View metadata, citation and similar papers at core.ac.uk

brought to you by CORE

provided by ethesis@nitr

STABILITY ANALYSIS AND DESIGN OF DC-DC CONVERTERS WITH INPUT FILTER

A thesis submitted in partial fulfillment of the requirement for the degree of

M.Tech Dual Degree
In
Electrical Engineering

Specialization: Control and Automation

BY

AMMULA.V.SIDDHARTHA

710EE3080

DEPARTMENT OF ELECTRICAL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA -769008.

STABILITY ANALYSIS AND DESIGN OF DC-DC CONVERTERS WITH INPUT FILTER

A thesis submitted in partial fulfillment of the requirement for the degree of

M.Tech Dual Degree
In
Electrical Engineering

Specialization: Control and Automation

BY

AMMULA.V.SIDDHARTHA (710EE3080)

Under the guidance of

Prof. S. SAMANTA

DEPARTMENT OF ELECTRICAL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY

ROURKELA -769008.

DEPARTMENT OF ELECTRICAL ENGINEERING

NATIONAL INSTITUTE OF TECHNOLOGY, ROURKELA

CERTIFICATE

This is to certify that the thesis entitled "STABILITY ANALYSIS AND DESIGN OF DC-DC CONVERTERS WITH INPUT FILTER" by AMMULA.V.SIDDHARTHA (710EE3080), in partial fulfillment of the requirements for the award of M.tech Dual Degree in ELECTRICAL ENGINEERING with specialization in CONTROL AND AUTOMATION during session 2010-2015 in the Department of Electrical Engineering, National Institute of Technology Rourkela, is a true work completed by him under our watch and direction. To the best of our insight, the matter encapsulated in the thesis has not been submitted to some other University/Institute for the grant of any Degree or Diploma

Date:

Place-Rourkela

Prof. S. Samanta

Department of Electrical Engineering

National Institute of Technology, Rourkela.

Acknowledgement

With a deep sense of gratitude, I want to express my sincere appreciation and respect to my guide, Prof. SUSOVON SAMANTA, for being the corner stone of my project. It was his relentless inspiration and direction amid times of questions and vulnerabilities that has helped me to go ahead with this undertaking. In this content I also thank Mr. ANUP KUMAR PANDA, Head of the Department, Electrical Engineering, NIT Rourkela. I also want to thank PhD students under Samanta sir for helping me in the project. I likewise need to thank my parents. I want to impart this snippet of joy to my guardians and relative .They rendered me enormous support during the whole tenure of my stay in NIT Rourkela. Finally, I would like to thank all whose direct and indirect support helped me to completing my semester project report in time. I would like to thank our department for giving me the opportunity and platform to make my effort a successful one.

AMMULA.V.SIDDHARTHA
710EE3080

ABSTRACT

At the point when an Input filter is added to the converter it decreases the electromagnetic Interference (EMI) of power input of converter and enhances the performance of load. Electromagnetic Interference (EMI) is disturbance because of either electromagnetic induction or electromagnetic radiation discharged from outside source that influence the electrical circuit. The EMI may interfere with or decrease the performance of the electrical circuit. Thus an input filter is for the most part used to lessen the electromagnetic interference in power source side of a converter. The Input filter added to converter to diminish the electromagnetic interference may change the system transfer function, which may bring about instability and influence the performance of the converter. In this way, input filter ought to be such that it will diminish the electromagnetic interference and it ought not to influence the performance and the stability of the system. Different stability criteria are considered in this undertaking to outline an input filter without influencing the performance and the stability of the system. One such criterion is Middlebrook's stability criterion which is chiefly utilized for designing input filter for DC-DC converters. The Middlebrook Criterion was at first proposed to investigate how the stability of a feedback-controlled switching converter is influenced by the addition of an input filter. Its objective is to ensure stability of the system, as well as to guarantee that converter dynamics are not changed by the presence of an input filter. The Middlebrook Criterion gives a basic design-oriented sufficient stability condition imposing a small-gain condition on the minor loop gain. In this thesis the design of input filter for various converters using Middlebrook Criterion is studied.

CONTENTS

Chapter 1 Introduction **11**

1.1 Thesis Objective 11

1.2 Literature review 12

1.2.1 Middlebrook's stability criterion 13

1.2.2 Derivation of Middlebrook's stability criterion 14

1.3 Approach/Methods 19

Chapter 2 Canonical Models of DC-DC converters **20**

2.1 Introduction 21

2.2 Canonical Model of Boost Converter 23

Chapter 3 Derivation of $z_N(s)$& $z_D(s)$ of DC-DC Converters for

Middlebrook's stability criterion **25**

3.1 Introduction 26

3.2 $z_N(s)$& $z_D(s)$ of Boost Converter 31

Chapter 4 Stability Analysis and Converter Transfer Functions

of DC-DC Converters with input filter **31**

4.1 Stability analysis and simulation of buck converter 32

with input filter before and after damping

4.2 Converter transfer functions of buck converter before and after addition of input filter. 42

4.5 Simulation and Stability Analysis of Cascaded Buck-Sepic Converter 46

Conclusion **48**

Bibliograhy **48**

LIST OF FIGURES

S.NO	NAME OF FIGURES
FIG 1.1	STABILITY CRITERIA BOUNDARIES
FIG.1.2	ADDITION OF INPUT FILTER TO A CONVERTER
FIG.2.1	STANDARD CANONICAL MODEL FORM
FIG.2.2	SMALL SIGNAL MODEL OF A BOOST CONVERTER
FIG.2.3	CANONICAL MODEL OF A BOOST CONVERTER
FIG.3.1	CANONICAL MODEL OF A BOOST CONVERTER
FIG.3.2	$z_D(s)$ FOR BOOST COVERTER
FIG.3.3	$z_N(s)$ FOR BOOST CONVERTER
FIG.4.1	SIMULATION OF BUCK CONVERTER WITH INPUT FILTER WITHOUT DAMPING
FIG.4.2	UNDAMPED FILTER
FIG.4.3	SIMULATION OF BUCK CONVERTER WITH INPUT FILTER AFTER DAMPING
FIG.4.4	FREQUENCY RESPONSE AFTER DAMPING THE INPUT FILTER.
FIG. 4.5	CONVERTER TRANSFER FUNCTION BEFORE ADDING INPUT FILTER
FIG.4.6	CONTROL-TO-O/P TRANSFER FUNCTION BEFORE AND AFTER ADDING INPUT FILTER

FIG.4.7	SIMULATION OF BUCK CONVERTER WITH INPUT FILTER NOT OBEYING MIDDLEBROOKS CRITERION
FIG.4.8	SIMULATION OF BUCK-SEPIC CONVERTER

CHAPTER-1

INTRODUCTION

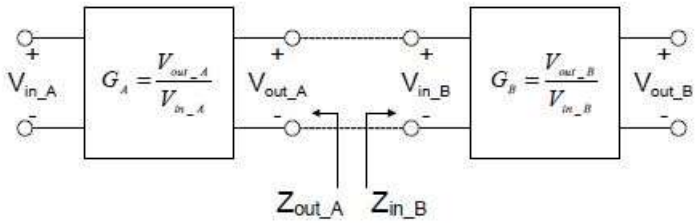
At the point when an Input filter is added to the converter it decreases the electromagnetic Interference (EMI) of power input of converter and enhances the performance of load. Electromagnetic Interference (EMI) is disturbance because of either electromagnetic induction or electromagnetic radiation discharged from outside source that influence the electrical circuit. The EMI may interfere with or decrease the performance of the electrical circuit. Thus an input filter is for the most part used to lessen the electromagnetic interference in power source side of a converter. The Input filter added to converter to diminish the electromagnetic interference may change the system transfer function, which may bring about instability and influence the performance of the converter. In this way, input filter ought to be such that it will diminish the electromagnetic interference and it ought not to influence the performance and the stability of the system. The input filter of a converter decreases the ripple voltage and current seen by the power source, and it can be utilized to decrease the rate of change of current also. The size of the i/p filter is dictated by the ripple current rating of the i/p capacitors and the dI/dt needed by the input line.

1.1 THESIS OBJECTIVE

The converter may become unstable or the system performance may be altered after the addition of an input filter which is not required, so the main objective of this study is to do Stability analysis and check variation in system perfomance of DC-DC converters with input filter.

1.2 LITERATURE REVIEW

The cascaded two individually stable systems is shown in the fig.



The total input to output transfer function of the above system is

$G_{AB} = \frac{v_{out_B}}{v_{in_A}}$

$= G_A G_B (Z_{in_B} / (Z_{in_B} + Z_{out_A}))$

$= G_A G_B \; 1/(1+T_{LG})$

Where the minor loop gain T_{LG} $is\ defined\ as\ T_{LG} = Z_{out_A} / Z_{in_B}$

Since G_A & G_B are stable transfer functions, minor loop gain term is the one responsible for stability. Therefore, a fundamental and adequate condition for stability of the system can be gotten by applying the Nyquist Criterion to T_{LG}, i.e. the interconnected system is stable if and only if the Nyquist contour of T_{LG} does not enclose the (-1, 0) point.

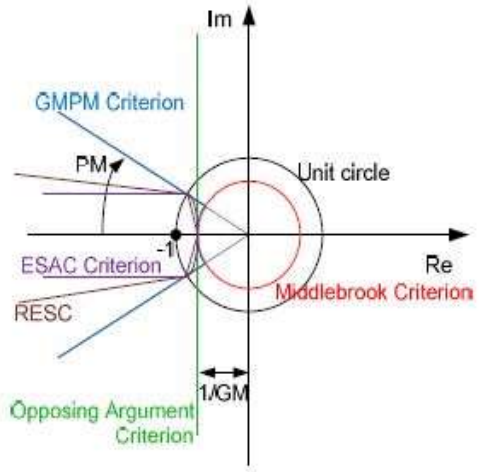


FIG 2.1 Stability Criteria Boundaries

1.2.1THE MIDDLEBROOK CRITERION

The Middlebrook Criterion was at first proposed to explore how the stability of a feedback-controlled switching converter is influenced by the addition of an input filter. Its objective is not only to guarantee system stability, but also to ensure that converter dynamics are not changed by the introduction of an input filter.

The Middlebrook Criterion gives a basic design-oriented sufficient stability condition imposing small-gain condition on the minor loop gain:

$$\|z_0\| \ll \|z_i\| \text{ or equivalently } \|T_{LG}\| = \| z_0/z_i\| \ll 1$$

The above comparison defines a forbidden region for T_{LG} in the s plane that lies outside the unit circle centered in (0, 0). Assuming that z_i is known, a practical design rule for the input filter output impedance z_0 imposes that the resulting minor loop gain must always lay inside a circle with radius equal the inverse of the desired gain margin (*GM*):

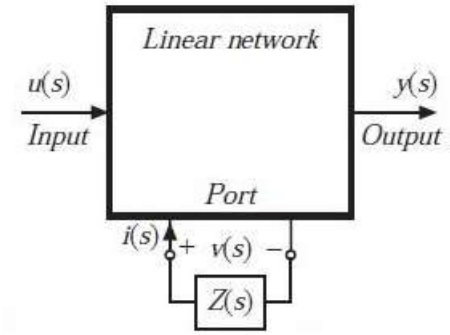
$$\|T_{LG}\| = \| z_0/z_i\| = 1/GM \text{ with } GM>1$$

The Middlebrook Criterion likewise indicates how the properties of the converter are modified by the addition of an input filter. In particular, by the utilization of the Middlebrook Extra Element Theorem the output impedance of the input filter z_0 can be viewed as an extra element. The subsequent loop gain is given by

$$T' = T\,(1+z_0/z_N)/(1+z_0/z_D)$$

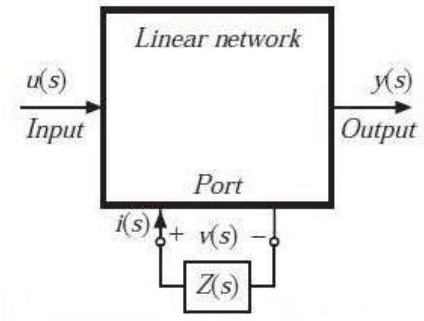
1.2.2 DERIVATION OF MIDDLEBROOK EXTRA ELEMENT THEOREM :

Original system:



$$G_{old}(s) = \frac{y(s)}{u(s)}|i(s) = 0$$

With extra element :



$$G(s) = \frac{y(s)}{u(s)} \quad (1)$$

$$v(s) = -i(s)z(s) \quad (2)$$

There are two independent quantities u(s) and i(s) dependent quantities, v(s) and y(s) can be expressed as functions of independent inputs.

$$y(s) = G_{old}(s)u(s) + G_i(s)i(s) \quad (3)$$

$$v(s) = G_v(s)u(s) + z_D(s)i(s) \quad (4)$$

With $G_{old}(s) = \frac{y(s)}{u(s)}|i(s) = 0$

$$z_D(s) = \frac{v(s)}{i(s)}|u(s) = 0$$

$$G_i(s) = \frac{y(s)}{i(s)}|u(s) = 0$$

$$G_v(s) = \frac{v(s)}{u(s)}|i(s) = 0$$

Now eliminate v(s) & i(s) from euations (1),(2),(3) & (4)

$$G(s) = \frac{y(s)}{u(s)} = G_{old}(s) - \frac{G_v(s)G_i(s)}{Z(s) + z_D(s)}$$

Now eliminate $G_v(s)$ & $G_i(s)$ and express interms of impedances measured at the port.

In the presence of i/p u(s) , inject current i(s) at the port, adjust i(s) in such a way that causes o/p y(s) to be nulled to zero.

$$z_N(s) = v(s)/i(s)|y(s) \to 0, y(s)\ nulled\ to\ zero$$

Nulling:

$y(s) = G_{old}(s)u(s) + G_{\mathrm{i}}(s)i(s)$

When y(s) is nulled to zero

$G_{old}(s)u(s) + G_{\mathrm{i}}(s)i(s) \rightarrow 0$

So the o/p is nulled when i(s) is chosen to satisfy

U(s)|y(s)→0 = $-\frac{G_{\mathrm{i}}(s)}{G_{old}(s)}\, i(s)|y(s) \rightarrow 0$

Substituting this equation into equation (4)

i.e. $v(s) = G_v(s)u(s) + z_D(s)i(s)$

we get $v(s)|\mathrm{y}(\mathrm{s}) \rightarrow 0 = G_v(s)u(s)|\mathrm{y}(\mathrm{s}) \rightarrow 0 + z_D(s)i(s)|\mathrm{y}(\mathrm{s}) \rightarrow 0$

$$= \left(-\frac{G_{\mathrm{i}}(s)G_v(s)}{G_{old}(s)} + z_D(s)\right) i(s)|\mathrm{y}(\mathrm{s}) \rightarrow 0$$

$v(s)|\mathrm{y}(\mathrm{s}) \rightarrow 0 = z_N(s)i(s)|\mathrm{y}(\mathrm{s}) \rightarrow 0$

$$= \left(-\frac{G_{\mathrm{i}}(s)G_v(s)}{G_{old}(s)} + z_D(s)\right) i(s)|\mathrm{y}(\mathrm{s}) \rightarrow 0$$

Hence $z_N(s) = z_D(s) - \frac{G_{\mathrm{i}}(s)G_v(s)}{G_{old}(s)}$

Now eliminate $G_v(s)$ & $G_i(s)$ from expression G(s) using $z_N(s)$ result:

G(s)= $G_{old}(\mathrm{s}) - \frac{z_D(s) - z_N(s)}{z_D(s) + z_N(s)}\, G_{old}(\mathrm{s})$

G(s)= $G_{old}(\mathrm{s})\frac{1+\frac{z_N(s)}{Z(s)}}{1+\frac{z_D(s)}{Z(s)}}$

Addition of an Input Filter to a Converter:

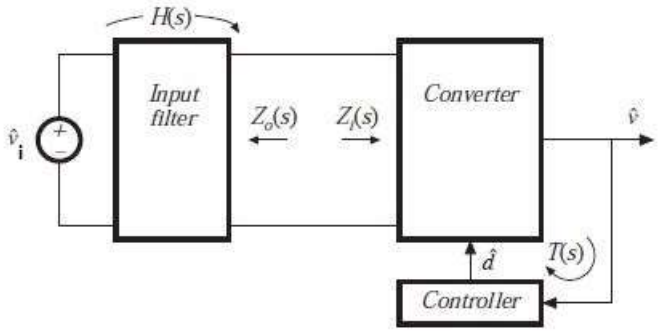
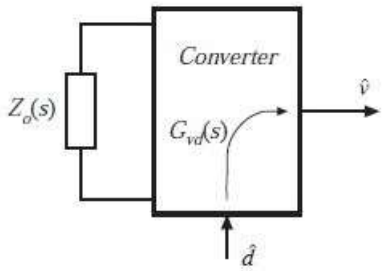


FIG.1.2 Addition Of Input Filter To A Converter

The small-signal transfer functions of a converter are modified by the addition of an i/p filter changes the small-signal transfer functions of a converter

Control-to-output transfer function, $G_{vd}(s) = \frac{\hat{v}(s)}{\hat{i}(s)} \mid \hat{v}_i(s) = 0$



When $\hat{v}_i = 0$. Input filter becomes impedance $z_o(s)$, added to the converter input port.

With no i/p filter the original transfer function is $G_{vd}(s)|z_o(s) = 0$

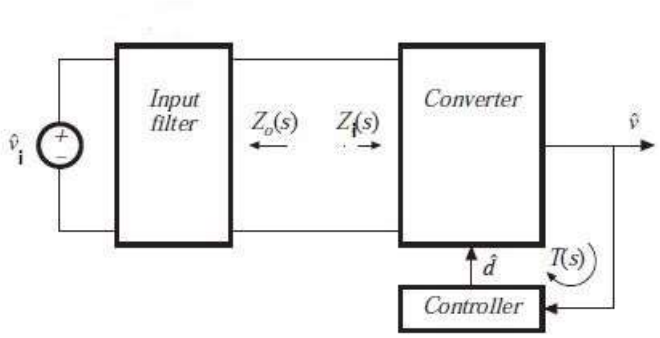
In the presence of i/p filter the control-to-output transfer function can be written as

$$G_{vd}(\mathrm{s}) = (G_{vd}(\mathrm{s})|z_o(s) = 0)\frac{1 + \frac{z_o(\mathrm{s})}{z_N(s)}}{1 + \frac{z_o(s)}{z_D(s)}}$$

From the above equation we can say that the input filter does not essentially change the control-to-output transfer function when

$$\|z_o\| \ll \|z_N\| \text{ , and}$$

$$\|z_o\| \ll \|z_D\|$$



$$z_D(s) = z_i(s)|\hat{d}(s) = 0$$

$$z_N(s) = z_i(s)|\hat{v}(s) = 0$$

1.3 APPROACH/ METHODS

Middlebrook stability criteria is used for designing input filters for various converters which assure stability as well as performance of the converter after adding input filter. So for using middlebrook criteria at first we need to find out output impedance of the input filter$z_o(\mathrm{s})$, $z_D(s)$ and $z_N(s)$ of different converters. After checking that the stability condition of the system is satisfied. i.e.

$$\|z_o\| \ll \|z_N\| \text{ , and}$$

$$\|z_o\| \ll \|z_D\|$$

We need to evaluate the converter transfer functions of different converters and see the performance of the system after addition of input filter. In this project I have evaluated $z_D(s)$, $z_N(s)$ and converter transfer functions of different converters,for which we need to find out small signal model of a converter from the small signal model we need to convert it to canonical model and then from superposition theorem we can find out $z_D(s)$, $z_N(s)$ and converter transfer functions of different converters. Detailed stability analysis of buck converter is provided in this thesis.

CHAPTER-2

Canonical Forms of DC-DC Converters

2.1 INTRODUCTION

Since all the PWM dc-dc converters perform similar basic functions, we can see that the equivalent circuit models have the same structure. Hence, the canonical circuit model of Fig.2.1 can speak to the physical properties of PWM dc-dc converters. The essential function of a dc-dc converter is change of dc voltage and current levels, ideally with the 100% efficiency. This function is spoken to in the model by a ideal dc transformer, denoted by the transformer symbol having a horizontal line. The dc transformer model has effective turns ratio equal to conversion ratio M(D). It complies with all of the usual properties of transformers, aside that it can pass dc voltages and currents. Although dc voltages cannot be passed by the conventional magnetic-core transformers, we are regardless allowed to characterize an ideal dc transformer symbol; utilization of this symbol in demonstrating dc-dc converter properties is defended because its predictions are correct. Small ac variations in source voltage Vi(t) are additionally changed by conversion ratio M(D). Henceforth a sinusoidal line is added to dc transformer symbol, to indicate that it likewise correctly represents how small-signal ac variations pass through the converter.

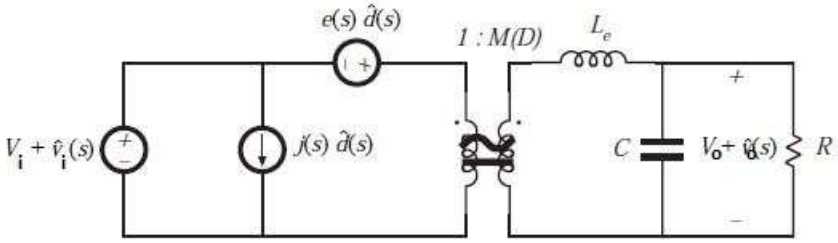


FIG.2.1 STANDARD CANONICAL MODEL FORM

Small ac variations in duty cycle d(t) excite ac variations in the converter currents and voltages. This is demonstrated by e(s)d and j(s)d generators of Fig. 2.1. As a rule, both current source and a voltage source are needed. The converter capacitors and inductors are important to low-pass filter the switching harmonics, also to low-pass filter ac variations. The canonical model along these lines contains an effective low-pass filter. Figure 2.1 outlines the two-pole low-pass filter of buck, boost, and buck-boost converters; complex converters having extra capacitors and inductors, such as the Cuk and SEPIC, contain correspondingly complex effective low-pass filters. The element values in the effective low-pass filter don't fundamentally coincide with physical element values in the converter. By and large, the element values, terminal impedance and transfer functions of the effective low-pass filter can vary with the quiescent operating point.

In general canonical model can be solved for two types of transfer functions: Gvd(s) and Gvg(s) . Gvg(s) will be input to output transfer function. Gvd(s) is the control-to-output transfer function.

Knowing that the above canonical form has all sources in input circuit and the effective filter circuit in the o/p, implies we can take our former hard work with AC models and re-develop them to fit canonical form above

To find $z_D(s)$ and $z_N(s)$ of different converters first we need to evaluate the canonical model from the small signal model of the converter. In this chapter the canonical models of different converters is evaluated.

2.2 Canonical Model of Boost Converter

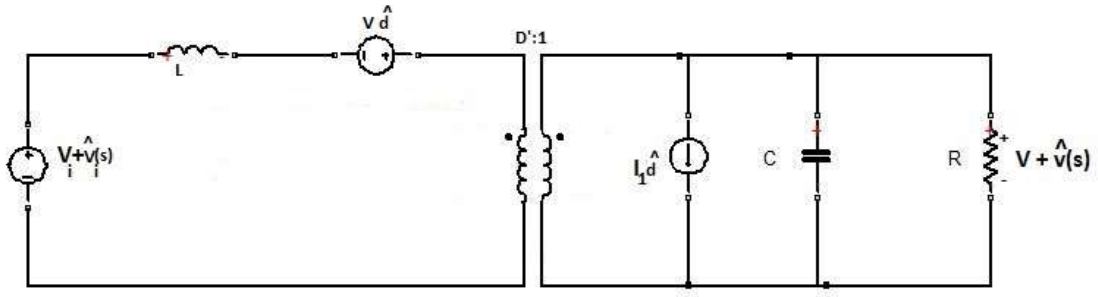


FIG.2.2 small signal model of a boost converter

By manipulating the small signal model of boost converter and get the standard canonical form

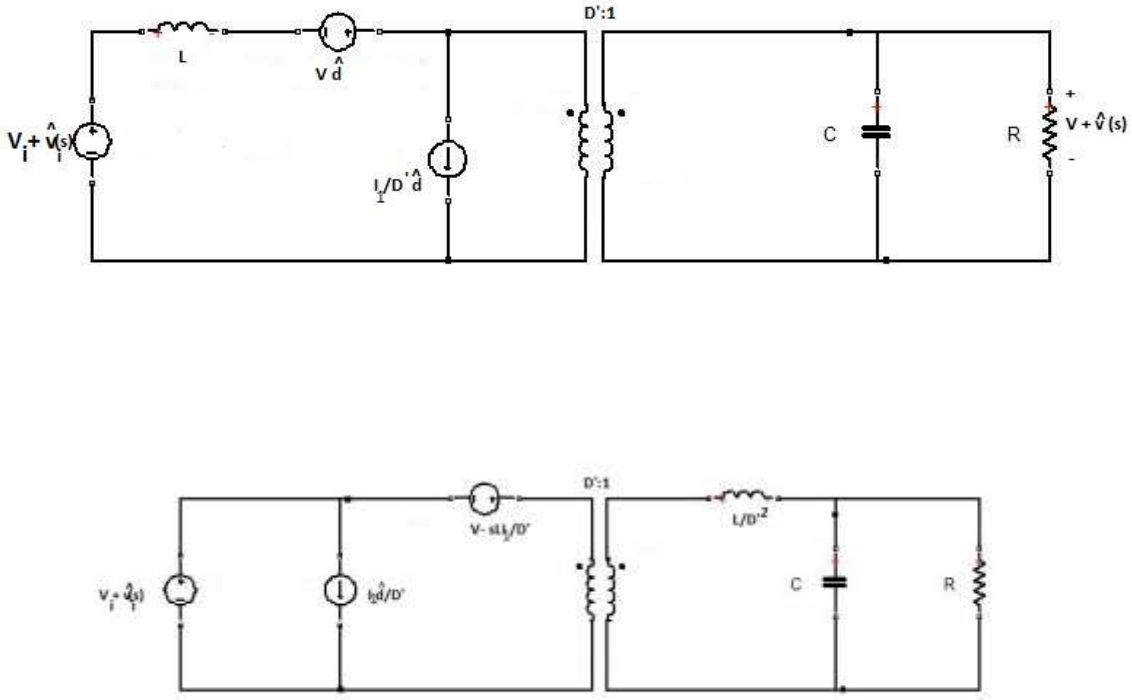


FIG.2.4 Canonical model of a boost converter

CHAPTER-3

Derivation of $z_N(s)$& $z_D(s)$ of DC-DC Converters for Middlebrook's stability criterion

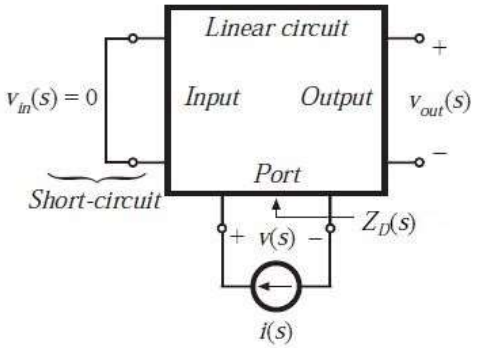
3.1 Introduction

For using middlebrook criteria

$$i.e\ G_{vd}(s) = (G_{vd}(s)|z_o(s) = 0)\frac{1 + \frac{z_o(s)}{z_N(s)}}{1 + \frac{z_o(s)}{z_D(s)}}$$

we need to find out output impedance of the input filter $z_o(s)$, $z_D(s)$ and $z_N(s)$ of different converters.

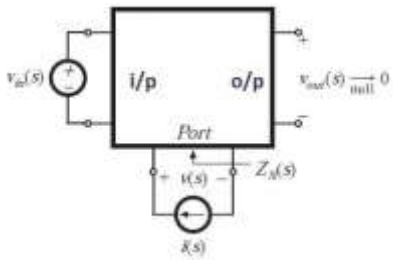
To find $z_D(s)$:



$$z_D(s) = \frac{v(s)}{i(s)}\ |V_{in}(s) = 0$$

$z_D(s)$ is the driving-point impedance proportionate to the Thevenin-equivalent impedance at the port where the additional component is joined. It is found by setting independent sources to zero, and injecting a current i(s) at the port.

To find $z_N(s)$:



Z_N is the impedance seen at the port where the additional component is included when the O/P is nulled. In the vicinity of the input $V_{in}(s)$, a current $i(s)$ is injected at the port. This current $i(s)$ is balanced such that the output $v_{out}(s)$ is nulled to zero. Under these conditions, $z_N(s)$ is the proportion of V(s) to i(s).

Note: nulling is not the same as shorting

From the canonical models evaluated in the previous chapter the $z_D(s)$ and $z_N(s)$ of the converters required for evaluating Middle brook's criterion are found in this chapter

3.2 $z_N(s)$& $z_D(s)$ of Boost Converter

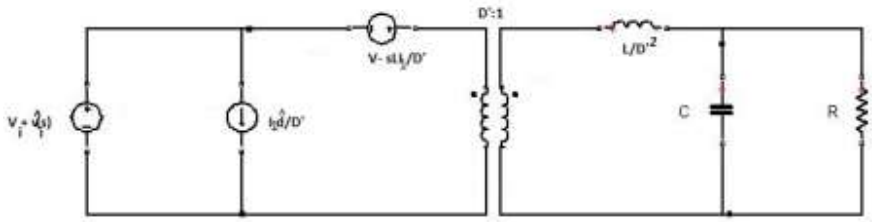


FIG.3.1 Canonical Model of a Boost Converter

To find $z_D(s)$ *for Boost converter*

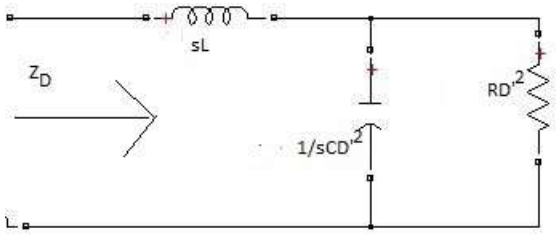


FIG.3.2 $\boldsymbol{z_D(s)}$ for Boost coverter

$$z_D(s) = sL + \frac{RD'^2}{(1+sCR)}$$

$$= \frac{sL + s^2LCR + RD'^2}{1+sCR}$$

$$= \frac{D'^2R\left(1 + \frac{sL}{D'^2R} + \frac{s^2LC}{D'^2}\right)}{1+sCR}$$

To find $z_N(s)$ *for Boost converter* :

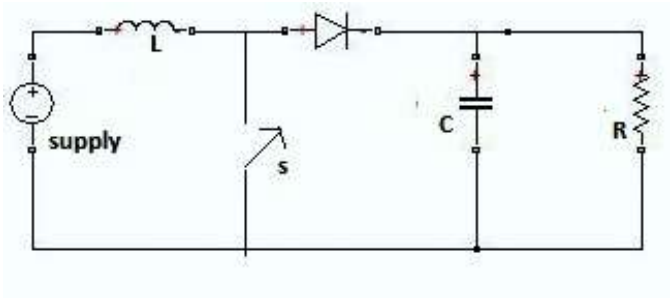
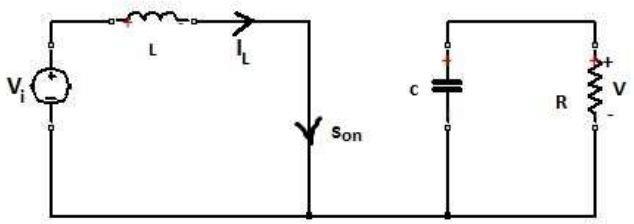


FIG.3.3 $z_N(s)$ for Boost Converter

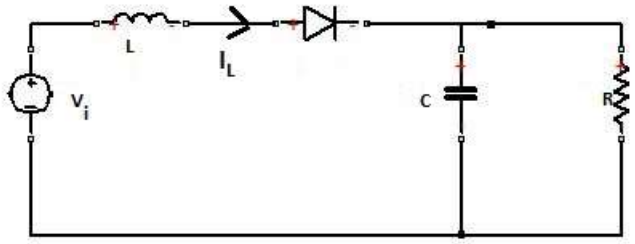
During Ton



$$V_L = L\,\frac{\mathrm{d}i}{\mathrm{d}t} = V_i$$

$$i_C = C\,\frac{\mathrm{d}V_c}{\mathrm{d}t} = -\frac{V}{R}$$

During Toff



$$V_L = L\,\frac{\mathrm{d}i}{\mathrm{d}t} = V_i - V$$

$$i_C = C\,\frac{\mathrm{d}V_c}{\mathrm{d}t} = i(t)\ -\frac{V}{R}$$

Inductor voltage waveform

$$<< V_L(t) > T_s = d(t) < V_i > T_s + d'(t) < V_i - V > T_s$$

$$= L \frac{d}{dt} < i(t) > T_s$$

Average capacitor Current

$$< i_C(t) > T_s = C \frac{\mathrm{d}}{\mathrm{d}t} < v(t) > T_s = -\frac{d(t)(< v(t) > T_s)}{R} + D' < I - \frac{V}{R} > T_s$$

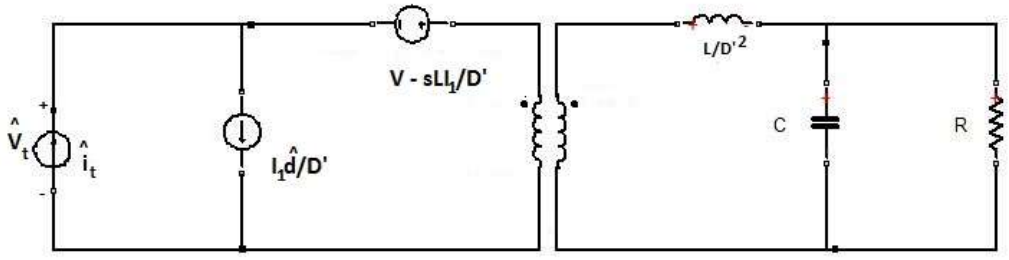
Linearizing about quiescent point

$0 = DV_i + D'V_i - D'V$

D'V= V_i (1)

$$0 = -\frac{DV}{R} + D'I - \frac{D'V}{R}$$

$$I = \frac{V}{D'R} \quad (2)$$



Using superposition theorem

$$\hat{\imath}_t = -\frac{I_1 \hat{d}}{D'} = -\frac{V}{D'^2 R}\hat{d}$$

$$and \quad \hat{V}_t = V - \frac{sLI_1}{D'}$$

From equation (2)

$$\hat{V}_t = V - \frac{sL}{D'}\left(\frac{V}{D'R}\right)$$

$$\hat{V}_t = V\left(1 - \frac{sL}{D'^2 R}\right)$$

$$z_N(s) = \frac{\hat{V}_t}{\hat{\imath}_t} = \frac{V\left(1 - \frac{sL}{D'^2 R}\right)}{-\frac{V}{D'^2 R}} = -D'R\left(1 - \frac{sL}{D'^2 R}\right)$$

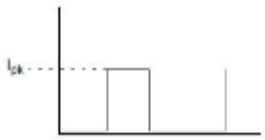
CHAPTER-4

Stability Analysis and Converter Transfer Functions of DC-DC Converters with input filter

4.1 Stability Analysis And Simulation Of Buck Converter With Input Filter Before And After Damping

The i/p filter of a converter reduces the ripple voltage and current seen by the power source, and it can be utilized to decrease the rate of change of current also. The size of the i/p filter is dictated by the ripple current rating of the i/p capacitors and the dI/dt needed by the input line.

A buck converter draws current in approximately rectangular pulses .



Filtering the current drawn by a buck can be accomplished by including low-ESR capacitors to the i/p of the converter. The voltage seen on the line is then Ipk x ESR. At point when a fast load step happens on the o/p of the converter, it furthermore shows up as a transient on the input of the converter.

The energy can't come uncertainly from the i/p capacitor: the i/p current must increment. Including an i/p inductor to the channel can control dI/dt.

Selecting the Input capacitor

For the estimations of inductance and capacitance commonly essential for a buck converter i/p filter the impedance of inductance at switching frequency is exceptionally high compared with impedance of capacitance. In this way, basically the majority of the AC current originates from the capacitors.

Since capacitors have ESR, AC current going through them offers ascend to self-heating ($P = {I_{RMS}}^2 \times ESR$)

This sets a breaking point on the amount of AC current that can be gone through a given capacitor without overheating it, subject to its ESR and package size. At last, this self-heating reasons capacitors to fall flat. Note that just capacitors that have ESR evaluated at 100kHz ought to be utilized for the input filter! It is commonplace to utilize capacitors that have an evaluated existence of no less than 2000 hours; better constructed converters will utilize 5000 hour parts.

Instead of determining thermal resistance in °C/W for each capacitor package, producers commonly indicate a maximum RMS ripple current. The ripple current rating is function of temperature, and the temperature utilized for the assessment ought to be the normal surrounding temperature the capacitor will see over its working life.

DC average of current is

$$I_{avg} = I_{Pk} \times dc$$

$During\ the\ on-time\ current\ is\ I_{Pk}$ - I_{avg} = I_{Pk}× (1-dc) $and\ during\ the\ off-time\ current\ is$ $-I_{avg}$ = $-I_{Pk}$× dc

We now find the RMS Value by squaring this waveform

$During\ on-time\ the\ current\ squared\ is\ I_{Pk}^2(1-\mathrm{d}c)^2$

$and\ during\ off-time\ it\ is\ (-I_{Pk}\,\mathrm{d}c)^2$

Including these together for their particular times, time = dc for the on-time and (1 - dc) for the off-time, (the *mean* part) gives after algebra

$$I_{Pk}^2(1-dc)^2 \text{ dc} + I_{Pk}^2\, dc^2 \text{ (1-dc)} = I_{Pk}^2 \text{ (} dc - dc^2)$$

At last, taking the square root gives the RMS current as:

$$I_{RMS} = I_{Pk}\sqrt{dC - dc^2} \quad (1)$$

Sample datasheet of capacitors:

Capacitor Value (µF)	Voltage (V)	Current (Arms)	ESR (mΩ)
330	16	4.58	17
820	6.3	4.04	14
220	6.3	3.9	15
680	6.3	5.2	10
470	6.3	1.6	15
150	6.3	1.5	40
680	6.3	3.8	18
1500	6.3	4.8	12

From the current rating of the capacitor we can find the suitable capacitor for input filter.

Selecting the Input Inductor

The i/p inductance may be dictated by the dI/dt requirement and the input capacitors that have been chosen. Fundamentally, a load step on the o/p must interpret into a load step on the input; the relative impedances of the capacitor and inductor decide how quick the current in the inductor rises. The systematic expression for the dI/dt is extremely intricate, however luckily it isn't required.

The maximum dI/dt happens when the greatest voltage is connected across the inductor. Since one end of the inductor is assumed altered at the DC i/p voltage, this happens when the least voltage shows up at the flip side, at the capacitor. Be that as it may, the capacitor sees its minimum voltage when the load step first happens, as a result of its ESR.

The minimum voltage on the capacitor is

$$V_{c,min} = V_{in} - I_{Pk} \times ESR$$

The voltage across the inductor is thus $V_{L,max} = V_{in} - (V_{in} - I_{Pk} \times ESR)$

$$= I_{Pk} \times ESR$$

Thus, the dI/dt of the inductor is

$$\frac{\mathrm{dI}}{\mathrm{dt}} = (I_{Pk} \times ESR)/L$$

Let us assume the maximum $\frac{\mathrm{dI}}{\mathrm{dt}}$ be 0.1A/msec .

$$L = \frac{(I_{Pk} \times ESR)}{\frac{\mathrm{dI}}{\mathrm{dt}}} \qquad (2)$$

$\frac{\mathrm{dI}}{\mathrm{dt}}$ in the above equation must be less than 0.1A/msec to find the required value of input inductor.

For BUCK CONVERTER WITH PARAMETERS

L = 32 µH, C = 58.59 µF, fs = 100 KHz, Vin = 12v,D= 0.4, Load R = 1 ohm

From the above equations (1) and (2) we can find the values of input capacitor (C_f) and inductor(L_f) which in this case are $C_f = 470\mu F \; and \; L_f = 0.18\mu H$

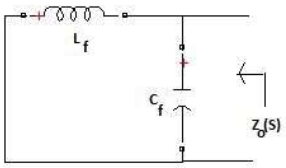


FIG. UNDAMPED INPUT FILTER

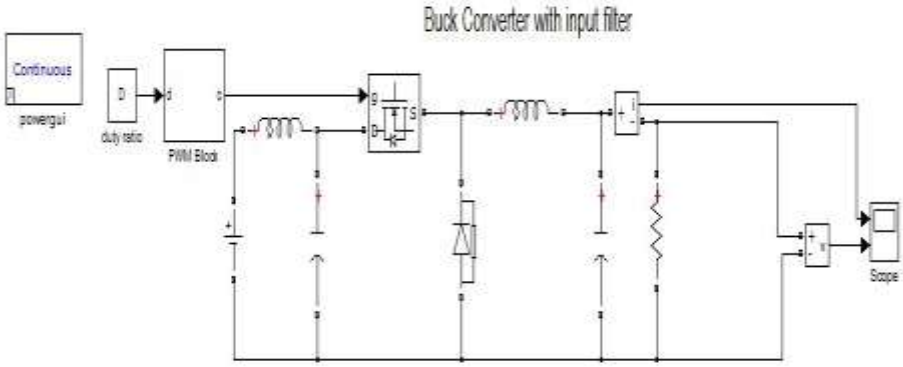
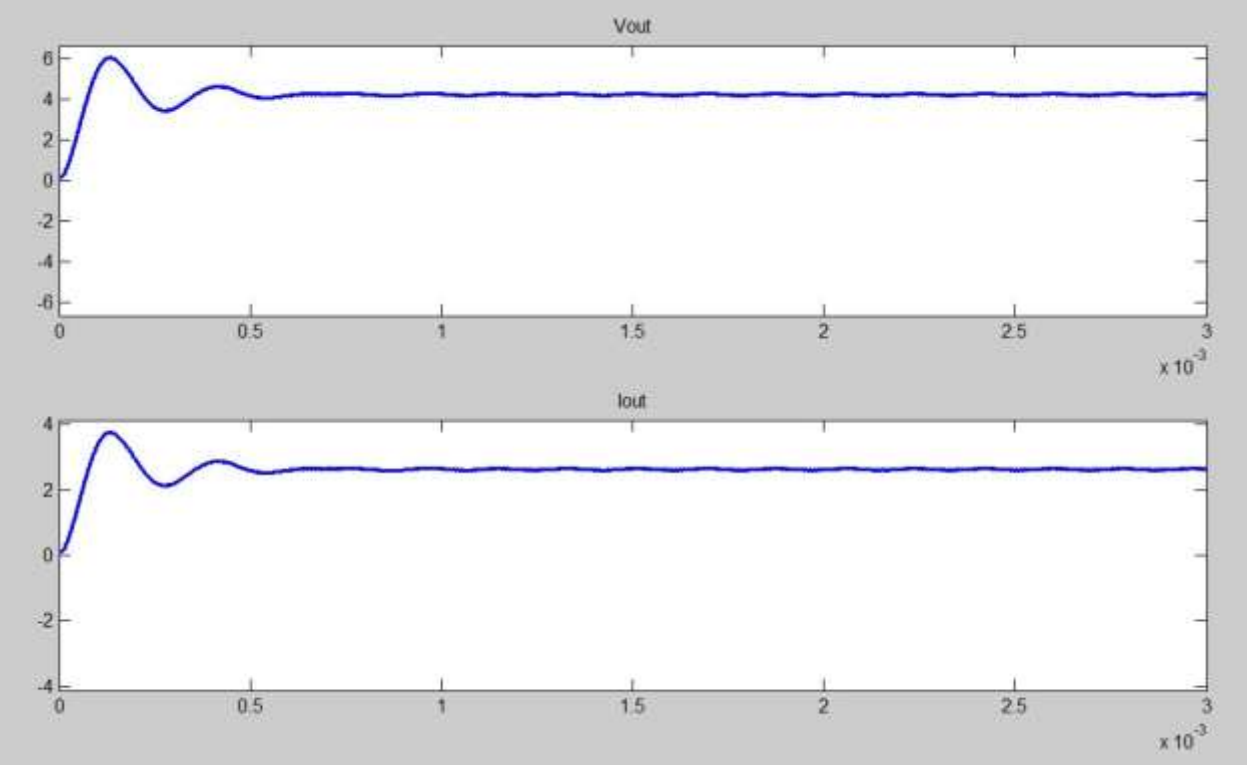


Fig.4.1 simulation of buck converter with input filter without damping



num = [0.18*(10^-6),0];

den = [0.18*470*(10^-12),0,1];

Zo = tf (num, den);

>> num1 = [18.75*(10^-9),200*(10^-6),10];

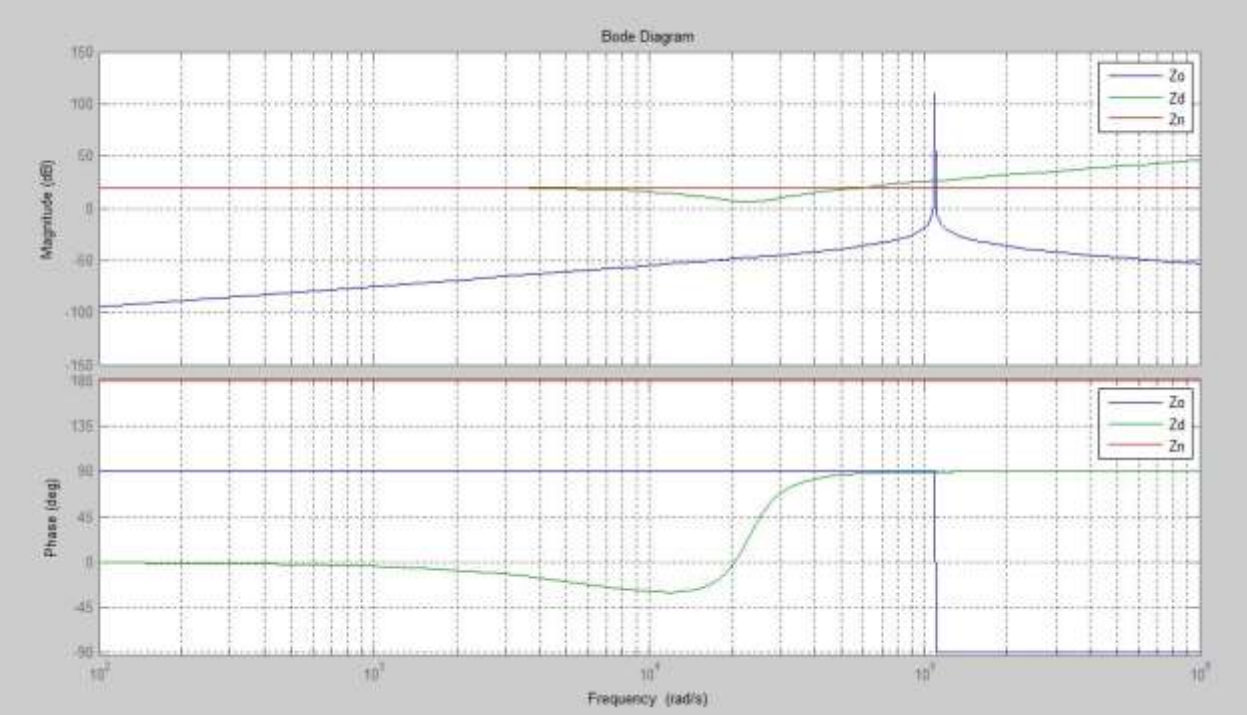
den1 = [93.744*(10^-6),1];

Zd = tf (num1, den1);

num2 = [-10];

Zn = tf (num2);

bode(Zo, Zd, Zn)



From the above graph it is seen that the filter met required Inequalities everywhere except at resonant frequency, so we need to damp the input filter.

Damping the input filter:

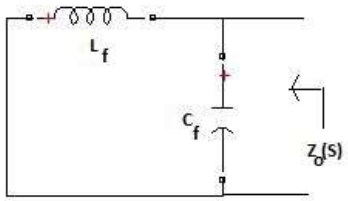
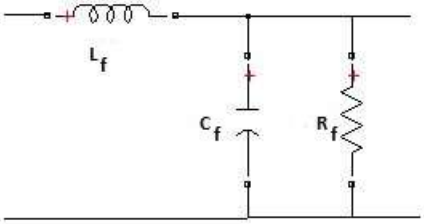
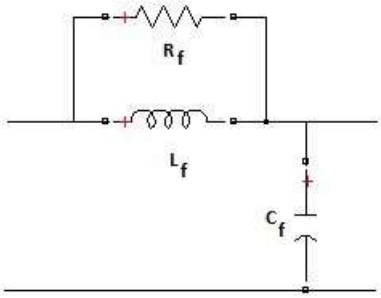


FIG.4.2 Undamped Filter

Two possible approaches :



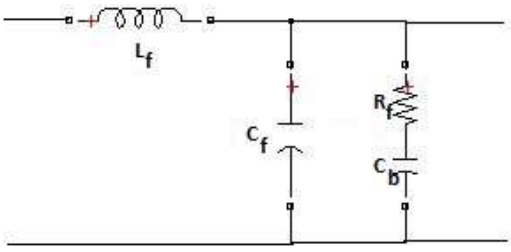


To meet the requirement $R_f \ll \| z_N \|$:

$$R_f \ll \frac{R}{D^2}$$

power loss in R_f is $\frac{V_i^2}{R_f}$ which is more than the load power

A solution for this is adding a dc blocking capacitor c_b. Choose c_b so that its impedance is smaller than R_f at the filter resonant frequency.



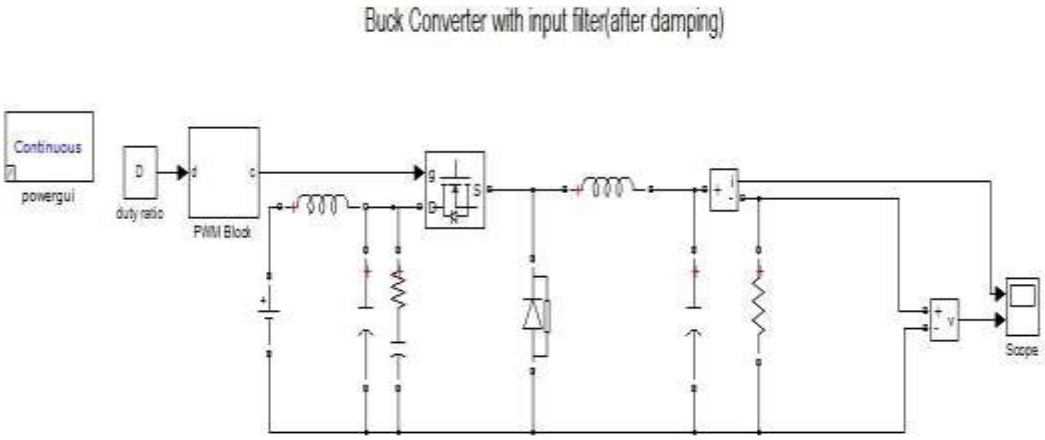
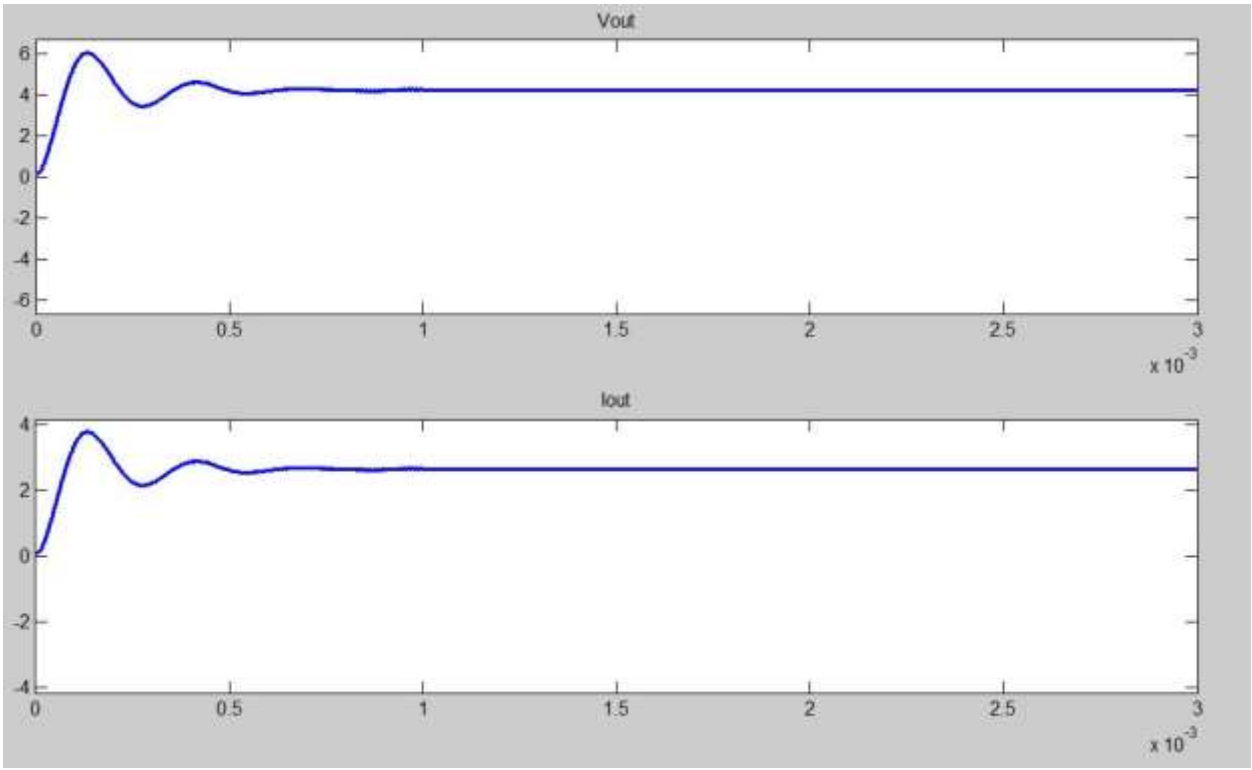


Fig.4.3 simulation of buck converter with input filter after damping



num = [0.18*4700*(10^-12),0.18*(10^-6),0];

den = [0.18*4700*58.59*(10^-18),0.18*(10^-6)*4758.59*(10^-6),4700*(10^-6),1];

Zo = tf (num, den);

num1 = [18.75*(10^-9),200*(10^-6),10];

den1 = [93.744*(10^-6),1];

Zd = tf (num1, den1);

num2 = [-10];

Zn = tf (num2);

>> bode(Zo, Zd, Zn)

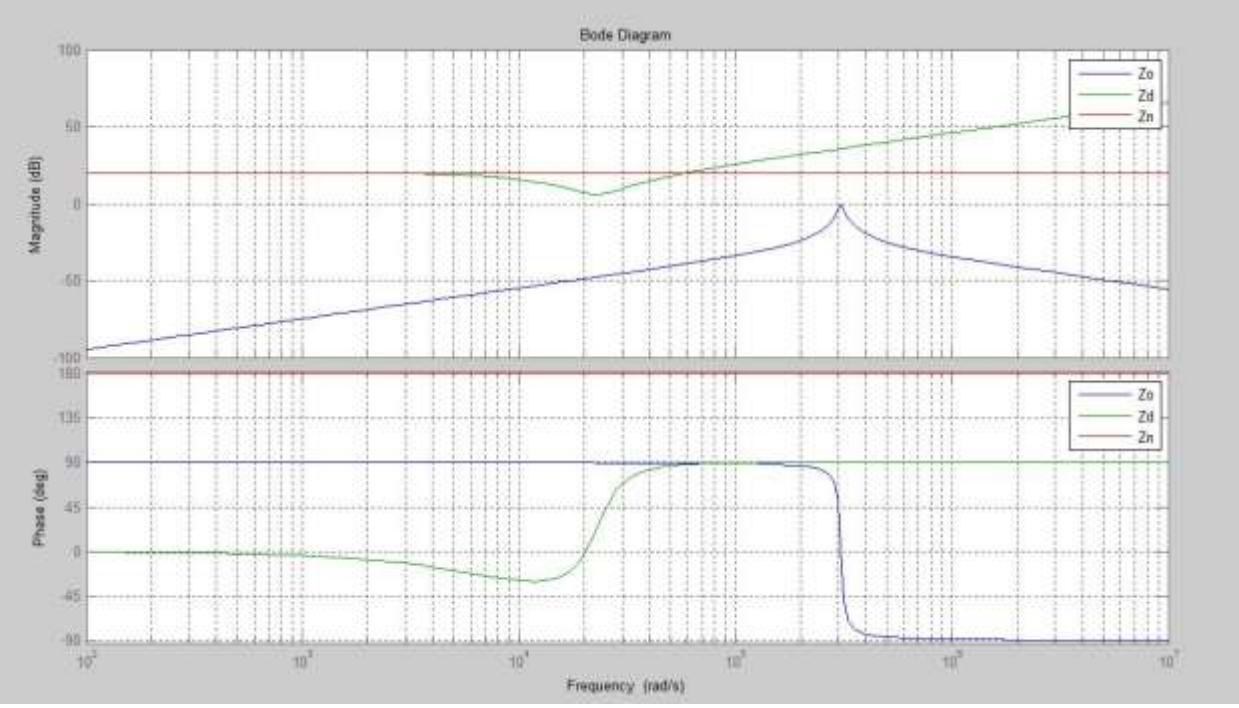


FIG.4.4 Frequency response after damping the input filter.

4.2 converter transfer functions of buck converter before and after addition of input filter

numt=[12];

dent=[1.875*(10^-9),20*(10^-6),1];

Gvd=tf(numt,dent);

>> bode(Gvd)

Where Gvd is control-to-output transfer function of buck converter.

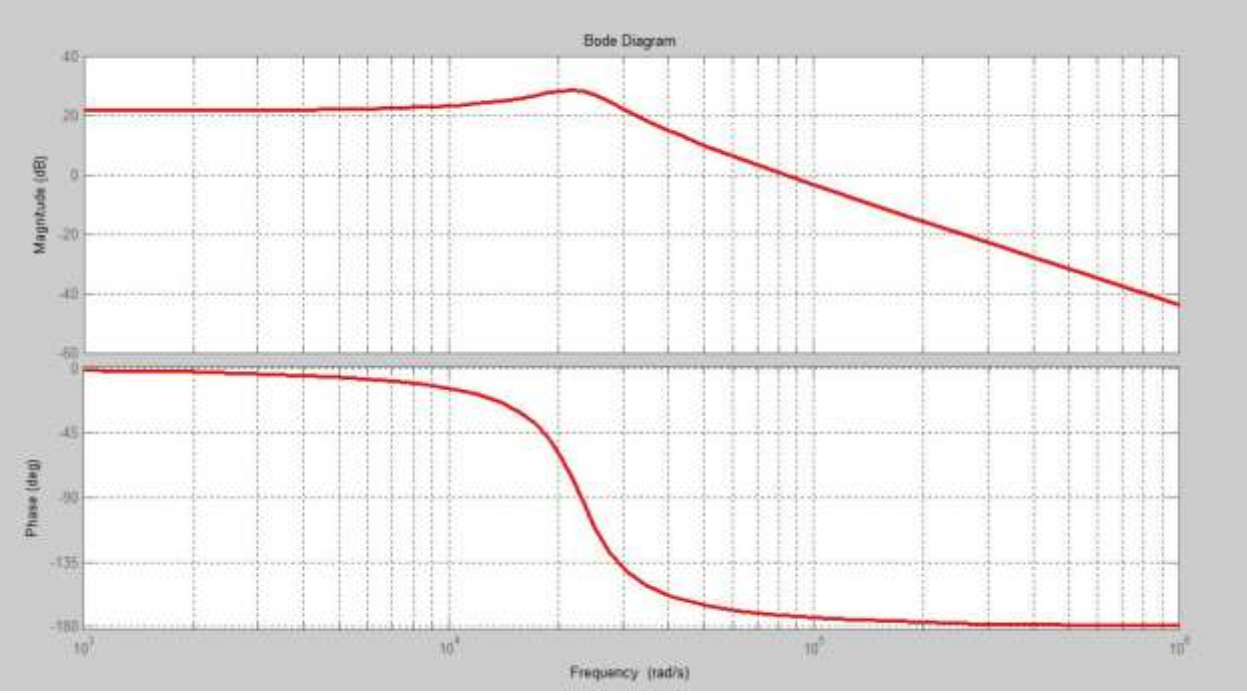


FIG. 4.5 CONVERTER TRANSFER FUNCTION BEFORE ADDING INPUT FILTER

numt=[12];

dent=[1.875*(10^-9),20*(10^-6),1];

Gvd=tf(numt,dent);

Gvdn=(Gvd)*(1+Zo/Zn)/(1+Zo/Zd);

bode(Gvdn,'r',Gvd,'y')

Gvd – control-to-output converter transfer function before adding input filter

Gvdn – control-to-output converter transfer function after adding input filter

Where the relation between transfer functions is Gvdn = Gvd $\frac{1+\frac{z_o(s)}{z_N(s)}}{1+\frac{z_o(s)}{z_D(s)}}$

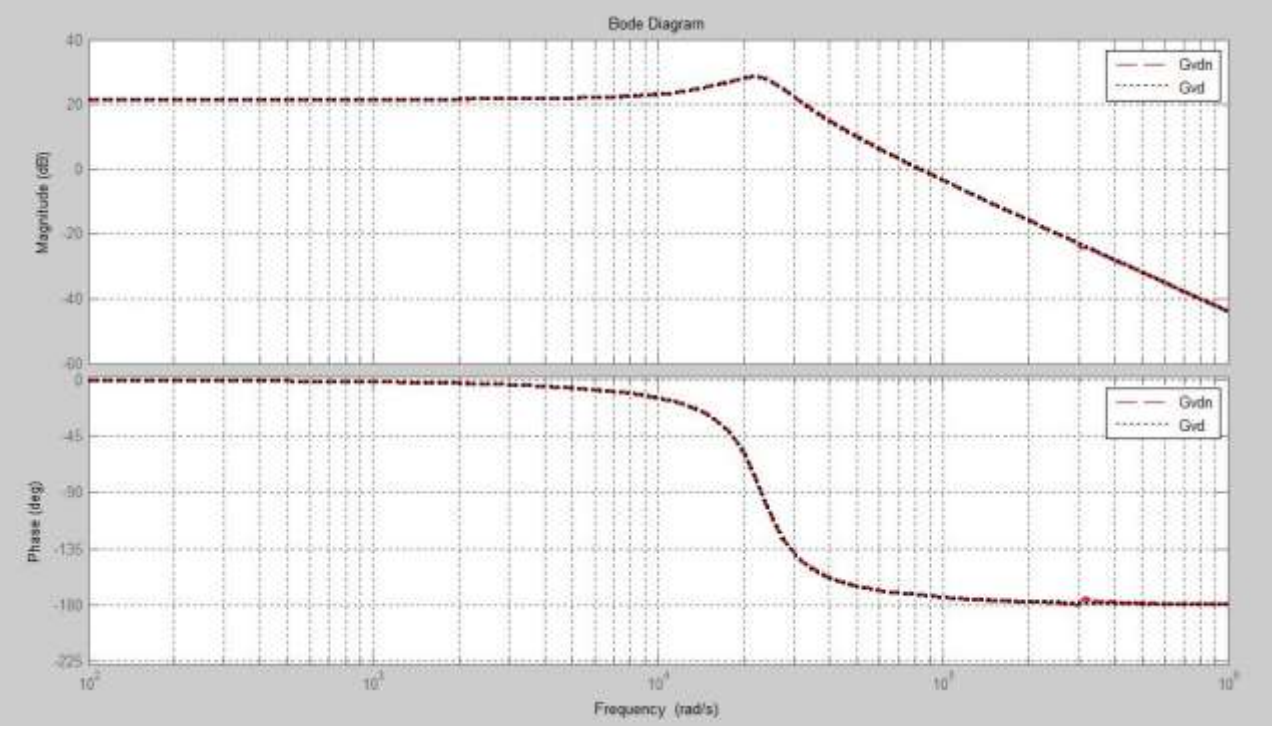


FIG.4.6 CONTROL-TO-O/P TRANSFER FUNCTION BEFORE AND AFTER ADDING INPUT FILTER

Buck converter with input filter not obeying Middlebrook's criterion :

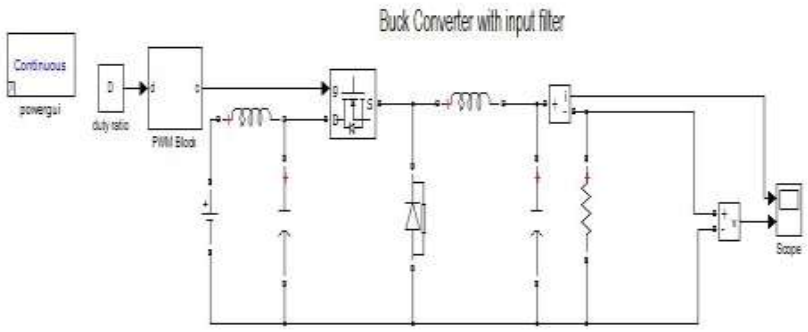
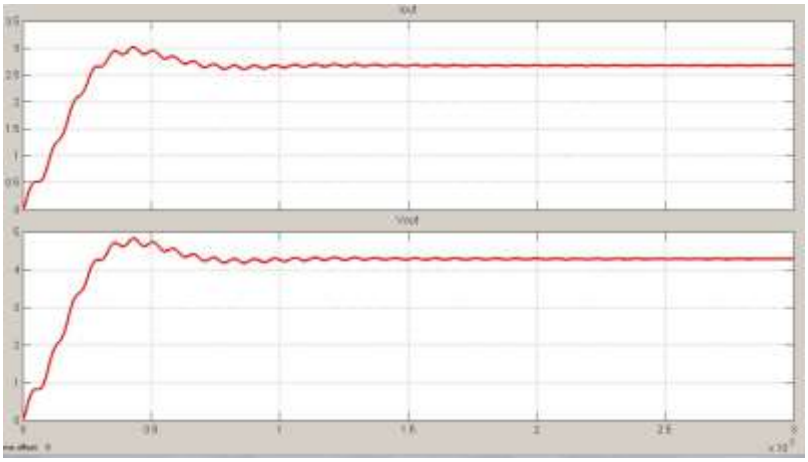


FIG.4.7 Simulation of Buck converter with input filter not obeying Middlebrooks criterion



num = [1*(10^-3),0];

den = [1*(10^-9),0,1];

Zo = tf (num, den);

>> num1 = [18.75*(10^-9),200*(10^-6),10];

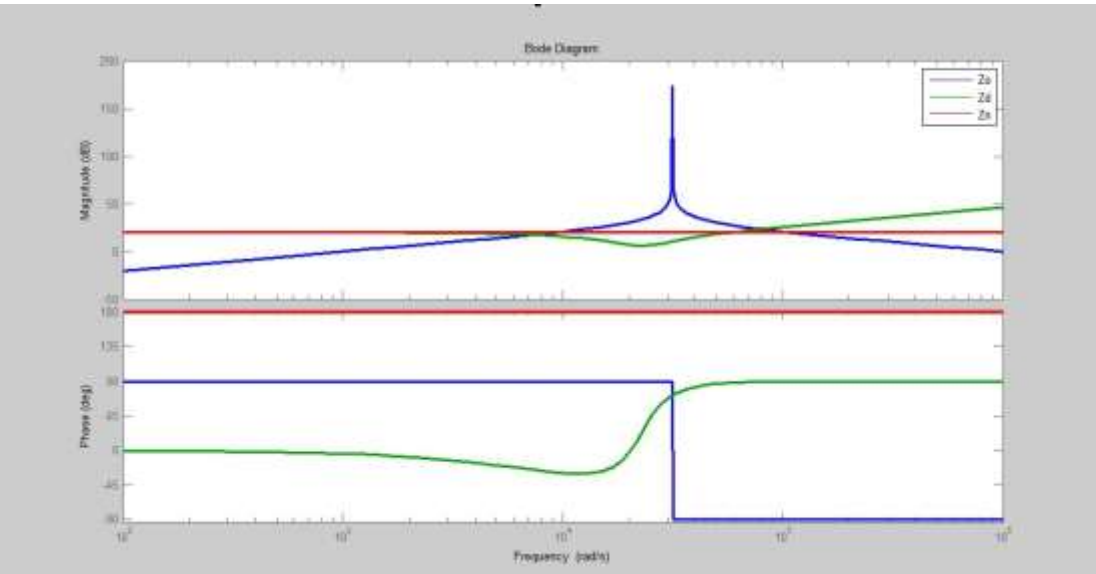
den1 = [93.744*(10^-6),1];

```
Zd = tf ( num1, den1 );

num2 = [-10];

Zn = tf ( num2);

bode(Zo, Zd, Zn)
```



4.7 Simulation and Stability Analysis of BUCK-SEPIC Converter

Cascaded BUCK-SEPIC Converter 30V-15V-5V

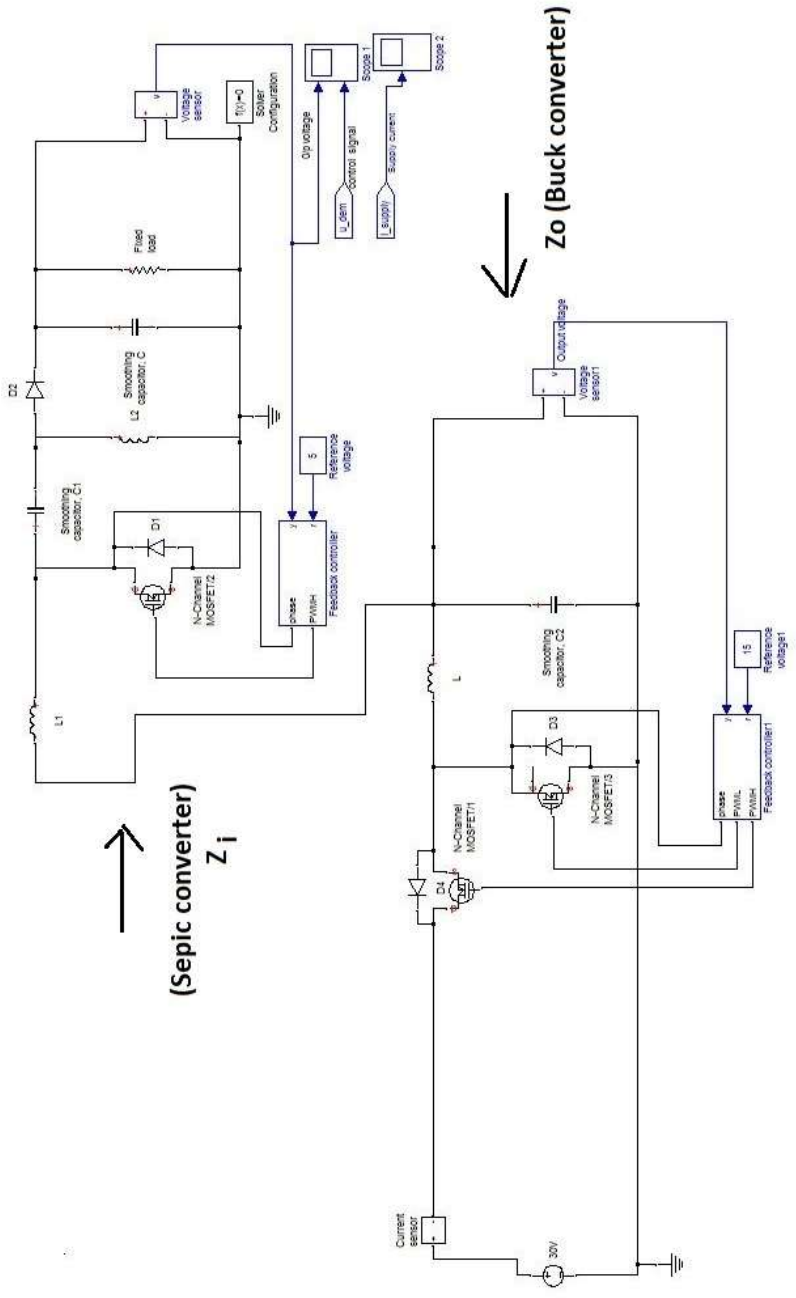
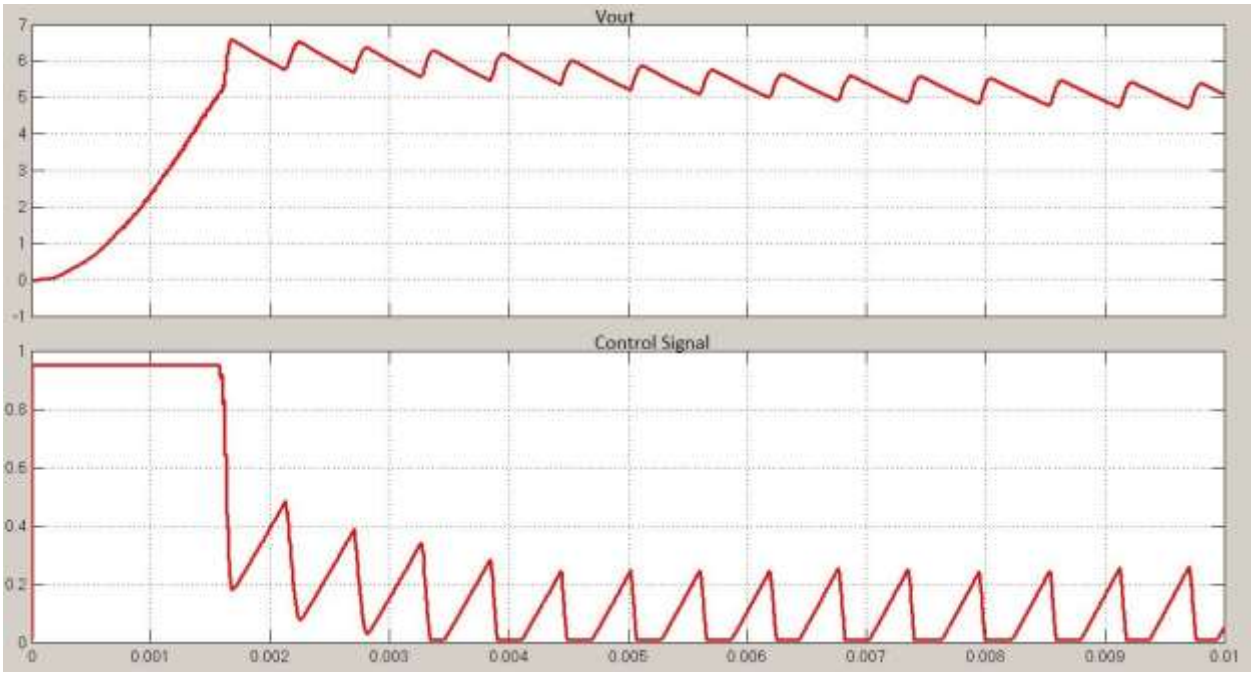
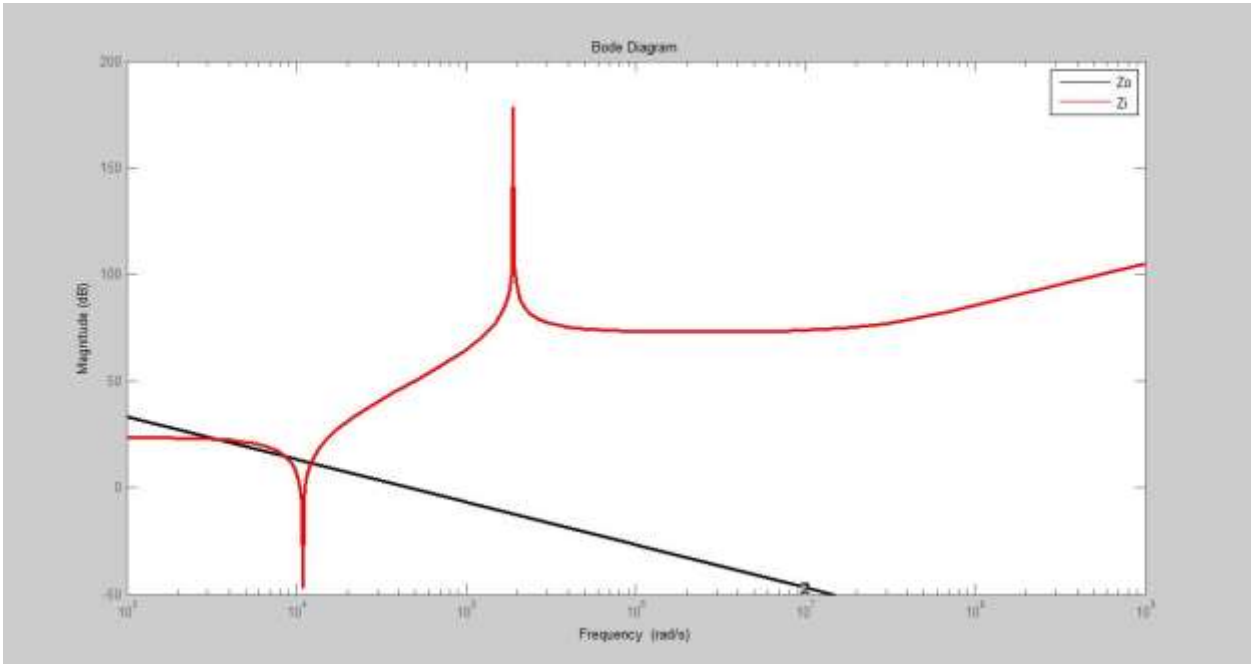


FIG.4.10 Simulation of Buck-Sepic Converter

When two independently stable buck and sepic converters are cascaded together, to do the stability analysis treat buck as an extra element added to the sepic converter and check the middlebrook's criterion.





CONCLUSION

The mathematical analysis of different DC-DC converters for applying MiddleBrook stability criteria is done. Simulations of buck , buck-boost and cascaded buck-sepic converters with input filter are obtained in MATLAB-Simulink environment. From the plots of system response and the variation in system transfer function before and after addition of input filter we can see that the middlebrook criterion for stability is satisfied and the performance of the system is good with the addition of input filter.

Bibliography

[1] R. Erickson and D. Maksimovi, Fundamentals of Power Electronics, second edition, Kulwer A cademic Publishers , 2001.

[2] A. Riccobono and E. Santi, "Comprehensive Review of Stability Criteria for DC Distribution Systems," *IEEE*, 2012.

[3] C. Cooper, "Choosing the right input caps for your buck converter," EE Times-India, september 2007.

[4] M.Sclocchi, "Input filter design for switching power supplies," National semi conductor corporation, 2010.

[5] V. Vorpérian, Fast analysis techniques for electrical and electronic circuits, Cambridge University Press, 2002.

[6] Vorpérian, "Analysis of Current -Mode Controlled PWM Converters Using the Model of the Current -Controlled PWM Switch," october 1990.

[7] A. Martin, "AN-2162 Simple Success with Conducted EMI From DC-DC Converters," Application Report SNVA489C, Novembber 2011- Revised April 2013.

[8] A. Riccobono and E. Santi, "Comprehensive Review of Stability Criteria for DC Distribution Systems," *IEEE*, 2012.

[9] "Application Bulletin AB-16 : Designing the Input Filter for a Pentium II Processor Converter," FairChild Semiconductor.