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New heuristics for the Fleet Size and Mix Vehicle Routing Problem with Time Windows

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In the Fleet Size and Mix Vehicle Routing Problem with Time Windows (FSMVRPTW) customers need to be serviced in their time windows at minimal costs by a heterogeneous fleet. In this paper new heuristics for the FSMVRPTW are developed. The performance of the heuristics is shown to be significantly higher than that of any previous heuristic approach and therefore likely to achieve better solutions to practical routing problems.
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Introduction

Although often assumed in theory, a trucking firm's vehicle fleet is rarely homogeneous. Vehicles differ in their equipment, carrying capacity, age and cost structure. The need to be active in different market segments (eg container and bulk transport) causes firms to buy vehicles with a container chassis, dump installation etc. Vehicles of different carrying capacity allow a dispatcher to maximize capacity utilization by deploying smaller vehicles in areas with a lower concentration of customers. Moreover, it is also possible to service customers requiring small vehicles because of accessibility restrictions (see eg Semet[1] and Rochat and Semet[2]). The differences in equipment, carrying capacity and the fact that vehicles might differ in age, causes them to have a different cost structure.

Contrary to the classical Vehicle Routing Problem with Time Windows (VRPTW), the objective of the Fleet Size and Mix VRPTW (FSMVRPTW) is to minimize both routing costs *and* vehicle costs (incurred by acquiring vehicles) of a heterogenous fleet. Although small FSMVRPTW problem instances (up to 80 customers) can be solved to optimality,[3] (meta)heuristic procedures are the only valid alternative for real-life applications, especially if additional side constraints are imposed. Liu and Shen[4] designed the first initial heuristics for the FSMVRPTW, yielding good feasible solutions. Their parallel savings heuristics are inspired by Solomon's[5] sequential insertion heuristics. Instead of linking routes, one route is inserted into another. Our approach to the FSMVRPTW is sequential insertion based. By extending Solomon's[5] sequential insertion heuristic I1 with vehicle insertion savings, based on Golden *et al*,[6] we obtain significantly better solutions.

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The paper is organized as follows. In the next section the FSMVRPTW is formulated. We then give a brief review of the FSMVRP(TW) literature and describe our sequential insertion heuristics for the FSMVRPTW. Finally computational results are reported and conclusions are made.

Problem formulation

In the FSMVRPTW heterogeneously capacitated vehicles located at a depot are required to service geographically scattered customers over a limited scheduling period (eg a day). The distance d_{ij} between each pair of customers is given. Each customer i has a known demand q_i to be serviced at time b_i chosen by the carrier. If time windows are hard, b_i is chosen within a time window, starting at the earliest time e_i and ending at the latest time l_i that customer i permits the start of service. In the soft time window case, a vehicle is allowed to arrive too late at a customer but a penalty is incurred. In both cases, a vehicle arriving too early at customer j, has to wait until e_j. In this paper, we will assume time windows are hard. If t_{ij} represents the direct travel time from customer i to customer j, and s_i the service time at customer i, then the moment at which service begins at customer j, b_j, equals $\max\{e_j, b_i + s_i + t_{ij}\}$ and the waiting time w_j is equal to $\max\{0, e_j - (b_i + s_i + t_{ij})\}$. A time window can also be defined for the depot in order to define a 'scheduling horizon' in which each route must start and end.[7]

The objective of the FSMVRPTW is to minimize the sum of travel costs and fixed vehicle costs of servicing the customers within the time window limits. The vehicle fleet consists of K different types of vehicles. a_k is the capacity of the vehicles of type k ($a_1 < a_2 < \cdots < a_K$). f_k is the fixed acquisition cost of a vehicle of type k ($f_1 < f_2 < \cdots < f_K$). Without loss of generality, the cost of travelling a unit of time or distance is assumed to be equal to one.

Because the Vehicle Routing Problem (VRP) is NP-hard, the FSMVRP and the FSMVRPTW are NP-hard by restriction. This implies that problems of real-life dimensions can only efficiently be solved by heuristic algorithms. Gheysens *et al*,[8] present a mathematical programming formulation for the Fleet Size and Mix Vehicle Routing Problem. This formulation is an extension of the standard VRP formulation, in that a second term is added to the objective function in order to cope with the fixed (or acquisition) cost of the vehicle fleet.

Literature review for the FSMVRP(TW)

In the literature, five types of heuristic approaches to the traditional FSMVRP are distinguished.[4,6]

Adaptations of the Clarke and Wright[9] *savings algorithm* start by generating a separate route for each customer. At each step, two routes are combined into one according to a savings criterion. For the FSMVRP, the concept of savings not only includes savings in routing costs, but also savings in fixed vehicle costs and so-called opportunity savings developed by Golden *et al.*[6] These opportunity savings, which are discussed in the next section, can result from replacing two vehicles (routes) by one—possibly larger—vehicle.

The *matching based savings heuristic* developed by Desrochers and Verhoog[10] is a parallel route building heuristic. The matching based savings algorithm concept for the classical VRP[11] considers the savings associated with all feasible combinations of two routes by using a weighted matching problem to select them. This algorithm is adapted to the FSMVRP by using the opportunity savings criteria of Golden *et al*,[6] which are discussed in the next section.

Giant tour algorithms[6] are examples of 'route first–cluster second' heuristics. They start by generating a single tour that visits all customers (for example by a travelling salesman problem (TSP) algorithm). This tour is then divided into sub-tours, until all problem constraints are satisfied. The sub-tours are contiguous segments of the original tour with the first and the last customer connected to the depot. In a subsequent step, the solution obtained by one of these algorithms can be enhanced through an improvement post-processor such as 2-opt[12] or Or-opt.[13]

A two-stage *general assignment based heuristic* is developed by Gheysens *et al.*[14] This heuristic uses Golden *et al*'s.[6] lower bound procedure to determine the fleet composition to be used in a generalized assignment heuristic[15] in the second phase.

Salhi and Rand[16] develop a seven-phase heuristic approach which tries to improve the current solution at each phase. Their improvement modules attempt to: (1) match the total demand of a route to an appropriate vehicle; (2) eliminate an entire route by inserting its customers in another route; (3) move customers from a certain route to another one if this means that the former route can be serviced by a smaller vehicle; (4) combine routes with smaller demand into larger ones; and (5) split large routes into smaller ones. Moreover, a relaxation procedure is implemented that permits a more flexible merging and splitting of the routes.

Given the complexity of all variants of the VRP, several *meta-heuristic procedures* have been proposed for the FSMVRP and other similar problems. Semet and Taillard[17] develop and implement a tabu search meta-heuristic for solving real-life VRPs. Their tabu search procedure is very flexible in that it allows for time windows, heterogeneous vehicles, vehicle-dependent utilization costs, accessibility and other restrictions. Rochat and Semet[2] develop a tabu search approach for an FSMVRPTW which takes drivers' breaks and accessibility restrictions into account. Brandão and Mercer[18] develop a tabu search procedure for the Multi-Trip Vehicle Routing and Scheduling Problem (MTVRSP), in which each vehicle can make several trips per day. Besides the constraints common to the FSMVRPTW, their algorithm allows for both weight and volume capacity restrictions on the vehicles. Moreover, access can be restricted for some vehicles to some customers, and driver's schedules have to respect maximum driving times.

Recently, several authors have pointed out the importance of the quality of initial heuristics on the performance of metaheuristics. Liu and Shen[4] conclude from the results reported by Garcia *et al*,[19] Thompson and Psaraftis,[20] and Potvin and Rousseau[21] that algorithms that only concentrate on improving a poor initial solution do not perform very well within a limited computation time. Louis *et al*[22] report on the impact of good initialization on solution quality and computational speed for genetic algorithms. Van Breedam[23] demonstrates the dependence of descent heuristics and tabu search on the quality of the initial solution.

To alleviate this problem, Liu and Shen[4] develop a number of insertion-based parallel savings heuristics capable of generating feasible solutions. Instead of merging individual routes, the insertion of each route—in its original or reversed order—is evaluated in all possible insertion places in all other routes for different parameter settings. To take possible savings in vehicle acquisition costs into account, Golden *et al*'s[6] savings criteria are modified. Solution quality can be enhanced by a composite improvement scheme. The heuristics developed in this paper generate feasible solutions of a higher quality at lower computational cost. They can be used to seed other (meta)-heuristic approaches to the FSMVRPTW.

A sequential insertion heuristic for the FSMVRPTW

In this section, three new heuristics are developed for the FSMVRPTW. First, the general outline of the heuristics is presented. Second, the vehicle savings criteria used in the first part, are elaborated on.

The general outline

Because Liu and Shen's[4] heuristics evaluate the insertion of each route—in its original or reversed order—in all possible insertion places in all other routes for different parameter settings, the heuristic is computationally expensive. We extend Solomon's[5] sequential insertion heuristic to build a straightforward and effective heuristic for the FSMVRPTW.

The sequential insertion heuristic starts by initializing the current route for the smallest vehicle type. Routes can be initialized with the customer farthest from the depot or the one with the earliest deadline. After starting the current route with the initialization criterion, the sequential insertion heuristic uses the insertion criterion $c_1(i, u, j)$ to calculate for each unrouted stop u the best place and associated cost for insertion between two adjacent customers i and j in the current partial route $(i_0, i_1, \ldots, i_m)$ in which i_0 and i_m represent the origin and destination location of the vehicle (eg the depot). Insertion criterion $c_1(i, u, j)$ has to take into account both the additional distance $c_{11}(i, u, j)$ and time $c_{12}(i, u, j)$ needed to serve customer u plus the possible change in vehicle costs. Solomon[5] equates the additional time needed, $c_{12}(i, u, j)$, to the difference between the new time at which service begins at customer j after inserting u, b_j^{new}, and the original start of service at j, b_j. We extend Solomon's[5] sequential insertion heuristic by adding a third component to the insertion criterion $c_1(i, u, j)$. The vehicle savings insertion $c_{13}(i, u, j)$ is equal to one of the adapted savings concepts defined in the next sub-section. The cheapest insertion cost and the associated insertion place is determined for each unrouted customer u as

$$c_1(i, u, j) = \min_p [c_1(i_{p-1}, u, i_p)], \quad p = 1, \ldots, m \tag{1}$$

in which

$$c_1(i, u, j) = \alpha_1 c_{11}(i, u, j) + \alpha_2 c_{12}(i, u, j) + \alpha_3 c_{13}(i, u, j) \tag{2}$$

with

$$c_{11}(i, u, j) = d_{iu} + d_{uj} - \mu d_{ij}, \quad \mu \geqslant 0$$

$$c_{12}(i, u, j) = b_j^{\text{new}} - b_j$$

$$c_{13}(i, u, j) = \text{ACS, AOOS, AROS}$$

Solomon[5] requires the weighting factors α_i to sum up to 1. We have removed this restriction to allow the components of the insertion criterion to be weighted by, eg, cost data.

In a second step, the customer that is best according to the selection criterion $c_2(i, u, j)$ is selected. The selected customer u^* is then inserted in the route between i and j.

$$c_2(i, u^*, j) = \max_u [c_2(i, u, j)] \quad u \text{ unrouted and feasible} \tag{3}$$

$$c_2(i, u, j) = \lambda(d_{0u} + t_{0u}) + s_u + F(q_u) - c_1(i, u, j), \quad \lambda \geqslant 0 \tag{4}$$

where s_u is the service time of customer u and $F(q_u)$ is the fixed cost of the smallest vehicle capable of moving a load q_u.

If no remaining unrouted customer has a feasible insertion place, a new route is initialized and identified as the current route.

The insertion criterion $c_1(i, u, j)$ looks for that insertion place that minimizes a weighted average of the additional distance and time needed to include a customer in the current partial route, taking into account the effect on vehicle costs. The weighting factors α_i are used to guide the heuristic to different (local) optima. The selection criterion $c_2(i, u, j)$ tries to maximize the benefit derived from inserting a customer in the current partial route rather than on a new, direct route. Following Gheysens *et al*,[8] $F(q_u)$ denotes the fixed cost of the smallest vehicle capable of moving a load q_u.

Specification of the vehicle savings insertion criteria

Golden *et al*[6] define three approaches to vehicle costs from a parallel savings perspective: Combined Savings (CS), Optimistic Opportunity Savings (OOS) and Realistic Opportunity Savings (ROS). The CS approach extends the Clarke and Wright[9] heuristic by taking the *immediate* vehicle cost savings by joining two sub-tours i and j. Let $F(z)$ be the fixed cost of the smallest vehicle that can service a demand of size z for a sub-tour. Then the combined savings $\bar{s}_{ij}$ are defined as

$$\bar{s}_{ij} = s_{ij} + F(z_i) + F(z_j) - F(z_i + z_j) \tag{5}$$

with

$$s_{ij} = c_{0i} + c_{0j} - c_{ij} \tag{6}$$

Both the OOS and the ROS heuristics extend the CS concept by evaluating the unused capacity of the vehicle servicing the combined sub-tours. The OOS heuristic s_{ij}^* assumes that in a future combination of routes, the smallest vehicle that can service the unused capacity, $P(z)$, can be absorbed.

$$s_{ij}^* = \bar{s}_{ij} + F(P(z_i + z_j) - z_i - z_j) \tag{7}$$

The ROS heuristic s'_{ij} expects that only the largest vehicle that fits in the unused capacity can be eliminated. To this end, $F'(z)$ is defined as the fixed cost of the largest vehicle whose capacity is less than or equal to z. The binary variable w makes sure that opportunity savings are only taken into account when the combination of two sub-tours requires a larger vehicle. If this is not the case, it is unnecessary to use opportunity savings to encourage the use of larger vehicles.

$$s'_{ij} = \bar{s}_{ij} + \delta(w) F'(P(z_i + z_j) - z_i - z_j) \tag{8}$$

in which

$$w = P(z_i + z_j) - P(\max\{z_i, z_j\}) \quad (9)$$

$$\delta(w) = \begin{cases} 0 & \text{if } w = 0 \\ 1 & \text{if } w > 0 \end{cases} \quad (10)$$

To adapt Golden *et al*'s[6] savings concepts for the insertion heuristic, the load of a vehicle and its maximum capacity are denoted by Q and $\bar{Q}$, respectively. The new load of the vehicle and its possibly new capacity after inserting a new customer is represented by Q^{new} and $\bar{Q}^{\text{new}}$, respectively.

The Adapted Combined Savings (ACS) concept is defined as the difference between the fixed costs of the vehicle capable of transporting the load of the route after and before inserting customer u, $(F(Q^{\text{new}}) - F(Q))$.

To reflect the original notion of Golden *et al*'s[6] OOS, the Adapted Optimistic Opportunity Savings (AOOS) concept extends the ACS by subtracting $F(\bar{Q}^{\text{new}} - Q^{\text{new}})$. This is the fixed cost of the smallest vehicle that can service the unused capacity $\bar{Q}^{\text{new}} - Q^{\text{new}}$.

The Adapted Realistic Opportunity Savings (AROS) concept takes the fixed cost of the largest vehicle smaller than or equal to the unused capacity, $F'(\bar{Q}^{\text{new}} - Q^{\text{new}})$, into account as opportunity saving. It only does so if a larger vehicle is required to service the current tour after a new customer has been inserted. The savings criteria are summarized in Table 1.

Computational results

Because we want to compare our heuristic's performance to Liu and Shen's[4] heuristics, we used the same Solomon[5] problem sets, vehicle capacities and costs (see Appendix). Note that because Liu and Shen[4] do not specify distance or time coefficients to value distance and time, they are implicitly valued at 1. Solomon's[5] problem sets for the VRPTW consist of 56 instances of 100 customers with randomly generated coordinates (set R), clustered coordinates (set C) or both (the so-called semi-clustered sets RC). The R1, C1 and RC1 problem sets have a smaller average number of customers per route than the R2, C2 and RC2 sets because of their shorter scheduling horizon and smaller vehicle capacities.

Table 1 Savings insertion criteria

Algorithm	Golden *et al*'s[6] savings formula
CW	$s_{ij} = c_{0i} + c_{0j} - c_{ij}$
CS	$\bar{s}_{ij} = s_{ij} + F(z_i) + F(z_j) - F(z_i + z_j)$
OOS	$s^*_{ij} = s_{ij} + F(P(z_i + z_j) - z_i - z_j)$
ROS	$s'_{ij} = \bar{s}_{ij} + \delta(\omega)F'(P(z_i + z_j) - z_i - z_j)$
Algorithm	Adapted savings insertion formula
ACS	$F(Q^{\text{new}}) - F(Q)$
AOOS	$[F(Q^{\text{new}}) - F(Q)] - F(\bar{Q}^{new} - Q^{\text{new}})$
AROS	$[F(Q^{\text{new}}) - F(Q)] - \delta(\omega)F'(\bar{Q}^{new} - Q^{\text{new}})$

An extended set of Solomon's[5] original parameter settings is used to test our heuristic. Solomon[5] uses two initialization criteria: the farthest unrouted customer and the customer with the earliest deadline, and four $(\mu, \lambda, \alpha_1, \alpha_2)$ settings: (1, 1, 1, 0), (1, 2, 1, 0), (1, 1, 0, 1) and (1, 2, 0, 1). By adding an additional term $c_{13}(i, u, j)$ to the insertion criterion, a new weighting factor α_3 is needed. As opposed to Solomon,[5] we no longer require that the weighting factors α_i sum up to 1 (see Equation (2)). The following α_i combinations are considered: (1, 0, 1), (0, 1, 1) and (1, 1, 1). In each of the three α_i combinations, $\alpha_3 = 1$ to allow different solutions for the different savings approaches. If $\alpha = (1, 1, 1)$ equal weights are given to the distance, time and vehicle savings related component of an insertion.

Liu and Shen[4] use the total schedule time of a solution (excluding the service times of the customers) to measure solution quality. Therefore we selected the run with the lowest schedule time of each of the 12 runs per problem instance. Liu and Shen[4] obtained the best results on Solomon's[5] problem instances with their modified heuristics $\text{MCS}_{-\lambda-\eta}$, $\text{MOOS}_{-\lambda-\eta}$ and $\text{MROS}_{-\lambda-\eta}$. The route shape parameter λ is due to Golden *et al*[6] and gives a different weight to the additional distance needed to combine two individual routes. The parameter η is used to control the construction of routes during the parallel construction.

Our sequential heuristics clearly dominate Liu and Shen's[4] best heuristics for cost structures A and B (see Tables 2 and 3 and the Appendix). In several cases the sequential insertion heuristic using ACS, AOOS or AROS is able to reduce total schedule time by more than 50%, even if an improvement heuristic was invoked ($\text{MCS}^*_{-\lambda-\eta}$, $\text{MOOS}^*_{-\lambda-\eta}$ and $\text{MROS}^*_{-\lambda-\eta}$). For cost structure C, cost differences with Liu and Shen[4] are smaller, but still significant. Our heuristics are clearly more robust than $\text{MCS}^*_{-\lambda-\eta}$, $\text{MOOS}^*_{-\lambda-\eta}$ and $\text{MROS}^*_{-\lambda-\eta}$. Liu and Shen's[4] modified heuristics' solution quality is highly dependent on the cost structure used. Our results are in line with Solomon's[5] results on the problem instances after removing service times from the published total schedule time. This comparison between the results is possible because for all problem sets except C1, the homogeneous vehicle fleet in Solomon[5] consists of the largest vehicle type from Liu and Shen.[4]

Because Liu and Shen[4] do not specify a time and distance coefficient, physical time and distance units are used in the analysis. In cost structure C the cost of possessing a vehicle of type A equals 5 units. Given that the implicit cost of one unit of time or distance equals 1 and that a vehicle can be used during 230 units of time (ie the length of the scheduling horizon in R1), cost structure C can be considered to be highly unusual.

To illustrate this point, consider Tables 4 and 5. The figures in Table 4 are averages of sample data from different companies of vehicles with different engine powers. They

Table 2 Comparison of our heuristic to Liu and Shen's[4] modified heuristics (total schedule time excluding service times)

Set	$MCS_{-\lambda-\eta}$	$MCS^*_{-\lambda-\eta}$	*ACS*	$\Delta\%^a$	$\Delta{*}\%^b$
R1A	4562	4398	1665.32	63.50	62.13
R1B	2155	2066	1617.10	24.96	21.73
R1C	1799	1716	1689.12	6.11	1.57
	$MOOS_{-\lambda-\eta}$	$MOOS^*_{-\lambda-\eta}$	*AOOS*	Δ%	Δ*%
R1A	4575	4401	1548.53	66.15	64.81
R1B	2152	2054	1574.66	26.83	23.34
R1C	1802	1700	1576.58	12.51	7.26
	$MROS_{-\lambda-\eta}$	$MROS^*_{-\lambda-\eta}$	*AROS*	Δ%	Δ*%
R1A	4564	4403	1556.14	65.90	64.66
R1B	2149	2068	1557.38	27.53	24.69
R1C	1788	1706	1557.85	12.87	8.68
Set	$MCS_{-\lambda-\eta}$	$MCS^*_{-\lambda-\eta}$	*ACS*	Δ%	Δ*%
C1A	8042	8007	1247.52	84.49	84.42
C1B	2803	2661	1163.78	58.48	56.27
C1C	1886	1749	1435.32	23.90	17.93
	$MOOS_{-\lambda-\eta}$	$MOOS^*_{-\lambda-\eta}$	*AOOS*	Δ%	Δ*%
C1A	8515	8295	1247.52	85.49	84.96
C1B	2626	2485	1126.01	58.48	54.69
C1C	1870	1705	1282.51	23.90	24.78
	$MROS_{-\lambda-\eta}$	$MROS^*_{-\lambda-\eta}$	*AROS*	Δ%	Δ*%
C1A	8042	8007	1166.09	85.50	85.44
C1B	2803	2661	1131.02	59.65	57.50
C1C	1886	1749	1155.45	38.74	33.94
Set	$MCS_{-\lambda-\eta}$	$MCS^*_{-\lambda-\eta}$	*ACS*	Δ%	Δ*%
RC1A	5483	5262	1777.62	67.58	66.22
RC1B	2366	2253	1780.94	24.73	20.95
RC1C	1926	1853	1887.07	2.02	−1.84
	$MOOS_{-\lambda-\eta}$	$MOOS^*_{-\lambda-\eta}$	*AOOS*	Δ%	Δ*%
RC1A	5539	5184	1686.95	69.54	67.46
RC1B	2359	2252	1697.06	28.06	24.64
RC1C	1933	1859	1744.71	9.74	6.15
	$MROS_{-\lambda-\eta}$	$MROS^*_{-\lambda-\eta}$	*AROS*	Δ%	Δ*%
RC1A	5429	5198	1665.04	69.33	67.97
RC1B	2342	2235	1680.55	28.24	24.81
RC1C	1929	1849	1689.92	12.39	8.60

[a](modified savings − adapted savings)/(modified savings) × 100.
[b]After invoking an improvement heuristic.

Table 3 Comparison of our heuristic to Liu and Shen's[4] modified heuristics (total schedule time excluding service times)

Set	$MCS_{-\lambda-\eta}$	$MCS^*_{-\lambda-\eta}$	*ACS*	$\Delta{*}\%^a$	$\Delta{*}\%^b$
R2A	3855	3809	1443.71	62.55	62.10
R2B	1915	1816	1456.78	23.93	19.78
R2C	1589	1513	1438.65	9.46	4.91
	$MOOS_{-\lambda-\eta}$	$MOOS^*_{-\lambda-\eta}$	*AOOS*	Δ%	Δ*%
R2A	4077	3975	1435.33	64.79	63.89
R2B	1924	1797	1431.49	25.60	20.34
R2C	1610	1530	1419.81	11.81	7.20
	$MROS_{-\lambda-\eta}$	$MROS^*_{-\lambda-\eta}$	*AROS*	Δ%	Δ*%
R2A	3855	3809	1426.52	63.00	62.55
R2B	1915	1816	1446.10	24.49	20.37
R2C	1589	1513	1445.27	9.05	4.48
Set	$MCS_{-\lambda-\eta}$	$MCS^*_{-\lambda-\eta}$	*ACS*	Δ%	Δ*%
C2A	7058	6717	821.38	88.36	87.77
C2B	2054	1978	821.38	60.01	58.47
C2C	1373	1288	811.16	40.92	37.02
	$MOOS_{-\lambda-\eta}$	$MOOS^*_{-\lambda-\eta}$	*AOOS*	Δ%	Δ*%
C2A	7354	3889	1072.28	85.42	72.43
C2B	2093	1970	931.89	55.48	52.70
C2C	1383	1300	828.13	40.12	36.30
	$MROS_{-\lambda-\eta}$	$MROS^*_{-\lambda-\eta}$	*AROS*	Δ%	Δ*%
C2A	7058	6717	1043.42	85.22	84.47
C2B	2054	1978	1043.42	49.20	47.25
C2C	1373	1288	1029.44	25.02	20.07
Set	$MCS_{-\lambda-\eta}$	$MCS^*_{-\lambda-\eta}$	*ACS*	Δ%	Δ*%
RC2A	5518	5324	1801.71	67.35	66.16
RC2B	2469	2339	1741.97	29.45	25.53
RC2C	2101	1994	1754.32	16.50	12.02
	$MOOS_{-\lambda-\eta}$	$MOOS^*_{-\lambda-\eta}$	*AOOS*	Δ%	Δ*%
RC2A	5381	5273	1800.82	66.53	65.85
RC2B	2432	2338	1783.61	26.66	23.71
RC2C	2066	1978	1741.75	15.69	11.94
	$MROS_{-\lambda-\eta}$	$MROS^*_{-\lambda-\eta}$	*AROS*	Δ%	Δ*%
RC2A	5518	5324	1804.56	67.30	66.11
RC2B	2462	2324	1770.23	28.10	23.83
RC2C	2101	1988	1962.27	16.12	11.35

[a](modified savings − adapted savings)/(modified savings) × 100.
[b]After invoking an improvement heuristic.

are calculated for firms respecting all statutory regulations with wage-earning truck drivers.

Given the traditional assumption from the VRPTW that one unit of distance equals one unit of time,[5] the figures from the above table have to be slightly modified to become comparable to Liu and Shen's[4] cost structure. Because a vehicle's fixed costs are expressed per hour, they have to be multiplied with the maximum statutory driving time (9 h) to obtain the daily cost of owning the vehicle. If we assume an average speed of 60 km/h, the hour and time coefficients are obtained as follows. The hour coefficient from Blauwens *et al*[24] is divided by 60 to approximate the time coefficient δ_t, expressing the opportunity cost of time. Indeed, in the long run the average opportunity cost of time equals the average cost of owning a vehicle. In the short run, the opportunity cost depends on the carrier's potential customers of that moment, making it higher during peak periods than during off-peak periods. The distance coefficient δ_d is equalled to the kilometre coefficient. Notice the level of the different cost components and the presence of pronounced economies of scale in Table 5. The cost of owning a vehicle with a carrying capacity of $2x$ costs a lot less than two times the costs of an x-ton vehicle. In Liu and Shen's[4] cost structure there are no economies of scale.

Table 4 Hour and kilometre coefficients in Euros for 1999[24]

Carrying capacity	*Hour coefficient*	*Kilometre coefficient*
Delivery van 0.5 t	16.03	0.10
Lorry 5 t	17.14	0.15
Lorry 8 t	18.06	0.17
Lorry 20 t	20.88	0.21
Truck and trailer 28 t	21.75	0.24

Table 5 Cost structure in Euros for 1999 based on Table 4

Carrying capacity	*Vehicle cost*	*Hour coefficient*	*Kilometre coefficient*
Delivery van 0.5 t	144.27	0.27	0.10
Lorry 5 t	154.26	0.29	0.15
Lorry 8 t	162.54	0.30	0.17
Lorry 20 t	187.92	0.35	0.21
Truck and trailer 28 t	195.75	0.36	0.24

Conclusions

This paper described three insertion-based heuristics for the FSMVRPTW. The heuristics are an extension of Solomon's[5] sequential insertion heuristic I1 with adapted formulations of Golden *et al*'s[6] savings concepts. Computational testing revealed that our new heuristics for the FSMVRPTW significantly outperform Liu and Shen's[4] best heuristics on the same problem set. Depending on the cost structure used, solution improvements of more than 50% can be easily attained. Because the solution improvements are the largest for the more realistic cost structures, the advantage of our heuristics over Liu and Shen's[4] can be expected to be important in real-life applications.

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Appendix

Liu and Shen's[4] problem set data

Vehicle	*Capacity*	*R1A*	*R1B*	*R1C*
A	30	50	10	5
B	50	80	16	8
C	80	140	28	14
D	120	250	50	25
E	200	500	100	50
Vehicle	*Capacity*	*C1A*	*C1B*	*C1C*
A	100	300	60	30
B	200	800	160	80
C	300	1350	270	135
Vehicle	*Capacity*	*RC1A*	*RC1B*	*RC1C*
A	40	60	12	6
B	80	150	30	15
C	150	300	60	30
D	200	450	90	45
Vehicle	*Capacity*	*R2A*	*R2B*	*R2C*
A	300	450	90	45
B	400	700	140	70
C	600	1200	240	120
D	1000	2500	500	250
Vehicle	*Capacity*	*C2A*	*C2B*	*C2C*
A	400	1000	200	100
B	500	1400	280	140
C	600	2000	400	200
D	700	2700	540	270
Vehicle	*Capacity*	*RC2A*	*RC2B*	*RC2C*
A	100	150	30	15
B	200	350	70	35
C	300	550	110	55
D	400	800	160	80
E	500	1100	220	110
F	1000	2500	500	250

References

1 Semet F (1995). A two-phase algorithm for the partial accessibility constrained vehicle routing. *Ann Opns Res* **61**: 45–65.
2 Rochat Y and Semet F (1994). A tabu search approach for delivering pet food and flour in Switzerland. *J Opl Res Soc* **45**: 1233–1246.
3 Haouari M (2001). On the effectiveness of column generation for time constrained routing problems. *Found Comput Decis Sci* **26**: 215–224.
4 Liu F-H and Shen S-Y (1999). The fleet size and mix vehicle routing problem with time windows. *J Opl Res Soc* **50**: 721–732.
5 Solomon M (1987). Algorithms for the vehicle routing and scheduling problems with time window constraints. *Opns Res* **35**: 254–265.
6 Golden B, Assad A, Levy L and Gheysens F (1984). The fleet size and mix vehicle routing problem. *Comput Opns Res* **11**: 49–66.
7 Potvin J and Rousseau J (1993). A parallel route building algorithm for the vehicle routing and scheduling problem with time windows. *Eur J Opl Res* **66**: 331–340.
8 Gheysens F, Golden B and Assad A (1984). A comparison of techniques for solving the fleet size and mix vehicle routing problem. *OR Spektrum* **6**: 207–216.
9 Clarke G and Wright W (1964). Scheduling of vehicles from a central depot to a number of delivery points. *Opns Res* **12**: 568–581.
10 Desrochers M and Verhoog T (1991). A new heuristic for the fleet size and mix vehicle routing problem. *Comput Opns Res* **18**: 263–274.
11 Desrochers M and Verhoog T (1989). A matching based savings algorithm for the vehicle routing problem. *Technical Report GERAD-89-04*. GERAD, École des Hautes Études Commerciales, Montréal.
12 Lin S and Kernighan B (1973). An effective heuristic algorithm for the travelling salesman problem. *Opns Res* **21**: 498–516.
13 Or I (1976). *Travelling salesman-type combinatorial problems and their relation to the logistics of regional blood banking.* PhD thesis, Department of Industrial Engineering and Management Sciences, Northwestern University.
14 Gheysens E, Golden B and Assad A (1986). A new heuristic for determining fleet size and composition. *Math Program Stud* **26**: 233–236.
15 Fisher M and Jaikumar R (1981). A generalized assignment heuristic for vehicle routing. *Networks* **11**: 109–124.
16 Salhi S and Rand G (1993). Incorporating vehicle routing into the vehicle fleet composition problem. *Eur J Opl Res* **66**: 313–360.
17 Semet F and Taillard E (1993). Solving real-life vehicle routing problems efficiently using tabu search. *Ann Opns Res* **41**: 469–488.

18 Brandão J and Mercer A (1997). A tabu search algorithm for the multi-trip vehicle routing and scheduling problem. *Eur J Opl Res* **100**: 180–191.
19 Garcia B, Potvin J-Y and Rousseau J-M (1994). A parallel implementation of the tabu search heuristic for vehicle routing problems with time window constraints. *Comput Opns Res* **21**: 1025–1033.
20 Thompson P and Psaraftis H (1993). Cyclic transfer algorithms for multivehicle routing and scheduling problems. *Opns Res* **41**: 935–946.
21 Potvin J-Y and Rousseau J-M (1995). An exchange heuristic for routing problems with time windows. *J Opl Res Soc* **50**: 1433–1446.
22 Louis SJ, Yin X and Yuan ZY (1999). Multiple vehicle routing with time windows using genetic algorithms. *Technical Report 171*. Department of Computer Science, University of Nevada.
23 Van Breedam A (2001). Comparing descent heuristics and metaheuristics for the vehicle routing problem. *Comput Opns Res* **28**: 289–315.
24 Blauwens G, De Baere P and Van de Voorde E (2002). *Handbook of Transport Economics*. De Boeck: Antwerp.

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