6.2

c) Find the Lebesgue ~~integral~~ integral of the following function on $(\mathbb{R}, Bor(\mathbb{R}), \lambda)$:

$$f(\omega) = \begin{cases} n; & \omega \in (\mathbb{Q}^c \cap [0,n]) \\ 0; & \text{otherwise} \end{cases}$$

We can write f as a simple function:

$$f = I_{\mathbb{Q}^c \cap [0,n]} \cdot n \quad ; \forall n \sim$$

By definition then:

$$\int f \, d\lambda = n \cdot \underbrace{\lambda\left(I_{\mathbb{Q}^c \cap [0,n]}\right)}_{= n - 0 = n} = n^2$$

because $\mathbb{Q}^c$ is dense in $\mathbb{R}$ and $[0,n]$ is dense in $\mathbb{R}$

$\Rightarrow$ $\mathbb{Q}^c \cap [0,n]$ dense in $\mathbb{R}$