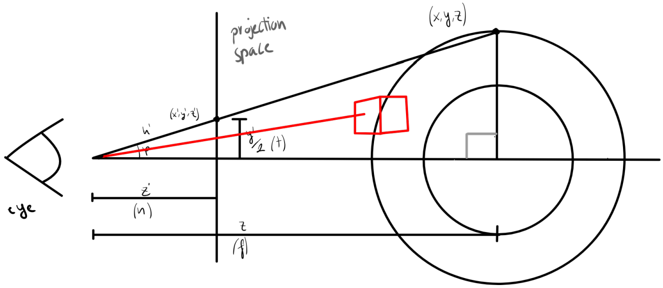
Projections

relative systems



$AR = \frac{y'}{x'}$

$\frac{y'}{z'} = \frac{y}{z} \Rightarrow y' = \frac{yz'}{z}$

$fov_y : 2\varphi$

$tg\varphi = \frac{y'}{2z'} = \frac{y}{2z}$

$\varphi(z') = arctg\left(\frac{y'}{2z'}\right)$

projections

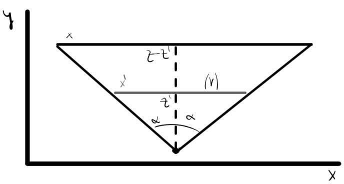
$$\begin{cases} y' = \frac{yz'}{z} \\ x' = \frac{xz'}{z} \end{cases}$$

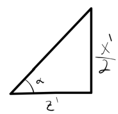
fov : α, y

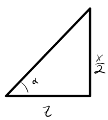
$t = \frac{y'}{2}$

$r = \frac{x'}{2}$

$$\begin{cases} \alpha_x = 2\alpha = 2\arctan\left(\frac{x'}{2z'}\right) \\ \varphi_y = 2\varphi = 2\arctan\left(\frac{y'}{2z'}\right) \\ z' = \frac{x'}{2tg2\alpha} = \frac{y'}{2tg2\varphi} \end{cases}$$







$$\left.\begin{aligned} tg\alpha &= \frac{x'}{2z'} \\ tg\varphi &= \frac{y'}{2z'} \end{aligned}\right\} z' = \frac{x'}{2tg\alpha} = \frac{y'}{2tg\varphi} \Rightarrow AR = \frac{tg\alpha}{tg\varphi}$$

$fov_x = 2\alpha$

$\alpha(z') = \arctan\left(\frac{x'}{2z'}\right)$

$tg\alpha = \frac{x'}{2z'} = \frac{x}{2z} \Rightarrow x' = \frac{xz'}{z}$

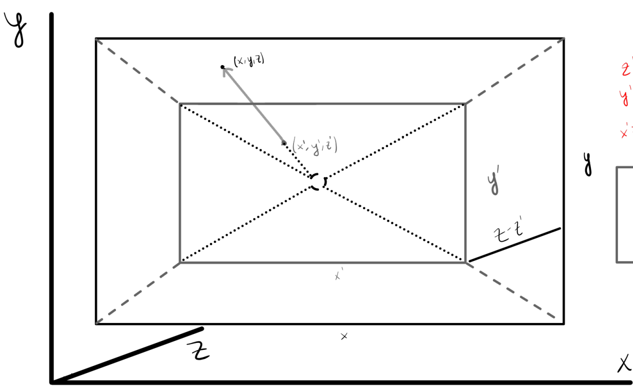
$$\left.\begin{aligned} AR &= \frac{x'}{y'} \\ y' &= 2z'tg\varphi \\ x' &= 2z'tg\alpha \end{aligned}\right\} \begin{aligned} &AR = \frac{x'}{2z'tg\varphi} \Rightarrow \frac{x'}{2z'} = ARtg\varphi \Rightarrow \\ &AR = \frac{2z'tg\alpha}{y'} \Rightarrow \frac{AR}{tg\alpha} = \frac{2z'}{y'} \Rightarrow \frac{y'}{2z'} = \frac{tg\alpha}{AR} \end{aligned}$$

$z'(\alpha) = \frac{x'}{2tg\alpha}$

$z'(\varphi) = \frac{y'}{2tg\varphi}$

$\alpha(z') = \arctan\left(ARtg\varphi\right) = \arctan\left(AR \cdot \frac{y'}{2z'}\right)$

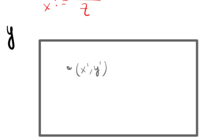
$\varphi(z') = \arctan\left(AR^{-1} tg\alpha\right) = \arctan\left(\frac{1}{AR} \cdot \frac{x'}{2z'}\right)$



$z' := K$ (const.)

$y' := \frac{K \cdot y}{z}$

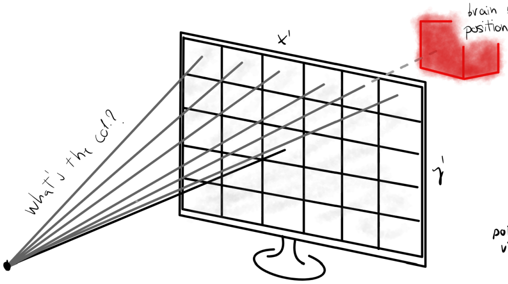
$x' := \frac{K \cdot x}{z}$

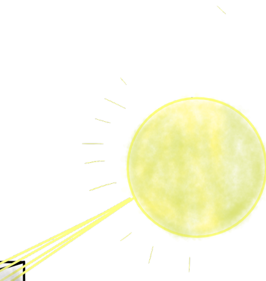


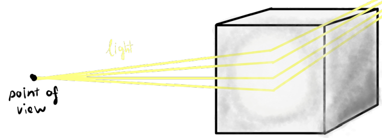
$(x', y') = \left(\frac{K \cdot x}{z}, \frac{K \cdot y}{z}\right)$

initial question

We perceive reflected light through the eyes and then our brain process those signals and create some sense of spatial position and depth along with the colour.







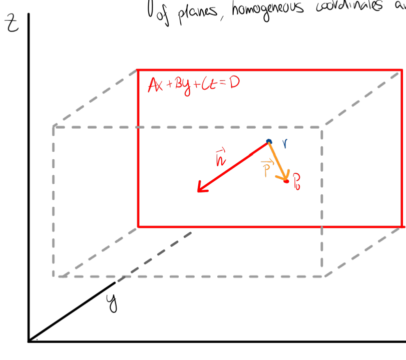
 pixel: 0-255

rgb (256^3 posible colors)

In the digital world we try to represent objects by computing continiously positions and perspectives and then showing that to the screen with its corresponding rgb value.

planes

first, let's take a look of the mathematical basis of planes, homogeneous coordinates and matrices.



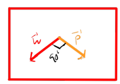
$\vec{n} = (A, B, C)$

$v = (v_x, v_y, v_t)$

$P_0(x_0, y_0, z_0)$

$\vec{p} = P_0 - v = (x_0 - v_x, y_0 - v_y, z_0 - v_t)$

$\vec{n} \perp \vec{p} \Rightarrow \vec{n} \cdot \vec{p} = 0$



$\vec{n} \cdot \vec{p} = 0$

$A\, p_x + B\, p_y + C\, p_t = 0$

$A(x_0 - v_x) + B(y_0 - v_y) + C(z_0 - v_t) = 0$

$Ax_0 - Av_x + By_0 - Bv_y + Cz_0 - Cv_t = 0$

$-Av_x - Bv_y - Cv_t + Ax_0 + By_0 + Ct_0 = 0$

$\underbrace{Cx_0 + Bx_0 + Cx_0}_{-D} = Av_x + Bv_y + Cv_t$

$\boxed{Av_x + Bv_y + Cv_t = -D}$

homogeneous coordinate system

allows matrix multiplipication (rotations, translations, ...) just by adding a new component w

$$\begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \Leftrightarrow \begin{pmatrix} x/w \\ y/w \\ z/w \end{pmatrix} \quad (w=1)$$

homogeneous vector — position vector

projection matrix

Start by considering the following system

$$p\begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} xz' \\ yz' \\ z^2 \\ z \end{pmatrix} \Leftrightarrow \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} : \begin{pmatrix} z' & 0 & 0 & 0 \\ 0 & z' & 0 & 0 \\ 0 & 0 & m_1 & m_2 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} xz' \\ yz' \\ z^2 \\ z \end{pmatrix}$$

$m_1 z + m_2 = z^2$

$-z^2 + m_1 z + m_2 = 0$

We get only 2 solutions, therefore we set 2 constraints: this is only true when $z = n$ or $z = f$

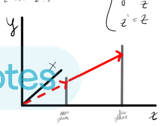
$(z = n)\ (z = f)$

$$\left.\begin{array}{l} m_1 n + m_2 = n^2 \\ m_1 f + m_2 = f^2 \end{array}\right\} \begin{array}{l} m_1 = f + n \\ m_2 = -fn \end{array}$$

We can't establish a linear relation between z and z', still works

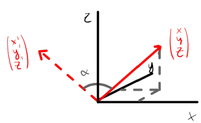
$$\left\{\begin{array}{l} x' = \frac{z'x}{z} \\ y' = \frac{z'y}{z} \\ z' = z \end{array}\right\}$$

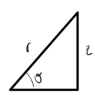
$$P = \begin{pmatrix} n & 0 & 0 & 0 \\ 0 & n & 0 & 0 \\ 0 & 0 & f+n & -fn \\ 0 & 0 & 1 & 0 \end{pmatrix}$$



matrices

· rotation matrix





example on z.

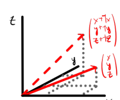
$$\left.\begin{array}{l} r = \sqrt{x^2 + z^2} \\ \theta = \arctan\left(\frac{z}{x}\right) \end{array}\right\} \begin{array}{l} x = r\cos\theta, \quad x' = r\cos(\theta + \alpha) \\ z = r\sin\theta, \quad z' = r\sin(\theta + \alpha) \end{array} \Bigg\} \begin{array}{l} \cos(\theta + \alpha) = \cos\theta\cos\alpha + \sin\theta\sin\alpha \\ \sin(\theta + \alpha) = \sin\theta\cos\alpha + \cos\theta\sin\alpha \end{array} \Bigg\} \begin{array}{l} x' = x\cos\alpha - y\sin\alpha \\ y' = x\sin\alpha - y\cos\alpha \end{array}$$

$$R_z = \begin{pmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad R_y = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix} \quad R_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{pmatrix}$$

$$R_z \cdot \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

· Translation matrix

$$T = \begin{pmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



· Scale matrix

$$S = \begin{pmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$