MAE 263F HW3

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***Abstract*— This work implements and evaluates a planar discrete-elastic-rod (DER) model of an aluminum tube under gravity with closed-loop end-effector control. Bending and axial stretching energies are discretized with guarded gradients/Hessians, and the dynamics are advanced by an implicit backward-Euler (BDF1) step solved with a damped (Levenberg–Marquardt) Newton method and a short backtracking line search. A rate-limited proportional controller commands the right-end position and orientation to guide the beam so that its middle node tracks a quarter-circle reference. We report stable convergence for small time steps, gravity/damping ramp-in, and moderate rate limits, along with middle-node tracking, control histories, and beam-shape snapshots.**

I. Introduction

Flexible continuum elements (beams, tubes) are central to soft/continuum robotics and structural dynamics. Robust simulation requires consistent elastic forces, well-conditioned Jacobians, and numerically stable time stepping under gravity and control actuation. We formulate a 2-D DER with bending and stretching, including small viscous damping, and drive the terminal pose (position and orientation) to steer the internal configuration; the middle node is tasked to follow a quarter-circle trajectory. The study emphasizes numerical safeguards—norm guards, Hessian regularization, and ramped loading—to ensure Newton convergence.

II. Methodology

A. Model and Energies

The beam is a thin aluminum tube of length $L = 1$m, discretized with $n_v = 19$nodes ($\Delta L = L/(n_v - 1)$). Axial area $A = \pi(R_o^2 - R_i^2)$and second moment $I = \frac{\pi}{4}(R_o^4 - R_i^4)$(with $R_o = 0.013$m, $R_i = 0.011$m, $Y = 70$GPa) give $EA = YA$and $EI = YI$. Stretching energy uses a Hookean extension measure $E_s = \frac{1}{2} EA\, \Delta L(1 - L_k/\Delta L)^2$. Bending follows the standard DER curvature binormal with guarded dot products; gradients and Hessians are assembled over segments/vertices. Numeric include ε-guards on norms and to avoid singularities.

B. Time Integration and Solve

States q(stacked $[x_i, y_i]$) advance implicitly by backward-Euler with mass lumped per node from density $\rho = 2700$kg/m ^{3}. The residual combines inertia, damping, elastic, and gravity forces; the tangent is the consistent mass/damping minus elastic Hessians. Newton updates solve reduced systems on free DOFs (left clamp, two rightmost nodes constrained to impose end position and orientation). We applied for a tiny LM term and a short backtracking line search. Gravity and damping are smoothly ramped during startups to prevent large initial residuals. Nominal parameters: $dt = 2 \times 10^{-3}$s, max 60 Newton iterations/step.

C. Control and Reference

The middle node is commanded to trace a quarter circle over the total horizon. A simple proportional law with rate limits generates desired right-end commands (x_c, y_c, θ_c). Position rates are limited relative to L; orientation rates are limited in rad/s to avoid impulsive changes that harm Newton convergence. The second-to-last node is constrained to enforce θ_c(end-segment orientation). We log middle-node trajectory vs. reference, control inputs vs. time, and beam-shape snapshots.

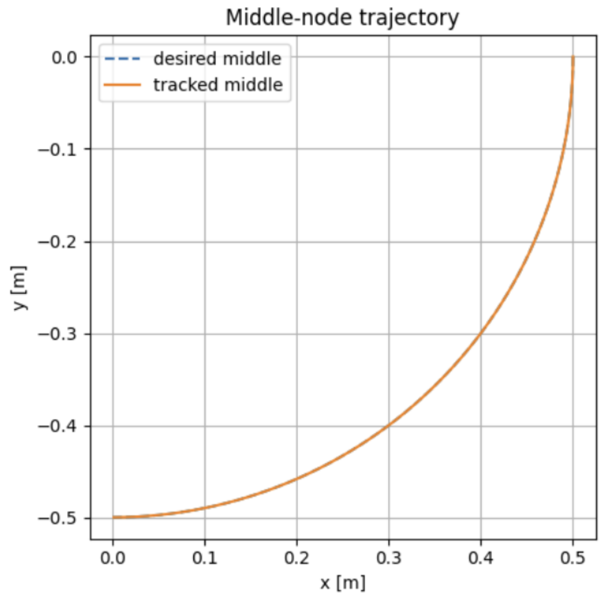
II. Results and Discussion

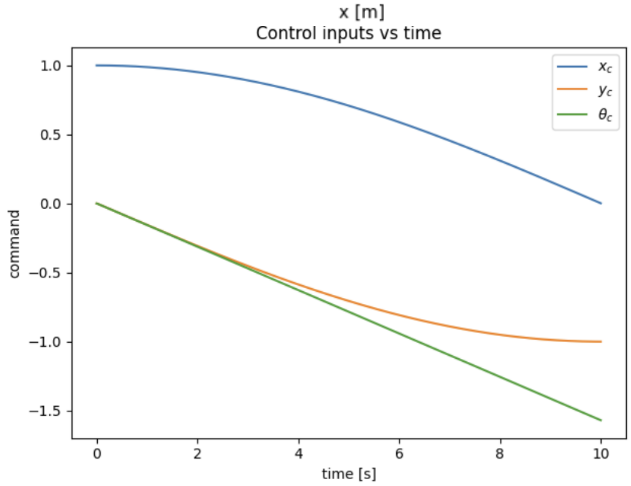
A. Convergence Behavior

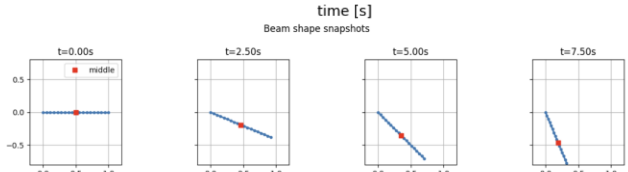
Without safeguards, Newton may stall early due to large gravity spikes, ill-conditioned tangents near collinear segments, or aggressive control steps. With (i) $\chi$and norm guards in curvature terms, (ii) LM regularization, (iii) a brief line search, (iv) gravity/damping ramp-in, and (v) end-command rate limits, the solver converges reliably for the stated tube and discretization. Practically, decreasing dtor relaxing rate limits increases robustness at the cost of runtime.

B. Tracking and shapes

Control histories show smooth, bounded (x_c, y_c, θ_c)consistent with the rate limits. The middle-node path follows the quarter-circle with a small lag typical of under-actuated flexible systems; lag grows if rate limits are very strict and shrinks as limits relax (until Newton stability becomes the bottleneck). Beam snapshots across five times illustrate progressive bending under gravity and control, with curvature concentrated near the driven end as expected when orientation is prescribed on the last segment.







III. CONCLUSION

We demonstrated a numerically robust DER simulation of an aluminum tube tracking a planar reference via end-effector control. Key enablers are physically consistent gradients/Hessians with guards, implicit integration with mild LM regularization and backtracking, and practical systems engineering—load ramp-in and control rate limiting. The result is stable convergence and credible middle-node tracking with interpretable control histories and beam shapes. Future work includes energy-consistent damping, adaptive time stepping, and model-based controllers that use the assembled Jacobians directly for improved tracking.

REFERENCES

[1] M. Khalid Jawed, MAE263F Lecture