

# Middle-mile Network Optimization

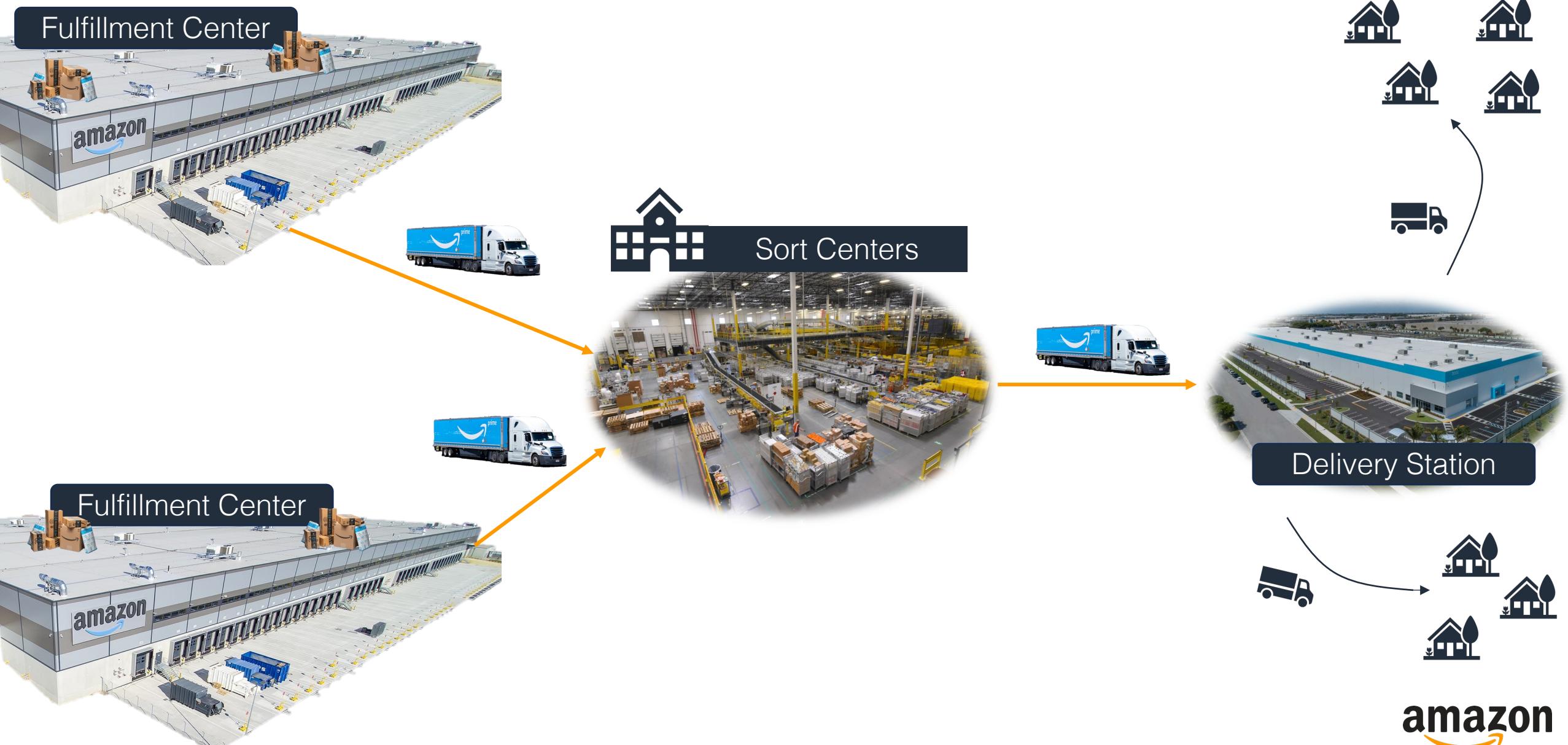
Minimizing costs while maximizing delivery speed

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Research Science Manager

Amazon Transportation Services (ATS), Science & Tech

# Middle-mile Transportation Network



# Middle-mile Optimization



Delivery Station (DS)



## Key decisions

1. Connectivity, i.e., buildings to connect.
2. Timing, i.e., trucks departure times.

## Objectives

1. Reduce cost.
2. Minimize carbon emissions.
3. Maximize delivery speed.

# Middle-mile Optimization



Fulfillment Center (FC)



Sort Centers (SC)

Delivery Station (DS)



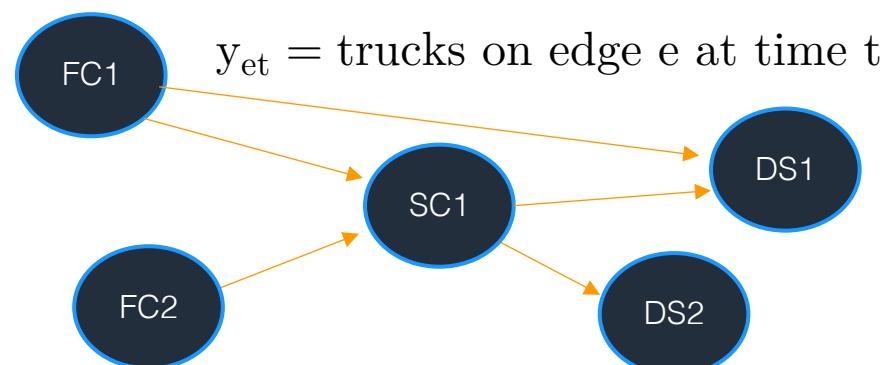
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## Objectives

1. Reduce cost.
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## Time-expanded network flow problem



Intractable at Amazon's scale!

# Why do we care about speed?

amazon Deliver to  
New York 10003 All french press chambord

All Best Sellers Prime Customer Service New Releases Find a Gift Today's Deals Books Gift Cards Kindle Books Pharmacy Fashion Amazon Basics Toys & Games Sell

Amazon Home Shop by Room Discover Shop by Style Home Décor Furniture Kitchen & Dining Bed & Bath Garden & Outdoor Home Improvement

< Back to results



Roll over image to zoom in

2 VIDEOS

**Bodum 1928-16US4 Chambord French Press Coffee Maker, 1 Liter, 34 Ounce, Chrome**

Visit the Bodum Store

★★★★★ 13,088 ratings

Amazon's Choice for "french press chambord"

List Price: \$53.50

Price: **\$34.99** & FREE Shipping. Details & FREE Returns

You Save: **\$18.51 (35%)**

Available at a lower price from other sellers that may not offer free Prime shipping.

Style: Glass Carafe

Glass Carafe Shatterproof SAN Carafe

Color: Chrome



Size: 34 Ounce

12 Ounce 17 Ounce 34 Ounce

51 Ounce

Material Borosilicate Glass, Stainless Steel, Plastic

Brand Bodum

Color Chrome

Capacity 1 Liters

Buy new: \$34.99 & FREE Shipping. Details & FREE Returns

Arrives: Monday, Jan 18 Details

Fastest delivery: Tomorrow Order within 7 hrs and 55 mins Details

In Stock.

Qty: 1

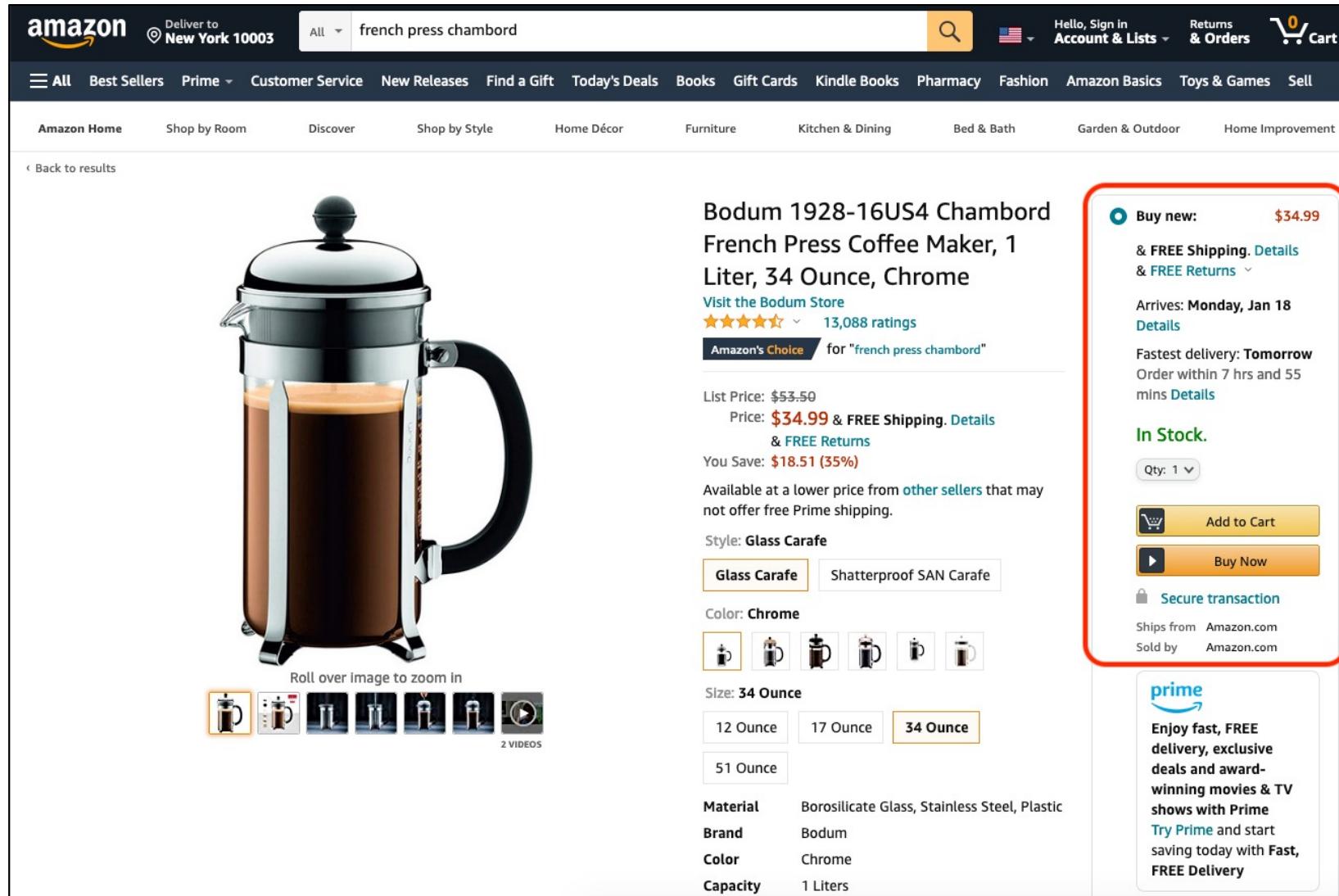
Add to Cart Buy Now

Secure transaction

Ships from Amazon.com Sold by Amazon.com

prime Enjoy fast, FREE delivery, exclusive deals and award-winning movies & TV shows with Prime Try Prime and start saving today with Fast, FREE Delivery

# Why do we care about speed?



The screenshot shows an Amazon product page for a Bodum 1928-16US4 Chambord French Press Coffee Maker. The product is a chrome-plated French press with a glass carafe and a black handle. It is filled with dark coffee. Below the main image is a zoomed-in view of the top part of the press. A callout box highlights the Prime delivery information.

**Bodum 1928-16US4 Chambord French Press Coffee Maker, 1 Liter, 34 Ounce, Chrome**

Visit the [Bodum Store](#)

★★★★★ 13,088 ratings

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**In Stock.**

Qty: 1 ▾

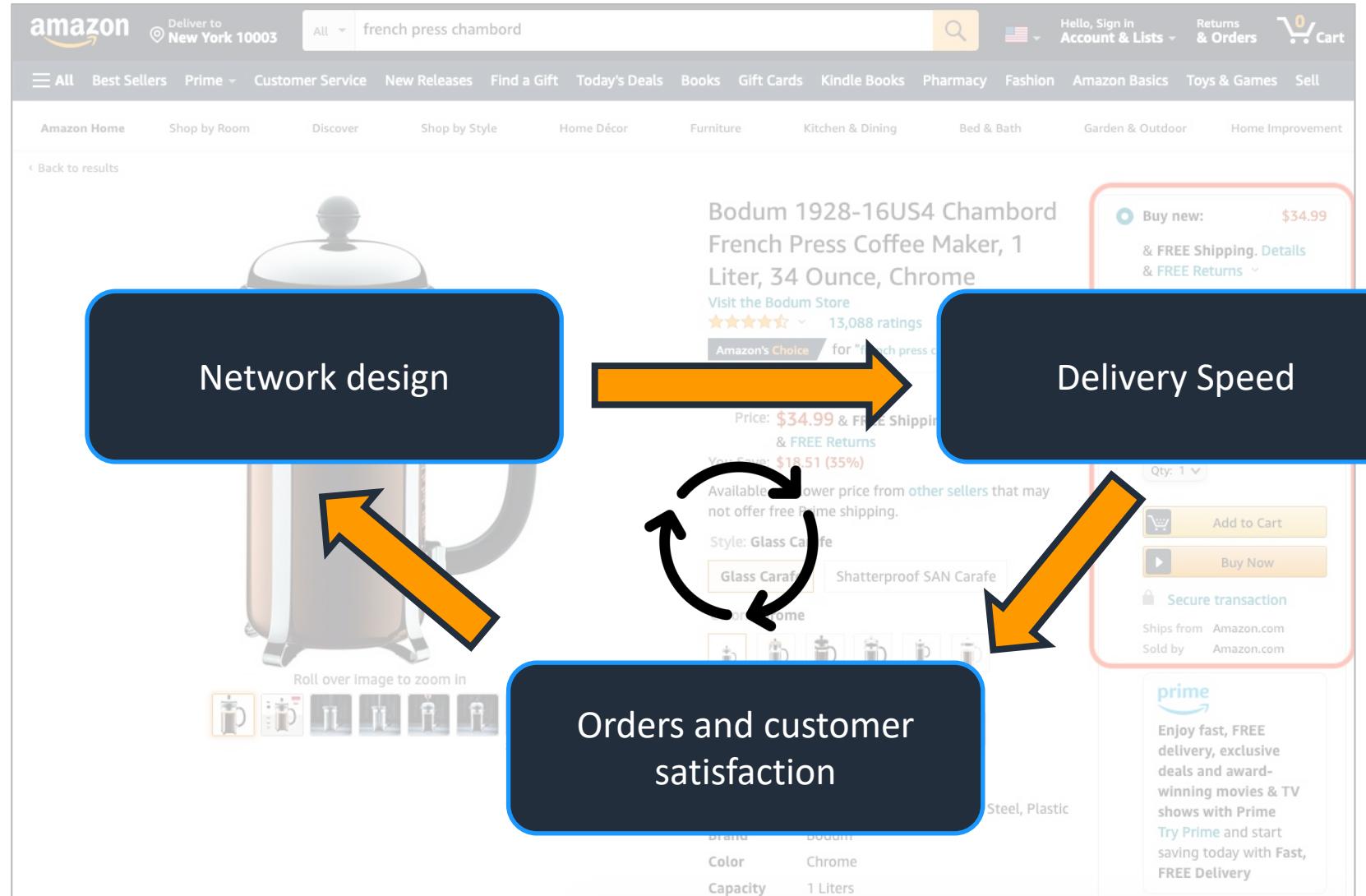
 Add to Cart  
 Buy Now

 Secure transaction

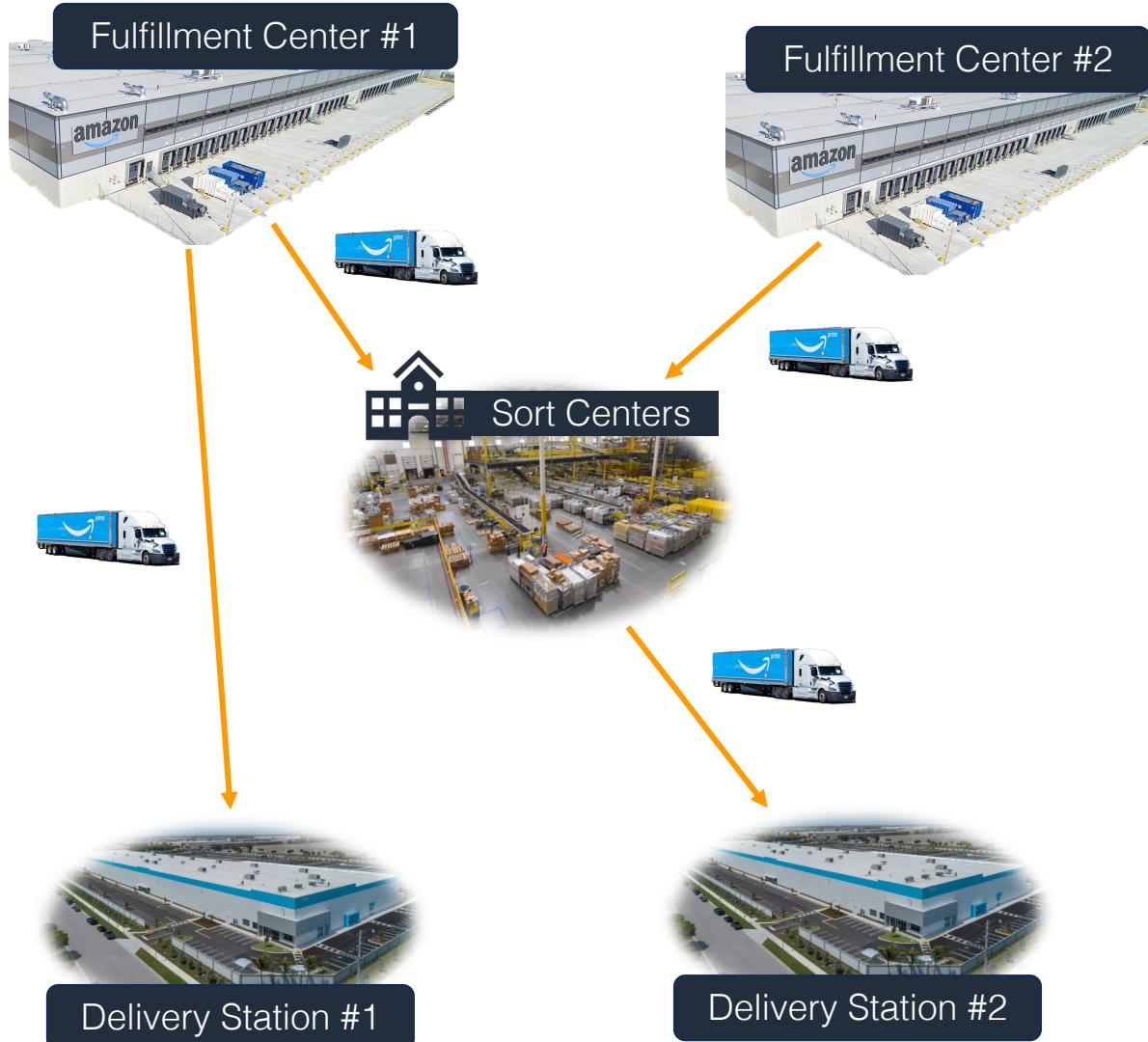
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Sold by Amazon.com

**prime**  
Enjoy fast, FREE delivery, exclusive deals and award-winning movies & TV shows with Prime  
[Try Prime](#) and start saving today with Fast, FREE Delivery

# Why do we care about speed?



# Network Design: Connectivity

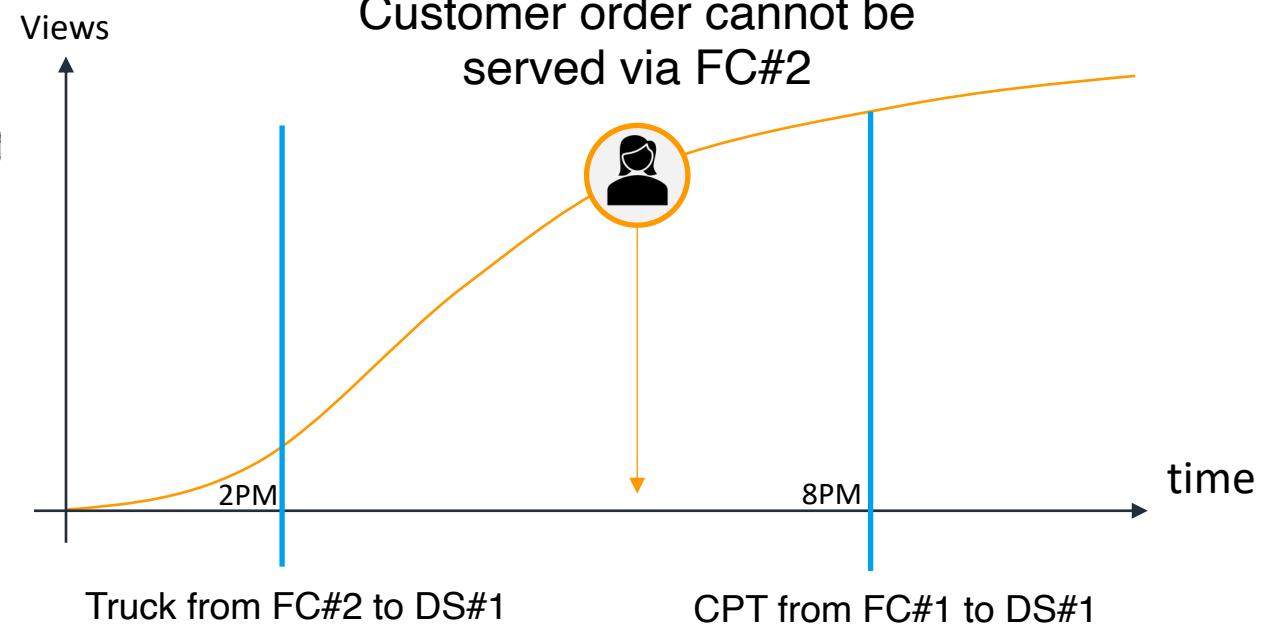


$$\begin{aligned}
 & \min_{\boldsymbol{p}, \boldsymbol{v}} \text{NetworkCost}(\boldsymbol{p}, \boldsymbol{y}) \\
 \text{s.t. } & (\boldsymbol{p}, \boldsymbol{y}) \in \text{FeasibleNetwork}
 \end{aligned}$$

Path vector variable

Trucks vector variable

# Network Design: Timing



$$\max_z \text{Speed}(z)$$

$$\text{s.t. } z \in \text{FeasibleSchedule}(p)$$

Truck vector variable

Dependency on connectivity

$$\begin{aligned} \min_{\mathbf{p}, \mathbf{y}, z} \quad & \text{NetworkCost}(\mathbf{p}, \mathbf{y}) - \text{Speed}(z) \\ \text{s.t.} \quad & (\mathbf{p}, \mathbf{y}) \in \text{FeasibleNetwork} \\ & z \in \text{FeasibleSchedule}(\mathbf{p}) \end{aligned}$$

## Why is this problem hard to solve?

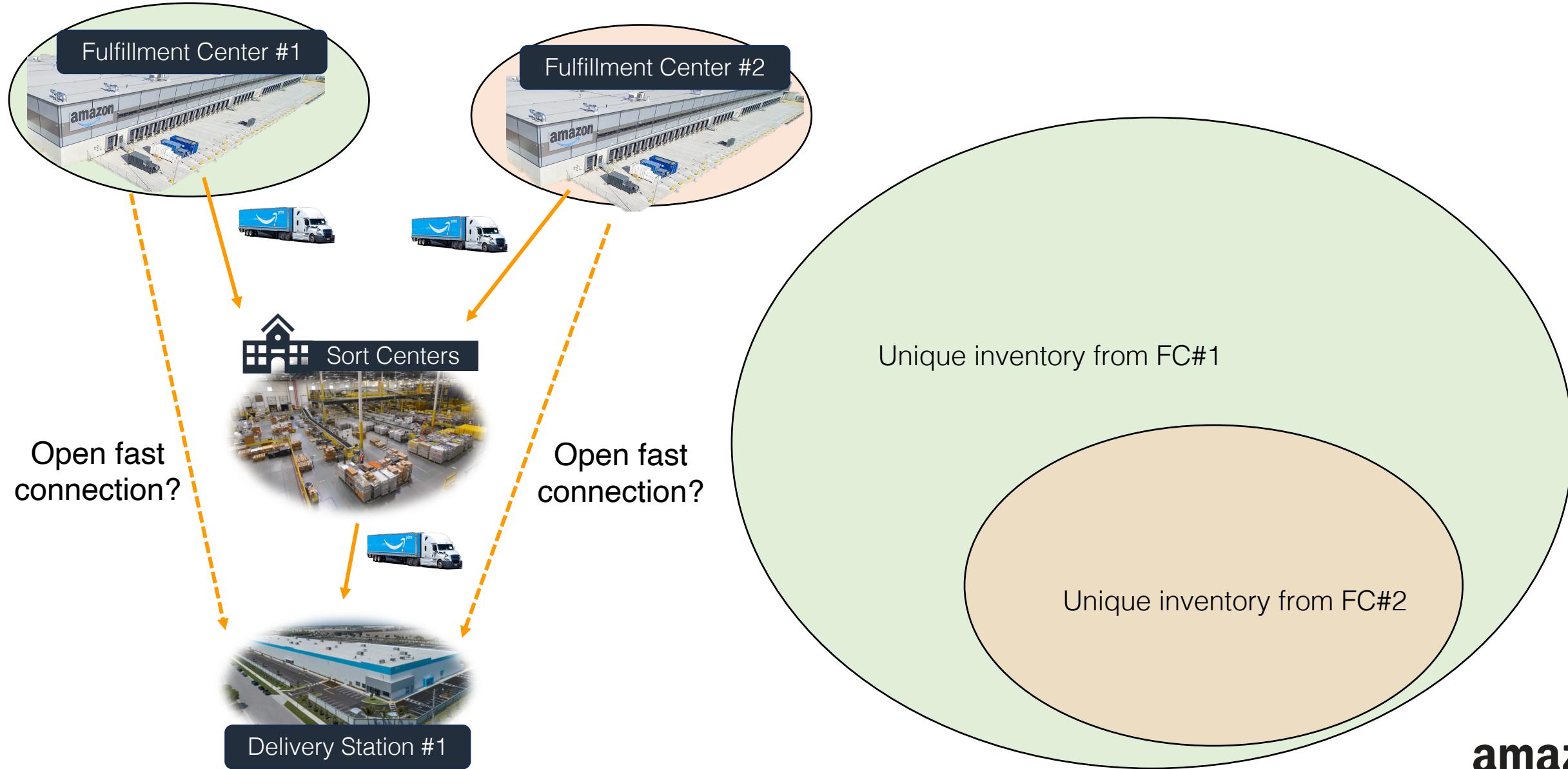
- (a) Feasibility: We must consider granular non-convex operational constraints, e.g., site opening hours.
- (b) Speed objective: Inventory at FCs impacts the speed given by expensive fast connections.
- (c) Scale: Billions of variables to model hourly decisions, e.g., when a truck should depart.
- (d) Uncertainty: Customers' demand is uncertain, thus we should minimize the expected cost.

$$\begin{aligned} \min_{\mathbf{p}, \mathbf{y}, z} \quad & \text{NetworkCost}(\mathbf{p}, \mathbf{y}) - \text{Speed}(z) \\ \text{s.t.} \quad & (\mathbf{p}, \mathbf{y}) \in \text{FeasibleNetwork} \\ & z \in \text{FeasibleSchedule}(\mathbf{p}) \end{aligned}$$

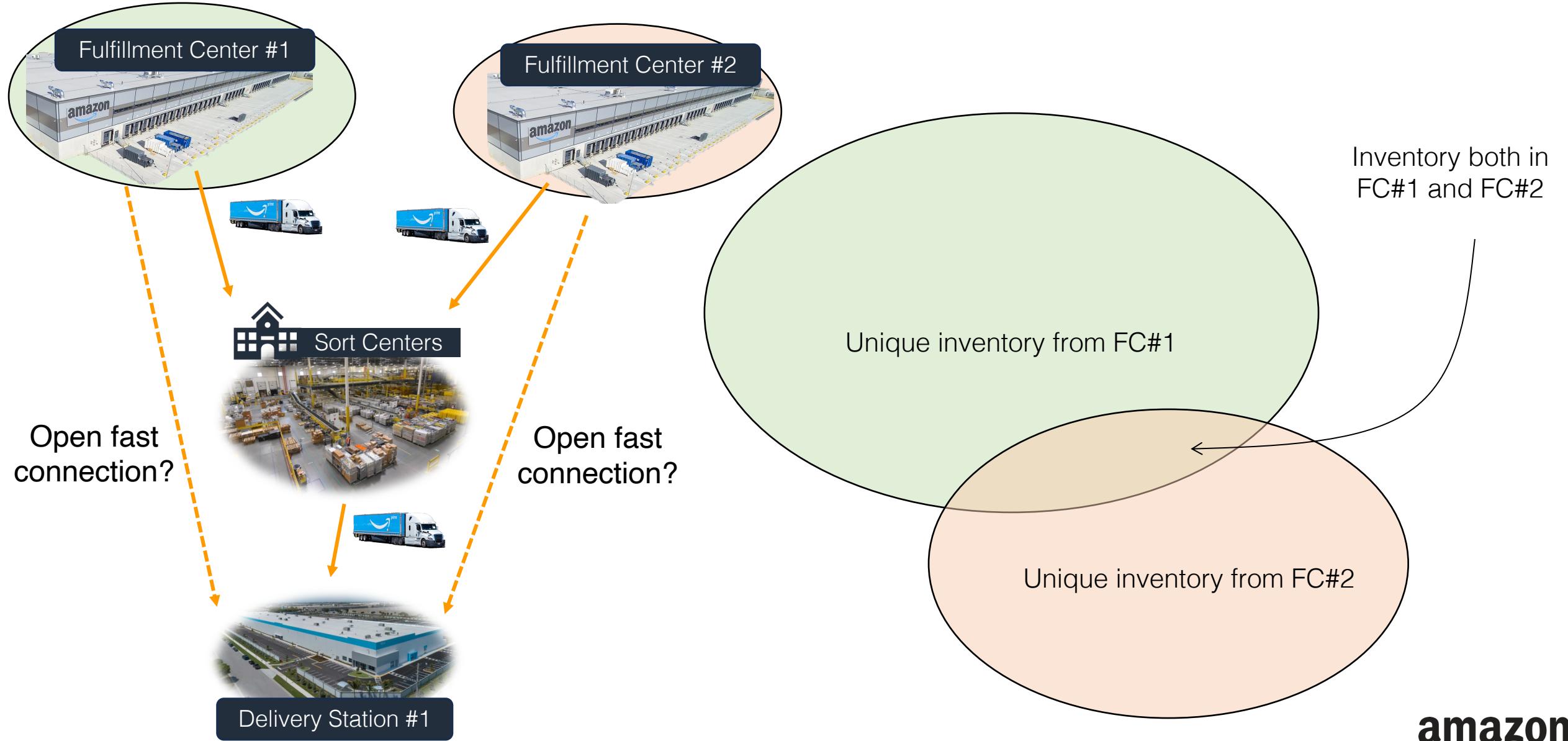
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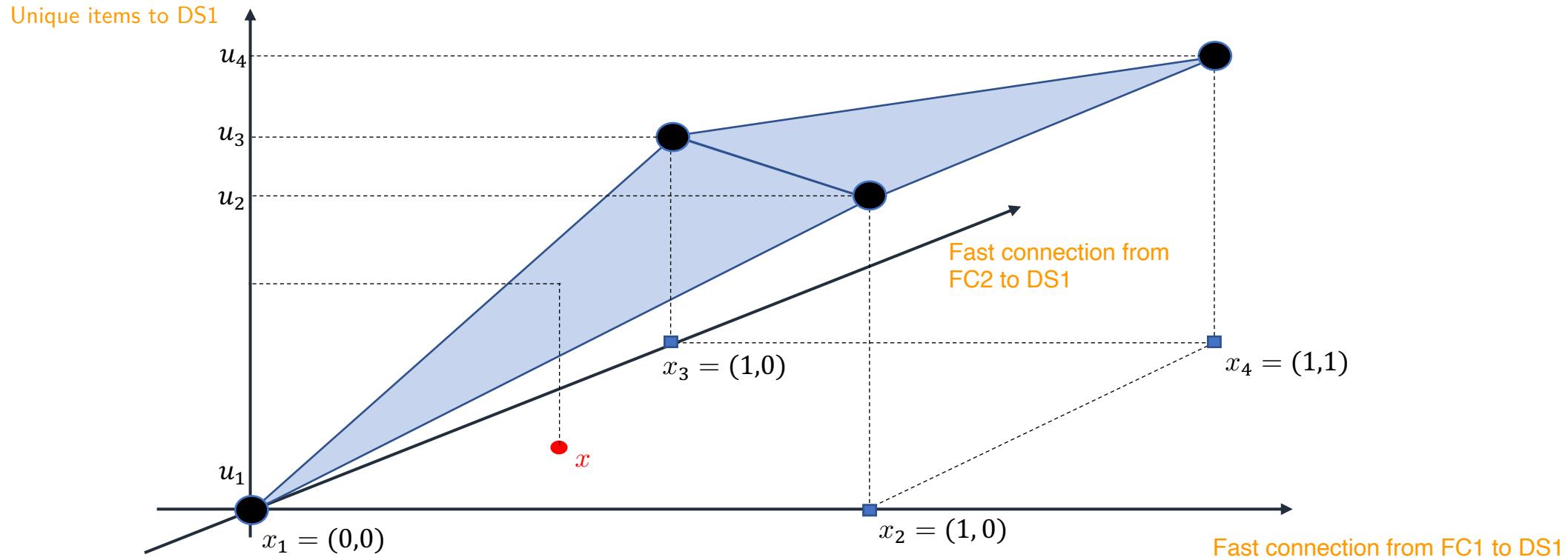
# Speed-aware Network Design



# Speed-aware Network Design



# Speed-aware Network Design



$$V(\mathbf{x}) = \max_{\alpha_i \geq 0} \quad \sum_i \alpha_i u_i$$

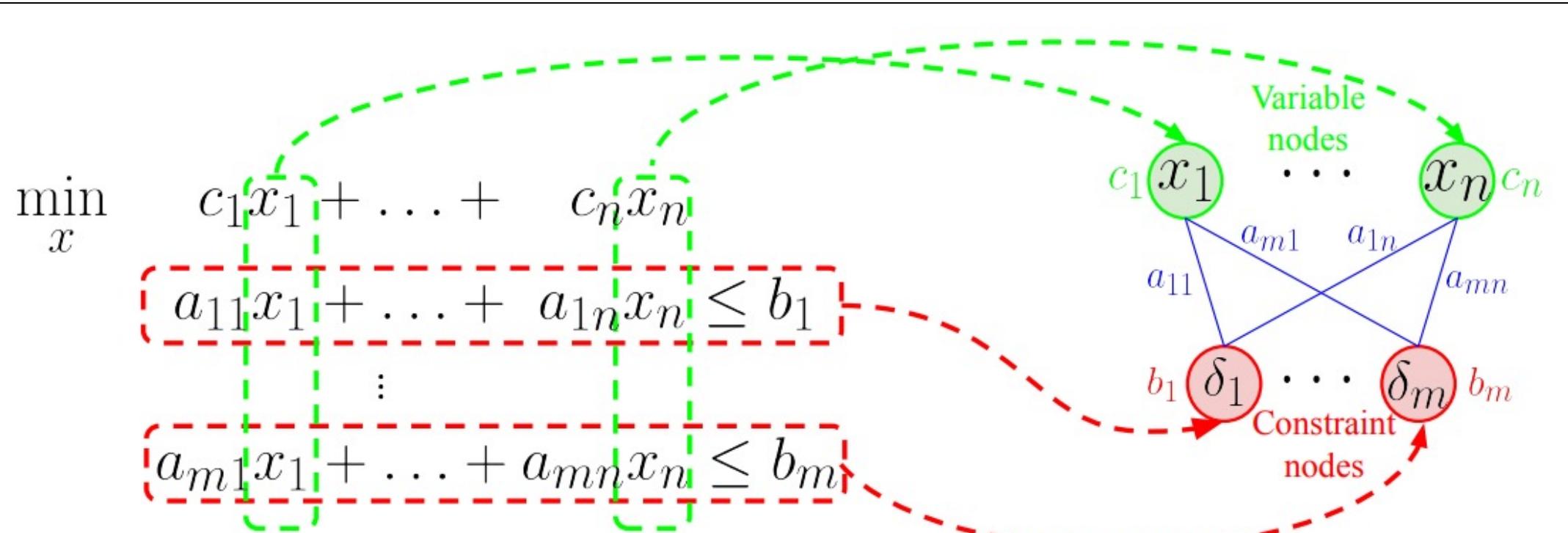
subject to  $\sum_i \alpha_i = 1, \sum_i \alpha_i x_i = \mathbf{x}$

$$\begin{aligned} \min_{\mathbf{p}, \mathbf{y}, z} \quad & \text{NetworkCost}(\mathbf{p}, \mathbf{y}) - \text{Speed}(z) \\ \text{s.t.} \quad & (\mathbf{p}, \mathbf{y}) \in \text{FeasibleNetwork} \\ & z \in \text{FeasibleSchedule}(\mathbf{p}) \end{aligned}$$

## Why is this problem hard to solve?

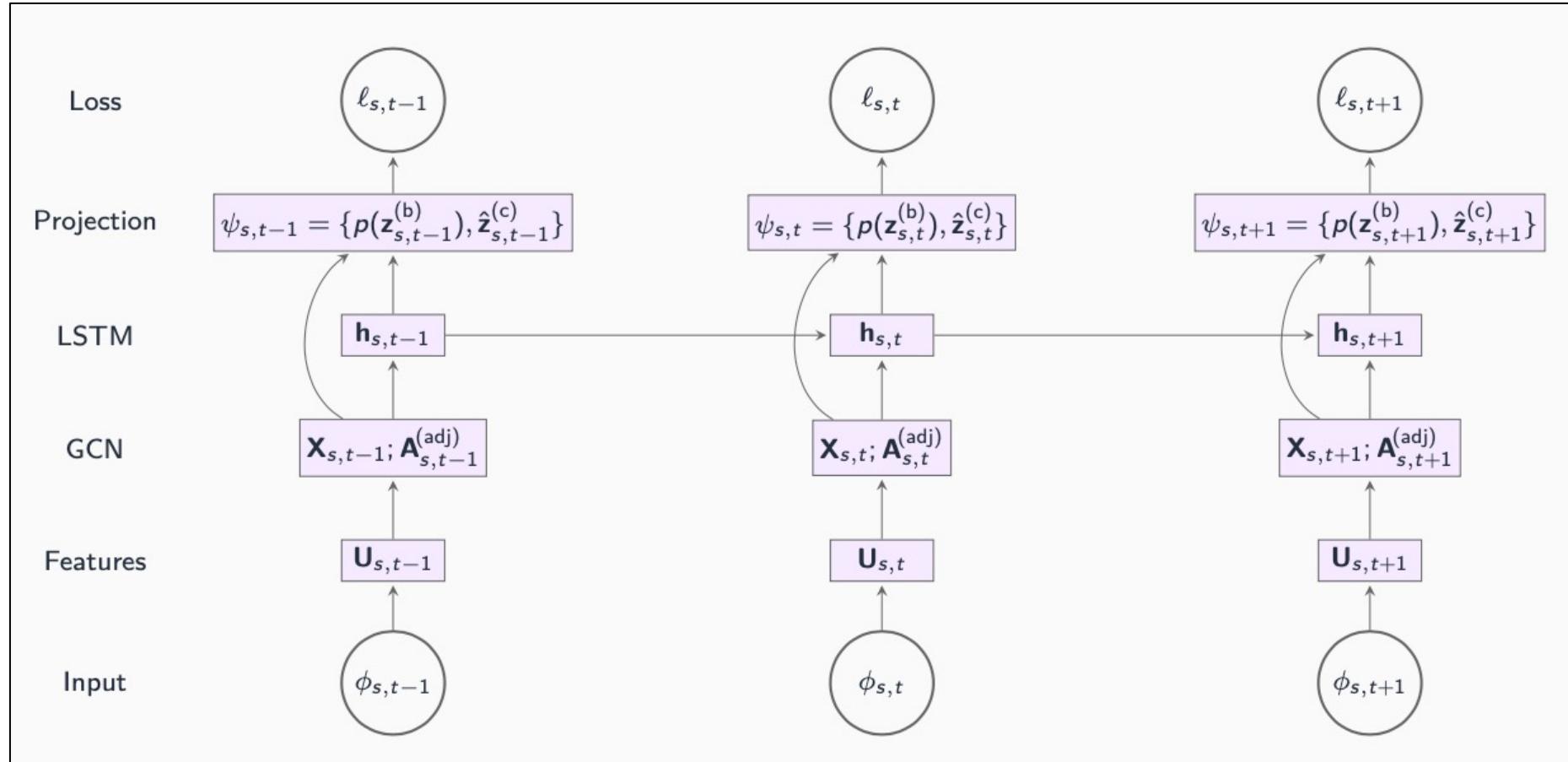
- (a) Feasibility: We must consider granular non-convex operational constraints, e.g., site opening hours.
- (b) Speed objective: Inventory at FCs impacts the speed given by expensive direct connections.
- (c) Scale: Billions of variables to model hourly decisions, e.g., when a truck should depart.
- (d) Uncertainty: Customers' demand is uncertain, thus we should minimize the expected cost.

# Solving MIPs with ML + Optimization



**Figure 3** Bipartite graph representation of a MIP used as the input to a neural network. The set of  $n$  variables  $\{x_1, \dots, x_n\}$  and the set of  $m$  constraints  $\{\delta_1, \dots, \delta_m\}$  form the two sets of nodes of the bipartite graph. The coefficients are encoded as features of the nodes and edges.

# Solving MIPs with ML + Optimization



**Key idea:**

1. Learn the probability that a variable will be active.
2. Fix variables that are active with high-probability to reduce problem dimensionality

**Table 1:** Datasets.

	path-routing	facility-location	travelling-salesman	energy-grid	revenue-maximization
# Integer variables	1044	1989	144	1000	10000
# Continuous variables	0	0	0	500	0
# Equality constraints	390	50	36	1	0
# Inequality constraints	36	39	4082	1010	10
Data	real	mixed	synthetic	synthetic	synthetic
Fraction of non-zeros in <b>A</b>	0.0333	0.0226	0.1414	0.6723	0.5011
Fraction of non-zeros in <b>b</b>	1.0	0.5	0.8333	0.9876	1.0
Fraction of non-zeros in <b>c</b>	0.9169	0.9804	0.9996	1.0	1.0
Fraction of non-zeros in <b>z*</b>	0.4298	0.0278	0.0833	0.7375	0.0166

# Solving MIPs with ML + Optimization

**Table 2:** Accuracy percentage (mean  $\pm$  std) over all instances of MIPnet vs. neural-diving. Bold indicates the best method (higher values are better).

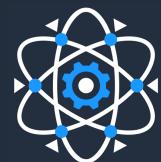
$\rho$	Method	path-routing	facility-location	travelling-salesman	energy-grid	revenue-maximization
30%	MIPnet	<b>0.9964<math>\pm</math>0.0007</b>	<b>1.0000<math>\pm</math>0.0000</b>	<b>0.9943<math>\pm</math>0.0027</b>	<b>1.0000<math>\pm</math>0.0000</b>	<b>1.0000<math>\pm</math>0.0000</b>
	neural-diving	0.9578 $\pm$ 0.0072	<b>1.0000<math>\pm</math>0.0000</b>	0.9642 $\pm$ 0.0053	<b>1.0000<math>\pm</math>0.0000</b>	<b>1.0000<math>\pm</math>0.0000</b>
40%	MIPnet	<b>0.9924<math>\pm</math>0.0018</b>	<b>1.0000<math>\pm</math>0.0000</b>	0.9936 $\pm$ 0.0024	<b>1.0000<math>\pm</math>0.0000</b>	<b>0.9997<math>\pm</math>0.0005</b>
	neural-diving	0.9083 $\pm$ 0.0933	0.9998 $\pm$ 0.0002	<b>0.9961<math>\pm</math>0.0012</b>	<b>1.0000<math>\pm</math>0.0000</b>	0.9991 $\pm$ 0.0013
50%	MIPnet	<b>0.9826<math>\pm</math>0.0022</b>	<b>1.0000<math>\pm</math>0.0000</b>	<b>0.9913<math>\pm</math>0.0029</b>	<b>0.9998<math>\pm</math>0.0000</b>	0.9997 $\pm$ 0.0004
	neural-diving	0.9002 $\pm$ 0.0066	0.9991 $\pm$ 0.0003	0.9552 $\pm$ 0.0046	0.9861 $\pm$ 0.0092	<b>1.0000<math>\pm</math>0.0000</b>
60%	MIPnet	<b>0.9649<math>\pm</math>0.0044</b>	<b>0.9998<math>\pm</math>0.0001</b>	<b>0.9867<math>\pm</math>0.0041</b>	<b>0.9972<math>\pm</math>0.0010</b>	<b>0.9997<math>\pm</math>0.0005</b>
	neural-diving	0.9199 $\pm$ 0.0271	0.9991 $\pm$ 0.0002	0.9761 $\pm$ 0.0277	0.9523 $\pm$ 0.0030	0.9980 $\pm$ 0.0028
70%	MIPnet	<b>0.9409<math>\pm</math>0.0048</b>	<b>0.9984<math>\pm</math>0.0005</b>	<b>0.9797<math>\pm</math>0.0046</b>	<b>0.9874<math>\pm</math>0.0011</b>	0.9995 $\pm$ 0.0007
	neural-diving	0.8926 $\pm$ 0.0399	0.9964 $\pm$ 0.0006	0.9669 $\pm$ 0.0220	0.9502 $\pm$ 0.0481	<b>1.0000<math>\pm</math>0.0000</b>

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Science & Tech

