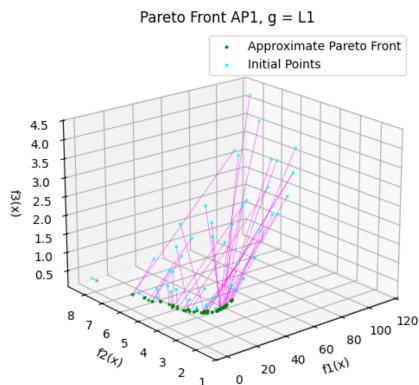
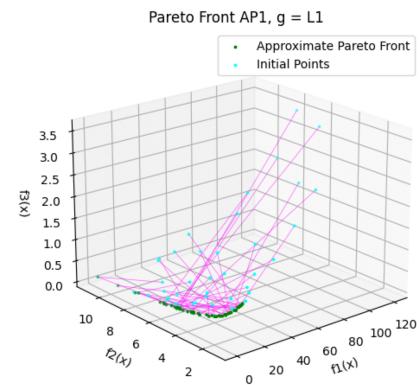


# 1. Optimal Pareto Front Comparisons (Proximal and ADMM)

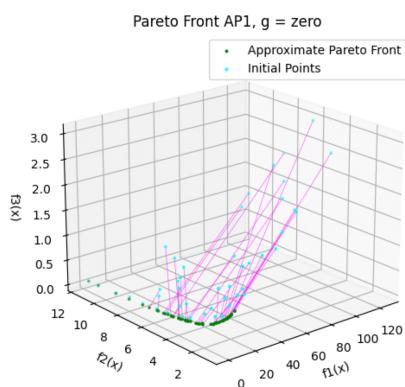
Proximal



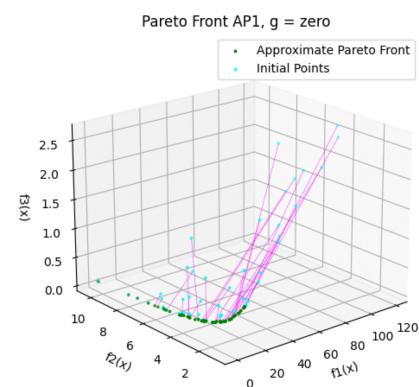
ADMM



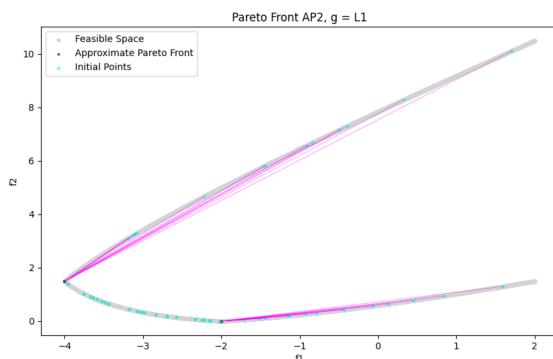
Proximal



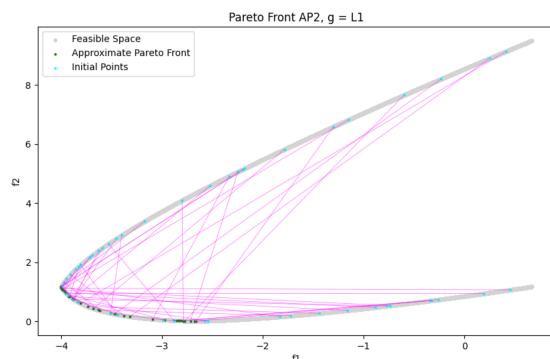
ADMM



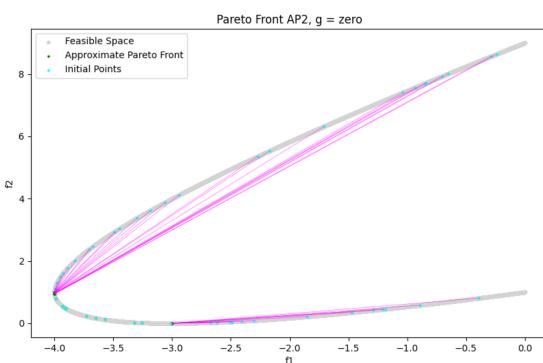
Proximal



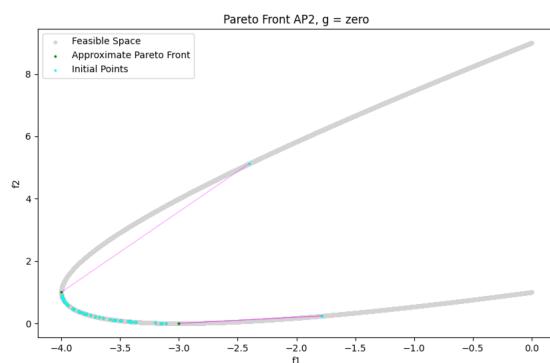
ADMM

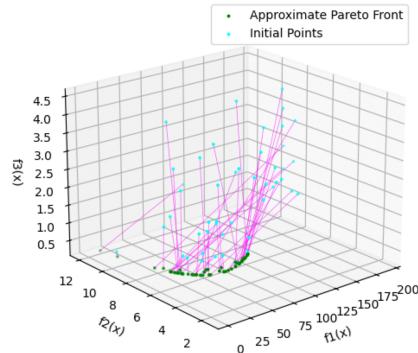
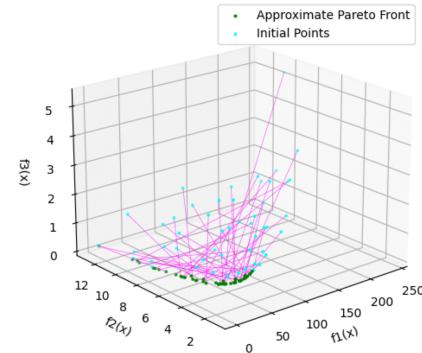
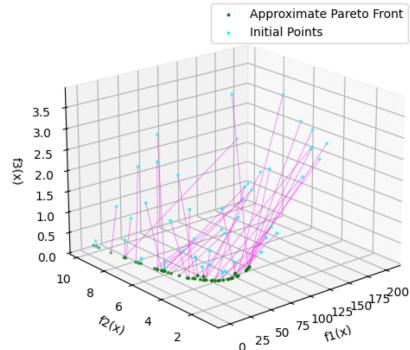
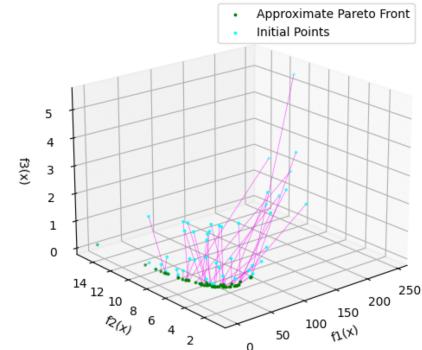
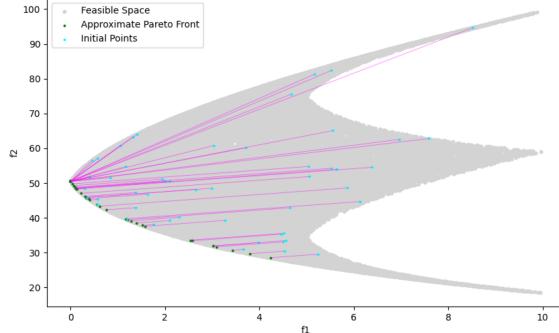
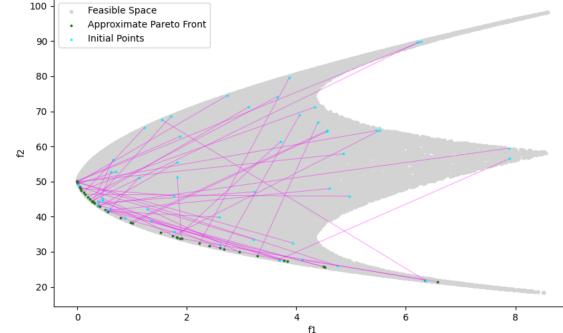
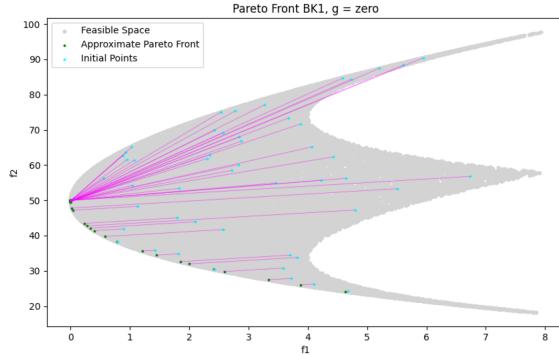
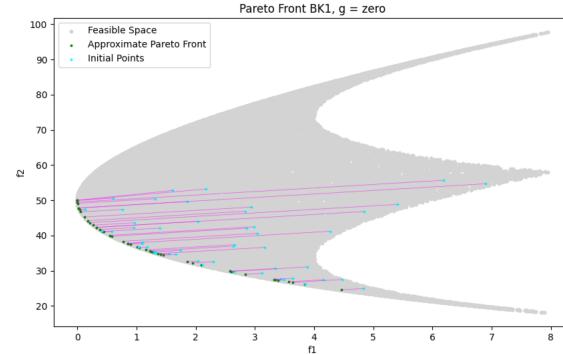


Proximal

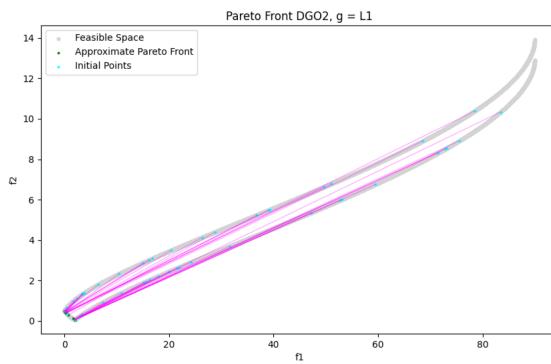


ADMM

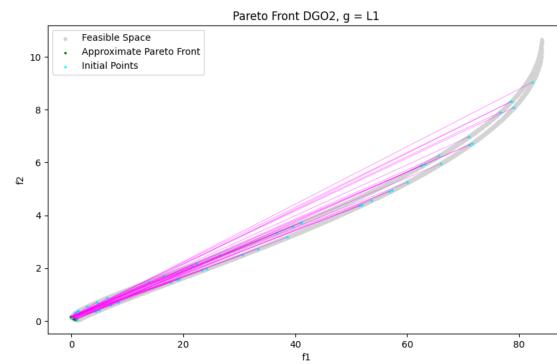


**Proximal**Pareto Front AP4,  $g = L1$ **ADMM**Pareto Front AP4,  $g = L1$ **Proximal**Pareto Front AP4,  $g = \text{zero}$ **ADMM**Pareto Front AP4,  $g = \text{zero}$ **Proximal**Pareto Front BK1,  $g = L1$ **ADMM**Pareto Front BK1,  $g = L1$ **Proximal**Pareto Front BK1,  $g = \text{zero}$ **ADMM**Pareto Front BK1,  $g = \text{zero}$ 

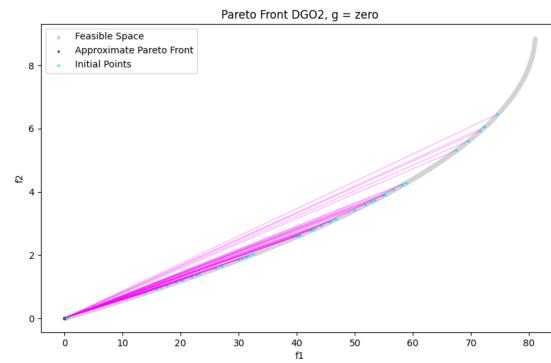
Proximal



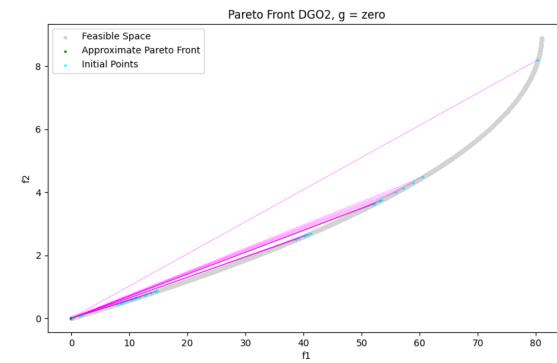
ADMM



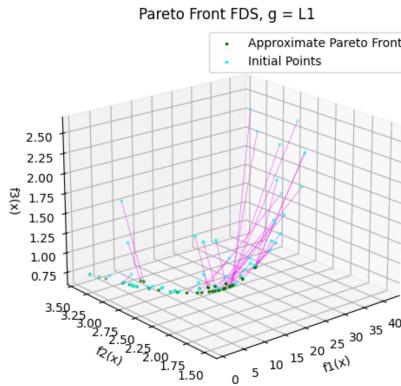
Proximal



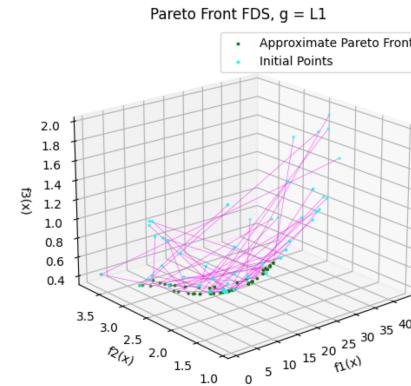
ADMM



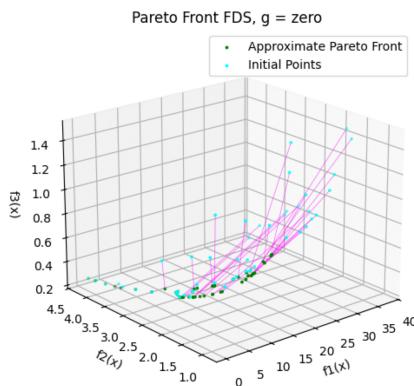
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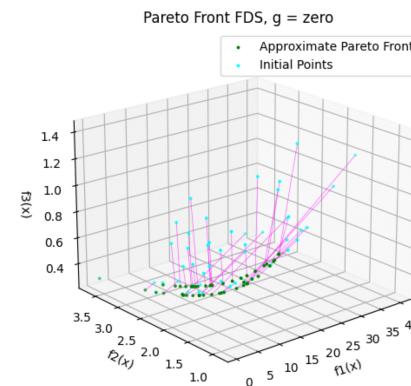
ADMM



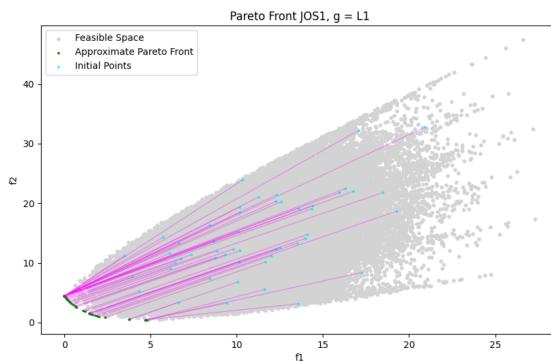
Proximal



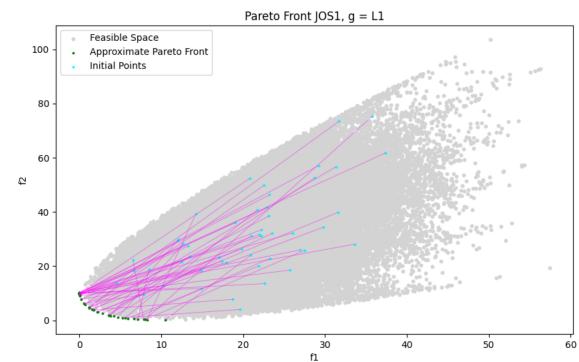
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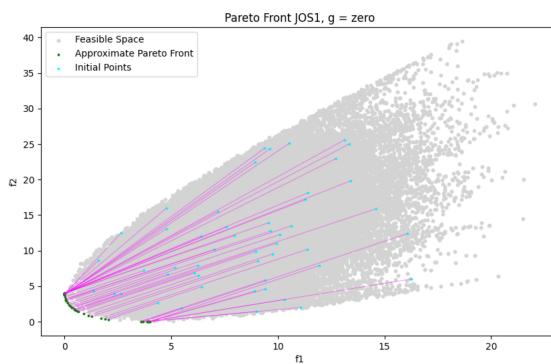
Proximal



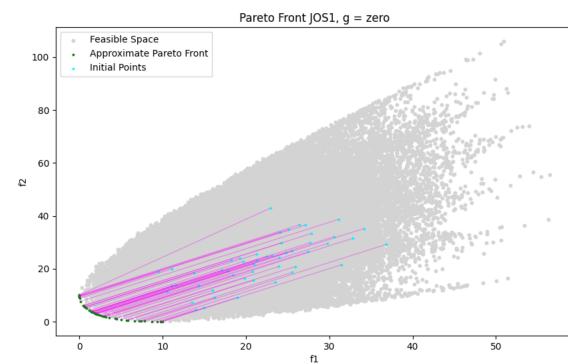
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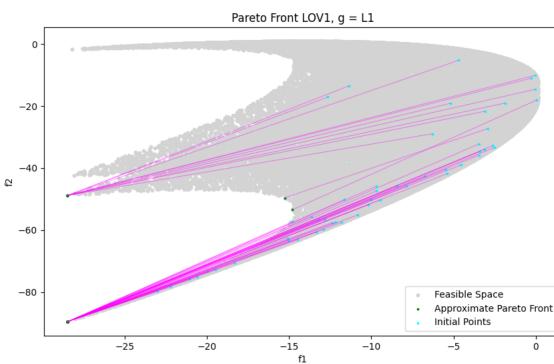
Proximal



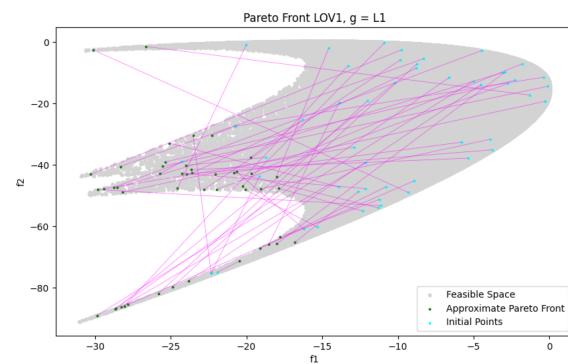
ADMM



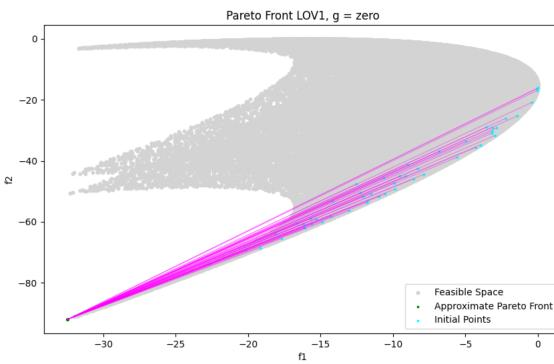
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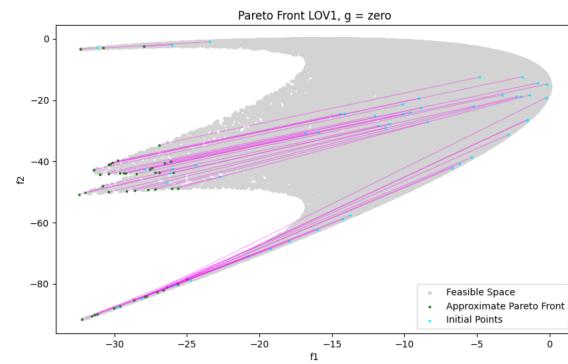
ADMM



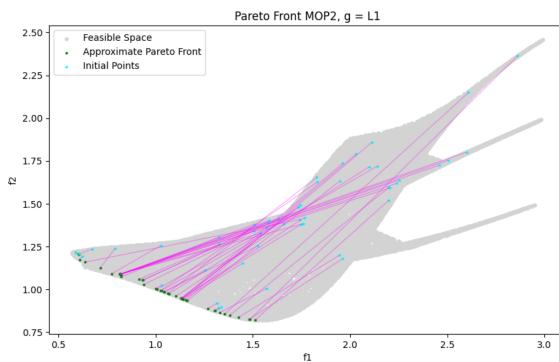
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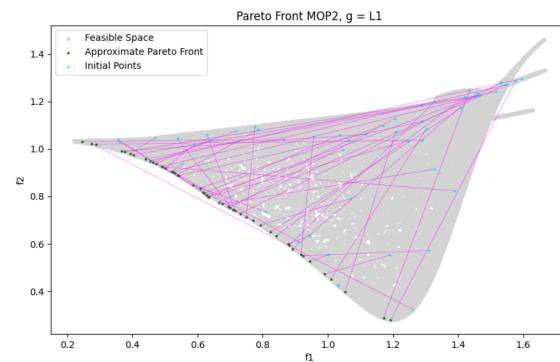
ADMM



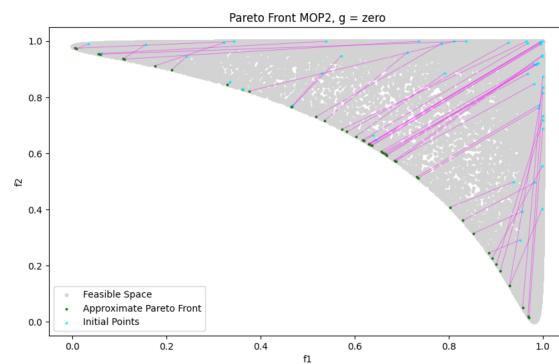
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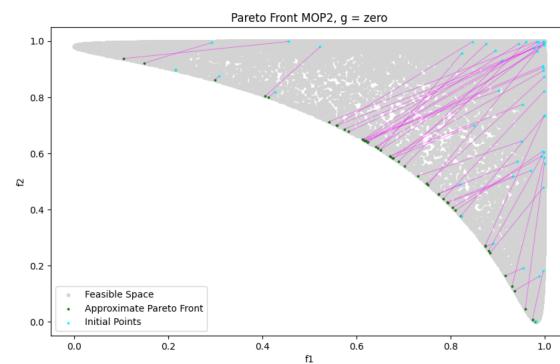
ADMM



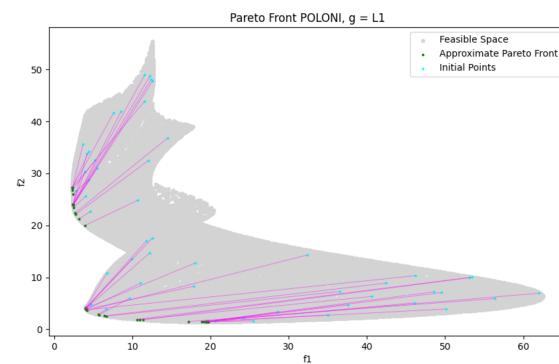
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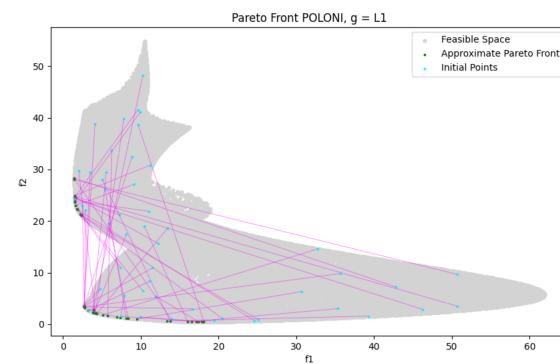
ADMM



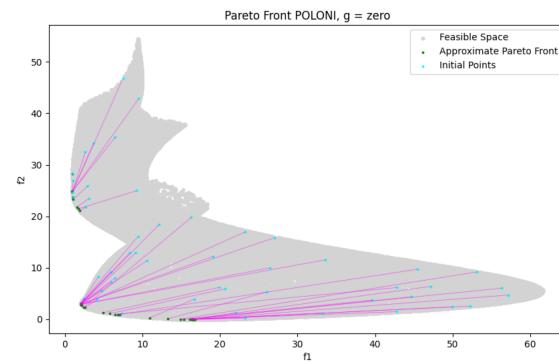
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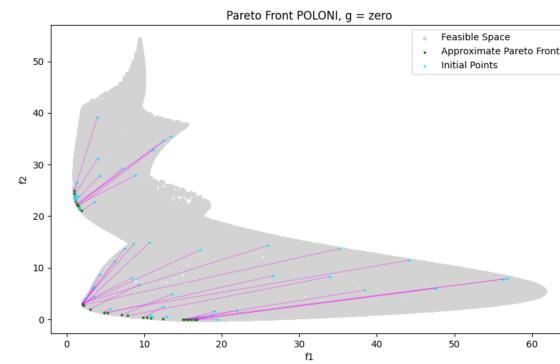
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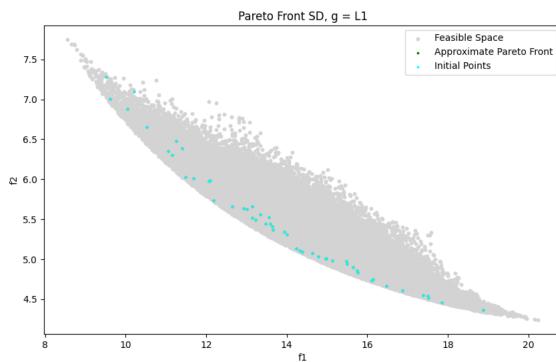
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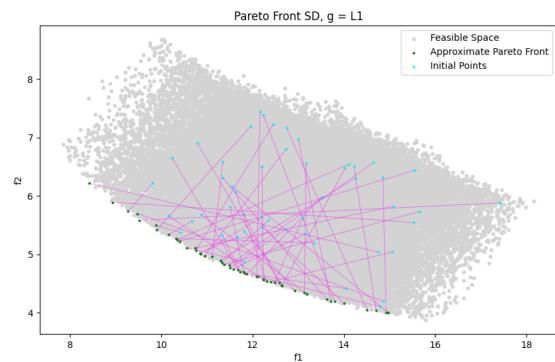
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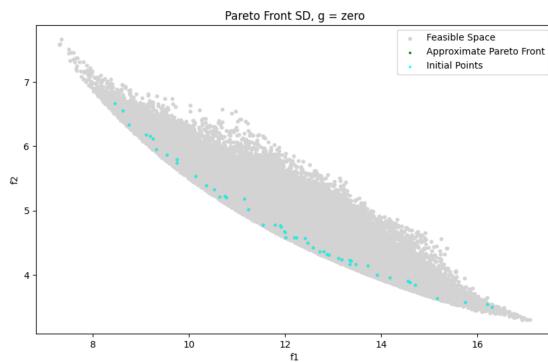
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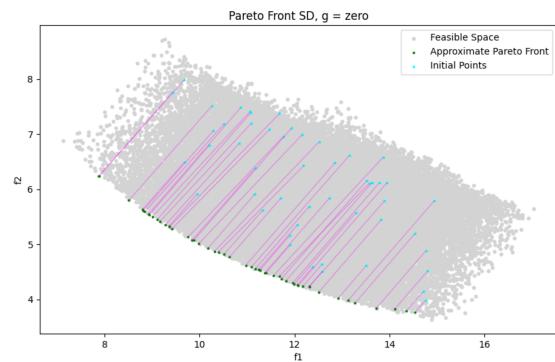
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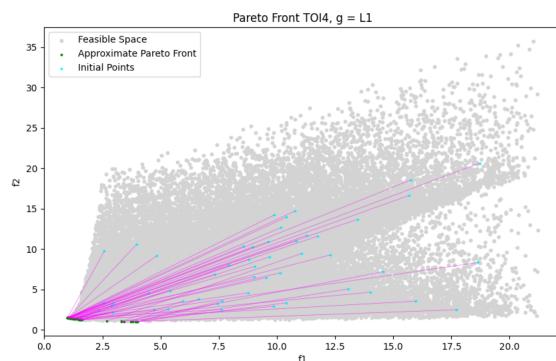
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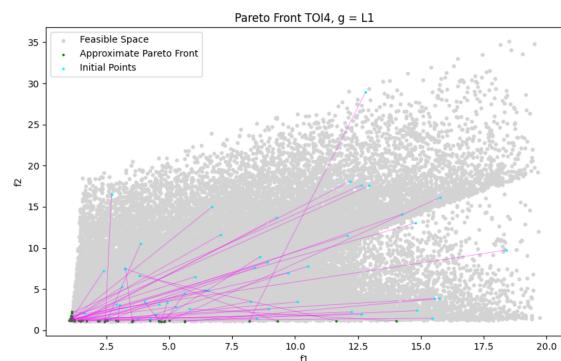
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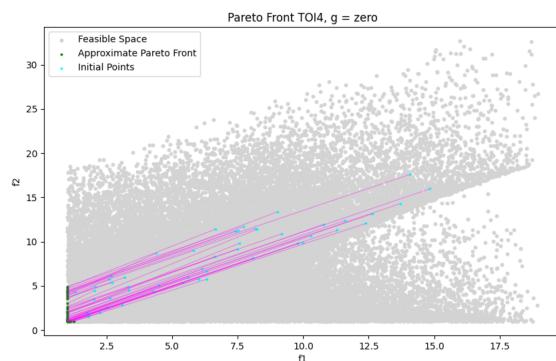
Proximal



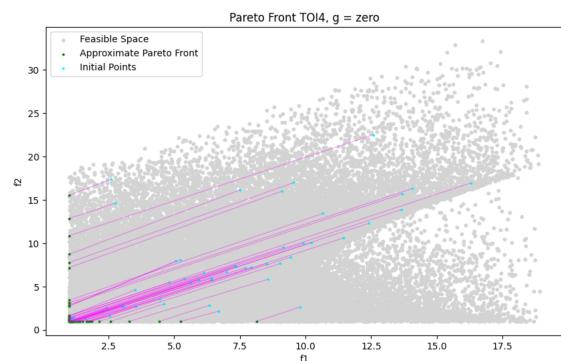
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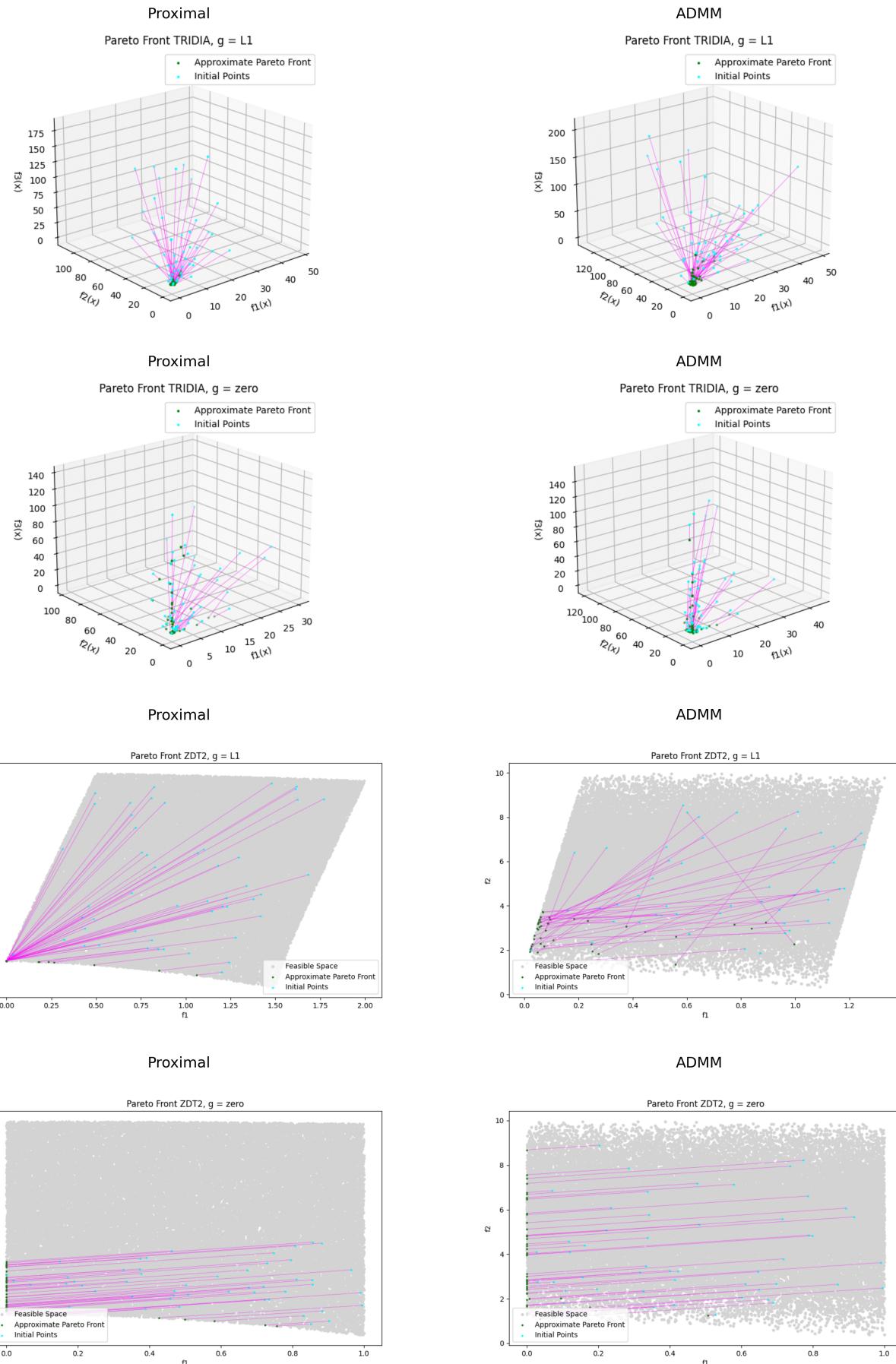


Proximal



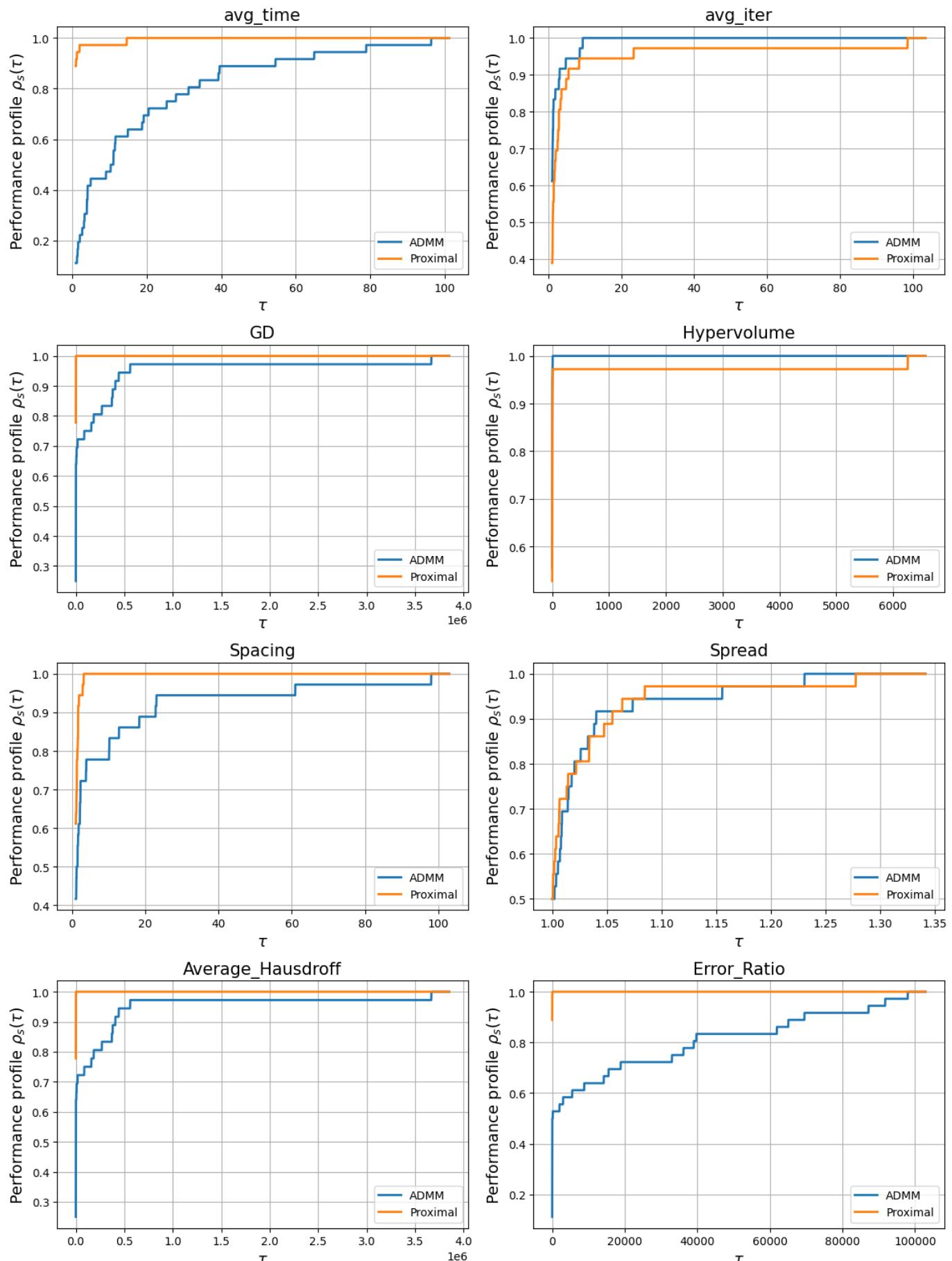
ADMM





## 2. Performance Profile Comparisons (Proximal and ADMM)

I am not quoting the metrics Convergence, R2, IGD and Epsilon as they get zero for some of the problems and hence comparison is not meaningful.



### 3. ADMM Results

Problem	$g(x)$	avg_time	avg_iter	GD	Hypervolume	Spacing	Spread	Average_Hausdroff	Error_Ratio
---------	--------	----------	----------	----	-------------	---------	--------	-------------------	-------------

Problem	g(x)	avg_time	avg_iter	GD	Hypervolume	Spacing	Spread	Average_Hausdroff	Error_Ratio
AP1	0	5.4548	52.945	0.1807	61.2402	0.1952	0.989	0.1807	0.02
	l1	32.9512	8.3029	1.8428	82.9541	1.78	1.0297	1.8428	0.1886
AP2	0	0.7098	5.935	0.0003	0.8207	0.0094	1.0149	0.0003	0.695
	l1	5.8501	2.5959	0.0093	1.1752	0.008	0.9952	0.0093	0.3973
AP4	0	11.1379	62.14	4.4237	339.7329	3.3648	1.0146	4.4237	0.03
	l1	58.8634	16.125	4.0807	286.4768	2.5217	0.9972	4.0807	0.0882
BK1	0	1.305	8.22	0.0011	105.4222	0.2097	1.007	0.0011	0.33
	l1	17.6961	4.8794	0.0492	185.2213	0.2194	1.0023	0.0492	0.3617
DGO2	0	0.8653	7.845	0.0	0.0	0.0	1.0103	0.0	0.98
	l1	14.4272	5.3571	0.869	0.0608	0.8236	1.2401	0.869	0.619
FDS	0	1.7768	18.19	1.5961	12.6091	1.5889	1.0504	1.5961	0.055
	l1	30.0002	7.9886	0.0833	11.5094	0.0917	0.9848	0.0833	0.142
JOS1	0	0.5422	9.28	0.0008	82.7005	0.0692	1.0073	0.0008	0.39
	l1	16.2095	6.9828	5.6059	85.3028	4.3661	1.162	5.6059	0.1552
LOV1	0	5.0589	10.44	24.9663	0.0	0.4723	0.9714	24.9663	0.99
	l1	20.2405	3.0357	25.5732	0.0	0.9412	0.9347	25.5732	0.9881
MOP2	0	1.7716	13.54	0.1176	0.2981	0.0115	1.0044	0.1176	0.095
	l1	17.9501	5.9563	0.2388	0.3029	0.0107	0.9924	0.2388	0.3279
POLONI	0	3.4793	11.83	4.5905	358.0754	0.4783	1.0198	4.5905	0.725
	l1	38.6743	9.8607	6.9201	412.6241	1.5742	1.0189	6.9201	0.6967
SD	0	3.3336	10.425	0.0996	10.3082	0.0416	0.9636	0.0996	0.615
	l1	51.8519	9.3846	0.1321	8.644	0.0738	0.9666	0.1321	0.5385
TOI4	0	0.7289	8.72	3.9327	0.6884	0.1408	1.0033	3.9327	0.955
	l1	47.6184	10.5321	3.8029	0.3303	0.4399	0.9722	3.8029	0.9174
TRIDIA	0	1.9624	8.02	21.0572	34590.35	4.1272	0.9726	21.0572	0.775
	l1	29.8219	13.8073	36.6798	9822.7196	4.7928	0.972	36.6798	0.8716
ZDT2	0	2.6944	9.49	0.6711	0.2204	0.0749	1.002	0.6711	0.96
	l1	21.147	3.0609	3.7147	0.2661	0.1007	0.9687	3.7147	0.9797

## 4. Iterations on Constrained JOS1 Problem

The problem I've tried to solve is a constrained version of the JOS1 problem, which is defined as follows:

```
\begin{aligned}
\min_{x, z} & F(x) + G(z) \\
\text{s.t.} & Ax + Bz = c
\end{aligned}

\text{where}
```

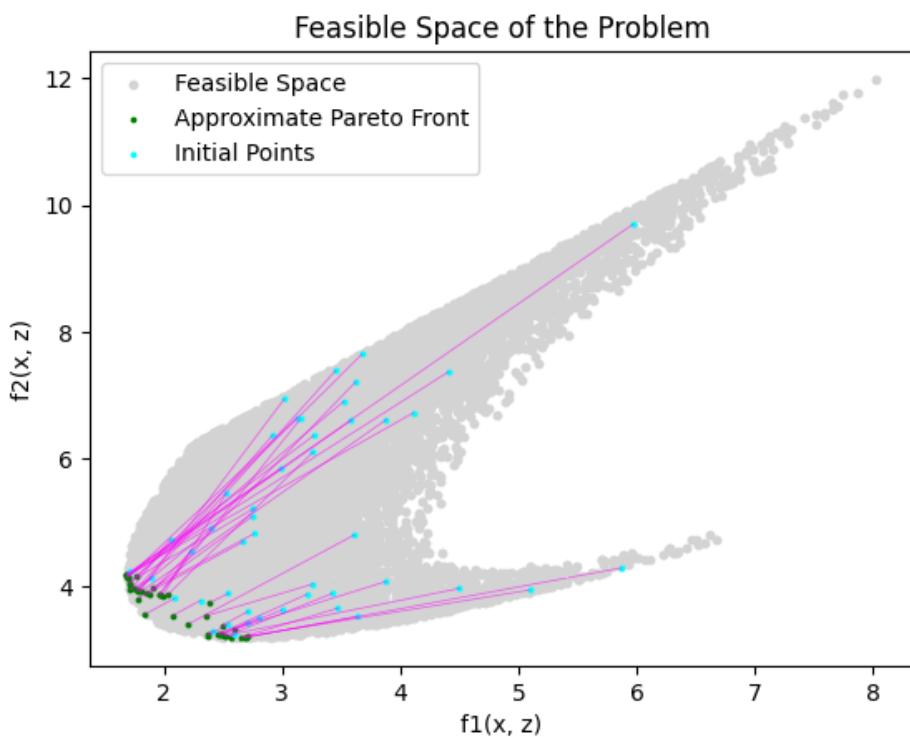
```

F(x) =
\begin{bmatrix}
f_1(x) \\
f_2(x)
\end{bmatrix}, \quad
G(z) =
\begin{bmatrix}
g_1(z) \\
g_2(z)
\end{bmatrix},

f_1(x) = \frac{1}{2} |x|^2, \quad
f_2(x) = \frac{1}{2} |x - 2|^2, \quad
g_1(z) = \frac{1}{2} |z|^2, \quad
g_2(z) = \frac{1}{2} |z - 2|^2, \quad

A =
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}, \quad
B =
\begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 1
\end{bmatrix}, \quad
C =
\begin{bmatrix}
2 \\
3
\end{bmatrix}.

```



The iterates on different initializations are:

### Point 1

x	z	dx	dz	t	F1	F2
---	---	----	----	---	----	----

x	z	dx	dz	t	F1	F2
[0.545151, 1.625365]	[1.029926, 0.949713, 0.424922]	[1.0, -1.0]	[-1.0, 1.0, 0.0]	0.25	2.541131	3.390976
[0.795151, 1.375365]	[0.779926, 1.199713, 0.424922]	[1.0, -1.0]	[-1.0, 1.0, -0.0]	0.03125	2.376024	3.225869
[0.826401, 1.344115]	[0.748676, 1.230963, 0.424922]	[0.901867, -0.901867]	[-0.901866, 0.901868, -1e-06]	0.007812	2.372964	3.222809
[0.833447, 1.337069]	[0.741631, 1.238009, 0.424922]	[0.195124, -0.195124]	[-0.195124, 0.195124, -0.0]	0.007812	2.372814	3.222659
[0.834971, 1.335544]	[0.740106, 1.239533, 0.424922]	[0.00229, -0.00229]	[-0.002291, 0.002289, 1e-06]	0.0625	2.372807	3.222652
[0.835115, 1.335401]	[0.739963, 1.239676, 0.424922]	[0.001145, -0.001145]	[-0.001146, 0.001144, 1e-06]	0.03125	2.372807	3.222652
[0.83515, 1.335365]	[0.739927, 1.239712, 0.424922]	[0.0, 0.0]	[0.0, 0.0, 0.0]	1.0	2.372807	3.222652

## Point 2

x	z	dx	dz	t	F1	F2
[0.166913, 1.80394]	[1.60388, 0.966853, 0.229207]	[1.0, -1.0]	[-1.0, 1.0, 0.0]	0.5	3.420916	3.87933
[0.666913, 1.30394]	[1.10388, 1.466853, 0.229207]	[0.0, -0.0]	[-1.0, -1.0, 1.0]	0.0625	2.783889	3.242303
[0.666913, 1.30394]	[1.04138, 1.404353, 0.291707]	[1.0, -1.0]	[-1.0, 1.0, -0.0]	0.0625	2.643403	3.226817
[0.729413, 1.24144]	[0.97888, 1.466853, 0.291707]	[0.0, 0.0]	[-1.0, -1.0, 1.0]	0.03125	2.634087	3.217501
[0.729413, 1.24144]	[0.94763, 1.435603, 0.322957]	[-0.0, -0.0]	[-1.0, -1.0, 1.0]	0.015625	2.568238	3.214152
[0.729413, 1.24144]	[0.932005, 1.419978, 0.338582]	[0.273256, -0.273256]	[-1.000001, -0.453489, 0.726745]	0.007812	2.536413	3.213577
[0.731548, 1.239305]	[0.924193, 1.416435, 0.34426]	[0.391298, -0.391313]	[-0.391343, 0.391268, 4.5e-05]	0.007812	2.524987	3.213507
[0.734605, 1.236248]	[0.921135, 1.419492, 0.34426]	[0.0942, -0.094186]	[-0.094156, 0.09423, -4.5e-05]	0.003906	2.524958	3.213478
[0.734973, 1.23588]	[0.920768, 1.41986, 0.34426]	[0.039664, -0.039669]	[-0.03968, 0.039653, 1.6e-05]	0.007812	2.524957	3.213477
[0.735282, 1.23557]	[0.920458, 1.42017, 0.34426]	[0.0, 0.0]	[0.0, 0.0, 0.0]	1.0	2.524957	3.213477

## Point 3

x	z	dx	dz	t	F1	F2
[0.160867, 1.621604]	[-0.130762, -0.5915, 1.969895]	[0.0, 0.0]	[1.0, 1.0, -1.0]	0.5	3.451469	7.39126

x	z	dx	dz	t	F1	F2
[0.160867, 1.621604]	[0.369238, -0.0915, 1.469895]	[1.0, -1.0]	[-1.0, 1.0, -0.0]	0.25	2.48039	5.420181
[0.410867, 1.371604]	[0.119238, 0.1585, 1.469895]	[-0.0, -0.0]	[1.0, 1.0, -1.0]	0.25	2.125021	5.064812
[0.410867, 1.371604]	[0.369238, 0.4085, 1.219895]	[1.0, -1.0]	[-1.0, 1.0, -0.0]	0.125	1.920732	4.360523
[0.535867, 1.246604]	[0.244238, 0.5335, 1.219895]	[0.0, 0.0]	[1.0, 1.0, -1.0]	0.125	1.836798	4.276589
[0.535867, 1.246604]	[0.369238, 0.6585, 1.094895]	[1.0, -1.0]	[-1.0, 1.0, -0.0]	0.0625	1.804965	3.994756
[0.598367, 1.184104]	[0.306738, 0.721, 1.094895]	[1.0, -1.0]	[-1.0, 1.0, 0.0]	0.03125	1.786436	3.976227
[0.629617, 1.152854]	[0.275488, 0.75225, 1.094895]	[0.0, 0.0]	[1.0, 1.0, -1.0]	0.015625	1.78303	3.972821
[0.629617, 1.152854]	[0.291113, 0.767875, 1.07927]	[0.665651, -0.665649]	[-0.331302, 0.999997, -0.334348]	0.015625	1.782347	3.940888
[0.640017, 1.142454]	[0.285936, 0.7835, 1.074046]	[0.001527, -0.001526]	[0.996947, 1.0, -0.998474]	0.000977	1.782015	3.930108
[0.640019, 1.142452]	[0.28691, 0.784477, 1.073071]	[0.084559, -0.084559]	[0.830883, 1.0, -0.915441]	0.000488	1.782012	3.928154
[0.64006, 1.142411]	[0.287316, 0.784965, 1.072624]	[0.119991, -0.119991]	[-0.107578, 0.132404, -0.012413]	0.007812	1.782011	3.92726
[0.640998, 1.141474]	[0.286475, 0.785999, 1.072527]	[0.001902, -0.001902]	[-0.001884, 0.001919, -1.8e-05]	0.0625	1.782009	3.927063
[0.641116, 1.141355]	[0.286357, 0.786119, 1.072526]	[0.0, 0.0]	[0.0, 0.0, 0.0]	1.0	1.782009	3.927061

## Point 4

This point does not converge as we can see in the pareto front also.

x	z	dx	dz	t	F1	F2
[-0.101985, 0.793916]	[0.764882, 0.868982, 1.337103]	[1.0, -1.0]	[-1.0, 1.0, -0.0]	0.125	1.88436	4.558566
[0.023015, 0.668916]	[0.639882, 0.993982, 1.337103]	[1.0, -0.0]	[-0.0, 1.0, -1.0]	0.0625	1.816635	4.490841
[0.085515, 0.668916]	[0.639882, 1.056482, 1.274603]	[1.0, -1.0]	[-1.0, 1.0, 0.0]	0.03125	1.802488	4.351693
[0.116765, 0.637666]	[0.608632, 1.087732, 1.274603]	[1.0, -0.0]	[-0.0, 1.0, -1.0]	0.015625	1.799228	4.348434
[0.13239, 0.637666]	[0.608632, 1.103357, 1.258978]	[0.772456, 0.22753]	[0.22754, 0.772465, -0.999995]	0.007812	1.798499	4.316455
[0.138425, 0.639443]	[0.61041, 1.109391, 1.251165]	[0.558231, 0.441769]	[0.441769, 0.558231, -1.0]	0.000977	1.798407	4.300737

x	z	dx	dz	t	F1	F2
[0.13897, 0.639875]	[0.610841, 1.109937, 1.250189]	[0.472458, 0.361759]	[0.36177, 0.472469, -0.834228]	0.000244	1.798405	4.298783
[0.139086, 0.639963]	[0.610929, 1.110052, 1.249985]	[0.453099, 0.342279]	[0.342291, 0.453111, -0.79539]	3.1e-05	1.798405	4.298375
[0.139099, 0.639973]	[0.61094, 1.110066, 1.249961]	[0.450491, 0.34003]	[0.340042, 0.450504, -0.790534]	1.5e-05	1.798405	4.298327
[0.139106, 0.639979]	[0.610945, 1.110073, 1.249949]	[0.450421, 0.338637]	[0.33865, 0.450434, -0.789071]	8e-06	1.798405	4.298303
[0.13911, 0.639981]	[0.610947, 1.110076, 1.249943]	[0.449773, 0.33808]	[0.338093, 0.449786, -0.787866]	0.0	1.798405	4.298291