NN HW 1 : 2.2, 2.3

(2.2) Show that $\mathcal{E} = \sigma(\mathcal{E})$ if and only if $\mathcal{E}$ is a sigma algebra.

We are proving $\mathcal{E} = \sigma(\mathcal{E}) \Leftrightarrow \mathcal{E}$ is a σ-alg. :

1) $\Rightarrow$: if $\mathcal{E}$ is its own σ-alg. then $\mathcal{E}$ is a σ-algebra //

2) $\Leftarrow$: ~~if $\mathcal{E}$~~ is $\mathcal{E}$ its smallest σ-algebra if $\mathcal{E}$ is a σ-algebra ?

YES : a) $\sigma(\mathcal{E})$ must contain ALL elements of $\mathcal{E}$, so $\boxed{\sigma(\mathcal{E}) \supseteq \mathcal{E}}$.

b) if $\mathcal{E}$ is a σ-algebra, then any other $\sigma(\mathcal{E})$ could only be bigger than $\mathcal{E}$, so we must take $\mathcal{E}$ as its $\sigma(\mathcal{E})$, i.e. $\mathcal{E} = \sigma(\mathcal{E})$

$\square$

(2.3) Let C and D be 2 collections of subsets on Ω s.t. $C \subset D$. Prove that $\sigma(C) \subseteq \sigma(D)$!

F is σ-alg if:

1) $\emptyset \in F$
2) $A \in F \Rightarrow A^c \in F$
3) $\{A_i\}_{i=1}^{\infty} \in F \Rightarrow \bigcup_{i=1}^{\infty} A_i \in F$

(•) $\sigma(C)$ and $\sigma(D)$ are both σ-algebras,

$\Rightarrow \sigma(C) \cap \sigma(D)$ is a σ-algebra.

(citing labs, problem 2.1)

(••) ~~[illegible]~~ ~~and~~ $C \subset \sigma(D)$

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$\boxed{C \subset (\sigma(D) \cap \sigma(C))}$

$\boxed{\text{(•) + (••)} \Rightarrow (\sigma(D) \cap \sigma(C)) \text{ is a candidate for } \sigma(C)}$

(∴) any F; F is σ-alg. and $F \not\subseteq (\sigma(D) \cap \sigma(C))$

* is bigger than $(\sigma(C) \cap \sigma(D)) \Rightarrow F$ can't be $\sigma(C)$

and $C \subseteq F$

$(\therefore) \Rightarrow (\sigma(C) \cap \sigma(D)) = \sigma(C) \Rightarrow \sigma(C) \subseteq \sigma(D)$

$\square$