World Applied Programming, Vol (1), No (3), August 2011. 118-131
ISSN: 2222-2510

Solution To Multi-Depot Vehicle Routing Problem Using Genetic Algorithms

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Abstract: The Multi-Depot Vehicle Routing Problem (MDVRP), an extension of classical VRP, is a NP-hard problem for simultaneously determining the routes for several vehicles from multiple depots to a set of customers and then return to the same depot. The objective of the problem is to find routes for vehicles to service all the customers at a minimal cost in terms of number of routes and total travel distance, without violating the capacity and travel time constraints of the vehicles. The solution to the MDVRP, in this paper, is obtained through Genetic Algorithm (GA). The customers are grouped based on distance to their nearest depots and then routed with Clarke and Wright saving method. Further the routes are scheduled and optimized using GA. A set of five different Cordeau's benchmark instances (p01, p02, p03, p04, p06) from the online resource of University of Malaga, Spain were experimented using MATLAB R2008b software. The results were evaluated in terms of depot's route length, optimal route, optimal distance, computational time, average distance, and number of vehicles. Comparison of the experimental results with state-of-the-art techniques shows that the performance of GA is feasible and effective for solving the multi-depot vehicle routing problem.

Key word: Multi-Depot Vehicle Routing Problem, Grouping, Routing, Scheduling, Genetic Algorithm

I. Introduction

The challenging strategy in the field of supply chain management and logistics industry is to optimize the product delivery from suppliers to customers thus satisfying constraints. Such problems are known as Vehicle Routing Problems (VRP), in which the vehicles leave the depot, serve customers assigned and upon completion of their routes return to the depot. Each customer is characterized by their own demand. Since the problem is related with only one depot, the VRP is also named Single-depot VRP [1]. In cases with more than one depot, VRPs are known as multi-depot VRPs (MDVRP). Single-depot VRPs are not suitable for practical situations though they have attracted researchers in a wide sense. In MDVRP, since there are a large number of depots, it is a difficult task for decision makers to determine which customers are served by which depots without exceeding the capacity constraints. Hence grouping is performed to cluster customers based on distance between the customers and the depots, prior to the routing and scheduling phases. Moreover, since MDVRPs are NP hard, exact methods are not suitable to obtain optimal solutions. Thus heuristic algorithms have been adopted to solve the MDVRPs at a faster rate thus providing computationally efficient solutions. The objective of the problem is aimed at minimizing the total cost of combined routes for a fleet of vehicles. Since cost is associated with distance, in general, the goal is to minimize the distance travelled by applying the bio-inspired Genetic Algorithm (GA).

II. Literature Survey

This section briefs the existing work related to MDVRP solutions by various heuristic methods. Research on MDVRPs is quite limited compared with the extensive literature on simple VRPs and their variants. Salhi et Al., [2] addressed a multi-level composite heuristic with two reduction tests. The initial feasible solutions were constructed in the first level, while the intra-depot and the inter-depot routes were improved in the second and third levels. Wu et Al., [3] reports a simulated annealing (SA) heuristic for solving the multi-depot location routing problem (MDLRP). Giosa et Al., [4] developed a "cluster first, route second" strategy for the MDVRP with Time Windows (MDVRPTW), an extension of the MDVRP. Considering the operational nature of the MDVRPTW, this paper, focuses more on the computational time. Haghani et Al., [5] presented a formulation for solving the dynamic vehicle routing problem with time-dependent travel times using Genetic Algorithm. Nagy et al. [6] proposed several enhancements to an integrated heuristic method for solving the MDVRP. Lee et al. [7]

handled the MDVRP by formulating the problem as deterministic dynamic programming (DP) with finite-state and action spaces, and then using a shortest path heuristic search algorithm. Creviera et Al., [8] proposed a heuristic combining tabu search method, and integer programming for multi-depot vehicle routing problem in which vehicles may be replenished at intermediate depots along their route.

Jeon et Al., [9] suggested a hybrid genetic algorithm (HGA) for MDVRP, which considers the improvement of generation for an initial solution, three different heuristic processes, and a float mutation rate for escaping from the local solution in order to find the best solution. Inorder to solve the MDVRP efficiently, two hybrid genetic algorithms (HGA1 and HGA2) were developed by Ho et Al [10]. Chen et Al [11] developed a hybrid genetic algorithm (GA) with simulated annealing for solving the MDVRP. Since the MDVRP integrates three hard optimization problems, three improvement heuristic techniques were introduced by Mirabi et Al [12]. These techniques outperformed Giosa's [4] method. In [13], Lau etAl., considered the cost due to the total traveling distance, and the cost due to the total traveling time for solving the MDVRP. They employed a stochastic search technique called fuzzy logic guided genetic algorithms (FLGA) to solve the problem. GA is effective to acquire the optimal or near-optimal solution in solving optimization problems but it has two major problems: (i) slow search speed and (ii) premature convergence. In this paper, we develop a solution for the MDVRP based on GA with best cost route crossover.

The report is organized as follows: Section III describes the MDVRP with an example and the mathematical model. The step by step procedure of implementing MDVRP using GA is explained in Section IV. The computational results for the benchmark instances are analysed in Section V. Section VI draws the conclusion and future scope of the application in this paper.

III. MDVRP MODEL

In a MDVRP, the number and locations of the depots are predetermined. Each depot is large enough to store all the products ordered by the customers. Each vehicle starts and finishes at the same depot. The location and demand of each customer is also known in advance and each customer is visited by a vehicle exactly once. Fig. 1 shows an example of the MDVRP with 2 depots and 10 customers. Since there are additional depots for storing the products, the decision makers have to determine depots through which the customers are served [10]. The decision making stages are classified into grouping, routing, scheduling and optimization as shown in Fig. 2. In grouping, customers are clustered based on distance between customers and depots. In the example, customers 1,5,9,4,8 are assigned to depot A while customers 7,10,3,6,2 are assigned to depot B.

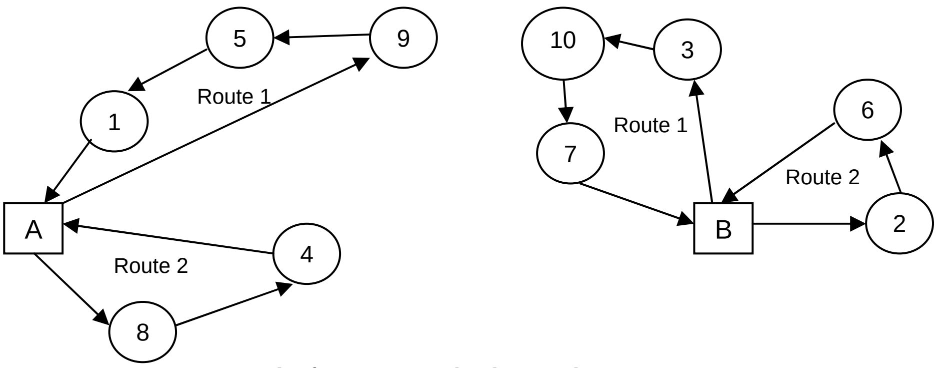

Figure 1. Example of an MDVRP with 2 depots and 10 customers

The customers of the same depot are assigned to several routes in the routing phase by Clarke and Wright saving method and each route is sequenced in the scheduling phase. The aim of routing is to minimize the number of routes without violating the capacity constraints. Since there are two depots the minimum number of routes can be limited to two. More number of routes increase the number of vehicles required thus reducing the quality of solutions. In depot A, customers 1,5,9 are in the first route, while customers 4 and 8 are served in the second route. Better routing and scheduling can result in shorter delivery distance, shorter time spent in serving all customers, higher level of efficiency and lower delivery cost. In general, the objective of the MDVRP is to minimize the total delivery distance or time spent in serving all customers. Lesser the delivery time, higher the customer satisfaction. Fewer vehicles mean that the total operation cost is less, thus the objective can also be minimizing the number of vehicles. Though there may be several objectives, the aim of MDVRP is to increase the efficiency of delivery.

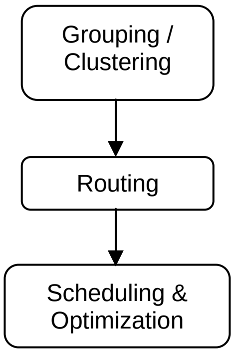

Figure 2. Decision making in MDVRP

The MDVRP is formulated with the objective of forming a sequence of customers on each vehicle route. The time required to travel between customers along with the depot and demands are known in advance. It is assumed that all vehicles have the same capacity, and each vehicle starts its travel from a depot, upon completion of service to customers, it has to return to the depot. The notations used and the mathematical model are as follows:

Sets:
I – Set of all depots
J – Set of all customers
K – Set of all vehicles

Indices:
i – depot index
j – customer index
k – route index

Parameters:
N – Number of vehicles
C_{ij} – Distance between point i and j, $i, j \in I \cup J$
V_i – Maximum throughput at depot i
d_i – Demand of customer j
Q_k – Capacity of vehicle (route) k

Decision variables:

$$x_{ijk} = \begin{cases} 1, if\ i\ immediately\ preceeds\ j\ on\ route\ k \\ 0, otherwise \end{cases}$$

$$z_{ij} = \begin{cases} 1, if\ customer\ j\ is\ allotted\ to\ depot\ i \\ 0, otherwise \end{cases}$$

U_{lk} – auxiliary variable for sub-tour elimination constraints in route k

Mathematical model

The objective function is to minimize the total distance of all vehicles given by Eqn. (1),

$$\min \sum_{i \in I \cup J} \sum_{j \in I \cup J} \sum_{k \in K} C_{ij} x_{ijk} \tag{1}$$

Each customer has to be assigned a single route according to Eqn. (2),

$$\sum_{k \in K} \sum_{i \in I \cup J} x_{ijk} = 1, j \in J \tag{2}$$

The capacity constraint for a set of vehicles is given by Eqn. (3),

$$\sum_{j \in J} d_j \sum_{i \in I \cup J} x_{ijk} \leq Q_k, k \in K \tag{3}$$

Eqn. (4) gives the new sub-tour elimination constraint set as,

$$U_{lk} - U_{jk} + N x_{ijk} \leq N - 1, \quad l, j \in J, k \in K \tag{4}$$

The flow conservation constraints are expressed as in Eqn. (5),

$$\sum_{j \in I \cup J} x_{ijk} - \sum_{j \in I \cup J} x_{jik} = 0, \quad k \in K, i \in I \cup J \tag{5}$$

Each route can be served atmost once according to Eqn. (6),

$$\sum_{i\in I}\sum_{j\in J} x_{ijk} \le 1, \quad k \in K \tag{6}$$

The capacity constraints for the depots are given in Eqn. (7) as,

$$\sum_{j\in J} d_i z_{ij} \le V_i, \quad i \in I \tag{7}$$

Constraints in Eq. (8) specify that a customer can be assigned to a depot only if there is a route from that depot going through that customer,

$$-z_{ij} + \sum_{u\in I\cup J}(x_{iuk} + x_{ujk}) \le 1, i \in I, j \in J, k \in K \tag{8}$$

The binary requirements on the decision variables are given by Eqns. (9) and (10)

$$x_{ijk} \in \{0,1\}, i \in I, j \in J, k \in K \tag{9}$$

$$z_{ij} \in \{0,1\}, i \in I, j \in J \tag{10}$$

The positive values of the auxiliary variable is defined in Eqn. (11) as,

$$U_{lk} \ge 0, l \in J, k \in K \tag{11}$$

IV. Genetic Algorithms

Genetic Algorithms (GA) is based on a parallel search mechanism, which makes it more efficient than other classical optimization techniques such as branch and bound, tabu search method and simulated annealing. The basic idea of GA is to maintain a population of candidate solutions that evolves under selective pressure. The GA can avoid getting trapped in a local optimum by tuning the genetic operators, crossover and mutation. Due to its high potential for global optimization, GA has received great attention in solving multi-depot vehicle routing problems. GA imitates the mechanism of natural selection and the survival of the fittest as witnessed in natural evolution. The general scheme of the genetic algorithm for MDVRP is explained in this section.

A. Chromosome Representation

The chromosomes for the solution of the MDVRP are encoded using path representation, in which the customers are listed in the order in which they are visited. In the example shown in Fig. 3, there are 6 customers designated 1-6. If the path representation for this instance is (0 2 4 1 0 3 6 5 0), then two routes are required by the vehicles to serve all the customers. The first route starts from the depot at 0 and travels to customers 2, 4 and 1, upon serving the vehicle returns back to the depot. Similarly, the second route starts from depot at 0, services customers 3, 6, 5 and returns to the depot. For *n* depots in the MDVRP, the chromosome consists of *n* links. The feasible solution for the optimization process is generated in three basic steps: Grouping, Routing and Scheduling.

Grouping – In this stage, the customers are assigned to each of the n links. The objective of the MDVRP is to minimize the total delivery time and hence customers are assigned to the nearest depots. In the example, there are two depots A and B, each customer c_i has to be assigned to a single depot exactly. This process of grouping is done based on the distance computation according to the following rule:

- If D(c_i,A) < D(c_i,B), then customer c_i is assigned to depot A
- If D(c_i,A) > D(c_i,B), then customer c_i is assigned to depot B
- If D(c_i,A) = D(c_i,B), then customer c_i is assigned to a depot chosen arbitrarily between A and B

In the above cases, $D(c_i,k) = \sqrt{(x_{c_i} - x_k)^2 + (y_{c_i} - y_k)^2}$, represents the distance between customer c_i and depot *k*.

Routing – The customers in the same link are assigned to several routes using Clarke and Wright Saving [14] method. The is based on the distance travelled by the vehicles for serving the customers. A saving matrix $S(c_i,c_j)$ is constructed for every two customers *i* and *j* in the same link. Further, the customers with large saving value are grouped in the same route without violating the capacity constraints. The saving matrix is constructed as,

$$S(c_i,c_j) = D(k,c_i) + D(k,c_j) - D(c_i,c_j) \tag{12}$$

Scheduling – Starting from the first customer, the delivery sequence is chosen such that the next customer is as close as to the previous customer. This process is repeated until all the unselected customers are sequenced. At the end of the scheduling phase, a feasible solution of the MDVRP example problem (Fig. 1) is constructed as shown in Fig. 3.

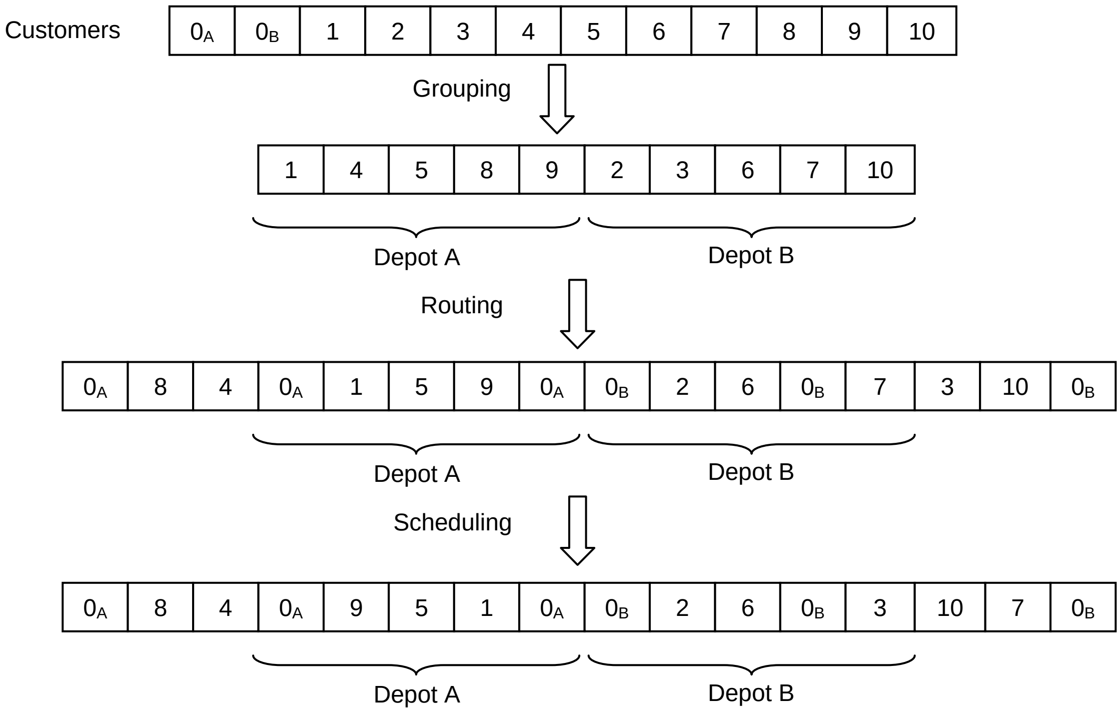

Figure 3. Chromosome representation and initial population

B. Fitness Evaluation

For the MDVRP, the objective function is to minimize the maximum delivery time spent among *n* depots. The delivery operations start at the same time in every depot, while it takes different time to complete serving the assigned customers. Some vehicles belonging to a depot may complete the delivery faster while other vehicles may complete their task in a shorter time. Thus the longest one among *n* depots is dominating time required to deliver all products to all customers. Let D_t be the total delivery time required by a depot *k* and let $\min(D_t)$ represent the minimum delivery time spent by all *n* depots, then

$$D_t = \sum_{k=1}^{m_k} \left[d[c(m_c), c(0)] + \sum_{i=1}^{m_c} d[c(i-1), c(i)] \right] \tag{13}$$

Where $d(a,b) = \dfrac{\sqrt{(x_b - x_a)^2 + (y_b - y_a)^2}}{V}$ is the travel time of a vehicle from customer a to b. V is the speed of the vehicle, *c(i)* is the location of the i^{th} customer, *c(0)* is the initial position of the depot, m_c is the number of customers in route *r*, m_k is the number of routes in depot *k*. Thus the fitness function is defined as

$$F = \sum_{p=1}^{psize} \min(D_t).$$

C. Selection

During each generation, the parents are selected for mating and reproduction. In this MDVRP application, we use tournament selection [15] to generate new individuals in the population. This selection strategy is based on fitness evaluation. The selection procedure is as follows:

Step 1: Select a set of *g* individuals from the population in a random manner to form the tournament set
Step 2: Choose a random number r_n in the range {0,1}
Step 3: If r_n < *threshold* select the fittest individual from the tournament set for reproduction
Else choose any two chromosomes in random from the tournament set for reproduction

Step 4: Apply elitism to guarantee that the best individuals are selected

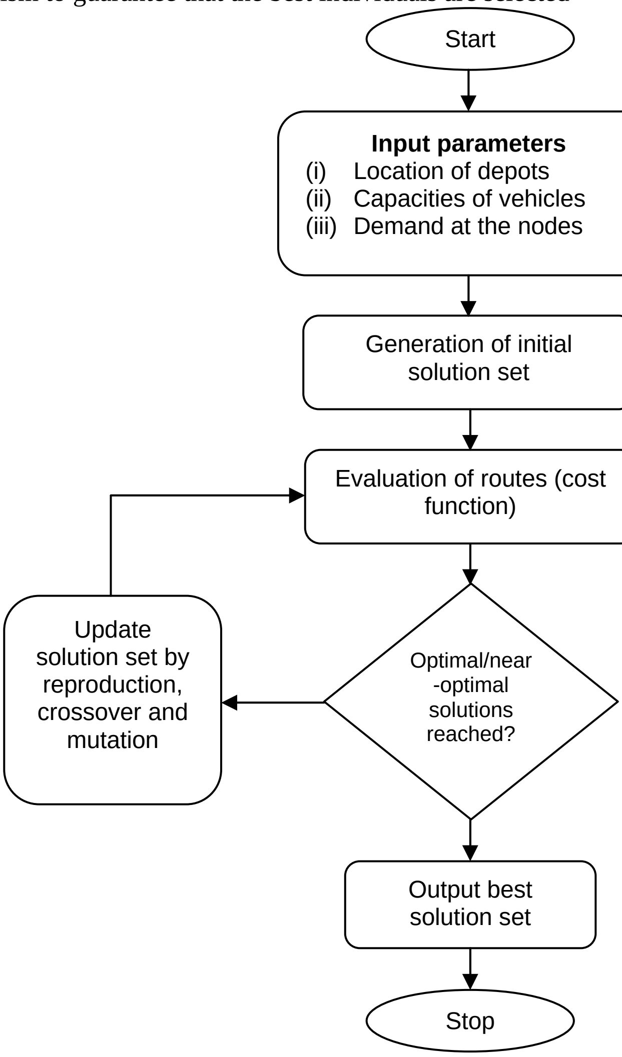

Figure 4. Flowchart of GA for MDVRP

D. Crossover

A problem specific crossover technique, the Best Cost Route Crossover (BCRC) developed by [16], for vehicle routing problem with time windows (VRPTW) is applied in this work for MDVRP with slight improvements. The steps involved in BCRC are shown below:

Step 1: Choose the parents from tournament selection
Step 2: Select a route from each parent in a random manner
Step 3: Remove all customers belonging to route 1 from parent 1
Remove all customers belonging to route 1 from parent 1
Step 4: For every customer belonging to route 1

- Compute the cost of insertion of route 1 into each location of parent 2 and store the costs in an ordered list.
- For each insertion location, check whether the insertion is feasible or not
- Generate a random number $r_n \in [0,1]$
- Choose the first feasible insertion location if r_n < *threshold*
- Else if if r_n > *threshold* choose the first entry in the ordered list, despite the feasibility

Step 5: Repeat Step 4 for customer belonging to route 2

E. Mutation

The inversion mutation is used in finding an MDVRP solution using GA. A substring is selected from the parent in a random manner and flips to form an offspring. The inversion mutation works on only one chromosome. The inversion operator is a mutation operation, which is used to increase the diversity of the population rather than to enhance the quality of the population. The general flowchart showing the implementation of MDVRP using GA is shown in Fig. 4.

V. COMPUTATIONAL RESULTS

The GA code was implemented in MATLAB R2008b on Intel core 2 Duo (1.73GHz), 3GB RAM PC. The performance of the MDVRP is evaluated using set of five Cordeau's instances namely p01, p02, p03, p04 and p06 taken from http://neo.lcc.uma.es/radi-aeb/WebVRP/ online resource of University of Malaga, Spain. The specifications of five Cordeau's instances such as p01 (4 depots and 50 customers), p02 (4 depots and 50 customers), p03 (5 depots and 75 customers), p04 (2 depots and 100 customers) and p06 (3 depots and 100 customers) are shown in Table I.

TABLE I. SPECIFICATIONS OF P01, P02, P03, P04 AND P06 CORDEAU'S INSTANCES

Parameters / Instances	P01	P02	P03	P04	P06
Total number of Customers	50	50	75	100	100
Total number of Depots	4	4	5	2	3
Total Number of Vehicles	32	20	35	24	30
Number of Vehicles in each Depot	8	5	7	12	10
Capacity of each Vehicle	80	100	140	100	100

TABLE II. ASSIGNMENT OF CUSTOMERS TO DEPOT FOR BENCHMARK INSTANCES

Depot	Customers Allotted	No. of customers
Problem instance: p01		
A(20,20)	4-13-17-18-19-25-40-41-42-44-45	11
B(30,40)	5-6-7-10-12-14-15-23-24-27-33-37-38-39-43-46-47-48-49	19
C(50,30)	1-2-8-9-11-16-21-22-26-28-29-30-31-32-34-50	16
D(60,50)	3-20-35-36	4
Problem instance: p02		
A(20,20)	4-13-17-18-19-25-40-41-42-44-45	11
B(30,40)	5-6-7-10-12-14-15-23-24-27-33-37-38-39-43-46-47-48-49	19
C(50,30)	1-2-8-9-11-16-21-22-26-28-29-30-31-32-34-50	16
D(60,50)	3-20-35-36	4
Problem instance: p03		
A(40,40)	3-4-5-6-9-12-15-17-18-20-25-26-27-29-30-32-34-37-39-40-44-45-47-48-50-51-55-60-67-68-70-75	32
B(50,22)	7- 8-13-35-46-52-57-58-72	9
C(55,55)	10-11-14-19-31-38-53-54-59-65-66	11
D(25,45)	2-16-21-24-28-33-36-49-62-63-69-71-73-74	14
E(20,20)	1-22-23-41-42-43-56-61-64	9
Problem instance: p04		
A(15,35)	2-5-6-7-8-13-14-15-16-17-18-21-22-23-36-37-38-40-41-42-43-44-45-46-47-48-52-53-56-57-58-59-60-61-72-73-74-75-82-83-84-85-86-87-89-91-92-93-94-95-96-97-98-99-100	55
B(35,55)	1-3-4-9-10-11-12-19-20-24-25-26-27-28-29-30-31-32-33-34- 35-39-49-50-51-54-55-62-63-64-65-66-67-68-69-70-71-76-77-78-79-80-81-88-90	45
Problem instance: p06		
A(15,20)	5-14-15-16-17-37-38-41-42-43-44-45-46-57-59-61-84-85-86-87- 91-92-93-95-96-97-98-99-100	29
B(50,20)	1-3-9-10-11-12-20-24-25-29-30-32-33-34-35-50-51-54-55-63- 64-65-66-68-70-71-76-77-78-79-80-81-90	33
C(35,35)	2-4-6-7-8-13-18-19-21-22-23-26-27-28-31-36-39-40-47-48-49-52-53-56-58-60-62-67-69-72-73-74-75-82-83-88-89-94	38

A. Grouping

In grouping, the customers are assigned to the adjacent depots so that the distance travelled by the vehicle is shorter. The customers are clustered based on the minimum distance between customers and depots. The Euclidean distance between the customer and the depot is computed, and based on the minimum distance, the Cordeau's instances p01, p02, p03, p04, p05 are grouped and the results are shown in Table II.

Fig. 5 indicates the initial customers and depot locations for the benchmark instance p04. The blue bullet indicates the location of customers (100 customers) and the red bullet represents the depot's location (2 depots). The grouping assignment for p04 is performed using the Euclidean distance formula and the resulting output is shown in Fig. 6. The customers are grouped such that they can either be served by depot A or depot B. The red bullet indicates customers assigned to depot A and blue bullet shows customers assigned to depot B.

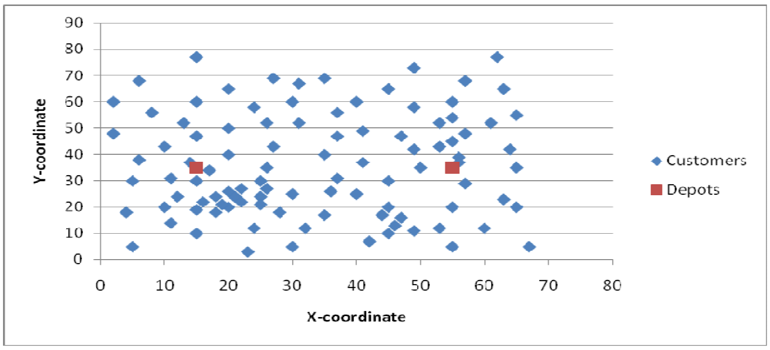

Figure 5. Initial Customer and Depots Locations for p04

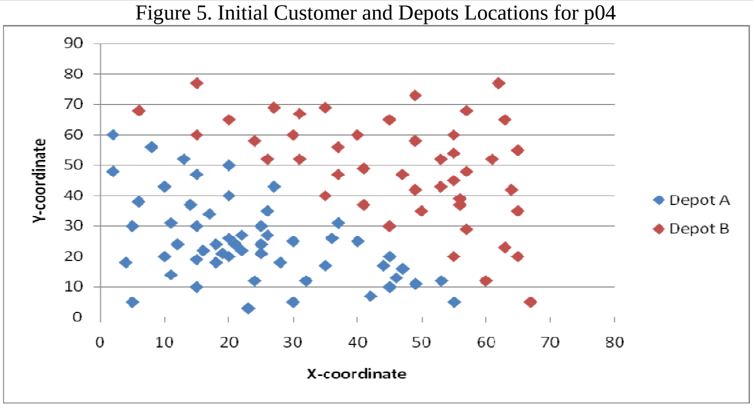

Figure 6. Grouping Assignment for p04

B. Routing

In routing phase, the customers in each group are divided into different routes. The aim of routing is to minimize the number of routes, or vehicles used, while not violating the vehicle capacity constraint. The Clark and Wright saving method is proposed to solve the routing. In this work, parallel version is used to compute the routes at a faster rate. It is worth noting that the number of routes may be reduced during the process of the parallel algorithm. For example, the two routes 0-1-2-0 and 0-3-4-0 will be combined into one route if the connection from depot A to B is established; in that case the resulting route becomes 0-1-2-3-4-0. Fully loaded percentage (FLP) is defined as the number of products loaded in each vehicle in every depot and is calculated according to Eqn. (14)

$$FL = \frac{Q - LQ}{Q} \times 100 \tag{14}$$

Where FL indicates the fully loaded percentage of each vehicle, Q stands for capacity of each vehicle, LQ represents the loading quantity of each vehicle during the distribution process. In the routing phase, the route of vehicles, fully loaded percentage, distance and the number of vehicles served by a depot are computed and the results are tabulated. The route allocation of customers for instance p01 using the Clark and Wright saving method is shown in Table III. The depots along with their coordinates are specified, and the set of routes followed by the customers with the distance is tabulated. It can be observed that for depot B located at (50, 22), there are 6 set of routes generated for 6 different vehicles. The total distance for the assigned vehicles to start from the depot B, serve all the customers and return to the depot B is 523.3 km.

TABLE III. ROUTE ALLOCATION OF CUSTOMERS FOR P01

Depot	Route	FLP (%)	Distance (km)	No. Vehicles
A(40,40)	A-4-13-17-A	85.85	57.65	5
	A-18-19-A	78.69	42.42	
	A-25-40-A	86.82	41.3	
	A-41-42-44-A	92.5	68.56	
	A-45-A	89.65	32.5	
B(50,22)	B-5-6-7-B	75.64	86.4	6
	B-10-12-14-B	74.78	94.56	
	B-15-23-24-27-B	93.75	112.45	
	B-33-37-38-B	83.5	56.85	
	B-39-43-46-47-B	93.5	107.64	
	B-48-49-B	64.85	65.4	
C(55,55)	C-1-2-C	78.51	56.92	5
	C-8 -9-11-C	86.84	84.6	
	C-16-21-22-26- 28-29-C	92.4	146.6	
	C-30-31-32-C	86.5	54.81	
	C-34-50-C	72.65	38.65	
D(25,45)	D-3-20-D	76.56	32.84	2
	D-35-36-D	85.6	29.76	

Table IV shows the results obtained for the benchmark instance p02. From the experiments carried out it is seen that 4 vehicles are served by depot A, 5 by depot B, 4 by depot C and 1 by depot D, respectively. The distances are computed using Euclidean distance and do not guarantee the optimal value.

TABLE IV. ROUTE ALLOCATION OF CUSTOMERS FOR P02

Depot	Route	FLP (%)	Distance (km)	No. Vehicles
A(40,40)	A-4-13-17-A	85.85	57.65	4
	A-18-19-A	78.69	42.42	
	A-25-40-41-A	86.82	41.3	
	A-42-44-45A	92.5	68.56	
B(50,22)	B-5-6-7-10-B	75.64	86.4	5
	B-12-14-15-B	74.78	94.56	
	B-23-24-27-33-B	93.75	112.45	
	B-37-38-39-43-46-B	83.5	56.85	
	B-47-48-49-B	93.5	107.64	
C(55,55)	C-1-2-8-C	78.51	56.92	4
	C-9-11-16-21-22-26-C	86.84	84.6	
	C-28-29-30-31-32C	92.4	146.6	
	C-34-50-C	86.5	54.81	
D(25,45)	D-3- 20-35-36-D	76.56	32.84	1

The routing algorithm was run for the p03 benchmark instance consisting of 5 depots, 75 customers and the results are shown in Table V. The highest fully loaded percentage of depot A is 96.82%, which implies that the vehicle serves customers through route A-20-25-26-27-29-A. Each route is followed by a vehicle and in depot A, 7 vehicles serve the customer requirements, while in depot B, 2 vehicles serve, in depot C, 3 vehicles, in depot D, 3 vehicles and in depot E, 2 vehicles serve the costumers following the routes as shown in the Table 8.5.

TABLE V. ROUTE ALLOCATION OF CUSTOMERS FOR P03

Depot	Route	FLP (%)	Distance (km)	No. Vehicles
A(40,40)	A-3-4-5-6-A	85.85	86.98	7
	A-9-12-15-17-18-A	78.69	104.42	
	A-20-25-26-27-29-A	96.82	148.4	
	A-30-32-34-37-39-A	93.5	136.8	
	A-40-44-45-47-A	89.65	106.13	
	A-48-50-51-55-60-A	93.65	158.54	
	A-67-68-70-75-A	68.51	93.54	
B(50,22)	B-7-8-13-35-46-52-B	95.64	108.58	2
	B-57-58-72-B	74.78	59.64	
C(55,55)	C-10-11-14-C	78.51	68.95	3
	C-19-31-38-53-C	86.84	96.64	
	C-54-59-65-66-C	90.4	83.58	
D(25,45)	D-2-16-21-D	76.56	96.68	3
	D-24-28-33-36-D	85.6	109.74	
	D-49-62-63-69-71-73-74-D	94.65	148.33	
E(20,20)	E-21- 22-23-41-42-43-E	91.84	101.56	2
	E-56-61-64-E	81.35	46.89	

The benchmark instance p04 consisting of 2 depots and 100 customers, considered as one of the large size problems is run using the routing algorithm and the results are shown in Table VI. It is observed that the total distance required by depot A with 11 vehicles to serve customers is 1433.41 km, and similarly depot B used 10 vehicles with a total distance of 1465.77 km, respectively such that all customers are served.

TABLE VI. ROUTE ALLOCATION OF CUSTOMERS FOR P04

Depot	Route	FLP (%)	Distance (km)	No. Vehicles
A (15,35)	A-2-5-6-7-8-13-A	90.62	135.67	11
	A-14-15-16-17-18-21-A	93.68	201.98	
	A-22-23-36-37-38-A	89.37	167.45	
	A-40-41-42-43-44-45-46-A	93.5	198.56	
	A-47-48-52-A	76.7	98.65	
	A-53-56-57-58-59-60-A	92.5	162.3	
	A-61-72-73-74-75-A	85.55	100.2	
	A-82-83-84-85-A	76.5	93.45	
	A-86-87-89-91-92-A	87.6	89.89	
	A-93-94-95-A	78.96	78.48	
	A-96-97-98-99-100-A	81.5	106.78	
B (35,55)	B-1-3-4-9-10-B	90.5	97.93	10
	B-11-12-19-20-24-25-B	94.5	189.77	
	B-26-27-28-29-30-B	93.5	208.79	
	B-31-32-33-34-B	88.6	144.76	
	B-35-39-49-B	87.6	86.65	
	B-50-51-54-55-62-63-B	92.5	234.92	
	B-64-65-66-67-B	76.5	104.87	
	B-68-69-70-71-76-B	96.61	156.17	
	B-77-78-79-80-81-B	95.5	173.17	
	B-88-90-B	58.67	68.74	

TABLE VII. ROUTE ALLOCATION OF CUSTOMERS FOR P06

Depot	Route	FLP (%)	Distance (km)	No. Vehicles
A (15,20)	A-5-14-15-16-17-A	93.75	89.81	6
	A-37-38-41-42-43-44-45-46-A	95	145.45	
	A-57-59-61-84-A	68.75	79.21	
	A-85-86-A	95	38.92	
	A-87-91-92-93-95-A	88.75	53.54	
	A-96-97-98-99-100-A	73.75	26.05	
B (50,20)	B-1-3-9-10-11-B	83.75	108.11	7
	B-12-20-24-25-29-30-B	83.75	176.1	
	B-32-33-34-35-50-51-B	98.75	108.12	
	B-54-55-63-64-65-B	73.75	167.11	
	B-66-68-70-B	82.5	74	
	B-71-76-77-78-79-80-B	92.5	117	
	B-81-90-B	56.25	44	
C (35,35)	C-2-4-6-7-8-13-18-C	97.5	217.72	9
	C-19-21-22-23-C	93.75	115.93	
	C-26-27-28-31-C	95	138.4	
	C-36-39-40-47-C	90	49.61	
	C-48-49-52-C	93.75	64.46	
	C-53-56-58-60-62-C	75	159.2	
	C-67-69-72-73-74-C	91.25	94.2	
	C-75-82-83-88-89-C	86.25	95.2	
	C-94-C	53.25	18	

The route allocation for customers in the p06 instance with 3 depots and 100 customers is shown in Table VII. Depot A is allocated with a set of 6 routes with a total distance of 432.98 km with loading capacities of 93.75%, 95%, 68.75%, 95%, 88.75%, and 73.75%, respectively. Similarly, depot B is allocated with 7 routes whose total distance is 794.44 km and depot C is allocated with 9 routes with a total distance of 952.72 km. The total distance obtained in the routing phase for the p04 instance are 1433.41 km for depot A and 1465.77 km for depot B, respectively. These values are obtained in the routing phase of the project using Clark and Wright saving method. The assignment of routes is shown in Fig. 7, where the blue line indicates the routing assignment of each vehicle in the depot A and the red line represents the routing assignment of each vehicle in the depot B. These routes are formed based on the capacity constraint of vehicle.

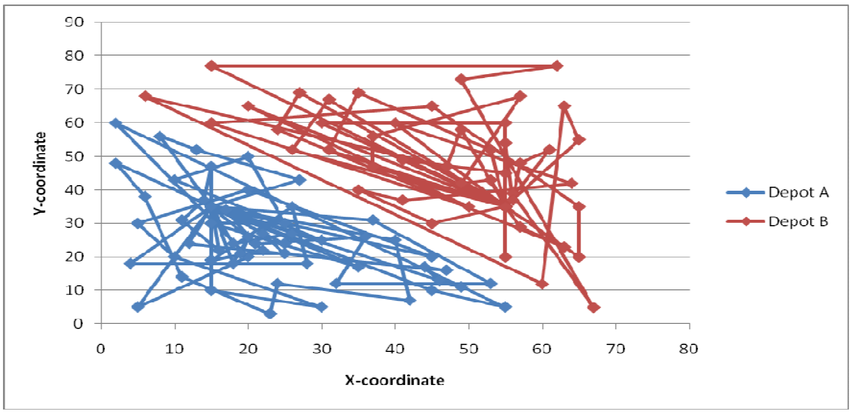

Figure 7. Routing Assignment for p04

C. GA for MDVRP

In GA, each chromosome is represented by the route of the each vehicle. The set of chromosomes form the initial population thus forming the search space. The population size decides the number of chromosomes in a single generation. A larger population size slows down the GA run, while a smaller value leads to exploration of a small search space. A reasonable range of the population size is between {50,100}. Based on the encoding of chromosomes in this project, the population size was set to 50. Genetic algorithms generate a new route sequence by selecting two individuals in the population to which the genetic operators crossover and mutation are applied. Best cost route crossover technique was applied to exploit a better solution for MDVRP. The choice of mutation was flip bit inversion type with a probability of 0.02, which explores a wider search space for the MDVRP in this project. The parameters used in solving MDVRP using GA for five benchmark problems are shown in Table VIII.

TABLE VIII. PARAMETER SETTINGS FOR GA BASED MDVRP

Parameters	Settings
Population size	based on the number of customers
Selection	Tournament selection
Crossover	Best Cost Route crossover
Mutation	Flip bit type
Crossover probability	0.6
Mutation probability	0.02
Elite	4

TABLE IX. OPTIMAL ROUTE USING GA FOR P01

Depot	No. Vehicles Available	No. customers served	No. vehicles required	Optimal Route
Instance p01				
A(20,20)	8	11	4	A-17-4-13-A, A-19-18-A, A-25-41-40-A, A-44-42-45-A
B(30,40)	8	19	5	B-7-10-5-6-B, B-12-14-15-B, B-23-27-33-37-24, B-39-38-46-43-47-B, B-48-49-B
C(50,30)	8	16	4	C- 8-2-9-1-C, C-28-26-11-21-16-22-C, C-29-31-32-30-C, C-50-34-C
D(60,50)	8	4	1	D-35-20-3-36-D
Instance p02				
A(20,20)	5	11	3	A-13-18-4-17-A, A-25-41-40-19-A, A-44-45-42-A
B(30,40)	5	19	5	B-6-7-10-5-B, B- 15-12-23-14-B, B-27-24-37-33-38-B, B- 46-39-47-43-48-B, B-49-B
C(50,30)	5	16	3	C-9-2-8-1-C, C- 11-16-21-29-28-26-22-C, C-34-50-31-32-30-C,
D(60,50)	5	4	1	D-3-20-35-36-D
Instance p03				
A(40,40)	7	32	6	A-9-3-6-5-4-A, A-18-17-20-15-26-12-25-A, A-34-27-37-29-30-32-A, A-40-39-44-47-45-A, A-48-60-67-55-50-51-68-A, A-75-70-A
B(50,22)	7	9	2	B-52-46-7-35-8-13-57-B, B-72-58-B
C(55,55)	7	11	3	C-19-14-11-10-C, C-59-65-31-38-53-54-C, C-66-C
D(25,45)	7	14	3	D-2-24-16-21-D, D-49-36-71-69-28-62-73-33-63-D, D-74-D
E(20,20)	7	9	2	E-61-22-42-41-56-23-43-1-E, E-64-E
Instance p04				
A(15,35)	12	55	10	A-8-7-6-13-2-14-5-A, A-22-15-16-17-18-21-A, A-38-43-41-23-40-36-37-42-A, A-45-46-47-44-A, A-57-56-58-53-52-48-A, A-73-72-74-59-61-60-A, A-82-83-84-85-75-A, A-91-86-89-87-92-A, A-97-95-94-96-93-A, A-99-98-100-A
B(35,55)	12	33	8	B-1-4-3-9-11-10-B, B-27-19-20-24-25-12-26-B, B-29-30-31-28-B, B-33-32-35-34-39-B, B-49-51-55-54-50-62-B, B-67-65-66-64-63-B, B-69-70-71-78-77-68-76-B, B-88-90-81-79-80-B
Instance p06				
A(15,20)	10	29	5	A-17-5-37-15-14-38-16-A, A-41-59-45-46-44-43-42-57-A, A-61-84-86-85-A, A-96-95-87-97-92-91-93-A, A-100-98-99-A
B(50,20)	10	33	5	B-24-12-3-20-9-11-10-1-B, B-34-29-25-30-32-33-35-B, B-55-65-64-63-51-50-54-B, B-70-66-71-68-76-B, B-81-79-78-80-77-90-B
C(35,35)	10	38	7	C-7-19-8-18-6-13-4-2-C, C-26-23-22-21-27-C, C-31-36-40-39-28-C, C-49-47-48-C, C-52-53-56-67-58-60-62-C, C-69-82-83-72-75-74-73-C, C-88-94-89-C

The scheduled routes are optimized using GA for the five benchmark problems and the number of customers serviced, number of vehicles required, depot route length and optimal route are evaluated. These parameters are tabulated in the Table IX, for five (p01, p02, p03, p04 and p06) benchmark instances, respectively. The optimized routes obtained using genetic algorithm for the p04 instance is shown Fig. 8. The routes passes through all the customer locations (shown as dots) starting from a source locations and ends at the same source after serving all customers.

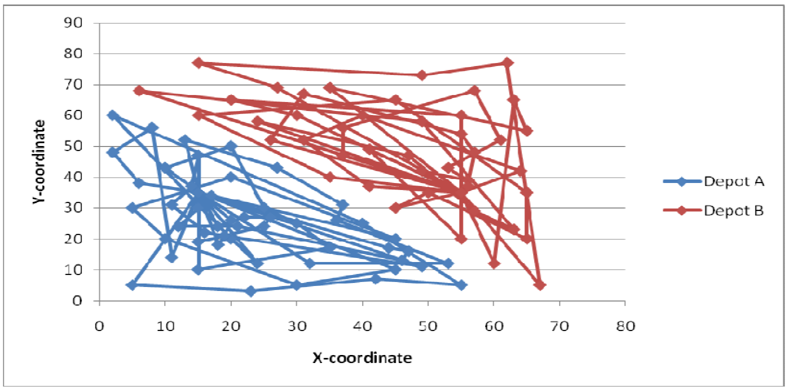

Figure 8. Optimized Routes using GA for p04

The computational results obtained through GA for the five benchmark instances are shown in Table X in terms of the number of customers, number of depots, best known distance, best optimal distance, computational time with respect to best optimal distance and average of best known and best optimal distances. The difference between the best known distance and optimal distances obtained by GA (P_d), is calculated by,

$$P_d = \frac{V_o - V_{bk}}{V_{bk}} \times 100 \tag{15}$$

Where V_o is the best optimal distance obtained by GA and V_{bk} is the best known distance. From the tabulated results it is found that the total delivery distance is decreased while optimizing the MDVRP using GA when compared to best known distance for all the benchmark instances considered in this work.

TABLE X. COMPUTATIONAL RESULTS FOR 5 BENCHMARK INSTANCES USING GA

Benchmark Instance Type	No. of customers	No. of Depots	Best known distance (km)	Best optimal distance (km)	Computational time (seconds)	Difference between best known and best optimal distances (%)
p01	50	4	576.87	598.45	4.0692	3.740877
p02	50	4	473.53	478.65	3.4207	1.081241
p03	75	5	641.19	699.23	6.8128	9.051919
p04	100	2	1001.59	1011.36	10.1081	0.975449
p06	100	3	876.50	882.48	9.3177	0.682259

D. Comparative Analysis

The performance of our GA on MDVRP instances are compared with Genetic Clustering (GC) [16] and GA in terms of the distance and the results are shown in Table XI. The proposed GA gave better results for all instances when compared with the work in [16]. When comparing with Genetic clustering for MDVRP proposed by [17], proposed GA yielded better distance values for instances p04 and p06.

TABLE XI. COMPARATIVE ANALYSIS

Benchmark Instance Type	No. of customers	No. of Depots	GA	GC [17]	GA [16]
p01	50	4	598.45	591.73	622.18
p02	50	4	478.65	463.15	480.04
p03	75	5	699.23	694.49	706.88
p04	100	2	1011.36	1062.38	1024.78
p06	100	3	882.48	976.02	908.88

VI. CONCLUSION

The effectiveness of the proposed techniques is tested by a set of five different Cordeau's benchmark instances namely p01, p02, p03, p04 and p06 in the MATLAB R2008b environment. The problem instances were initially grouped to assign the customers to their corresponding depots based on Euclidean distance. The customers of the same depot are assigned to several routes in the routing phase by Clarke and Wright saving method and each route is sequenced in the scheduling phase. The scheduled routes are optimized using GA. The simulation results of the proposed heuristic algorithms were compared in terms of depot's route length, optimal route, optimal distance, computational time, average distance, and number of vehicles. It was observed from the conducted experiments, the performance of GA was exceptional and the computational time proved that GA is much faster for solving of multi-depot vehicle routing problem. In future, efforts will be taken to impose more realistic constraints on the problem structure and large size real-time problems would be attempted by the proposed methodology.

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