "Asymmetric Effects of Monetary Policy in Regional Housing Markets," **American Economic Journal: Macroeconomics**, 14, 499–529.

Adam, K., P. Kuang, and A. Marcet (2012): "House Price Booms and the Current Account," **NBER Macroeconomics Annual**, 26, 77–122.

Adam, K. and A. Marcet (2011): "Internal Rationality, Imperfect Market Knowledge and Asset Prices," **Journal of Economic Theory**, 146, 1224–1252.

Adam, K., A. Marcet, and J. Beutel (2017): "Stock price booms and expected capital gains," **American Economic Review**, 107, 2352–2408.

Adam, K. and S. Merkel (2019): "Stock price cycles and business cycles," **Available at SSRN 3455237**.

Adam, K., O. Pfäuti, and T. Reinelt (2022): "Subjective Housing Price Expectations, Falling Natural Rates and the Optimal Inflation Target," .

Altavilla, C., L. Brugnolini, R. S. Gürkaynak, R. Motto, and G. Ragusa (2019): "Measuring Euro Area Monetary Policy," **Journal of Monetary Economics**, 108, 162–179.

Benigno, P. (2004): "Optimal Monetary Policy in a Currency Area," **Journal of International Economics**, 63, 293–320.

Bletzinger, T. and L. von Thadden (2021): "Designing QE in a fiscally sound monetary union," **European Economic Review**, 132, 103611.

Caines, C. and F. Winkler (2021): "Asset Price Beliefs and Optimal Monetary Policy," **Journal of Monetary Economics**, 123, 53–67.

Calza, A., T. Monacelli, and L. Stracca (2013): "Housing finance and monetary policy," **Journal of the European Economic Association**, 11, 101–122.

Catte, P., N. Girouard, R. Price, and C. André (2004): "Housing markets, wealth and the business cycle," .

Corsetti, G., J. B. Duarte, and S. Mann (2022): "One money, many markets," **Journal of the European Economic Association**, 20, 513–548.

Driscoll, J. C. and A. C. Kraay (1998): "Consistent Covariance Matrix Estimation with Spatially Dependent Panel Data," **Review of Economics and Statistics**, 80, 549–560.

Drudi, F., P. Koehler, C. Kok, G. Wolswijk, E. Stoess, K. Wagner, H. Hasko, N. Valckx, L. Bartiloro, P. E. Mistrulli, et al. (2009): "Housing finance in the euro area," .

Farhi, E. and I. Werning (2017): "Fiscal unions," **American Economic Review**, 107, 3788–3834.

Kaplan, G., K. Mitman, and G. L. Violante (2020): "The housing boom and bust: Model meets evidence," **Journal of Political Economy**, 128, 3285–3345.

Kekre, R. (2022): "Optimal Currency Areas with Labor Market Frictions," **American Economic Journal: Macroeconomics**, 14, 44–95.

Pica, S. (2021): "Housing Markets and the Heterogeneous Effects of Monetary Policy Across the Euro Area," **Available at SSRN**.

Schmitt-Grohé, S. and M. Uribe (2003): "Closing small open economy models," **Journal of International Economics**, 61, 163–185.

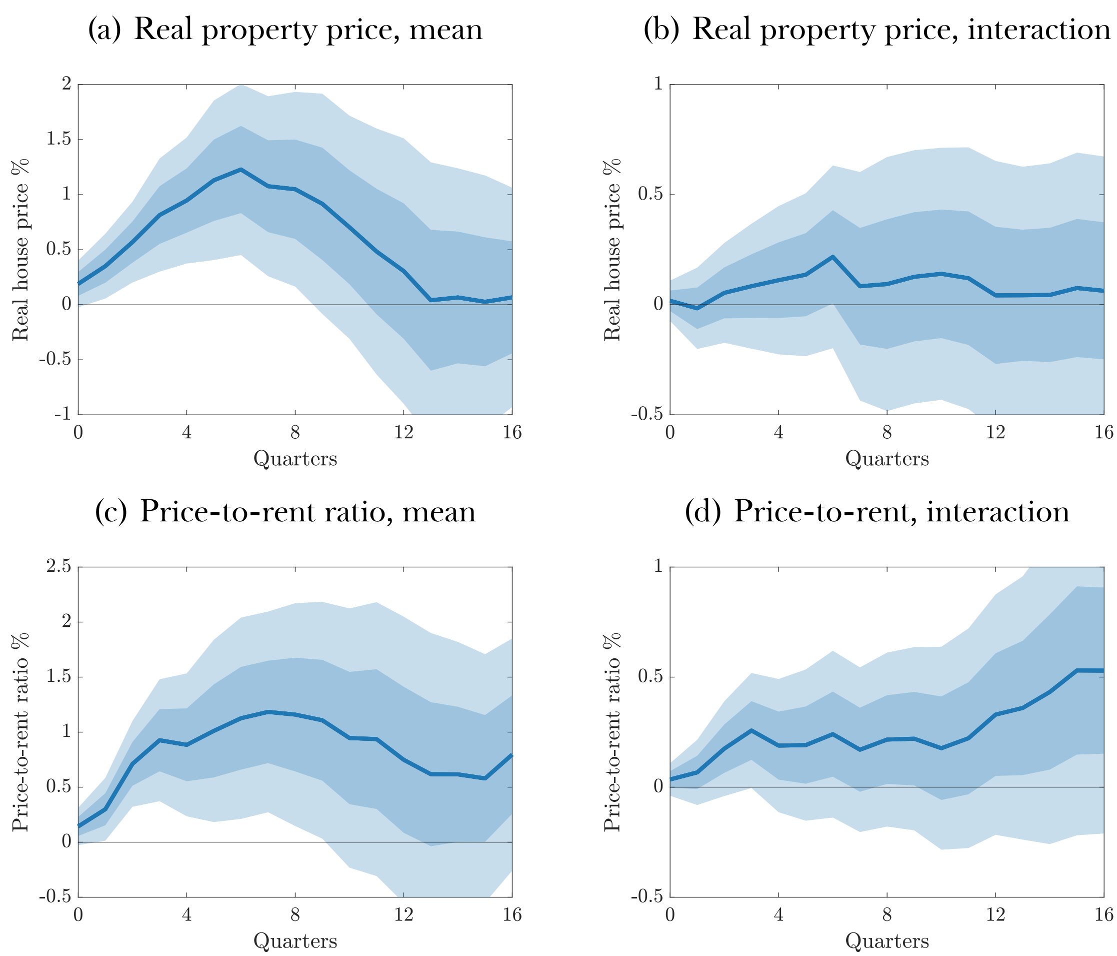
Winkler, F. (2020): "The role of learning for asset prices and business cycles," **Journal of Monetary Economics**, 114, 42–58.

Appendices

A Local projections: robustness exercise

Below we show responses to monetary policy shocks for house prices and price-to-rent ratios. We use different interaction terms that the literature has pointed to. Cross-country differences in adjustable rate mortgage shares, homeownership rates, and the share of wealthy hand-to-mouth households correlate positively with responses of GDP to monetary policy shock. Further, transaction costs, arising for instance from the taxation of housing sales, and loan-to-value (LTV) constraints are often mentioned in relation to cross-country differences in house price variations. None of the above-mentioned interaction terms leads to statistically significant results.

Figure A.1: Share of adjustable rate mortgages



Notes: Responses to expansionary monetary policy shock (1 std). One std is roughly 13bp. Interaction term: Share of adjustable rate mortgages (HFCS). CI: 68% and 95% (Driscoll and Kraay, 1998).

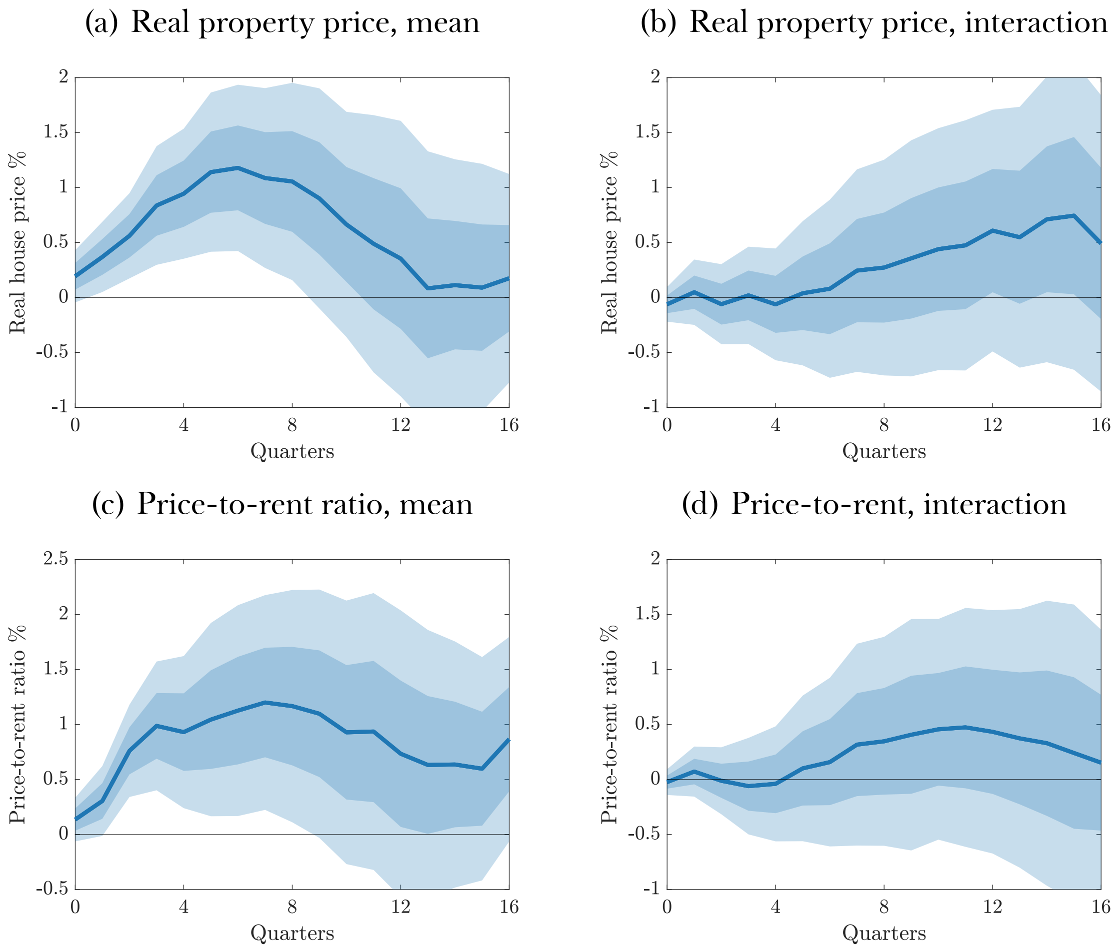
Figure A.2: Homeownership rate

Notes: Responses to expansionary monetary policy shock (1 std). One std is roughly 13bp. Interaction term: Homeownership rate (HFCS). CI: 68% and 95% (Driscoll and Kraay, 1998).

Figure A.3: Wealthy hand-to-mouth

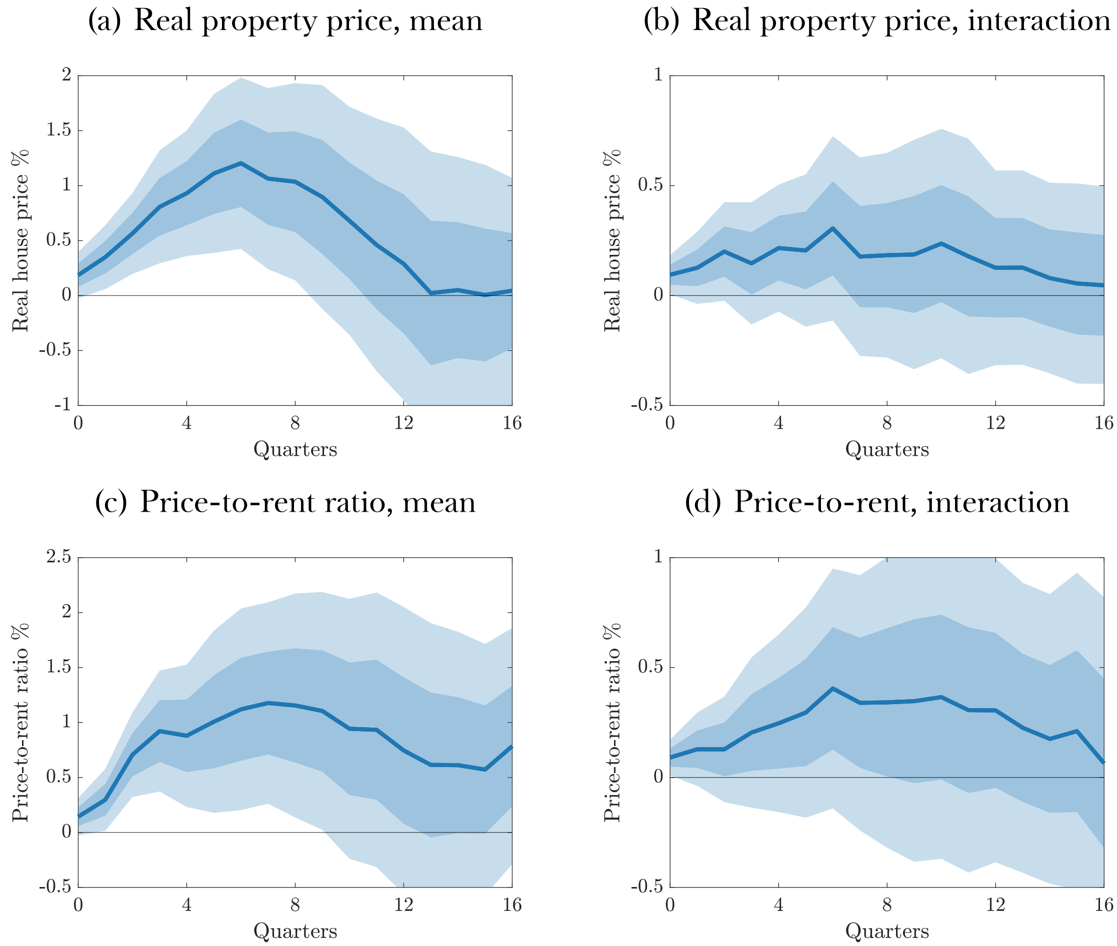
Notes: Responses to expansionary monetary policy shock (1 std). One std is roughly 13bp. Interaction term: Share of wealthy hand-to-mouth households (HFCS). CI: 68% and 95% (Driscoll and Kraay, 1998).

Figure A.4: Maximum applicable capital gains tax



Notes: Responses to expansionary monetary policy shock (1 std). One std is roughly 13bp. Maximum applicable capital gains tax (Drudi et al., 2009). CI: 68% and 95% (Driscoll and Kraay, 1998).

Figure A.5: LTV constraints



Notes: Responses to expansionary monetary policy shock (1 std). One std is roughly 13bp. Loan-to-value (LTV) constraints (Catte et al., 2004). CI: 68% and 95% (Driscoll and Kraay, 1998).

B Micro-founding the debt-elastic interest rate

In the model, households in country H receive on their bond holdings the effective nominal interest rate $1+i_{t-1}-\psi b_t$, with b_t being the real value of the aggregate bond holding in country H; households in country F receive the effective nominal rate $1+i_{t-1}-\psi b_t^*$. Moreover, the intermediation of bond positions entails a real cost $\gamma(1+\pi_t)^{-1}\frac{\psi}{2}(b_t)^2+(1-\gamma)(1+\pi_t)^{-1}\frac{\psi}{2}(b_t^*)^2$ of which $(1+\pi_t)^{-1}\frac{\psi}{2}(b_t)^2$ is paid by each consumer in H and $(1+\pi_t)^{-1}\frac{\psi}{2}(b_t)^2$ is paid by each consumer in F. In this Appendix we detail how these debt-elastic interest rates and the associated intermediation cost can be parsimoniously micro-founded. We achieve this by introducing two competitive bond clearing houses, one in each country, that represent the only access of households to financial markets and who incur a real cost that is quadratic in the size of their balance sheet. The specific market arrangement is as follows: households hold a consol and may hold liquid bonds.

Consol. Each household in H is endowed with $\bar{b} \in \mathbb{R}$ units of a non-marketable consol[13] that pays as a coupon $(\beta^{-1} - 1)(\gamma P_{t-1} + (1-\gamma)P^*_{t-1})P_t^{-1}$ units of H's consumption basket each period, per unit of consol. This implies that the nominal coupon rate, applied to the nominal coupon value $P_{t-1}\bar{b}$, is $(\beta^{-1} - 1)(\gamma P_{t-1} + (1-\gamma)P^*_{t-1})/P_{t-1}$; the real coupon rate applied to the real value $\bar{b}$ in turn is $(\beta^{-1} - 1)\left(\gamma + (1-\gamma)\frac{P^*_{t-1}}{P_{t-1}}\right)(1+\pi_t)^{-1}$. The situation in country F is symmetric: each household is endowed with $\bar{b}^*$ units of a consol that pays $(\beta^{-1} - 1)(\gamma P_{t-1} + (1-\gamma)P^*_{t-1})(P_t^*)^{-1}$ units of F's consumption basket each period, per unit of consol. $\bar{b}, \bar{b}^*$ are model parameters selected such that (i) $\gamma\bar{b} + (1-\gamma)\bar{b}^* = 0$ and (ii) all markets clear in the non-stochastic steady state with zero net inflation and terms-of-trade parity *without* households holding any liquid bonds. The latter fact ensures that there is no cost of financial intermediation in the steady state, shutting down this particular friction. The specific choice of the coupon payment scheme ensures two facts: (1) condition (i) implies that the nominal payments between H and F associated with the two consols exactly cancel out – whatever H/F receives as coupon payments on its consol endowment is paid for by F/H as a coupon service on its (endowed) short position of consol; and (2) the real coupon rates paid by/ to the consol endowment only depend on the real exchange rate and the inflation rates, not on the price levels. Households cannot trade their consol holdings.

Bonds. Household do have the possibility, though, to vary their position in the liquid bond. This liquid bond is a nominal, one-period, zero-coupon bond and the positions of the representative H-, respectively the representative F-household are denominated b_t, b_t^*. If a household wants to hold a net balance of liquid bonds different from zero, she has to go to one of the clearing houses in her country: In the H-country, there is a continuum of mass γ (respectively mass $1-\gamma$ in F) of competitive clearing houses buying and selling bonds from and to the government and from and to the respective country's citizens. Households themselves cannot directly buy/sell government bonds without having an account at the clearing house. The clearing house can costlessly buy/sell bonds but incurs an operating cost that is quadratic in the size of its balance sheet, making this a model of costly financial intermediation. Thus, the interest rate that each citizen gets on her bond holdings is determined by the nominal rate paid on government bonds and the aggregate holding of liquid bonds. Each clearing house is owned equally by all citizens of the respective country so that it pays its profits to those

[13] A consol is a type of bond that has infinite maturity and just keeps paying a constant or varying coupon perpetually.

citizens.[14] Consider an arbitrary clearing house in H (with symmetric arrangements in F). Denoting as $B_{c,t+1}$ the nominal value of the clearing house's net liabilities against H's citizens and as $B_{g,t+1}$ the nominal value of the clearing house's position in the government bond, the profit maximization program is:

$$\max_{B_{c,t+1},B_{g,t+1}\in\mathbb{R}} -(1+i_t^b)B_{c,t+1}+(1+i_t)B_{g,t+1}-\frac{\psi}{2}P_t^{-1}(B_{c,t+1})^2, \text{ s.t. } B_{c,t+1}=B_{g,t+1}$$

where i_t^b is the nominal rate clearing the market for household bond positions and i_t is the nominal rate on government bonds that is set by the monetary authority. $\frac{\psi}{2}P_t^{-1}(B_{c,t+1})^2 = \frac{\psi}{2}P_t(b_{c,t+1})^2$ is the nominal cost of intermediating – crucially, the real cost of intermediation does not directly depend on the price level. The first order conditions for this program are

$$\begin{aligned} 1+i_t^b+\psi P_t^{-1}B_{c,t+1} &= \mu_t, \\ 1+i_t &= \mu_t, \\ B_{c,t+1} &= B_{g,t+1}, \end{aligned}$$

where μ_t is the Lagrange multiplier on the balance-sheet constraint $B_{c,t+1} = B_{g,t+1}$. Market clearing in the household bond positions in H requires

$$\gamma B_{c,t+1} = \gamma P_t b_{t+1},$$

and market clearing in the government bond positions requires

$$\gamma B_{g,t+1} + (1-\gamma)B^*_{g,t+1} = 0,$$

so that by using the balance-sheet constraints $B_{c,t+1} = B_{g,t+1}$, $B^*_{c,t+1} = B^*_{g,t+1}$ and the clearing conditions for household bond positions in H and F we recover the market clearing condition for government bonds in the main model:

$$\gamma P_t b_{t+1} + (1-\gamma)P^*_t b^*_{t+1} = 0.$$

[14] In equilibrium, each clearing house makes a non-negative profit, and along the transition path back to the steady state after some shock, each clearing house makes a strictly positive profit. This fact is in principle incompatible with the notion of competitiveness (there is an incentive to open up more clearing houses or, equivalently, it is strictly profitable to split each clearing house). Therefore, it is better to interpret the program of the clearing house as reflecting capacity constraints: the here-presented program can be thought of as the inner problem of a profit maximization program with an additional factor (say, managerial effort) that which (i) makes the intermediation service production function exhibit constant returns to scale (instead of decreasing RTS), (ii) is provided by households, and (iii) is in perfectly inelastic supply. Under this way of modeling the clearing house, it behaves exactly as modeled here, it always makes zero profits, and households get as remuneration for providing the additional factor the amount that is the profit in the current way of modeling.

In sum, the aggregate conditions implied by this market arrangement are:

$$\begin{aligned} 1+i_t^b+\psi b_{t+1} &= 1+i_t, \\ 1+i_t^{b,*}+\psi b_{t+1}^* &= 1+i_t, \\ \gamma P_t b_{t+1}+(1-\gamma)P_t^* b_{t+1}^* &= 0. \end{aligned}$$

The nominal profits of the typical clearing house in H in equilibrium are:

$$\begin{aligned} \text{Profit}_{t+1} &= (i_t-i_t^b)B_{c,t+1}-\frac{\psi}{2}P_t^{-1}(B_{c,t+1})^2 \quad \text{with optimal } B_{c,t+1}=\frac{i_t-i_t^b}{\psi}P_t \\ &= (i_t-i_t^b)^2\psi^{-1}P_t-(i_t-i_t^b)^2\psi^{-1}P_t\cdot\frac{1}{2} \\ &= P_t\frac{\psi}{2}(b_{t+1})^2 \quad \text{using market clearing in the household bond positions.} \end{aligned}$$

Of the $1+i_t\%$ nominal interest collected on (paid for) its position of government bonds, each clearing house withholds $\psi b_{t+1}\%$ of the interest from its customers (respectively, charges $-\psi b_{t+1}\%$ of additional interest if $b_{t+1}<0$). Half of these $\psi b_{t+1}\%$ are used for covering the operating cost (by buying this amount of H's final basket and selling it in exchange for numéraire), and the other half is paid as profit to the owners of the clearing house (which, in equilibrium, are its customers).

C Derivations for subjective beliefs

Equation (4) is the result of the following calculations:

$$\begin{aligned} \mathbb{E}_t^{\mathcal{P}} q_{t+s} &= q_t\cdot\mathbb{E}_t^{\mathcal{P}}\frac{q_{t+s}}{q_t} = q_t\cdot\mathbb{E}_t^{\mathcal{P}}\exp\left(\ln q_{t+s}-\ln q_t\right) = q_t\cdot\mathbb{E}_t^{\mathcal{P}}\exp\left(\sum_{n=1}^{s}\Delta\ln q_{t+n}\right) \\ &= q_t\cdot\mathbb{E}_t^{\mathcal{P}}\exp\left(\sum_{n=1}^{s}\ln m_{t+n}\right)\cdot\underbrace{\mathbb{E}_t^{\mathcal{P}}\left[\prod_{n=1}^{s}e_{t+n}\right]}_{=\prod_n\mathbb{E}_t^{\mathcal{P}}e_{t+n}=1} \\ &= q_t\cdot\mathbb{E}_t^{\mathcal{P}}\exp\left(\sum_{n=1}^{s}\left[\sum_{j=0}^{n-1}\varrho^j\ln v_{t+n-j}+\varrho^n\ln m_t\right]\right) \\ &= q_t\cdot\mathbb{E}_t^{\mathcal{P}}\exp\left(\ln m_t\cdot\sum_{n=1}^{s}\varrho^n\right)\cdot\mathbb{E}_t^{\mathcal{P}}\underbrace{\exp\left(\sum_{n=1}^{s}\sum_{j=0}^{n-1}\varrho^j\ln v_{t+n-j}\right)}_{\sim\mathcal{N}} \end{aligned}$$

$$
\begin{aligned}
&= q_t \cdot \mathbb{E}_t^{\mathcal{P}} \exp\left(\varrho \frac{1-\varrho^s}{1-\varrho} \ln m_t\right) \cdot \exp(V), \quad V \propto \sigma_v^2 \\
\iff \mathbb{E}_t^{\mathcal{P}} q_{t+s} &= q_t \cdot \exp\left(\ln \overline{m}_t \cdot \varrho \tfrac{1-\varrho^s}{1-\varrho} + \frac{1}{2}\sigma^2 \big(\varrho \tfrac{1-\varrho^s}{1-\varrho}\big)^2\right) \cdot \exp(V), \quad V \propto \sigma_v^2
\end{aligned}
$$