PRACTICAL No: 1

Topic-

Limits and Continuity.

1. $\lim_{x\to a}\left[\dfrac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}}\right]$

$$\lim_{x\to a}\left[\frac{\sqrt{a+2x}-\sqrt{3x}}{\sqrt{3a+x}-2\sqrt{x}} \times \frac{\sqrt{a+2x}+\sqrt{3x}}{\sqrt{a+2x}+\sqrt{3x}} \times \frac{\sqrt{3a+x}+2\sqrt{x}}{\sqrt{3a+x}+2\sqrt{x}}\right.$$

$$\lim_{x\to a} \frac{(a+2x-3x)}{(3a+x-4x)} \frac{(\sqrt{3a+x}+2\sqrt{x})}{(\sqrt{a+2x}+\sqrt{3x}}$$

$$\lim_{x\to a} \frac{(a-x)(\sqrt{3a-x}+2\sqrt{x}}{(3a-3x)(\sqrt{a+2x})+\sqrt{3x}}$$

$$\frac{1}{3}\lim_{x\to a} \frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{a+2x}+\sqrt{3x})}$$

$$= \frac{1}{3}\lim_{x\to a} \frac{(a-x)(\sqrt{3a+x}+2\sqrt{x})}{(a-x)(\sqrt{2x+a}+\sqrt{3x})}$$

$$= \frac{1}{3}\,\frac{\sqrt{3a+a}+2\sqrt{a}}{\sqrt{2a+a}+\sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{\sqrt{4a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}.}$$

$$= \frac{1}{3} \times \frac{2\sqrt{a}+2\sqrt{a}}{\sqrt{3a}+\sqrt{3a}}$$

$$= \frac{1}{3} \times \frac{4\sqrt{a}}{2\sqrt{3a}}$$

$$= \frac{2}{3\sqrt{3}.}$$

2. $$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\right]$$

$$\lim_{y\to 0}\left[\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right]$$

$$\lim_{y\to 0}\frac{a+y-a}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$$

$$\lim_{y\to 0}\frac{y}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$$

$$=\frac{1}{\sqrt{a+0}\,(\sqrt{a+0}+\sqrt{a})}$$

$$=\frac{1}{\sqrt{a}\,(\sqrt{a}+\sqrt{a})}$$

$$=\frac{1}{\sqrt{a}\,(2\sqrt{a})}=\frac{1}{2a}$$

$$\lim_{x\to\pi/6}\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}$$

By substituting $x-\pi/6=h$

$x=h+\pi/6$

where $h\to 0$

$$\lim_{h\to 0}\frac{\cos(h+\pi/6)+\sqrt{3}\sin(h+\pi/6)}{\pi-6(h+\pi/6)}$$

$$\lim_{h\to 0}\frac{\cos h\cdot\cos\pi/6-\sin h\sin\pi/6-\sqrt{3}\sin\cos\pi/6+\cos h\sin\pi/6}{\pi-6\left(\frac{6h+\pi}{6}\right)}$$

$$\lim_{h\to 0}\frac{\cos h\cdot\frac{\sqrt{3}}{2}-\sin h\,1/2-\sqrt{3}(\sin h\,\sqrt{3}/2+\cos h\,\frac{1}{2}}{\pi-6h+\pi.}$$

$$\lim_{h\to 0}\frac{(\cos h\,\frac{\sqrt{3}}{2}+\sin h/2)-\sqrt{3}\left(1/2\cos h-\sqrt{3}/2\sin h\right)}{6h}$$

$$\lim_{h\to 0}\frac{\frac{\sqrt{3}}{2}\cos h+\frac{\sin h}{2}-\frac{\sqrt{3}\cos h}{2}+\frac{3\sin h}{2}}{6h}$$

$$\lim_{h\to 0}\frac{4\sin h}{2(6h)}$$

$$\lim_{h\to 0}\frac{4\sin h}{12h}$$

$$=1/3$$

4) $$\lim_{x\to 0}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}}\right]$$

By Rationalizing Numerator & Denominator both.

$$\lim_{\infty}\left[\frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]\times\left[\frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}}\times\frac{\sqrt{x^2+3}+\sqrt{x^2+1}}{\sqrt{x^2+3}+\sqrt{x^2+1}}\right]$$

$$\lim_{\infty}\left[\frac{(x^2+5-x^2+3)(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)(\sqrt{x^2+5}+\sqrt{x^2-3})}\right.$$

$$\lim_{\infty} \frac{1}{2} \frac{(\sqrt{x^2+3}+\sqrt{x^2+1})}{(\sqrt{x^2+5}+\sqrt{x^2+3})}$$

$$4 \lim_{\infty} \frac{\sqrt{x^2(1+3/x^2)}+\sqrt{x^2(1+1/x^2)}}{\sqrt{x^2(1+\frac{5}{x^2})}+\sqrt{x^2(1-3/x^2)}}$$

After applying limit we get,

$=4$

5. i) $f(x) = \frac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < x \leq \pi/2$

$= \frac{\cos x}{\pi - 2x}$, for $\frac{\pi}{2} < x < \pi$

at $x = \pi/2$

f at $x = \frac{\pi}{2}$ defined

ii) $\lim_{x\to\pi/2^+} f(x) = \lim_{x\to\pi/2^+} \frac{\cos x}{\pi - 2x}$

by substituting method,

$x - \pi/2 = h$.

where $h \to 0$

$$\lim_{h\to 0} \cos(h+\pi/2) / \pi - 2(h+\pi/2)$$

$$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - 2\left(\frac{2h+\pi}{2}\right)}$$

$$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{-2h}$$

$$\lim_{h\to 0} \frac{\cos h \cdot \cos \pi/2 - \sin h \sin \pi/2}{-2h}$$

$$\lim_{x\to 0} \frac{\cos h \cdot 0 - \sin h}{-2h}$$

$$\lim_{h\to 0} \frac{-\sin h}{-2h}$$

b) $\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2} \frac{\sin 2x}{\sqrt{1-\cos^2 x}}$

$$\lim_{x\to\pi/2} \frac{2\sin x \cos x}{\sqrt{2\sin^2 x}}$$

$$\lim_{x\to\pi/2} \frac{2\cos x}{\sqrt{2}}$$

$$\frac{2}{\sqrt{2}} \lim_{x\to\pi/2} \cos x$$

LHS $\neq$ RHL

$\therefore$ 'f' is not continous at $x = \pi/2$

5) ii) $f(x) = \frac{x^2-9}{x-3}$ $\quad 0 < x < 3$

$= x+3$ $\quad 3 \leq x \leq 6$

$= \frac{x^2-9}{x+3}$ $\quad 6 \leq x < 9$

at $x=3$ and $x=6$

$x=3$

i) $f(3) = \frac{x^2-9}{x-3} = 0$

'f' at $x=3$ defined

ii) $\lim_{x\to3^+} f(x) = \lim_{x\to3} x+3$

$f(3) = x+3 = \lim_{x\to3} x+3$

'f' is defined at $x=3$,

$\lim_{x\to3} f(x) = \lim_{x\to3^+} (x+3) = 6$

$\lim_{x\to3} f(x) = \lim_{x\to3} \frac{x^2-9}{x-3} \quad \frac{(x-3)(x+3)}{x-3}$

LHL = RHL.

'f' is continous at $x=3$.

2. $\lim_{x\to6} \frac{x^2-9}{x+3}$

$\lim_{x\to6^+} \frac{(x-3)(x+3)}{x+3}$

$\lim_{x\to6^+} (x-3) = 6-3=3$

$\lim_{x\to6^+} x+3 = 3+6=9$

LHL $\neq$ RHL

Functions is not continous.

6) i) $f(x) = \frac{1-\cos 4x}{x^2}$, $x<0$
$= k$, $x=0$ $\Big\}$ at $x=0$.

f is continous at $x=0$

$\lim_{x\to0} \frac{1-\cos 4x}{x^2} = k$

$\lim_{x\to0} \frac{2\sin^2 2x}{x^2} = k.$

$2\lim_{x\to0} \frac{\sin^2 2x}{x^2} = k.$

$2\lim_{x\to0} \left(\frac{\sin 2x}{x}\right)^2 = k$

$2(2)^2 = k$

$k=8.$

ii) $f(x) = (\sec^2 x)^{\cot^2 x}$, $x\neq0$
$= k$, $x=0$ $\Big\}$ $x=0$

$f(x) = (\sec^2 x)^{\cot^2 x}$

$\lim_{x\to0} (1+\tan^2 x)^{1/\tan^2 x}$

We know That,

$\lim_{x\to0} (1+px)^{1/px} = e.$

$\therefore e$

$\therefore x = e.$

$$f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x} \quad x \neq \pi/3$$
$$= k \quad x = \pi/3 \quad \Big\} \text{ at } x = \pi/3$$

$x - \pi/3 = h$

$x = h + \pi/3$

where. $h \to 0$.

$$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h\to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$$

$$\lim_{h\to 0} \frac{\sqrt{9} - \tan \pi/3 + \tan h}{1 - \tan \pi/3 \tan h}$$

$$\lim_{h\to 0} \frac{\dfrac{\sqrt{3}(1 - \tan\pi/3 \tan h) - (\tan\pi/3 + \tan h)}{1 - \tan\pi/3 \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3} - \sqrt{3} \times \sqrt{3}\tan h) - (\sqrt{3} + \tan h)}{1 - \tan\pi/3 \cdot \tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{\dfrac{(\sqrt{3} - 3\tan h) - (\sqrt{3} + \tan h)}{1 - \sqrt{3}\tan h}}{-3h}$$

$$\lim_{h\to 0} \frac{-4\tan h}{-3h(1 - \sqrt{3}\tan h)} = \frac{4}{3} \lim_{h\to 0} \left(\frac{\tan h}{h}\right)\left(\frac{1}{1 - \sqrt{3}\tan h}\right)$$

$$= \frac{4}{3} \frac{1}{1 - \sqrt{3}/0)} = \frac{4}{3}$$

7.) i) $f(x) = \dfrac{1 - \cos 3x}{x \tan x} \quad x \neq 0$ $\Big\}$ at $x = 0$
$\qquad\qquad\qquad\qquad x = 0$

$$f(x) = \frac{1 - \cos x}{x \tan x}$$

$$= 2\sin^2 \tfrac{3}{2} x \Big/ x \tan x$$

$$= \frac{2\sin^2 \frac{3x}{2} x^2}{2} \Big/ \frac{x \cdot \tan x}{x^2} x^2$$

$$= 2 \lim_{x\to 0} (3/2)^2 / 1$$

$$= \cancel{2} \times \frac{9}{4\cancel{2}} = \frac{9}{2}$$

$$\lim_{x\to 0} f(x) = \frac{9}{2} \qquad 9 = f(0)$$

$\therefore f$ is not at $x = 0$

Redline . function

$$f(x) = \frac{1 - \cos 3x}{x \tan x} \qquad x \neq 0$$

$$\frac{9}{2} \qquad x = 0$$

8.8..

Now, $\lim_{x\to 0} f(x) = f(0)$

'f' has removable discontinuity at $x=0$

ii) $f(x) = \frac{(e^{3x}-1)\sin x^\circ}{x^2}$ $\left.\begin{matrix} x\neq 0 \\ x=0 \end{matrix}\right\}$ at $x=0$

$= \pi/6$

at $x=0$

$\lim_{x\to 0} (e^{3x}-1) \sin\left(\frac{\pi x}{180}\right) / x^2$

$3 \lim_{x\to 0} \frac{e^{3x}-1}{3x} \lim_{x\to 0} \frac{\sin \pi x/180}{x}$

$3\log_e \frac{\pi}{180} = \frac{\pi}{60} = f(0)$

'f' is continous at $x=0$

8 $f(x) = \frac{e^{x^2}-\cos x}{x^2}$ $x=0$

is continous at $x=0$

'f' is continous at $x=0$

$\lim_{x\to 0} f(x) = f(0)$

$= \frac{e^{x^2}-\cos x}{x^2} = f(0)$

$\frac{e^{x^2}-\cos x-1+1}{x^2}$

$(e^{x^2}-1)+(1-\cos x)/x^2$

$\frac{e^{x^2}-1}{x^2} + \lim_{x\to 0} 2\frac{\sin^2 x/2}{x^2}$

$= \log_e + 2\left(\frac{\sin x/2}{x}\right)2$

Multiplying with 2 on Numerator & Denominator

$= 1+2\times\frac{1}{4} = \frac{3}{2} = f(0)$

9. $f(x) = \frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x}$ $x\neq \pi/2$

$x=\pi/2$

'f' is continous at

$\frac{\sqrt{2}-\sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2}+\sqrt{1+\sin x}}{\sqrt{2}+\sqrt{1+\sin x}}$

$= \frac{2-1+\sin x}{\cos^2 x(\sqrt{2}+\sqrt{1+\sin x})}$

$= \frac{1+\sin x}{1-\sin^2 x\ (\sqrt{2}+\sqrt{1+\sin x})}$

$= \frac{1}{2\sqrt{2+\sqrt{2}}}$

$= \frac{1}{2\times 2\sqrt{2}} = \frac{1}{4\sqrt{2}}$

- $f(x) = (\sec^2 x)^{\cot^2 x}$ $\left.\begin{matrix} x\neq 0 \\ x=0 \end{matrix}\right\}$ at $x=0$
  $= k$

$f(x) = (\sec^2 x)^{\cot^2 x}$

Using,

$\sec^2 - \tan^2 x - \sec^2 x = 1$

$\therefore \sec^2 x = 1+\tan^2 x$

and, $\cot^2 x = 1/\tan^2 x$

$\lim_{x\to 0} (1+\tan^2 x)/\tan^2 x$

We know,

$\lim_{x\to 0} (1+px)^{1/px} =$

$= e$

$\underline{k=e}$

I.S.

PRACTICAL No. 2

Topic- Derivation.

Q.1. Show following function defined from R to R:
i) $\cot x$. 2) $\operatorname{cosec} x$ 3) $\sec x$

1) $\cot x = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to a} \frac{\cot x - \cot a}{x-a}$

$= \lim_{x\to a} \frac{1/\tan x - 1/\tan a}{x-a}$

$= \lim_{x\to a} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$

put $x-a=h$, $x=a+h$. as $x\to 0$, $h\to 0$

$Df(h) = \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{(a+h-a)\tan(a+h)\tan a}$

$= \lim_{h\to 0} \frac{\tan a - \tan(a+h)}{h\times \tan(a+h)\tan a}$

$\tan A - \tan B = \tan(A-B)(1+\tan A\cdot \tan B)$

$= \lim_{h\to 0} \frac{\tan(a-a-h) - (1+\tan a + \tan(a+h))}{h\times \tan(a+h)\tan a}$

$= \lim_{h\to 0} -\frac{\tan h}{h} \times \frac{1+\tan a\tan(a+h)}{\tan(a+h)\tan a}$

$= -1 \times \frac{\tan^2 a+1}{\tan^2 a}$

$= -\frac{\sec^2 a}{\tan^2 a} = -\frac{1}{\cos^2 a} \times \frac{\cos^2 a}{\sin^2 a}$

$= -\operatorname{cosec}^2 a.$

$Df(a) = -\cos^2 a$

f is differentiable $\forall a\in R$.

2) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$Df(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}$

$= \lim_{x\to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x-a}$

$= \lim_{x\to a} \frac{1/\sin x - 1/\sin a}{x-a}$

$= \lim_{x\to a} \frac{\sin a - \sin x}{(x-a)\sin a \sin x}$.

put $x-a=h$, $x=a+h$, as $x\to a$, $h\to 0$

$Df(h) = \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$

$= \lim_{h\to 0} \frac{\sin a - \sin(a+h)}{(a+h-a)\sin a \sin(a+h)}$

$= \lim_{h\to 0} 2\cos\left(\frac{a+a+h}{2}\right) \sin\left(\frac{a-a-h}{2}\right) / h\times \sin a\cdot \sin(a+h)$

$= \lim_{h\to 0} -\frac{\sin h/2}{h/2}\times\frac{1}{2} \times 2\cos\left(\frac{2a+h}{2}\right) / \sin a \sin(a+h)$

$= \frac{-1}{2}\times 2\cos\left(\frac{2a}{2}\right) / \sin(a+0)$

$= -\frac{\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$

3. $\sec x$

$f(x) = \sec x$

$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$

$= \lim_{x \to a} \frac{\sec(x) - \sec(a)}{x-a}$

$\lim_{x \to a} \frac{1/\cos x - 1/\cos a}{x-a}$

$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a) \cos a \cos x}$ $x-a=h$, $x = a+h$

$Df(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \cos a (\cos(a+h))}$

$= \lim_{h \to 0} -2\sin\left(\frac{C+D}{2}\right) \sin\left(\frac{C-D}{2}\right)$

$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right) \sin\left(\frac{a-a-h}{2}\right)}{\cos a \cdot \cos(a+h) \times -h/2}$

$= \tan a \sec a$

2 If $f(x) = 4x+1 \quad x \leq 2$
$= x^2+5 \quad x > 0$ at $x=2$, then
find f' is differentiable or not

If $f(x) = 4x+7 \quad x < 3$
$= x^2+3x \quad x \geq 3$ at $x=3$ then,
find f is differentiable or not.

Q.4 If $f(x) = 8x-5 \quad x \leq 2$
$= 3x^2-4x+7 \quad x \leq 2$ at $x=2$. then,
find f is differentiable or not?

Q.2) LHD

$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^-} \frac{4x-1-(4\times 2+1)}{x-2}$

$= \lim_{x \to 2^-} \frac{4x-1-9}{x-2}$

$= \lim_{x \to 2} \frac{4x-8}{x-2}$

$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)} = 4$

$Df(2^-) = 4$

RHD - $Df(2^+) - \lim_{x \to 2^+} x^2+5-9/x-2$

$= \lim_{x \to 2^+} \frac{x^2+4}{x-2}$

$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{(x-2)}$

$= 2+2$
$= 4$

RHD = LHD.

'f' is differentiable at $x = 2$.

Q.3. RHD

$Df(3^+) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-(3^2+3 \cdot 3+1)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$

$= \lim_{x \to 3^+} x^2+6x-3x-18/x-3$

$= \lim_{x \to 3^+} \frac{x(x+6) - 3(x+6)}{x-3}$

$= \lim_{x \to 3^+} \frac{(x+6)(x+3)}{(x+3)}$

$= 3+6 = 9$

$Df(3^+) = 9$

$LHD = Df(3^-)$

$= \lim_{x \to 3^-} f(x) - f(3)/x-3$

$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3} = \lim_{x \to 3^-} \frac{4x-12}{x-3} = \lim_{x \to 3} 4(x-3)/x-3$

$Df(3^+) = 4$

$RHD \neq LHD$

'f' is not differentiable.

Q.4) $f(2) = 8 \times 2 - 5 = 16 - 5 = 11$

$RHD = Df(2^+) = \lim_{x \to 2^+} \frac{f(x) - f(2)}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 4x - 4}{x-2}$

$= \lim_{x \to 2^+} \frac{3x^2 - 6x + 2x - 4}{x-2}$

$= \lim_{x \to 2^+} \frac{3x(x-2) + 2(x-2)}{x-2}$

$= \lim_{x \to 2^+} (3x+2)(x-2)/x-2$

$= 3 \times 2 + 2 = 8$

$LHD = Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$

$\lim_{x \to 2^-} \frac{8x - 5 - 11}{x-2}$

$\lim_{x \to 2^-} \frac{8(x-2)}{(x-2)}$

$Df(2^-) = 8$

$LHD = RHD$

'f' is differentiable at $x = 3$

PRACTICAL · NO · 3.

Topic - Application of Derivative.

1) Find the intervals in which function in increasing or decr..

$f(x) = x^3 - 5x - 11$; $f(x) = x^2 - 4x$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f(x) = x^3 - 27x + 5$

$f(x) = 69 - 24x - 9x^2 + 2x^3$

Solⁿ

$f(x) = x^3 - 5x - 11$

$f'(x) = 3x^2 - 5$

f is increasing iff $f(x) > 0$

$3x^2 - 5 > 0$

$3(x^2 - 5/3) > 0$

$(x - \sqrt{5/3}) - (x + \sqrt{5/3}) > 0$

$+$ $-$ $+$

$-\sqrt{5/3}$ $\sqrt{5/3}$

$x \in (-\infty, -\sqrt{5/3}) \cup (\sqrt{5/3}, \infty)$

and f is decreasing iff $f'(x) > 0$

$3x^2 - 5 < 0$

$3(x^2 - 5/3) < 0$

$(x - \sqrt{5/3}) \cdot (x + \sqrt{5/3}) < 0$

$+$ $-$

$-\sqrt{5/3}$ $+\sqrt{5/3}$

$x \in (-\sqrt{5/3}, \sqrt{5/3})$

2. $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$f(x)$ is increasing iff $f'(x) > 0$

$2x - 4 > 0$

$2(x - 2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

and f' is descending iff $f'(x) < 0$

$2x - 4 < 0$

$2(x - 2) < 0$

$x - 2 < 0$

$x \in (-\infty, 2)$

3) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

f is increasing iff $f'(x) > 0$

$6x^2 + 2x - 20 > 0$

$3x^2 + x - 10 > 0$

$3x^2 + 6x - 5x - 10 > 0$

$3x(x + 2) - 5(x + 2) > 0$

$(x + 2)(3x - 5) > 0$

$+$

-2 $5/3$

$x \in (-\infty, -2) \cup (5/3, \infty)$

and f is decreasing iff $f'(x) < 0$

$6x^2 - 2x - 20 < 0$

$2(3x^2 + x - 10) < 0$

$3x^2 + x - 10 < 0$

$3x^2 + 6x - 5x - 10 < 0$

$\cancel{3x(x+2)} - 5(x+2) < 0$

$(x+2) - 5(x+2) < 0$

$(x + 2)(3x - 5) < 0$

$+$ $-$ $+$

-2 $5/3$

$x \in (-2, 5/3)$

4) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

f is decreasing iff $f'(x) > 0$

$3(x^2 - 9) > 0$

$(x-3)(x+3) > 0$

$+ \quad - \quad +$

$-3 \quad 3$

$x \in (-\infty, -3) \cup (3, \infty)$

and f' is decreasing iff $f'(x) < 0$

$3x^2 - 27 < 0$

$3(x^2 - 9) < 0$

$(x-3)(x+3) < 0$

$+ \quad - \quad +$

$-3 \quad 3$

$x \in (-3, 3)$

5) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

$\because f'$ is increasing iff $f'(x) > 0$

$6x^2 - 18x - 24 > 0$

$6(x^2 - 3x - 4) > 0$

$x(x-4) + 1(x-4) > 0$

$(x-4)(x+1) > 0$

$+ \quad - \quad +$

$-1 \quad 1$

$x \in (-\infty, -1) \cup (4, \infty)$

and f' is descending iff $f'(x) > 0$

$6x^2 - 18x - 24 < 0$

$6(x^2 - 3x - 4) < 0$

$x^2 - 4x - x - 4 < 0$

$(x-4)(x+1) < 0$

$+ \quad - \quad +$

$-1 \quad 1$

$x \in (-1, 4)$

2) Find intervals in which functions is concave upwards

$y = 3x^2 - 2x^3$; $y = x^3 - 27x + 5$

$y = x^4 - 6x^3 + 12x^2 + 5x + 7$; $y = 69 - 24x - 9x^2 + 2x^3$

$y = 2x^3 + x^2 - 20x + 4$.

Sol^n 1) $y = 3x^2 - 2x^3 = f(x)$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is continous upward if $f''(x) > 0$

$(6 - 12x) > 0$

$12(6/12 - x) > 0$

$x - 1/2 > 0 \quad \therefore x > 1/2$

$\therefore f''(x) > 0 \quad x \in (1/2, \infty)$

2) $y = x^4 - 6x^3 + 12x^2 + 5x + 7$.

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$f''(x) = 12x^2 - 36x + 24$

f is concave upward if $f''(x) > 0$

$12x^2 - 36x + 24 > 0$

$12(x^2 - 3x + 2) > 0$

$x^2 - 2x - x + 2 > 0$

$x(x-2) - 1(x-2) > 0$

$(x-2)(x-1) > 0$

$+ \quad - \quad +$

$1 \quad 2$

$x \in (-\infty, 1) \cup (2, \infty)$

3) $y = x^3 - 27x + 5$.

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upward iff $f''(x) > 0$

$6x > 0$

$x \in (0, \infty)$

4) $y = 69 - 24x - 9x^2 + 2x^3$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f' is concave upwards iff $f''(x) > 0$

$12x - 18 > 0$

$12\left(x - \frac{18}{12}\right) > 0$

$x - \frac{3}{2} > 0 \Rightarrow x > \frac{3}{2}$

$x \in \left(\frac{3}{2}, \infty\right)$

5) $y = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 12$

f' is concave upward iff $f''(x) > 0$

$f''(x) > 0$

$12x + 2 > 0$

$12\left(x + \frac{2}{12}\right) > 0$

$x + \frac{1}{6} > 0$

$x < \frac{-1}{6}$

$f''(x) \ngtr 0$

There exist interval $\left(\frac{-1}{6}, \infty\right)$

PRACTICAL No. 4.

Topic : Application of derivative & Newton's Method.

a) Find max & min value of following

i) $f(x) = x^2 + \frac{16}{x^2}$

$f(x) = 3 - 5x^3 + 3x^5$

$f(x) = x^3 - 3x^2 + 1 \quad \left[-\frac{1}{2}, 4\right]$

$f(x) = 2x^3 - 3x^2 - 12x + 1 \quad [-2, 3]$

b) Find the root of following equation by Newton's (Take 4 Iteration) correct upto 4 decimal.

$f(x) = x^3 - 3x^2 - 55x + 9.5$ (take $x_0 = 0$)

$f(x) = x^3 - 4x - 9$ in $[2, 3]$

$f(x) = x^3 - 1.8x^2 - 10x + 17$ is $[1, 2]$

Q.1) $f(x) = x^2 + 16/x^2$

$f'(x) = 2x - 32/x^3$

Now consider, $f'(x) = 0$

$2x - 32/x^3 = 0$

$2x = 32/x^2$

$x^4 = 32/2$

$x = \pm 2$

$f'(x) = 2 + 96/x^4$

$f''(x) = 2 + \frac{96}{2^4} = 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

f' has min. value at $x = 2$

$f(2) = 2^2 + 16/2^2$

$= 4 + 16/4$

$= 4 + 4 = 8$

f' has minimum value at $x = 2$

$f''(-2) = 2 + 96$

$= 2 + 96/16$

$= 2 + 6$

$= 8 > 0$

Functions reaches minimal value at $x=2$, and $x=-2$

) $f(x) = 3-5x^3+3x^5$

$f'(x) = -15x^2+15x^4$

Consider, $f'(x)=0$

$-15x^2+15x^4=0$

$x^2=1$

$x=\pm 1$

$f''(x) = -30x+60x^2$

$f(1) = -30+60$

$= 30>0$ has min value at $x=1$

$f(1) = 3-5(1)^3+3(1)^5$

$= 6-5 = 1$

$f''(-1) = -30(-1)+60(-1)^3$

$= 30-60$

$= -30<0$ has max value at $x=-1$

$f(-1) = 3-5(-1)^3+3(-1)^5$

$= 3+5-3 = 5$

f has max value 5 at $x=1$, and has the min value 1 at $x=1$

$f(x) = x^3-3x^2+1$

$f'(x) = 3x^2-6x$

Consider, $f'(x)=0$

$3x^2-6x=0$

$3x(x-2)=0$

$3x=0$ OR $x=2$

$x=0$

$f''(x) = 6x-6$

$f''(0) = 0$

$= -6<0$ has max value at $x=0$.

3) $f(x) = x^3-3x^2+1$

$f'(x) = 3x^2-6x$

Consider, $f'(x)=0$

$3x^2-6x=0$

$3x(x-2)=0$

$3x=0$ OR $x=2$

$x=0$

$f''(x) = 6x-6$

$f''(0) = 6(0)-6$

$= -6<0$ $\therefore$ f has max value at $x=0$

$f(0) = 0^3-3(0)^2+1 = 1$

$f''(2) = 6(2)-6 = 12-6 = 6$ min value at $x=2$

$f(2) = (2)^3-(3)(2)^2+1$

$= 8-3(4)+1 = -3$ max value 1 at $x=0$ and f has min value -3 at $x=2$.

4) $f(x) = 2x^3-3x^2-12x+1$

$f'(x) = 6x^2-6x-12$

Consider, $f'(x)=0$

$6x^2-6x-12=0$

$x^2-x-2=0$

$x^2+x-2x-2=0$

$x(x+1)-2(x+1)=0$

$x=2$ OR $x=-1$

$f''(x) = 12x-6$

$f''(2) = 12(2)-6$

$= 18>0$ min value at $x=2$

$f(2) = 2(2)^3-3(2)^2-12(2)+1$

$= -19$

$f''(-1) = 12(-1)-6$

$= -12-6 = -18<0$

f has max value at $x=-1$

$f(-1) = 2(-1)^3-3(-1)^2-12(1)+1$

$= 8$

f has ~~max~~ min value -1 at $x=2$

Q2) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\quad x_0 = 0 \rightarrow$ given

i) $f'(x) = 3x^2 - 6x - 55$

By Newton's Method,

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = x_0 - f(x_0)/f'(x_0)$

$x_1 = 0 + 9.5/55$

$x_1 = 0.1727$

$f(x_1) = 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f'(x) = 3(0.1727)^2 - 6(0.1727) - 55$

$= -55.9467$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 0.1712$

$f(x_2) = 0.0050 - 0.0879 - 9.416 + 9.5$

$= 0.0011$

$f'(x_2) = 3(0.1712)^2 - 6(0.1712) - 55$

$= -55.9393$

$x_3 = x_2 - f(x_2)/f'(x_2)$

$= 0.1712$

2) $f(x) = x^3 - 3x^2 - 55x + 9.5$ $\quad x_0 \rightarrow 0 \rightarrow$ given

$f'(x) = 3x^2 - 6x - 55$

By Newton's Method

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = 0 + 9.5/55$

$= 0.1727$

2) $f(x) = x^3 - 4x - 9$ $\quad [2,3]$

$f'(x) = 3x^2 - 4$

$f(2) = 23 - 4(2) - 9$

$= 8 - 8 - 9$

$= 9$

$f(3) = 3^3 - 4(3) - 9$

$= 27 - 12 - 9$

$= 6.$

Let $x_0 = 3$. be initial approx .

By Newton's Method.

$x_{n+1} = x_n - f(x_n)/f'(x_n)$

$x_1 = 3 - 6/23$

$= 2.7392$

$f(x_1) = (2.7329)^3 - 4(2.7392) - 9$

$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$

$= 22.5096 - 4$

$= 18.5096$

$x_2 = x_1 - f(x_1)/f'(x_1)$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071)$

$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$

$= 17.9851.$

$= 2.7071 - 0.0056 = 2.7015$

$f(x_3) = (2.7015)^3 - 4(2.7015) - 9$

$= 19.758 - 10.806 - 9$

$= -0.0091$

$f'(x_3) = 3(2.7015)^3 - 4 = 17.8943$

$f(4) = 2.7015 + 0.0901/17.8443$

$= 2.7065$

3) $f(x) = x^3 - 1.8x^2 - 10x + 17$ $\quad [1,2]$

$f'(x) = 3x^2 - 3.6x - 10$

$= (1)^3 - 1.8(1)^2 - 10(1) + 17$

$= 6.2$

$f(2) = (2)^3 - 1.8(2)^2 - (10)(2) + 17$
$= -2.2$

Let $x_0 = 2$ be initial approx.

By Newton's Method,

$x_{n+1} = x_n - f(x_n)/f'(x_n)$
$= x_0 - f(x_0)/f'(x_0)$
$= 2 - 0.4230$
$= 1.577$

$f(x_1) = (1.577)^3 - 18(1.577)^2 - 10(1.577) + 17$
$= 0.6755$

$f'(x) = 3(1.577)^2 - 3.6(1.577) - 10$
$= -8.2164$

$x_2 = x_1 - f(x_1)/f'(x_1)$
$= 1.577 + 0.6755/8.2164$
$= 1.6592$

$f(x_2) = (1.6592)^3 - 1.8(1.6592)^2 - 10(1.6592) + 17$
$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) - 10$
$= -7.7143$

$f(x_3) = (1.6618)^3 - 1.8(1.6618)^2 - 10$
$= -7.6977$

$x_4 = x_3 - f(x_3)/f'(x_3)$
$= 1.6618$

PRACTICAL NO 5

Topic - Integration

1) $\dfrac{dx}{\sqrt{x^2+2x-3}}$

$= \int \dfrac{1}{\sqrt{x^2+2x-3}} dx$

$= \int \dfrac{1}{\sqrt{x^2+2x-4}} dx$

$= a^2+2ab+b^2 = (a+b)^2$

$= \int \dfrac{1}{\sqrt{(x+1)^2-4}} dx$

Substitute put $x = 1 = t$

$dx = \frac{1}{t} \times dt$ where $t-1$, $t = x+1$

$\int \dfrac{1}{\sqrt{t^2-4}} dt$

using $\int \dfrac{1}{\sqrt{x^2-a^2}} dx = \ln(1x + \sqrt{x^2-a^2})$

$= \ln(1x+1+\sqrt{(x+1)^2-4})$
$= \ln(1x+1+\sqrt{x^2+2x-3})$
$= \ln(1+x+\sqrt{x^2+2x-3}) + c$

2) $\int (4e^{3x}+1)\,dx$

$= 4\int e^{3x}dx + \int 1 dx$ # $\int e^{ax} = \frac{1}{a} \times e^{ax}$
$= \dfrac{4e^{3x}}{3} + x + c$

3) $\int 2x^2 - 3\sin(x) + 5\sqrt{x}\,dx$

$= \int 2x^2 - 3\sin(x) + 5^{1/2}\,dx$

$= \int 2x^2\,dx - \int 3\sin\cos x + \int 5x^{3/2}\,dx)$

$= \frac{2x^3}{3} + \cos x + \frac{10\sqrt{x}}{3} + c$

$= 2x^3 + 10\sqrt{x}/3 + 3\cos x + c$

4) $\int x^3 + 3x + 4/\sqrt{x}\,dx$

split the dominator.

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2} + 3x^{1/2} + \frac{4}{x^{1/2}}\,dx$

$= \int x^{5/2}\,dx + \int 3x^{1/2}\,dx + \int \frac{4}{x^{1/2}}\,dx$

5) $\int t^7 x \sin(2t^4) t$

put $u = 2t^4$

$du = 2 \times 4t^3$

$\int t^7 x \sin(2t^4) \times \frac{1}{2\times 4} \times \frac{1}{2\times 4}\,du$

$= \int t^4 \sin(2t^4) \times \frac{1}{8}\,du - \left(\frac{t^4 x \sin(2t^4)}{8}\right) du$

Substitute

$\int \frac{4/2 \times \sin(4)}{2} / 8\,du$

$\int \frac{4 \times \sin(4)}{16}\,du$

$= \frac{1}{16} \int 4 \times \sin(4)\,du$

$\int u\,dv$

$= \frac{1}{16}(u \times (-\cos(u)) - \int -\cos(u)\,du$

$= \frac{1}{16} \times (4 \times (-\cos u)) + \int \cos(u)\,du$

$= \frac{1}{16} \times (4 \times (-\cos(4)) + \sin(4)$

Return the sub $u = 2t^4$

$= \frac{1}{16} \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4))$

$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t)^4 + c}{16}$

6) $\int \sqrt{x}(x^2 - 1)\,dx$

$= \int \sqrt{x}\,x^2 - \sqrt{x}\,dx$

$= \int x^{5/2} - x^{1/2}\,dx$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx$

$I_1\ \frac{x^{5/2+1}}{5/2+1} = \frac{2x^3\sqrt{x}}{7}$

$I_2 = \frac{x^{1/2+1}}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3}$

$\frac{2\sqrt{x^3}}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x^3}}{3} + c$

$\int \frac{\cos x}{\sqrt[3]{\sin(x^2)}}\,dx$

$= \int \left(\frac{\cos x}{\sin x^{\frac{2}{3}}}\right) dx$

put $t = \sin(x)$

$t = \cos x.$

$\int \frac{\cos(x)}{\sin(x)^{3/2}} \times \frac{1}{\cos x(t)}$

$= \frac{1}{\sin x^{3/2}} dt$

$= \frac{1}{t^{2/3}} dt$

$I = \int \frac{1}{t^{2/3}} dt = \frac{-1}{(2/3-1) t^{2/3-1}}$

$= \frac{-1}{(2/3+1) t^{2/3-1})}$

$= 3\sqrt{T}$

x) $\int \frac{x^2-2x}{x^3 3x^2+1} dx$

put $x^3 - 3x^2+1 = dt$

$I = \int \frac{x^2 2x}{x^3 3x^2+1} \times \frac{1}{3x^2 \times 3x \times 2x} dt$

$= \int \frac{x^2-2x}{x^3 3x^2+1} \times \frac{1}{3} dt$

$= \int \frac{1}{3(x^3-3x^2+1)} dt = \int \frac{1}{3t} dt$

$= 1/3 \int 1/t \, dt \quad \int \frac{1}{x} dt = \ln|x|$

$= 1/3 \times \ln(|x^3 - 3x^2+1|) + C$

?

PRACTICAL No. 6.

Aim-

Application of Integration and Numerical Integration.

Q1) Find the length of the following curve -

1. $x = t\sin t$, $y = 1-\cos t$ $t \in [0, 2\pi]$
2. $y = \sqrt{4-x^2}$ $x \in [-2, 2]$
3. $y = {}^{3/2}$ in $[0, 4]$
4. $x = 3\sin t$, $y = 3\cos t$ $\in [0, 2\pi]$
5. $x = \frac{1}{6} y^3 + \frac{1}{2y}$ on $y \in [1, 2]$

Q2) Using Simpson's Rule solve the following

1) $\int_0^2 e^{x^2} dx$ with $n = 4$

2) $\int_0^4 x^2 dx$ with $n = 4$

3) $\int_0^{\pi/3} \sqrt{\sin x} \, dx$ in $n = 6$.

7.1 .

Q.1

1) $y = \sqrt{4-x^2}$ $x \in [-2,2]$

$\frac{dy}{dx} = \frac{1}{2\sqrt{4-x^2}} \cdot (-2x)$

$= \frac{-2x}{\sqrt{4-x^2}}$

$L = \int_{-2}^{2} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_{-2}^{2}\sqrt{1+\left(\frac{-x}{\sqrt{4-x^2}}\right)^2}\, dx$

$= \int_{-2}^{2}\sqrt{\frac{4-x^2+x^2}{4-x^2}}\, dx$

$= \int_{-2}^{2} \sqrt{\frac{2}{\sqrt{4-x^2}}}\, dx$

$= \int_{-2}^{2}\frac{2}{\sqrt{4-x^2}}\, dx$

$= 2[\sin^{-1}(x/2)]_{-2}^{2}$

$= \sin^{-1}(1) - \sin^{-1}(-1)$

$= 2[\pi/2 - (\pi/2)]$

$= 2[\cancel{\pi}/2 + \pi/2]$

$= 2\pi$

2) if $= x^{3/2}$ in $[0,4]$

$\frac{dy}{dx} = \frac{3}{2}x^{3/2-1}$

$L = \int_0^4 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

$= \int_0^4\sqrt{1+\left(\frac{3\sqrt{x}}{2}\right)^2}\, dx$

$= \int_0^4 \sqrt{(1+9x/4)}\, dx$

$= 1/2 \int_0^4 \sqrt{4+9x}\, dx$

$= \frac{1}{2} \left[\frac{(4+9x)^{1/2+1}}{1/2+1}\right]_0^4 dx$

$= \frac{1}{2}\left[\frac{(4+9x)^{3/2}}{3/2}\times\frac{1}{9}\right]_0^4$

$= \frac{1}{27}\left[(4+9x)^{3/2}\right]_0^4$

$= \frac{1}{27}(4+0)^{3/2} - (4+36)^{3/2}]$

$= \frac{1}{27}((4)^{3/2} - (40)^{3/2}]$

$= \frac{1}{27}[(4)^{3/2} - 8]$ units

3) $x = 3\sin t$ $y = 3\cos t$

$\frac{dx}{dy}\frac{dy}{dx} = 3\cos t$; $\frac{dy}{dx} = -3\sin t$

$= \int_0^{2\pi} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= \int_0^{2\pi}\sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$

$= \int_0^{2\pi}\sqrt{9\sin^2 t + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} \sqrt{9(1)}\, dt$

$= \int_0^{4\pi} 3dt$

$= 3\int_0^{2\pi} dt$

$= 3[x]_0^{2\pi}$

$= 3(2\pi - 0)$

$= 6\pi.$

4) $\frac{dx}{dy} = \frac{1}{6}\frac{d}{dy}(y^3) - 1\cdot\frac{1}{2}\frac{dx}{dy}(1/y)$

$= \frac{y^2}{2} - 1/2y^2$

$= \frac{y^2 - 1}{2y^2}$

$L = \int_1^2 \sqrt{1 + \left(\frac{dx}{dy}\right)^2}$

$= \int_1^2 \sqrt{1 + \left(\frac{y-1}{2y^2}\right)^2}\, dy$

$= \int_1^2 \sqrt{1 + \left(\frac{y^2 - 1}{4y^4}\right)}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{4y^4}}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)^2}{(2y^2)^2}}\, dy$

$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$

$= \int_1^2 \frac{y^4}{2y^2}\, dy = \int_1^2 \frac{1}{2y^2}\, dy$

$= 1/2 \int_1^2 y\, dy - 1/2 \int \frac{1}{y^2}\, dy$

$= 1/2 \left[y^3/3 - y^2 \right]_1^2$

$= \frac{1}{2}\left[\frac{x}{3} - 1/2 - 1/3 + 1 \right]$

$= 1/2 \left[\frac{7}{3} + 1/2 \right] = 17/12$

5) Curve length $\int^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$

$dt = \int_0^{2\pi} \sqrt{(1-\cos t)^2 (\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{1 - 2\cos t + \cos^3 t + \sin^2 t + dt}$

$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\, dt$

$= \int_0^{2\pi} 2\sin 1/2\, dt$

$= \left[-9\cos 1/2 \right]_0^{2\pi}$

$= (-4\cos\pi) + 4\cos 0$

$= 8$ units.

Q.2) $\int_0^2 e^{x^2} dx$ with $n = 4$.

$a=0$, $b=2$, $n=4$

$h = \frac{b-a}{n} = 0.5$

x	0	0.5	1	1.5	2
	1	1.284	2.718	9.487	54.598

$\int_0^2 e^{x^2} = \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + x_4\right]$

$= \frac{0.5}{3}\left[(1) + 4(1.284) + 2(2.718) + 4(9.487) + 54.598\right]$

$= \frac{0.5}{3}\left[104.118\right]$

$= 17.353$

4) $\int_0^4 x^2 dx$, $n=4$

$a=0$, $b=4$, $n=4$

$h = \frac{b-a}{n} = \frac{4-0}{4} = 1$

x	0	1	2	3	4
	0	1	4	9	16

$\int_0^4 x^2 dx = \frac{h}{3}\left[0 + 4(1) + 2(4) + 4(9) + 16\right]$

$= \frac{1}{3}\left[0 + 4 + 8 + 36 + 16\right]$

$= \frac{1}{3}\left[12 + 36 + 16\right]$

$= \frac{1}{3}\left[64\right] = 21.333$

5) $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ with $n=6$

$h = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \frac{\pi/3}{6} = \frac{\pi}{18}$

x	0	$\frac{\pi}{18}$	$\frac{2\pi}{18}$	$\frac{3\pi}{18}$	$\frac{4\pi}{18}$	$\frac{5\pi}{18}$	$\frac{6\pi}{18}$ $(\pi/3)$
y	0	0.4167	0.7071	0.8017	0.8752	0.9306	
	y_0	y_1	y_2	y_3	y_4	y_5	

$\int_0^{\pi/3} \sqrt{\sin x}\, dx = \frac{h}{3}\left[0.4167 + 0.9306 + 4(0.4167 + 0.7071 + \right.$
$\left. + 2(0.5848 + 0.8017)\right]$

$= \pi/54 \left[1.3473 + 4(1.999) + 2(1.3865)\right]$

$= \pi/54 \times 12.1163$

$= \int_0^{\pi/3} \sqrt{\sin x}\, dx = 0.7049.$

PRACTICAL NO·7

Topic - Differential Equation

Q.1) Solve the following equation.

1) $x\frac{dy}{dx} + y = e^x$

$\frac{dy}{dx} + \frac{1}{x}y = \frac{e^x}{x}$

$P(x) = \frac{1}{x} \cdot Q(x) = \frac{e^x}{x}$

$IF = e^{\int 1/x\, dx}$

$= e^{\ln x}$

$IF = x$

$y(IF) = \int Q(x)(IF)\,dx + c$

$= \int \frac{e^x}{x} x \cdot dx + c$

$= \int e^x dx + c$

$xy = e^x + c$

2) $e^x\frac{dy}{dx} + 2e^x y = 1$

$\frac{dy}{dx} + \frac{2e^x}{e^x}y = \frac{1}{e^x}$ (÷ by e^x)

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2 \cdot Q(x) = e^{-x}$

$\int P(x) = dx$

$IF = e^{\int 2dx}$

$= e^{2x}$

$y(IF) = \int Q(x)(IF)\,dx + c$

$ye^{2x} = \int e^{-x} + 2x\,dx + c$

$= \int e^x dx + c$

$y \cdot e^{2x} = e^x + c$

3) $x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$x\frac{dy}{dx} = \frac{\cos x}{x} - 2y$

$\therefore \frac{dy}{dx} + \frac{2y}{x} = \frac{\cos x}{x^2}$

$P(x) = 2(x) \cdot Q(x) = \cos x / x^2$

$IF = e^{\int P(x)\,dx}$

$= e^{\int 2/x\, dx}$

$= e^{2/x}x$

$= \ln x^2$

$IF = x^2$

$y(IF) = \int Q(x)(IF)\,dx + c$

$= \int \frac{\cos x}{x^2} x^2 dx + c$

$= \int \cos x + c$

$x^2y = \sin x + c.$

4) $x\frac{dy}{dx} + 3y = \frac{\sin x}{x^2}$

$\frac{dy}{dx} + \frac{3y}{x} = \frac{\sin x}{x^3}$ ($\div$ by x on bs)

$P(x) = 3/x \quad Q(x) = \sin x/x^3$

$= e^{\int P(x)dx}$

$= e^{\int P(x)dx}$

$= e^{3/x}$ x

$= e^{\ln x^3}$

$IF = x^3$

$y(IF) = \int Q(x)(IF)\,dx + c$

$= \int \frac{\sin x}{x^3} x^3\,dx + c$

$= \int \sin x\,dx + c$

$x^3 y = -\cos x + c$

5) $e^{2x}\frac{dy}{dx} + 2e^{2x}y = 2x$

$\frac{dy}{dx} + 2y = \frac{2x}{e^{2x}}$

$P(x) = 2 \quad , Q(x) = 2x/e^{2x} = 2xe^{-2x}$

$IF = e^{\int P(x)dx}$

$= e^{\int 2dx}$

$= e^{2x}$

$y(IF) = \int Q(x)(IF)\,dx + c$

$= \int 2xe^{-2x} e^{2x}\,dx + c$

$= \int 2x\,dx + c$

$ye^{2x} = x^2 + c$

6) $\sec^2 x \tan y\,dx + \sec^2 y \tan x\,dy = 0$

$\sec^2 x \cdot \tan y\,dx = -\sec^2 y \tan x\,dy$

$\frac{\sec^2 x\,dx}{\tan x} = -\frac{\sec^2 y\,dy}{\tan y}$

$\int \frac{\sec^2 x\,dx}{\tan x} = -\int \frac{\sec^2 y\,dy}{\tan y}$

$\log|\tan x| = -\log|\tan y| + c$

$\log|\tan x - \tan y| = c$

$\tan x \cdot \tan y = e^c$

7) $\frac{dy}{dx} = \frac{2x+3y-1}{6x+9y+6}$

Put $2x + 3y = v$

$2 + 3\frac{dy}{dx} = \frac{dv}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{dv}{dx} - 2\right)$

$\frac{1}{3}\left(\frac{dv}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+2}\right)$

$\frac{dv}{dx} = \frac{v-1}{v+2} + 2$

$\frac{dv}{dx} = \frac{v-1+2v+4}{v+2}$

$= \frac{3v+3}{v+2}$

$= \frac{3(v+1)}{v+2}$

$\int \left(\frac{v+2}{v+1}\right) dv = 3\,dx$

$= \int \frac{v+1}{v}\,dx + \int \frac{1}{v+1}\,dv = 3x.$

$$v + \log|x| = 3x + c$$
$$2x + 3y + \log|2x + 3y + 1| = 3x + c$$
$$3y = x - \log|2x + 3y + 1| + c$$

8.) $\frac{dy}{dx} = \cancel{\frac{2x+3y-1}{6x+4y+2}}\ \sin^2(x-y+1)$

Put $x-y+1=v$

Diff w.r.t. x

$x-y+1=v$

$$1-\frac{dy}{dx} = \frac{dv}{dx}$$

$$1-\frac{dv}{dx} = \frac{dy}{dx}$$

$$1-\frac{dv}{dx} = \sin^2 v$$

$$\frac{dv}{dx} = 1-\sin^2 v$$

$$\frac{dv}{dx} = \cos^2 v$$

$$\frac{dv}{\cos^2 v} = dx$$

$$\int \sec^2 v\,dv = \int dx$$

$$\tan v = x + c$$

$$\tan(x+y-1) = x + c$$

AV
01/2020

PRACTICAL NO. 8

Aim -

Using Euler's Method find following

1.) $\frac{dy}{dx} = y + e^x - 2$ $\quad y(0) = 2$, $h = 0.5$ find $y(2)$

2.) $\frac{dy}{dx} = 1 + y^2$, $y(0) = 0$, $h = 0.2$, find $y(1)$

3.) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$; $y(0) = 1$, $h = 0.2$ find $y(1)$

4.) $\frac{dy}{dx} = 3x^2 + 1$, $y(1) = 2$, find $y(2)$
for $h = 0.5$, & $h = 0.25$.

5.) $\frac{dy}{dx} = \sqrt{xy} + 2$, $y(1)$; find $y(1.2)$ with $h = 0.2$.

$\frac{dy}{dx} = y+e^x-2$

$f(x,y) = y+e^x-2$, $y_0 = 2$, $x_0 = 0$, $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	1	2.5
1	0.5	2.5	2.487	3.57435
2	1	3.5743	4.2925	5.3615

$y_{n+1} = y_n + h\, f(x_n y_n)$

n	x_n	y_n	$f(x_n y_n)$	y_{n+1}
3	1.5	5.3615	7.8431	9.28305
4	2	9.2831		

By Euler's Formula,

$y(2) = 9.2831$

$\frac{dy}{dx} = 1+y^2$

$f(x,y) = 1+y^2$, $y_0 = 0$, $x_0 = 0$, $h = 0.2$

Using Euler's Iteration formulae.

$y_{n+1} = y_n + h\, f(x_n y_n)$

n	x_n	y_n	$f(x_n y_n)$	y_{n+1}
0	0	0	1	0.2
1	0.2	0.2	0.104	0.408
2	0.4	0.408	1.1665	0.6419
3	0.6	0.6419	1.4113	0.9236
4	0.8	0.9236	1.8503	1.2942
5	1	1.2942		

By Euler's Formulae

$y(1) = 1.2942$

3) $\frac{dy}{dx} = \sqrt{\frac{x}{y}}$ $y(0) = 1$, $x_0 = 0$, $h = 0.2$

Using Euler's Iteration Formulae,

$y_{n+1} = y_n + hf(x_n y_n)$

n	x_n	y_n	$f(x_n y_n)$	y_{n+1}
0	0	1	0	0
1	0.2	0		
2	0.4			
3	0.6			
4	0.8			
5	1			

4) $\frac{dy}{dx} = 3x^2+1$ $y_0 = 2$, $x_0 = 1$ $h = 0$

. For $h = 0.5$

Using Euler's Iteration Method,

$y_{n+1} = y_n + hf(x_n y_n)$

n	x_n	y_n	$f(x_n y_n)$	y_{n+1}
0	1	2	4	28.5
1	1.5	4	49	
2	2	28.5		

By Using Euler's Method,

$y(2) = 28.5$.

For $h = 0.25$

n	x_n	y_n	$f(x_n\, y_n)$	y_{n+1}
0	1	2	4	3
1	1.25	3	5.6875	4.4219
2	1.5	4.4219	7.75	6.3594
3	1.75	6.3594	10.1815	8.9048
4	2	8.9048		

By Euler's Formula.

$y(2) = 8.9048$

5) $\frac{dy}{dx} = \sqrt{xy} + 2$ $\quad y_0 = 1$, $x_0 = 1$, $h = 0.2$

Using Euler's Iteration formula,

$y_{n+1} = y_n + h f(x_n\, y_n)$

n	x_n	y_n	$f(x_n\, y_n)$	y_{n+1}
0	1	1	3	1.6
1	1.2	1.6		

By Euler's Formula,

$y(1.2) = 1.6$.

PRACTICAL No. 9

Topic - Limit & partial order derivative

① Evaluate the following limits.

i) $\lim\limits_{(x,y) \to \cancel{(-4,-1)}\,(2,0)} \cancel{\frac{x^3 - 3y + y^2 - 1}{xy + 5}}$ $\quad \frac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$.

Applying limit.

$$\frac{(-4)^3 - 3(-1) + (-1)^2 - 1}{(-4)(-1) + 5}$$

$$\frac{(0+1)(2)^2 + (0)^2 - 4(2)}{2 + 3(0)}$$

$$\frac{1(4-8)}{2} = -\frac{4}{2} = -2$$

ii) $\lim\limits_{(x,y)\,(-4,-1)} \frac{x^3 - 3y + y^2 - 1}{xy + 5}$

Applying limit,

$$\frac{(0+1)(2)^2 - (0)^2 - 4(2)}{2 + 3(0)}$$

$$\frac{1(4-8)}{2} = -\frac{4}{2} = -2$$

iii) $\lim\limits_{(x,y,z) \to (1,1,1)} \frac{x^2 - y^2 - z^2}{x^3 - x^2 yz}$.

$\lim\limits_{(x,y,z) \to (1,1,1)} \frac{(x)^2 - (yz)^2}{x^2(x - yz)}$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{(x+yz)(x-zy)}{x^2(x-yz)} \quad [\because (a^2)-(b^2) = (a+b)(a-b)]$$

$$\lim_{(x,y,z)\to(1,1,1)} \frac{x+yz}{x}$$

$$\frac{1+(1)(1)}{(1)^2} = 2$$

2) Find f_x, f_y for each of the following

$f(x,y) = xy e^{x^2+y^2}$

$f(x) = \frac{\partial f}{\partial x}$

$= \frac{\partial (xy e^{x^2+y^2})}{\partial x}$

$= y \frac{\partial (x \cdot e^{x^2+y^2})}{\partial x}$

$= y \left[x \cdot \frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2} \cdot \frac{d}{dx}(x) \right]$

$\left[\because \frac{d}{dx}(uv) = uv' + v \cdot u' \right]$

$= y \left[x \cdot e^{x^2+y^2} \cdot 2x + e^{x^2+y^2}(1) \right]$

$= y : e^{x^2+y^2} [2x+1]$

Now,

$f(y) = \frac{\partial f}{\partial y}$

$= \partial(xy e^{x^2+y^2})/\partial y$

$= x \cdot \frac{\partial}{\partial y}(y \cdot e^{x^2+y^2})$

$= x\left[y \cdot \frac{d}{dy}(e^{x^2+y^2}) + e^{x^2+y^2} \frac{d}{dy}(y)\right] \quad \left[\because \frac{d(uv)}{dx}\right]$

$= x[2y^2 \cdot e^{x^2+y^2} + e^{x^2+y^2}]$

$= x \cdot e^{x^2+y^2}[2y^2+1]$

(ii) $f(x,y) = e^x \cos y$.

$f(x) = e^x \cos y$

$f(y) = e^x \cdot \frac{\partial}{\partial y}(\cos y)$

$= e^x(-\sin y)$

$= -e^x \sin y$

~~(iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$~~

~~$f(x) = e^x(-\sin y)$~~

~~$f(y) = e^x(-\sin y)$~~

~~$= e^x(-\sin y)$~~

(iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(x^3y^2 - 3x^2y + y^3 + 1)}{\partial x}$

$= 3x^2y^2 - 3(2x)y$

$= 3x^2y^2 - 6xy$

~~$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(x^3y^2 - 3x^2y + y^3 + 1)}{\partial y}$~~

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(x^3y^2 - 3x^2y + y^3 + 1)}{\partial y}$

$f(y) = x^3(2y) - 3(1)x^2 + 3y^2$

$= 2x^3y - 3x^2 + 3y^2$

(iii) Using defⁿ find value of $f_x \cdot f_y$ at $(0,0)$ for $f(x,y) = \frac{2x}{1+y^2}$

$$f_x(a,b) = \lim_{h \to 0} \frac{f(a+h,b) - f(a,b)}{h}$$

where $(a,b) = (0,0)$

$$f(x)(0,0) = \lim_{h \to 0} \frac{f(h,0) + f(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{2h-0}{2} = 2$$

Similarly,

$$f_y(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{0-0}{h} = 0$$

$f_x = 2$, $f_y = 0$

$$f(x)(0,0) = \lim_{h \to 0} \frac{f(h,0) - f(0,0)}{h}$$

$$= \lim_{h \to 0} \frac{2h-0}{h} = 2$$

$f(x)(0,0)$

Q.4) Find all second order pd of 'f'. Also verify wheather $f_{xy} = f_{yx}$

i) $f(x,y) = y^2 - xy / x^2$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial (y^2 - xy)}{x^2}$$

$$= \frac{x^2 \cdot \frac{d}{dx}(y^2 - xy) - (y^2 - xy) \cdot \frac{d}{dx}(x^2)}{(x^2)^2} \quad \left[\because \frac{d}{dx}\left(\frac{u}{v}\right)\right]$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$-x^2y - 2xy^2 + 2x^2y / x^4 = x(xy - 2y^2)/x^4$

$f(x) = xy - 2y^2 / x^3$

$$f(y) = \frac{\partial f}{\partial y} = \frac{\partial (y^2 - xy)}{x^2}$$

$$= \partial \left(\frac{y^2}{x^2} - \frac{xy}{x^2}\right) / \partial y$$

$$= \partial \left(\frac{y^2}{x^2} - \frac{y}{x}\right) / \partial y$$

$$= \frac{1}{x^2} 2y - \frac{1}{x}$$

$$f(y) = \frac{2y - x}{x^2}$$

$f(xx) = \partial(xy - 2y^2)/\partial x$

$$= x^3 \frac{d}{dx}(xy - 2y^2) - (xy - 2y^2)\frac{d}{dx}(x^3) \Big/ (x^3)^2$$

$= x^3(y) - (xy)y^2)(3x^2)/6$

$$= \frac{6x^2y^2 - 2x^3y}{x^6} = x^2(6y^2 - 2xy)/x^6$$

$= 6y^2 - 2xy / x^4$

$f(yy) = \partial(2y - x)/\partial y$

$$= \frac{1}{x^2} \partial(2y - x)/\partial y = \frac{1}{x}(2) = \frac{2}{x^2}$$

$$f(xy) = \frac{\partial \left(\frac{xy - 2y^2}{x^3}\right)}{\partial y} \quad \cancel{= \frac{1}{x}(2) = \frac{2}{x}}$$

$$f(xy) = \partial\left(\frac{y}{x^2} - \frac{2y^2}{x^3}\right) / \partial y$$

$$= \frac{1}{x^2} - \frac{1}{x^3} \quad \partial(2y)$$

$$= \frac{1}{x^2} - \frac{4y}{x^3} = \frac{x^3 - 4y}{x^4}$$

$$f(yx) = \partial\left(\frac{2y - x}{x^2}\right) / \partial x$$

$$= \frac{\partial\left(\frac{2y}{x^2} - \frac{x}{x^2}\right)}{\partial x} = \partial\left(\frac{2y}{x^2} - \frac{1}{x}\right) / \partial x$$

$$= 2y\left(\frac{-2}{x^3}\right) - \left(\frac{-1}{x^2}\right)$$

$$= \frac{-4y}{x^3} + \frac{1}{x^2}$$

$$= \frac{x^2(x - 4y)}{x^6}$$

$$= \frac{x - 4y}{x^4}$$

$$f(x,y) = f(yx) = \frac{x - 4y}{x^4}$$

Hence verified

ii) $f(x,y) = x^3 + 3x^2y^2 - \log(x^2+1)$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(x^3 + 3x^2y^2 - \log(x^2+1))}{\partial x}$$

$$f(x) = 3x^2 + 6xy^2 - 2x / x^2+1$$

$$f(x) = 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$$

$$f(y) = \frac{\partial f}{\partial y} = \partial(x^3 + 3x^2y^2 - \log(x^2+1)) / \partial y$$

$$= 0 + 3(2y)(x^2) + 0$$

$$= 6x^2y$$

$$f(xx) = \frac{\partial fx}{\partial x} = \frac{\partial}{\partial x}\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right)$$

$$= 6x + 6y^2(1) - 2\left[\frac{x^2+1(1) - x(2x)}{x^2+1}\right]$$

$$\left[\because \frac{d\, u/v}{dx}\right]$$

$$= 6x + 6y^2 - 2\left(\frac{x^2+1-2x^2}{(x^2+1)^2}\right)$$

$$= 6x + 6y^2 - 2\left(\frac{-x^2+1}{(x^2+1)^2}\right)$$

$$f(yy) = \frac{\partial fy}{\partial y} = \frac{\partial(6x^2y)}{\partial y}$$

$$= 6x^2(1) = 6x^2$$

$$f(xy) = \partial\left(3x^2 + 6xy^2 - \frac{2x}{x^2+1}\right) / \partial y$$

$$= 0 + 6x(2y)$$

$$= 12xy$$

$$f(xy) = f(yx) = 12xy$$

Hence verified

(iii) $f(x,y) = \sin(xy) + e^{x+y}$

$$f(x) = \frac{\partial f}{\partial x} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial x}$$

$= \cos(xy)(y) + e^{x+y}(1)$

$= y\cos xy + e^{x+y}$

$f(y) = \frac{\partial f}{\partial y} = \frac{\partial(\sin(xy) + e^{x+y})}{\partial y}$

$= x\cos xy + e^{x+y}$

$f(xx) = \frac{\partial fx}{\partial x} = \frac{\partial(y\cos xy + e^{x+y})}{\partial x}$

$= y\cos xy(y) + e^{x+y}(1)$

$= y^2\cos xy + e^{x+y}$

$f(yy) = \frac{\partial fy}{\partial y} = \frac{\partial(x\cos xy + e^{x+y})}{\partial y}$

$= x\cos xy(x) + e^{x+y}(1)$

$= x^2\cos xy + e^{x+y}$

$f(xy) = \frac{\partial fx}{\partial y} = \frac{\partial(y\cos xy + e^{x+y})}{\partial y}$

$f(xy) = \frac{\partial fx}{\partial y} = \frac{\partial(y\cos xy + e^{x+y})}{\partial y}$

$= y[\sin(xy)(x) + \cos xy(1)] + e^{x+y}(1).$

$\left[\because \frac{d}{dx}(uv)\right]$

$= -xy\sin(xy) + \cos xy + e^{x+y}$

$f(y,x) = \frac{\partial fy}{\partial x} = \frac{\partial(x\cos xy + e^{x+y})}{\partial y}$

$f(y,x) = \frac{\partial fy}{\partial x} = \frac{\partial(x\cos xy + e^{x+y})}{\partial x}$

$= \cos xy(1) + x(-\sin(xy))(y) + e^{x+y}$

$= -xy\sin(xy) + \cos(xy) + e^{x+y}$

$f(xy) = f(yx) = -xy\sin x(xy) + \cos(xy) + e^{x+y}$

5) Find the linearization of $f(x,y)$ at given:

i) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$= \sqrt{(1)^2+(1)^2}$

$= \sqrt{2}$

$f(x) = \frac{1}{2\sqrt{x^2+y^2}}$ $\therefore 2x = \frac{x}{\sqrt{x^2+y^2}}$

$fx(1,1) = \frac{1}{\sqrt{(1)^2+(1)^2}} = \frac{1}{\sqrt{2}}$

$L(x,y) = f(a,b) + fx(a,b)(x-a) + fy(a,b)(y-b)$

$= \frac{2 + x - 1 + (y-1)}{\sqrt{2}}$

$= \frac{2 + x + y - 2}{\sqrt{2}} = \frac{x+y}{\sqrt{2}}$

ii) $f(x,y) = 1 - x + y\sin x$ at $\left(\frac{\pi}{2}, 0\right)$

$f\left(\frac{\pi}{2}, 0\right) = 1 - \frac{\pi}{2}$

$f(x) = -1 + y\cos x$

$= 1$

$f_x(\frac{\pi}{2}, 0) = -1 + 0\cos(\frac{\pi}{2})$

$= -1$

$= 1$

$L(x,y) = f(a,b) + f_x(a,b)\cdot(x-a) + f_y(a,y)(y-b)$

$= 1 - \frac{\pi}{2} - x + \frac{\pi}{2} + y$

$= y - x + 1$

iii) $f(x,y) = \log x + \log y$ – at $(1,1)$

$\rightarrow f(1,1) = \log(1) + \log(1)$

$= 0 + 0$

$f(x) = \frac{1}{x}$ $\qquad f(y) = \frac{1}{y}$

$f_x(1,1) = 1$ $\qquad f_y(1,1) = 1$

$L(x,y) = f(a,b) + f_x(a,b)\cdot(x-a) + f_y(a-b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x - 1 + y - 1$

$= x + y - 2.$

PRACTICAL NO. 10

Q.1 Find derivatives (directional) of following funct^n at given & in the direction of given vector.

$\Rightarrow f(x,y) = x + 2y - 3 \qquad a = (1,-1), \; u = 3i - j$

Here,

$u = 3i - j$ is not a unit vector

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{10}}\cdot(3,-1)$

$\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$f(a) = f(1,-1) = (1) + 2(-1) - 3 = 1 - 2 - 3 = -4$

$f(a+hu) = f(1,-1) + h\left(\frac{3}{\sqrt{10}}, \frac{-1}{\sqrt{10}}\right)$

$= f\left(1 + \frac{3}{\sqrt{10}}\right), \left(-1 - \frac{h}{\sqrt{10}}\right)$

$f(a+hu) = \left(1 + \frac{3}{\sqrt{10}}\right) + 2\left(-1 - \frac{h}{\sqrt{10}}\right) - 3$

$= 1 + \frac{3}{\sqrt{10}} - 2 - \frac{2h}{\sqrt{10}} - 3$

$f(a+hu) = -4 + \frac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h \to 0} \frac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \frac{4 + h\sqrt{10} + 4}{h}$

$= \frac{1}{\sqrt{10}}$

ii) $f(x) = y^2 - 4x + 1 \quad a = (3,4) \cdot u = i + 5j$

Here $u = i + 5j$ is not a Unit Vector

$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

Unit vector along u is $\frac{u}{|u|} = \frac{1}{\sqrt{26}} = (1,5)$

$= \left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 = 5.$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= f\left(3 + \frac{h}{\sqrt{26}}, 4 + \frac{5h}{\sqrt{26}}\right)$

$f(a+hu) = 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 - \frac{4h}{\sqrt{26}} + 1$

$= \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - \frac{4h}{\sqrt{26}} + 5$

$= \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} + 5$

$D_u f(a) = \lim_{h \to 0} \frac{25h^2}{26} + \frac{36h}{\sqrt{26}} \cancel{+5} \cancel{-5} / h.$

$= \frac{25h}{26} + \frac{36}{\sqrt{26}}$

iii) $2x + 3y \quad a = (1,2) \quad u = (3i + 4j)$

Here $u = 3i + 4j$ is not Unit Vector

$|\bar{u}| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) = 2(1) + 3(2) = 8$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a+hu) = 2\left(\frac{1+3h}{5}\right) + 3\left(2 + \frac{4h}{5}\right)$

$= 2 + \frac{6h}{5} + 6 + \frac{12h}{5} = \frac{18h}{5} + 8$

$D_u f(a) = \lim_{h \to 0} \frac{18\cancel{h}}{5} \cancel{+8} \cancel{-8} / \cancel{h}$

$= \frac{18}{5}$

Q2) Find gradient vector for following function at given point.

i) $f(x,y) = x^y + y^x \Rightarrow a(1,1)$

$f_x = y \cdot x^{y-1} + y^x \log y$

$f_y = x^y \log x + x y^{x-1}$

$\nabla f(x,y) = (f_x, f_y)$

$= (y x^{y-1} + y^x \log y, x^y \log x + 2y^{x-1})$

$f(1,1) = (1+0, 1+0)$

$= (1,1)$

ii) $f(x,y) = (\tan^{-1} x) y^2 \cdot a = (1,-1)$

$f(x) = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1} x$

$\nabla f(x,y) = (f_x, f_y)$

$= \left(\frac{y^2}{1+x^2}, 2y \tan^{-1} x\right)$

$f(1,-1) = \left(\frac{1}{2}, \frac{\pi}{4}(-2)\right)$

$= \left(\frac{1}{2}, \frac{-\pi}{2}\right)$

iii) $f(x,y,z) = xyz - e^{x+y+z}$ $\quad a = (1,-1,0)$

$f_x = yz - e^{x+y+z}$

$f_y = xz - e^{x+y+z}$

$= xy - e^{x+y+z}$

$\nabla f(x,y,z) = f_x, f_y, f_z$

$= yz - e^{x+y+z}, xz - e^{x+y+z}, xy - e^{x+y+z}$

$y(1,-1,0) = (-1)(0) - e^{(1+(-1)+0)}, (1)(0) - e^{1+(-1)+0}$

$(1)\cdot(-1) - e^{1+(-1)+0}$

$= (0 - e^{0}, 0 - e^{0}, -1 - e^{0}) = (-1, -1, 2)$

Q.3) Find equation of tangent & normal to each of the following using curves at given points.

i) $x^2\cos y + e^{xy} = 2$ at $(1,0)$

$f_x = \cos y \, 2x + e^{xy} y$

$f_y = x^2(-\sin y) + e^{xy}\cdot x$

$(x_0 y_0) = (1,0) \therefore x_0 = 1, y_0 = 0$

equation of tangent,

$f(x)(x - x_0) = \cos 0 \, 2(1) + e^{0} 0$

$= 1(2) + 0 = 2$

$f_y(x_0 y_0) = (1)^2(\sin 0) + e^{0} 1$

$= 0 + 1 = 1$

$2(x-1) + 1(y-0) = 0$

$2x - 2 + y = 0$

$2x + y - 2 = 0$ required of tangent

eq of Normal

$= ax + by + c = 0$

$(1) 1 + 2(y) + d = 0$

$1 + 2y + d = 0$ at $(1,0)$

$= 1 + 2(0) + d = 0$

$d + 1 = 0$

$d = -1$

ii) $x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

$f_x = 2x + 0 - 2 + 0 + 0$

$= 2x - 2$

$f_y = 0 + 2y - 0 + 3 + 0$

$= 2y + 3$

$(x_0 y_0) = (2,-2) \therefore x_0 = 2 \; y_0 = -2$

$f_x(x_0 y_0) = 2(-2) + 3 = -1$

$= 2(2) - 2 = 2$

eq of tangent,

$f_x(x - x_0) + f_y(y - y_0) = 0$

$2x - 2 - y - 2 = 0$

$2x - y - 4 = 0 \rightarrow$ eq of tangent

eqn of Normal

$= ax + by + c = 0$

$bx + ay + d = 0$

$= -1(x) + 2(y) + d = 0$

$-x + 2y + d = 0$ at $(2,-2)$

$-2 - 4 + d = 0$

$-6 + d = 0$

$d = 6$

Q 4) Find the eq^n of tangent & normal line to each of the following surface.

i) $x^2 - 2yz + 3y + xz = 7$ at $(2, 1, 0)$

$f_x = 2x - 0 + 0 + z$

$f_x = 2x + z$

$f_y = 0 - 2z + 3 + 0$

$= 2z + 3$

$f_z = -2y + z \cdot x$

$(x_0 y_0 z_0) = (2, 1, 0) = x_0 = 2,\ y_0 = 1,\ z_0 = 0$

$f_x(x_0 y_0 z_0) = 2(2) + 0 = 4$

$f_y(x_0 y_0 z_0) = 2(0) + 3 = 3$

$f_z(x_0 y_0 z_0) = -2(1) + 2 = 0$

eq of tangent,

$f_x(x_0 - x_0) + f_y(y_0 - y_0) + f_z(z_0 - z_0) = 0$

$\therefore 4x - 8 + 3y - 3 = 0$

$\therefore 4x + 3y - 11 = 0 \rightarrow$ required eq of tangent.

Eq^n of normal at $(4, 3, -11)$

$$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$$

$$= \frac{x-2}{4} = \frac{y-1}{3} = \frac{z+1}{0}$$

ii) $3xyz = -x - y + z = -4$ at $(1, -1, 2)$

$3xyz - x - y + z + 4 = 0$ at $(1, -1, 2)$

$f_x = 3yz - 1 - 0 + 0 + 0 = 3yz - 1$

$f_y = 3xz - 0 - 1 + 0 + 0 = 3xz - 1$

$f_z = 3xy - 0 - 0 + 1 + 0 = 3xy + 1$

$(x_0 y_0 z_0) = (1, -1, 2)$ $\quad x_0 = 1,\ y_0 = -1,\ z_0 = 2$

$f_x = 3(-1)(2) - 1 = 7$

$f_y = 3(+1)(2) - 1 = 5$

$f_z = 3(1)(-1) + 1 = -2$

eq^n of tangent

$-7(x-1) + 5(y+1) - 2(z-2) = 0$

$-7x + 7 + 5y + 5 - 2z + 4 = 0$

$-7x + 5y - 2z + 16 = 0$ required eq^n of tang

Eq^n of normal at $(-7, 5, 2)$

$x - x_0 / f_x = y - y_0 / f_y = z - z_0 / f_z.$

$$= \frac{x-1}{-7} = \frac{y+1}{5} = \frac{z-2}{2}$$

Q 5) Find the local maxima & minima for the following funct

i) $f(x, y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 - 0 = 6x - 3y + 6$

$f_y = 2y - 3x - 4$

$f_x = 0,\quad 6x - 3y + 6 = 0$

$2x - y + 2 = 0$

$2x - y = -2 \quad — (1)$

$f_y = 0$

$2y - 3x - 4 = 0$

$2y - 3x = 4 \quad — (2)$

Multiplying eq^n (1) by (2)

$2y - 3x = 0\ 4$

$x = 0$

Substituting value of 'x' in eq (1)

$2(0) - y = -2$

$y = 2$

Critical point are $(0,2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here $r > 0$ = $6(2) - (-3)^2$

$= 3 > 0$

'f' has maximum at $(0,2)$

$3x^2 + y^2 - 3xy + 6x - 4y$ · at $(0,2)$

$0 + 4 - 0 - 0 - 8 = -4$

ii) $f(x,y) = 2x^4 + 3x^2y - y^2$

$f_y = 3x^2 - 2y$

$f_x = 8x^3 + 6xy$

$f_x = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0$ — (1)

$f_y = 0$

$3x^2 - 2y = 0$ — (2)

Multiplying eq^n (1) with 3. ; (2) with 4

$12x^2 + 9y = 0$

$-12x^2 - 8y = 0$

$y = 0$

Substitute value of y in eq (1)

$4x^2 + 3(0) = 0$

$x = 0$

Critical point is $(0,0)$

$r = f_{xx} = 24x^2 + 6x$

$t = f_{yy} = 0 - 2 = -2$

$s = f_{xy} = 6x - 0 = 6x = 6(0) = 0$

r at $(0,0)$, $24(0) + 6(0) = 0$

$f(x,y)$ at $(0,0)$

$2(0)^4 + 3(0)^2(0) - (0)$

$= 0$

$rt - s^2 = 0(-2) - (5)^2$

$= 0$

$r = 0$ & $rt - s^2 = 0$

iii) $f(x,y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0$ ∴ $2x + 2 = 0$

$x = -1$

$f_y = 0$ $-2y + 8 = 0$

$y = 4$

Critical point is $(-1, 4)$

$r = f_{xx} = 2$

$t = f_{yy} = -2$

$s = f_{xy} = 0$

$r > 0$.

$rt - s^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4$

$= -4 < 0$

$f(x,y)$ at $(-1, 4)$

$(-1)^2 - (4)^2 + 2(-1) + 8(4) - 70$

$1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= \cancel{300}\ 47 - 70$

$= -33$

AK
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