TSP Art

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Abstract

Bosch and Herman recently described how to use the traveling salesman problem (TSP) to construct a continuous line drawing based on a user-supplied image. They create a distribution of "cities" that approximates the darkness of the source image, and pass the cities to a heuristic TSP solver. We discuss their method and present alternative algorithms for city distribution that yield more attractive line drawings.

1. Introduction

Halftoning is the process of reproducing a continuous tone image using only two colours. For example, when printing a black and white photograph in a newspaper, the two available colours are the white paper and the black ink. The halftoning process distributes small points of ink in such a way that from a distance, the eye is fooled into seeing continuous shades of gray.

The goal of halftoning is to communicate an image, and most techniques seek to minimize any visual effects that could distract from that goal. If the points of ink happen to line up into patterns, those patterns become noticeable and can obscure the original image.

From an artistic point of view, however, these patterns unlock an exciting medium where an image and an overt means of representing it can co-exist. Artistic Screening [3] is one example where small user-defined shapes take the place of the simple dots usually used in halftoning.

Recently, Bosch and Herman presented a halftoning technique that approximates a source image using a single closed path that never intersects itself. Their technique is based on the traveling salesman problem (TSP). Given a collection of points in the plane (the "cities"), the TSP asks for a closed path that visits every city and is as short as possible. An optimal TSP solution is guaranteed to be a simple, closed polygon. Finding the optimal solution is a well known to be intractable, but heuristic solvers can discover very high quality tours that are almost always simple. By choosing a distribution of cities whose density approximates the darkness of a source image, Bosch and Herman create a continuous line drawing that is a recognizable rendering of the image.

After describing Bosch and Herman's grid-based method for generating TSP instances in Section 2, we present two new methods motivated by stippling techniques in computer graphics. The first is based on Secord's weighted Voronoi stippling (Section 3). It can be seen as a direct extension of Bosch and Herman's work. The second is based on ordered dithering (Section 4). We also describe some simple stylizations that can be applied when rendering (Section 5) and conclude with ideas for future work (Section 6).

2. Bosch and Herman's grid-based method

Bosch and Herman introduced the notion of TSP art and presented a simple grid-based algorithm for distributing cities in a TSP instance [2].

Let a $w \times h$ image I be given as a two-dimensional grid of grayscale values $I_{i,j} \in [0, 1)$, where 0 represents black and 1 represents white. Choose an integer $k \geq 1$. Divide the image into a grid of $\lfloor \frac{w}{k} \rfloor \times \lfloor \frac{h}{k} \rfloor$ square cells k pixels on a side, and let $\mu_{i,j}$ be the mean grayscale value in the cell (i, j).

Choose an integer γ corresponding to the maximum desired number of cities in a cell. The actual number of cities $g_{i,j}$ in cell (i, j) is then computed from $\mu_{i,j}$ via the formula $g_{i,j} = \gamma - \lfloor (\gamma + 1)\mu_{i,j} \rfloor$. This formula splits the range $[0, 1)$ into $\gamma + 1$ equal intervals corresponding to all possible numbers of cities from zero to γ. The algorithm then places $g_{i,j}$ cities in cell (i, j), with locations chosen uniformly at random.

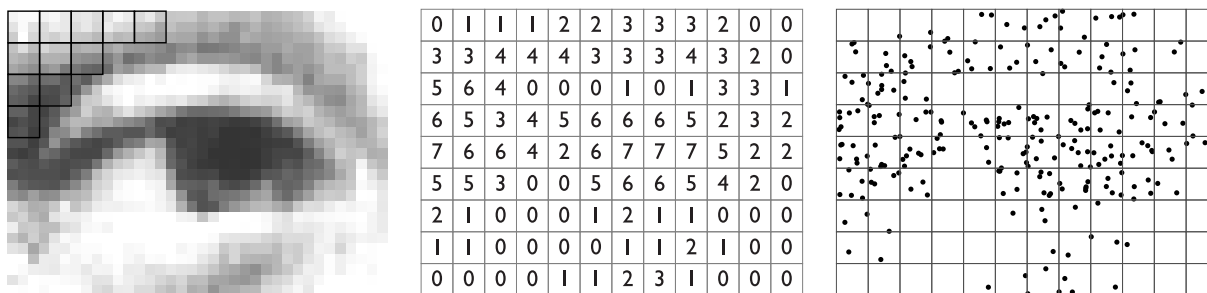| 0	1	1	1	2	2	3	3	3	2	0	0
3	3	4	4	4	3	3	3	4	3	2	0
5	6	4	0	0	0	1	0	1	3	3	1
6	5	3	4	5	6	6	6	5	2	3	2
7	6	6	4	2	6	7	7	7	5	2	2
5	5	3	0	0	5	6	6	5	4	2	0
2	1	0	0	0	1	2	1	1	0	0	0
1	1	0	0	0	0	1	1	2	1	0	0
0	0	0	0	1	1	2	3	1	0	0	0

Figure 1 Creating a TSP instance with the grid-based method of Section 2. The source image is shown on the left, with part of the sampling grid superimposed. The middle frame shows the number of cities computed for each cell. The final distribution of cities appears on the right.

Figure 1 illustrates the construction of a TSP instance on a fragment of da Vinci's Mona Lisa. The 36×27 sub-image on the left is broken into a grid of 12×9 cells, each containing 9 pixels. The $g_{i,j}$ values for the cells are shown in the middle, and one possible distribution of cities on the right. The cities are packed more closely where the source image is darker. With some judicious squinting, one can start to see the original image in the pointillist pattern on the right.

The resulting distribution of cities needs to be turned into a TSP tour. Bosch and Herman use the Lin-Kernighan heuristic solver from the Concorde TSP package by Applegate et al. [1]. It finds a tour that is probably not optimal, but that is almost always simple. The tour can then be rendered in Postscript.

Figures 2(b) and 3(a) show the effect of running Bosch and Herman's construction technique on a reference gradient image and on the Mona Lisa's face. In both cases we use $\gamma = 9$. Although the source image is clearly visible in both of these line drawings, a significant amount of contrast is lost. The placement of cities as given above ignores the uniform darkening and contrast reduction caused by the lines in the tour. We compensate for this loss of contrast by distributing $\lfloor \frac{1}{3} g_{i,j}^2 \rfloor$ cities in each cell instead of $g_{i,j}$. This compensation function was determined from measurement of a small set of sample drawings. The modified distribution produces better TSP drawings, as demonstrated in the gradient of Figure 2(c) and the Mona Lisa of Figure 3(b).

3. City placement via stippling

The first step in the approach of the previous section is to distribute points within a rectangle so that they approximate the tone of a source image. When seen from the point of view of an artist covering a page with small points of ink, this distribution is nothing more than a stippling technique. Artists and illustrators
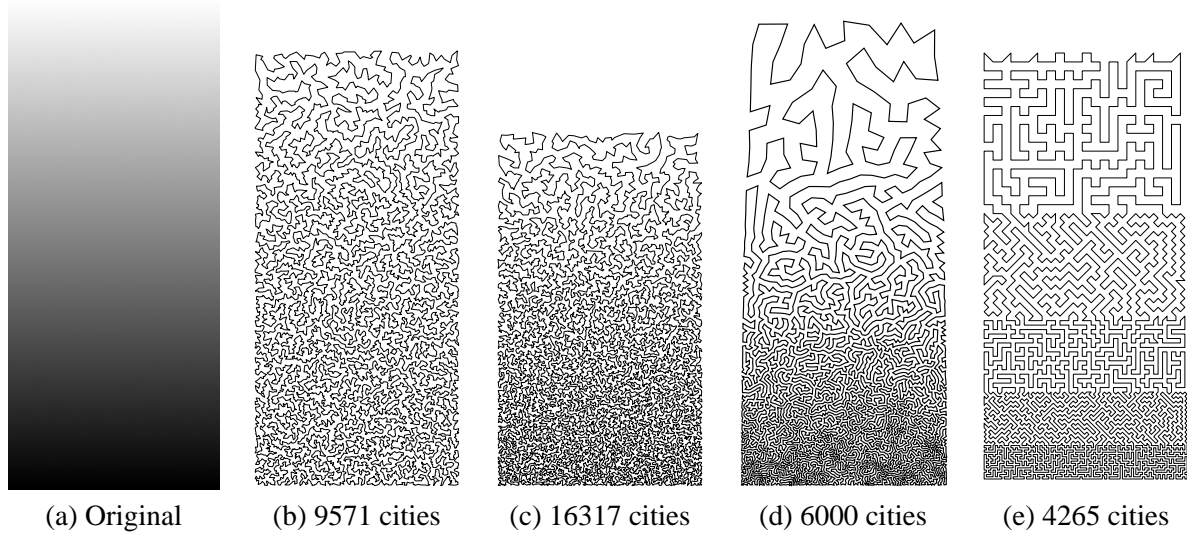
(a) Original   (b) 9571 cities   (c) 16317 cities   (d) 6000 cities   (e) 4265 cities

Figure 2 TSP tours computed for a gradient using various point distribution methods. The simple grid-based method of Section 2 is used in (b), and the improved version with higher contrast in (c). Weighted Voronoi stippling, explained in Section 3 is used in (d). Finally, (e) uses ordered dithering as in Section 4.
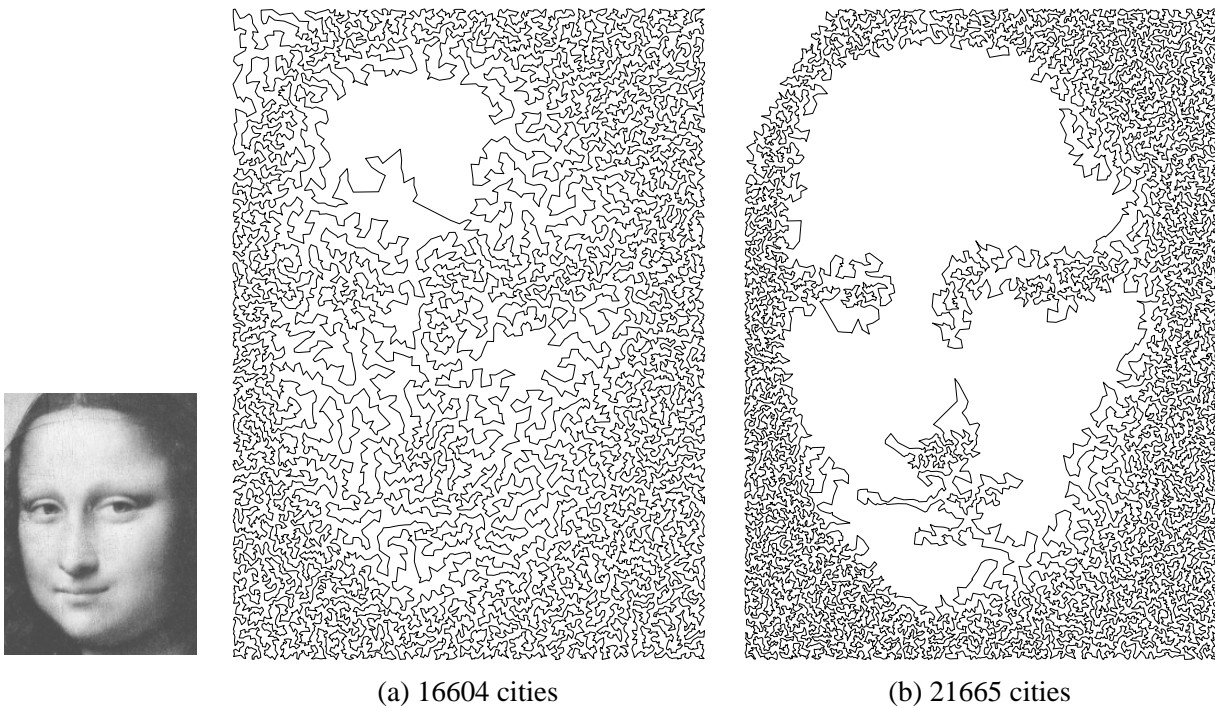

(a) 16604 cities   (b) 21665 cities

Figure 3 Mona Lisa drawn using grid-based city layouts as described in Section 2.

have used stippling for well over a century as a way to suggest continuous tone with a single colour of ink. Given that the grid-based point distribution is just a kind of stippling, it seems natural to ask whether other stippling algorithms could produce attractive TSP instances.

Several projects in computer graphics have undertaken to assist in the creation of stippled drawings.

(a) 10000 cities   (b) 10000 cities
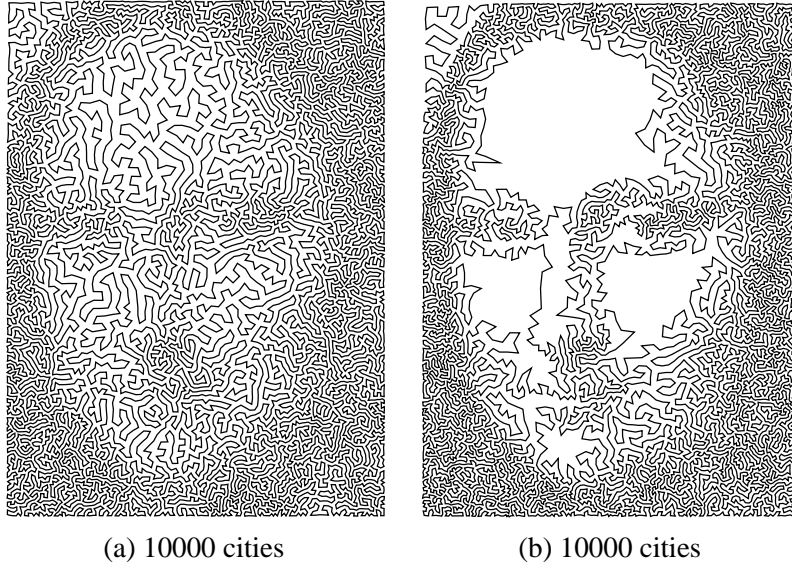
Figure 4 Mona Lisa drawn using stippled city layouts as described in Section 3. To produce the drawing on the right, the contrast of the source image was boosted before stippling.

Secord's weighted Voronoi stippling [4] is fast and automatic, and produces high-quality stippled drawings. He starts with an initial point distribution and improves it using an iterative relaxation process. At each step in the iteration, he computes the Voronoi diagram of the points, and moves each point to the centre of mass of its Voronoi region. The tone of the source image is used to assign non-uniform mass to space, causing points to migrate towards darker parts of the image.

Because the grid-based method distributes points uniformly, it produces unappealing stippled drawings. There is no control over the distance of points to their nearest neighbours. Close-together points will produce short, jagged edges in the TSP tour. The relaxation algorithm used in weighted Voronoi stippling will tend to space points out as evenly as possible while still approximating image tone.

But how do the TSP tours produced by these point distribution algorithms differ? Figures 2(d) and 4(a) show the gradient and Mona Lisa drawings with points distributed via weighted Voronoi stippling. The evenness of the points shows through in the resulting tour, which has a tidier, more organized appearance. Weighted Voronoi stippling avoids the short edges produced with the grid-based approach, meaning that there are no cities whose contribution to the final drawing is negligible. For the same reason, fewer cities are necessary with stippling than with the grid-based method to achieve drawings of comparable quality.

As with the grid-based method, the basic stippling technique will produce a distribution of cities that loses contrast in the final drawing. We cannot explicitly manipulate the local density of cities as before, so we boost the contrast of the image before passing it to the stippling algorithm. The algorithm is also very sensitive to the least amount of darkness in the image. We induce city-free regions by rounding a range of bright grays up to pure white. A line drawing created from an enhanced version of the Mona Lisa image is shown in Figure 4(b).

4. City placement via ordered dithering

A very different aesthetic can be achieved by arranging cities in a fixed pattern. We use an ordered dithering algorithm to obtain TSP imagery where the tour contains obvious, interesting patterns.
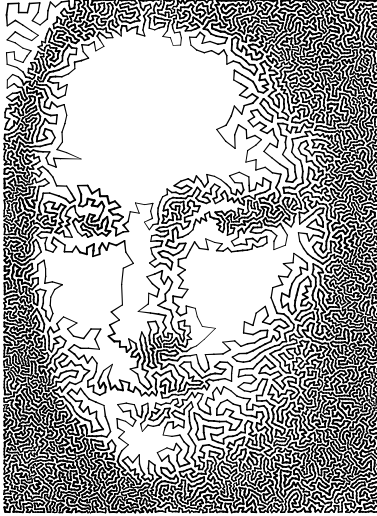
Figure 5 Alternative rendering styles for the drawing in Figure 4(b). On the left, line width is controlled by local density of cities. On the right, cities are interpreted as control points for a cubic B-spline curve.
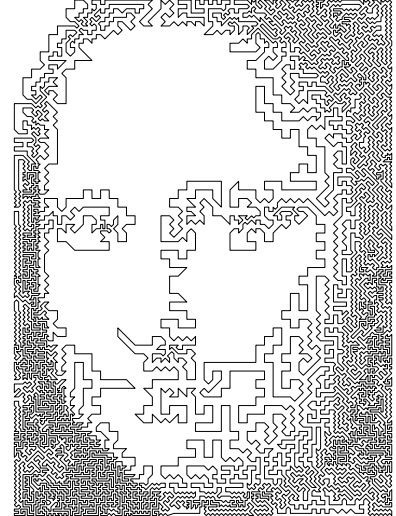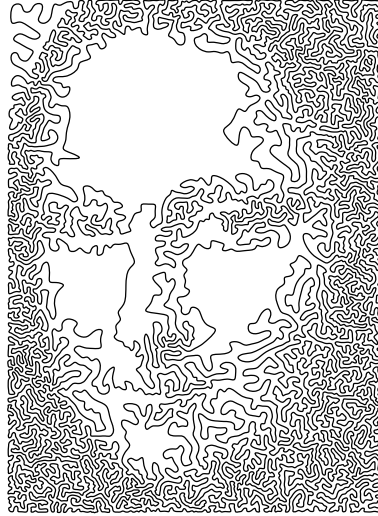
Figure 6 Mona Lisa dithered using the method of Section 4. The tour contains 10459 cities.

Dithering is a simple form of halftoning that works well on devices with fixed raster grids, such as computer monitors. Pixels are selectively set to black or white so that the overall gray level approximates the original image. Dithering can be seen as a specialized kind of stippling, where sample points must be chosen from a fixed lattice.

Ordered dithering begins like the grid-based point distribution method: the source image is divided into cells and the average darkness computed in each cell. But instead of placing points uniformly at random in the cells, ordered dithering uses a fixed set of points arranged in a grid pattern. For example, if we wish to place a number of points from zero to nine (corresponding to $\gamma = 9$ above), we might choose ten 3×3 matrices, each containing a different number of black pixels.

In practice, ordered dithering is frowned upon. Most dithering algorithms seek to suppress unwanted order in the pattern of black pixels, since it distracts the eye from the underlying image being represented. However, this order turns out to be interesting aesthetically, especially when used as a basis for TSP art. We use a 4×4 dithering pattern that has more structure than necessary: it only produces square grids of points, oriented horizontally and at 45 degrees. The dither matrices for our pattern are given in Figure 7. When turned into TSP art, the grids of points yield maze-like patterns at different sizes. A drawing of the gradient is shown in Figure 2(e), and one of the Mona Lisa is shown in Figure 6.

5. Rendering

Once the cities are placed and the tour is computed, the last step is simply to render the tour as a continuous closed path. We have investigated two modifications to this basic rendering style. The first is to treat the cities as B-spline control points and render the tour as a smooth curve. The resulting path still resembles the source image but has no sharp corners. The other modification is to vary the thickness of the line. When using weighted Voronoi stippling to place cities, we can tie the thickness of the line to the local density of the cities (which can easily be estimated from the size of each city's Voronoi region). Figure 5 shows the Mona Lisa rendered in these two styles.
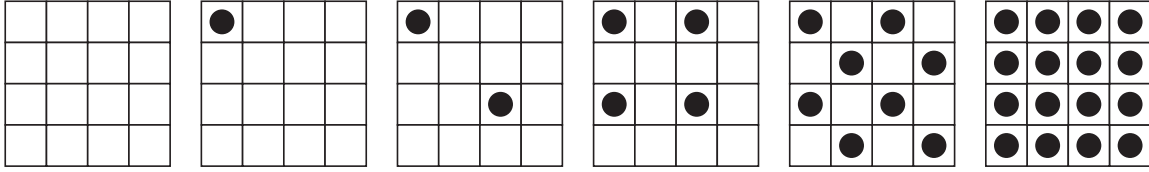
Figure 7 The dithering grid used for the city distribution method of Section 4.

6. Conclusions

TSP art relies on two optimizations that run sequentially. First, we place cities using some form of stippling. Then, we link the cities together using a heuristic solver for the traveling salesman problem. Weighted Voronoi stippling shows that the quality of the result is strongly influenced by the first of these optimizations. In what other ways does the quality of the optimizations affect the result? For example, given a fixed choice of cities, can a viewer detect at a glance that one tour is closer to optimal than another? That is, does geometric optimality have an aesthetic quality? We are also interested in whether the placement of cities and the TSP solver could somehow cooperate to produce better final images.

There are many ways to link cities together to form interesting paths. The TSP solver attempts to satisfy a global constraint of tour length. To satisfy that goal, the solver will tend to move from a city to a close neighbour. In terms of capturing the source image, this local property of taking small steps seems more important than the global property of minimizing the whole tour. If that is true, then many other paths besides TSP tours might be used to link points together into interesting patterns.

Figures 8 and 9 show a variety of examples drawn using the city distribution and rendering styles described in this paper.

TSP drawings resemble mazes. We ask whether the method presented here could be modified to produce mazes that reproduce the tone of a source image.

Acknowledgments

References

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[3] Victor Ostromoukhov and Roger D. Hersch. Artistic screening. In *SIGGRAPH '95: Proceedings of the 22nd annual conference on Computer graphics and interactive techniques*, pages 219–228. ACM Press, 1995.

[4] Adrian Secord. Weighted voronoi stippling. In *NPAR '02: Proceedings of the 2nd international symposium on Non-photorealistic animation and rendering*, pages 37–43. ACM Press, 2002.
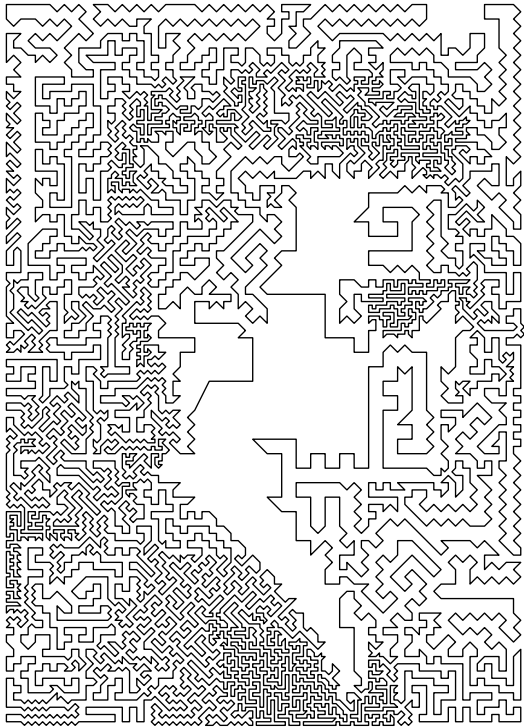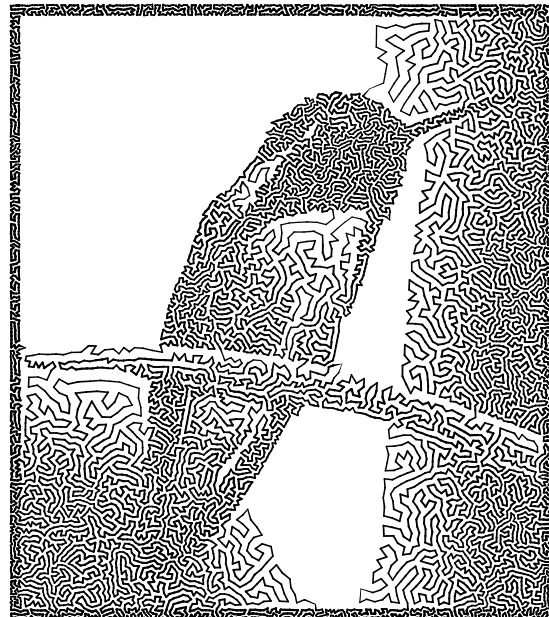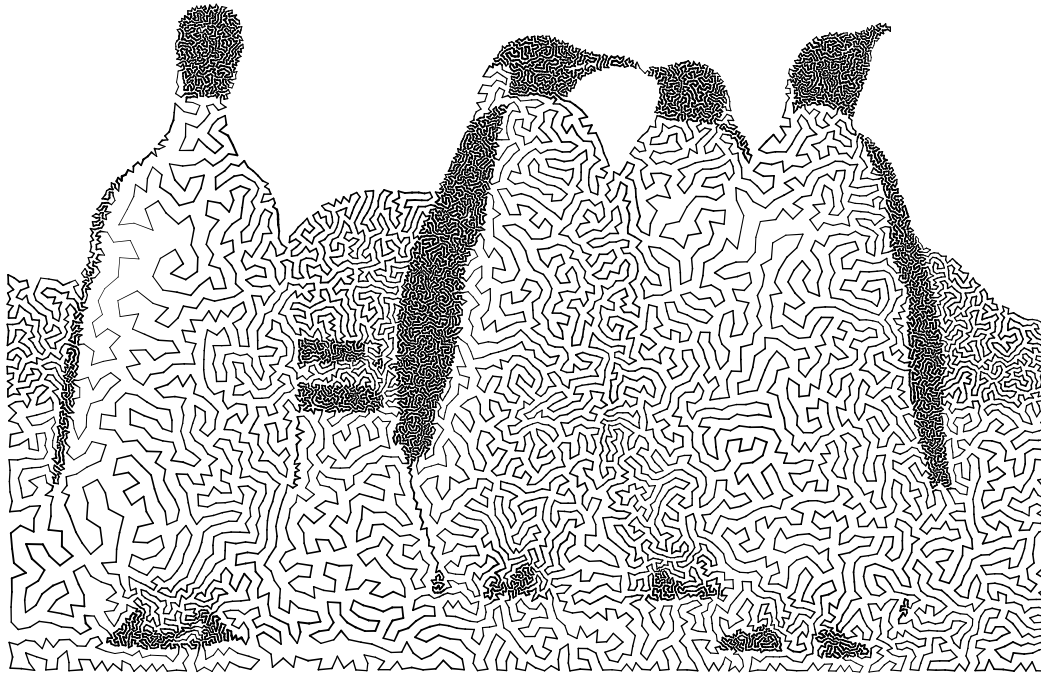
Chris (8408 cities)
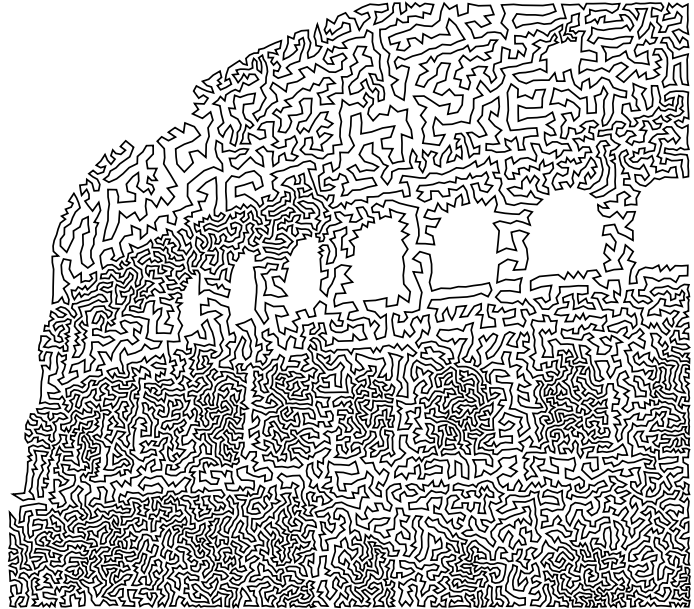
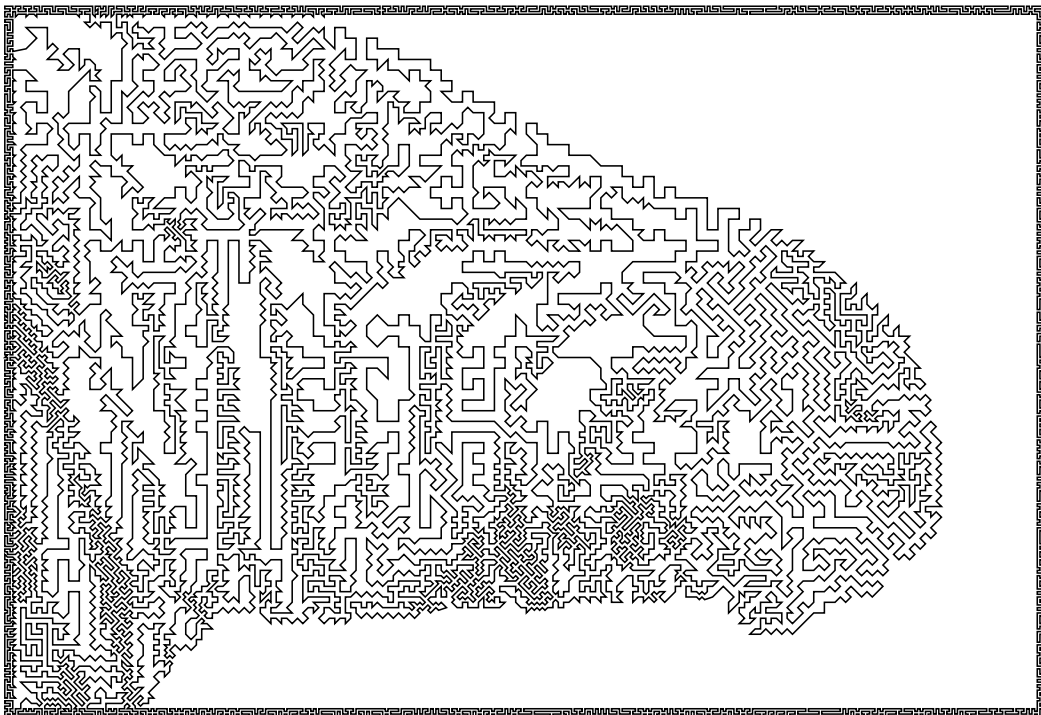Coliseum (11999 cities)
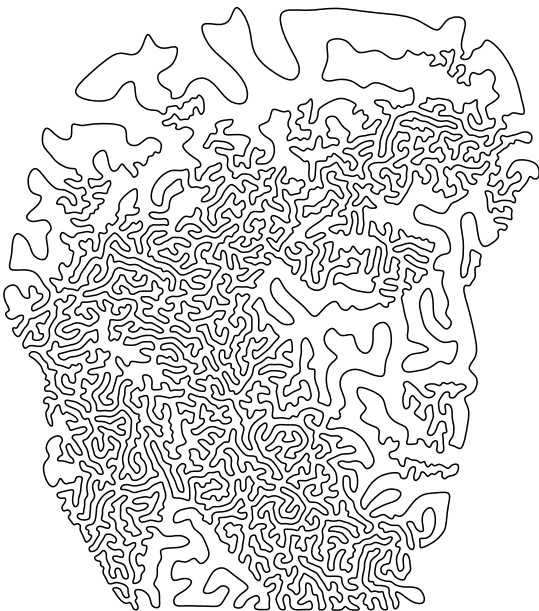
David (5000 cities)

Hummingbird (10000 cities)

Figure 8 Example drawings produced using the various distribution techniques and rendering styles described in the paper.

Penguins (15000 cities)

Zebra (14491 cities)

Figure 9 More example drawings.