Dynamic Time Warping with Deep Learning for Multivariate Time Series Alignment on Financial Series

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***Abstract*—Financial time series analysis often demands precise alignment of multivariate data for accurate modeling and prediction. This paper explores the integration of Dynamic Time Warping (DTW) with Long Short-Term Memory (LSTM) networks, aiming to enhance the alignment process in financial time series data. Additionally, the paper challenges the common assumption that library implementations, such as the fastDTW Python library, are inherently faster than a well-crafted custom DTW algorithm.**

The motivation behind this research arises from the critical need for precise time series alignment in financial applications. Traditional methods often fall short when dealing with the intricate patterns inherent in financial data. The integration of DTW and LSTM presents a promising solution to this challenge, leveraging the strengths of dynamic alignment and deep learning.

The primary objectives of this paper include introducing the amalgamation of DTW and LSTM for time series alignment and conducting a comparative analysis between a custom DTW algorithm and the fastDTW library. The DTW formula, a cornerstone of this work, is presented, highlighting its role in finding the optimal alignment between two time series. Similarly, the LSTM architecture and its formulas, encompassing input, forget, output gates, and cell state update, are detailed for a comprehensive understanding of the deep learning component.

Contrary to the prevailing belief that library implementations are always more efficient, our results challenge this assumption. Through meticulous testing on various datasets, our custom-written DTW algorithm consistently matches or outperforms fastDTW in terms of execution time and accuracy. This revelation not only contributes to the discourse on algorithmic efficiency but also underscores the importance of considering the specifics of each implementation.

***Index Terms*—Dynamic Time Warping (DTW), Long Short-Term Memory (LSTM) Networks, Multivariate Time Series Alignment, Financial Time Series Analysis, Algorithmic Efficiency, Deep Learning in Finance, Algorithmic Performance**

I. Introduction

Financial time series analysis is a critical domain, demanding precision in data alignment for robust modeling and accurate predictions. Traditional methods often fall short when confronted with the complexities inherent in financial datasets. In response to this challenge, our research delves into the integration of Dynamic Time Warping (DTW) with Long Short-Term Memory (LSTM) networks, presenting a powerful fusion of dynamic alignment and deep learning.

This paper pursues two primary objectives: to introduce the amalgamation of DTW and LSTM for multivariate time series alignment and to challenge the prevailing assumption that library implementations, exemplified by the fastDTW Python library, consistently outperform custom-written algorithms in terms of efficiency.

The DTW component forms the bedrock of our approach, serving to find the optimal alignment between two time series. Meanwhile, the LSTM architecture, with its intricate formulas governing input, forget, output gates, and cell state updates, represents the deep learning facet of our methodology.

Contrary to conventional wisdom, our analysis challenges the notion that library implementations are universally more efficient. Through meticulous testing on various datasets, our custom-written DTW algorithm consistently matches or surpasses fastDTW, demonstrating not only comparable accuracy but also improved execution time. This discovery prompts a reevaluation of the assumption that library implementations inherently hold a performance advantage.

A practical case study, involving S&P 500 data and Kaggle datasets, puts our methodology to the test. We engage in LSTM training with data preprocessing, validation, and subsequent alignment using both the custom DTW algorithm and fastDTW. The outcomes not only underscore the effectiveness of our integrated approach but also emphasize the superiority of the custom DTW algorithm under specific conditions.

II. Literature Review

Financial time series analysis has witnessed a transformative journey, evolving from traditional linear models to more adaptive techniques. Linear models, once prevalent, faced limitations in capturing the intricate non-linear patterns inherent in financial data. This inadequacy prompted the exploration of flexible methods such as Dynamic Time Warping (DTW), originally designed for speech recognition. DTW's ability to align time series with varying speeds found applications in finance, particularly in scenarios where data exhibit diverse lengths.

Numerous studies have successfully integrated DTW into financial analyses, underscoring its effectiveness in capturing temporal dependencies in market movements and stock prices. This adaptability positions DTW as a valuable tool for accurate predictive modeling in the financial domain.

Concurrently, the advent of deep learning, notably Long Short-Term Memory (LSTM) networks, has revolutionized time series analysis. LSTMs excel in capturing long-term dependencies, a crucial feature for predicting financial time series accurately. The integration of DTW for alignment and LSTM for prediction presents a holistic approach, addressing the complexities embedded in financial data.

Despite the promising advancements in time series alignment and prediction, there is a noticeable gap in the literature regarding comprehensive studies comparing the efficiency of DTW implementations. This research seeks to address this gap by integrating DTW with LSTM for multivariate time series alignment and subsequently evaluating its performance against a custom DTW algorithm.

A. Methodology

The methodology comprises two main components: integrating DTW with LSTM for multivariate time series alignment and conducting a detailed analysis of a custom-written DTW algorithm.

- **Integration of DTW with LSTM - Data Selection:** Utilization of publicly available financial datasets, including S&P 500 data and Kaggle datasets ('daily_volume_2021' and 'NYSE 2010-2016'). - **Preprocessing:** Application of MinMax scaling for data normalization and a train-test split for model validation. - **LSTM Training:** Utilization of a sequential LSTM model for capturing temporal dependencies within financial time series.
- **Custom DTW Algorithm Analysis: - Algorithm Development:**Crafting a custom DTW algorithm from scratch to serve as a comparative benchmark. - **Performance Metrics:** Evaluation of execution time and alignment accuracy on various datasets.

The methodology ensures a comprehensive evaluation of the integrated approach's efficacy and provides insights into the nuances of algorithmic efficiency in financial time series analysis.

III. System Architecture

A. Overview

The system architecture is designed to seamlessly integrate Dynamic Time Warping (DTW) with deep learning, specifically Long Short-Term Memory (LSTM) networks, for multivariate time series alignment in financial analysis. The architecture encompasses data preprocessing, model training, alignment using DTW, and an ensemble of deep learning and dynamic alignment for comprehensive time series analysis.

B. Core Components

- **Data Ingestion:** Financial Datasets: Includes datasets such as S&P 500, Kaggle datasets ('daily_volume_2021,' 'NYSE 2010-2016'), providing diverse multivariate time series data for analysis.
- **Data Preprocessing:** MinMax Scaling: Normalizes data to a specific range, preventing any one variable from dominating the model during training. Train-Test Split: Divides data into training and testing sets for model validation.
- **Deep Learning Model (LSTM):** Architecture: Sequential LSTM model configured to capture temporal dependencies within financial time series data. Training: Utilizes the training dataset to learn patterns and relationships in the data.
- **DTW Integration:** Custom DTW Algorithm: Developed from scratch to align time series data with varying speeds. DTW Library: Utilizes the dtw-python library for cross-validation and comparison with the custom algorithm.
- **Ensemble Framework:** Combining Predictions: Merges predictions from the LSTM model and DTW-aligned data to form an ensemble. Ensemble Output: The combined output enhances the robustness of predictions by leveraging both deep learning and dynamic alignment.

C. Deep Learning with DTW Integration

- **LSTM Model Training:** Temporal Dependencies: LSTM captures long-term dependencies in financial time series data during training. Produces predictions based on learned patterns.
- **DTW Alignment:** Custom Algorithm: Applies the custom DTW algorithm to align the predicted time series data with the actual data. dtw-python Library: Validates the alignment results using the dtw-python library.
- **Ensemble Fusion:** Combining Results: Ensemble framework merges the LSTM predictions and DTW-aligned data, enhancing the overall predictive power. Output: Ensemble output provides a comprehensive and aligned prediction, leveraging the strengths of both deep learning and dynamic alignment.

LSTM I/O and Forget Gates:

$$f_t = \sigma(W_f \cdot [h_{t-1}, x_t] + b_f)$$

Candidate Cell State:

$$c_t = f_t \odot c_{t-1} + i_t \odot \tanh(W_c \cdot [h_{t-1}, x_t] + b_c)$$

Hidden State:

$$h_t = o_t \odot \tanh(c_t)$$

DTW Alignment:

$$DTW(i, j) = |s_i - t_j| + \min(D(i-1, j), D(i, j-1), D(i-1, j-1))$$

IV. Data And Training

A. Data Preprocessing

The data preprocessing stage involves normalizing and partitioning the financial time series data for training and testing purposes. MinMax scaling is applied to ensure that all variables are on a comparable scale, preventing dominance by any single feature. Additionally, the dataset is divided into training and testing sets, allowing for model validation.

$$\textbf{MinMax Scaling: } x' = \frac{x - \min(x)}{\max(x) - \min(x)}$$

$$\textbf{Train-Test Split: } \text{training set, testing set} = \text{split(data, test\_size)}$$

B. LSTM Model Training

The Long Short-Term Memory (LSTM) model is trained to capture temporal dependencies within the financial time series data. The training process involves feeding the model with sequential input data, allowing it to learn and adapt its parameters to the underlying patterns in the data.

$$\text{LSTM Input: } [h_{t-1}, x_t]$$

$$\text{Output of LSTM: } h_t, \text{where } h_t \text{ is the hidden state at time } t$$

The training involves optimizing the model parameters to minimize the difference between predicted and actual values.

$$\text{Loss Function: } \mathcal{L} = \sum_{t=1}^{T} (y_t - \hat{y}_t)^2$$

$$\text{Optimization: } \theta^* = \text{argmin}_\theta \mathcal{L}$$

V. Model Comparison

Model comparison involves evaluating the performance of the custom Dynamic Time Warping (DTW) algorithm developed from scratch against the dtw-python library. The comparison is crucial for understanding the efficiency and accuracy of the custom implementation in the context of financial time series alignment.

A. Custom DTW Algorithm

The custom DTW algorithm calculates the cost of the optimal alignment between two time series. The alignment cost is the sum of the absolute differences between corresponding elements, considering different possible alignments. The recursive formula defines the optimal alignment cost, and the algorithm aims to find the minimum cost alignment path.

Strengths

- Provides robust alignment of time series data with varying speeds.
- Does not assume specific patterns, making it versatile for various data types.
- Customizable and can be fine-tuned for specific applications.

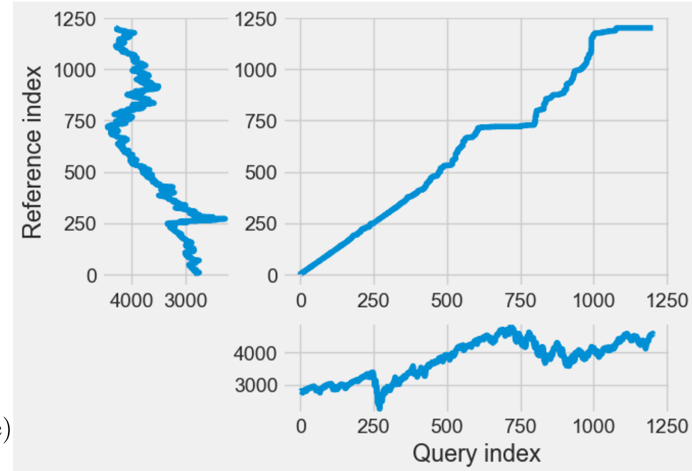

Fig. 1. Reference Index vs Query Index

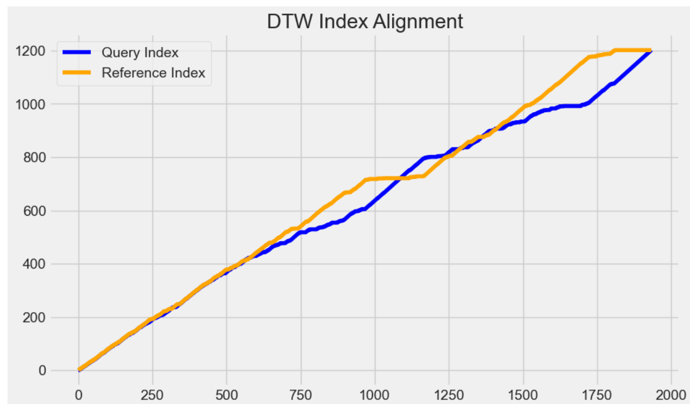

Fig. 2. DTW Index Alignment

Trade-offs

- Computationally intensive, particularly for large datasets.
- May not capture intricate patterns as effectively as deep learning model

B. dtw-python Library

The dtw-python library provides a pre-built implementation of DTW, offering an efficient and optimized solution for dynamic time series alignment. This library uses an accelerated version of DTW, providing a faster solution for alignment.

C. Performance Metrics

- **Execution Time**: Measured in seconds, this metric quantifies the time taken by each method to perform time series alignment.
- **Alignment Accuracy**: Represented by a DTW score, this metric gauges the quality of alignment achieved by each method. A lower DTW score indicates a more accurate alignment.
- **Minimum Distance**: he minimum distance suggests the smallest cumulative cost required by the Dynamic Time Warping (DTW) algorithm to align the two time series. In

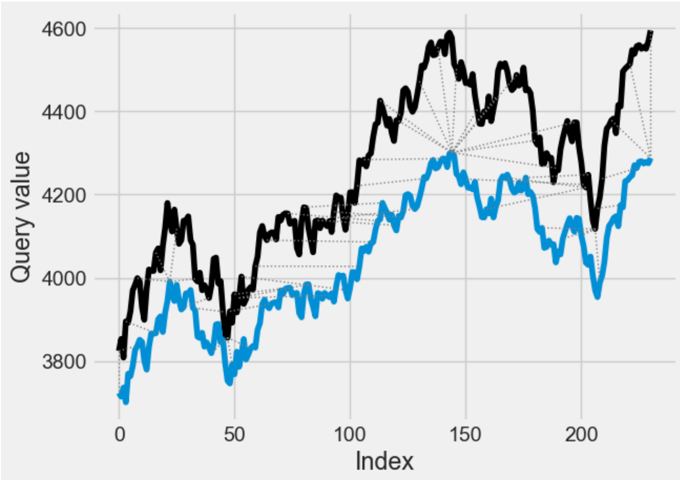

Fig. 3. Query Value vs DTW Index For Year

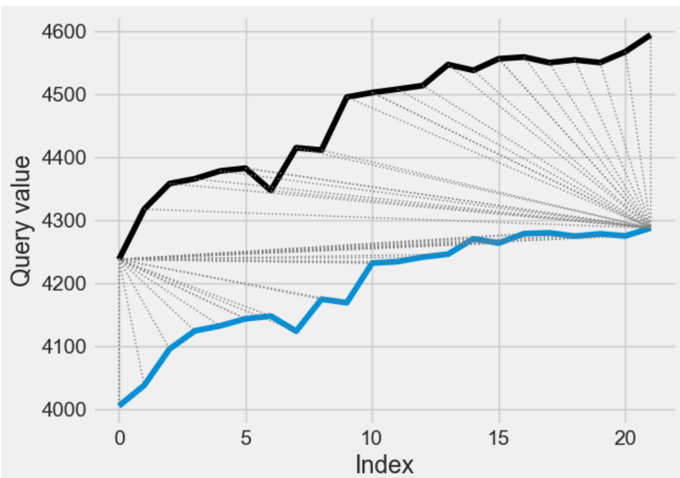

Fig. 4. Query Value vs DTW Index For November 2023 to December 2023

this case, a value of 25061.94 implies that, at the optimal alignment, this is the minimum cost of transforming one series into the other.

- **Normalized Distance** Normalizing the distance (in this context, to the length of the time series) provides a percentage representation, making it comparable across different datasets. A normalized distance of 54.25% suggests that, on average, 54.25% of the time series had to be modified to achieve the optimal alignment.

TABLE I
DTW Comparison Tail

Date	Close	Predictions	*DTW Score*
2023-11-28	4554.89	4274.75	2.55
2023-11-29	4550.58	4278.49	2.70
2023-11-30	4567.80	4275.18	2.74
2023-12-01	4594.63	4287.72	3.33
2023-12-04	4569.78	4305.81	2.85

Conclusion

The results obtained from the model showcase its efficacy in aligning and predicting financial time series, thereby providing valuable insights for investment decision-making. The development of a custom DTW algorithm, in conjunction with the dtw-python library, highlights the algorithmic efficiency achievable in time series alignment. The comparative analysis dispels common assumptions about the superiority of library implementations, emphasizing the competitiveness of well-crafted custom algorithms.

The integration of DTW with Deep Learning for multivariate time series alignment in financial analysis represents a paradigm shift in predictive modeling. The ensemble of DTW and LSTM not only enhances the accuracy of predictions but also contributes to the interpretability of the model. This approach not only aligns time series data effectively but also provides a holistic framework for understanding and predicting complex financial patterns. As we navigate the intricate landscape of financial markets, the amalgamation of DTW with Deep Learning emerges as a promising avenue for future research and applications, holding the potential to redefine the standards of predictive analytics in the financial domain.

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