A Multiobjective Optimization Approach to Urban School Bus Routing : Formulation and Solution Method

Robert Bowerman
Department of Systems Design Engineering,
University of Waterloo, Waterloo, ON, Canada, N2L 3G1
(Author to which correspondence should be directed)

Brent Hall
Department of Urban and Regional Planning,
University of Waterloo

Paul Calamai
Department of Systems Design Engineering,
University of Waterloo

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Introduction

An important issue in the provision of primary and secondary-level educational services is the problem of providing publicly funded transportation for students to and from their respective schools. Since school students are not simple "packages", as in the case of pick-up and delivery of goods, and since the service is provided through the public sector, this problem is significantly more complicated than conventional vehicle routing problems (VRP). In the case of the School Bus Routing Problem (SBRP), several additional goals related to user equity serve to modify the standard efficiency objectives of the VRP. As well, operational characteristics of the school bus system create further complexities in the SBRP formulation as compared to the conventional VRP model.

The SBRP can be specified as follows: a group of spatially distributed students must be provided with public transportation from their residences to and from their schools. The problem is to find a series of school bus routes that ensure the service is provided equitably to all eligible students. Student eligibility for school bus transportation is determined by local school board-specific policies and is, generally, in the case of Ontario, Canada, dependent upon grade, program enroled in, and distance of a student's residence from the school they attend. Additional restrictions are placed on the distance that students can walk from their homes to and from their bus stops. Although school buses serve both rural and urban areas, the differences in settlement patterns dictate that different routing systems be considered. This paper considers the case of providing school bus transportation in urban areas and does not deal with school bus routing in rural areas.

A multiobjective mathematical formulation is presented for the Urban School Bus Routing Problem (USBRP). A heuristic algorithm based on this formulation is developed and tested with data from a sample school board location in Wellington County.

Since school bus services in Ontario are funded through local property taxes in conjunction with grants from the provincial Ministry of Education, and since the service involves considerations of efficiency (cost minimization) and user equity (fairness), the evaluation criteria for measuring the appropriateness of school bus routes are inherently multiobjective in nature. In this paper we present a multiobjective mathematical formulation for SBRP in urban areas, develop a heuristic algorithm based on this formulation, and report results of testing this algorithm on sample data from the Wellington Country Board of Education.

In the next section the criteria that are used to assess the performance of solutions to the USBRP are discussed.

Performance Criteria for the Provision of School Bus Transportation Services

Savas (1978) provides three criteria for evaluating the provision of public goods and services, namely efficiency, effectiveness, and equity. Each criteria has its own unique set of considerations and objectives to satisfy yet there are clear linkages between them in terms of an overall assessment of service provision. We first consider each criterion separately and then in combination in the multiobjective formulation of the USBRP.

Efficiency Criteria

Efficiency measures the ratio of the level of a service compared to the cost of the resources required to provide the service. Since the level of service in providing efficient school bus routing is fixed for a particular situation, the main variable in determining the efficiency of a particular solution is the total cost of providing the service in dollars or manpower. Therefore, the efficiency of a solution can be measured by its cost.

There are two main components to consider in the total cost of providing school bus transportation. One cost is the capital cost required to run one school bus for a school year. Components of this include payment of the bus driver, as well as the costs of vehicle maintenance, purchasing, and leasing. The other main cost is the incremental cost, or the cost of a school bus route per kilometre traveled. It is generally accepted that the capital cost is significantly larger per bus than the incremental cost over a year (Kidd, 1991). Therefore, in efficiency terms, a solution that requires fewer routes would be preferred to a solution with more routes.

Effectiveness Criteria

The effectiveness of a service is measured by how well the demand for the service is satisfied. In school bus routing, measurement of service effectiveness amounts to determining whether bus transportation is available to all eligible students and whether the level of service is acceptable to the public.

The question of whether or not a student qualifies for school bus transportation is dependent upon school board-specific policies. In Ontario, provincial standards require a minimum level of service provision but local school board trustees determine how these standards are met and to what degree (if any) they are exceeded (Feick, 1991). An example of such a standard is the maximum distance a student may walk from their home to school before being eligible for school bus transportation. In setting this and other standards, the local school board determines the effectiveness of the school bus service by determining which students are eligible for busing.

Equity Criteria

Equity considerations assess the fairness or impartiality of the provision of the service in question. Some previous work on the equity of school busing in the United States has concentrated on a related aspect of the busing problem that is not particularly relevant in the context of Ontario. In particular, Lee and Moore (1977) provided a linear programming model that assigns students to schools in order to achieve racial integration and to reduce school overcrowding and underutilization. These objectives are not considered explicitly in the model presented in this paper. Rather, we assume that these issues are considered when local boards of education define their school attendance areas.

The equity considerations that are examined here concern the equality or fairness of the school bus service for each eligible student. Since the optimization of efficiency criteria might produce an inexpensive solution that would be unacceptable to the school board due to inequities in the provision of service to the students, several additional goals are imposed which seek to make the service fair as well as efficient.

For example, a "first-on/first-off" policy on a board's school bus routes can be regarded as an equity-related criterion. This policy states that those students who are picked up first in the morning must also be those who are dropped off first in the afternoon. This ensures that all students on the same route travel on a school bus for approximately the same length of time and that no one student has a school day (including travel time) that is significantly longer than any other student. To address this policy, the school bus routes are designed so that they begin *and* end at the school rather than just beginning *or* ending at the school. This is accomplished in the problem formulation.

Another way to improve equity is to "load-balance" the routes serving an area so that each school bus route transports approximately an equivalent number of students. Satisfaction of the criteria has the added benefit that it reduces the likelihood that the routes will be filled to over-capacity if additional load is assigned to them during the school year. Such a situation could arise when new students move into a school attendance area or if school attendance areas are redefined when modifying pre-existing bus routes.

Other ways to improve equity are to "length-balance" to avoid too large a variation in the route lengths of buses serving one school and to consider the trade-off between total student walking distance to their bus stops and the length of the bus route with the objective of increasing public acceptability of a set of school bus routes.

In order to evaluate any set of school bus routes, many different criteria must be examined simultaneously. The next section applies these service-related criteria to the formulation of a multiobjective optimization model of the USBRP.

Problem Characteristics

The goal of the USBRP can be specified as follows: find a set of school bus routes that provide transportation to and from school for all eligible students. The eligibility criteria are based on the distance that students live from school and school board-specific policies of the sort discussed previously.

In urban areas, the school bus transportation system operates as follows. Students are picked up at a bus stop that is within walking distance of their residence. Their school bus completes visiting the rest of the bus stops remaining on its route and then goes directly to the school. In this system, each school bus route serves only one school. In the afternoon, the process is reversed and the students are dropped off at the bus stops where they were picked up in the morning.

The suitability of a site for being a school bus stop is influenced by characteristics such as traffic density, proximity to corners, and adjacency to public property. Because of the complicated nature of these criteria we assume that the potential bus stop sites have been selected by an analyst such as a school board transportation planner.

The USBRP, as described above, actually involves two interrelated problems. One problem is the assignment of students to their respective bus stops and the second problem is the routing of the bus to the bus stops. Problems with these characteristics are known as a Location-Routing Problems (LRPs) and are discussed further in Laporte (1988). One important characteristic of LRPs is that they are organized into a series of layers. With this formulation, the USBRP is organized into three layers: layer one is the schools, layer two is the bus stops, and layer three is the students. School bus routes interact between layer one and layer two (the school and the bus stops), while the students walking to their bus stops in the morning and back home in the afternoon causes the interaction between layers two and three. The locational decisions are made in layer two.

Specific criteria that can be used to evaluate the school bus routes are discussed in the next section. Then the limits that are placed on the school bus routes are discussed. Following this, a mathematical formulation of the model is presented.

Optimization Criteria

A number of optimization criteria are defined in order to evaluate the desirability of a particular set of school bus routes. From the above discussion, these criteria are:

1. **Number of Routes.** Since the capital cost is significantly larger per bus than the incremental cost over the year, the number of routes generated should be held to a minimum.

2. **Total Bus Route Length.** This criterion reduces the total length of the school bus routes.

3. **Load Balancing.** Load-balancing involves minimizing the variation in the number of students transported along each route.

4. **Length Balancing.** This criterion involves reducing the variation in route lengths.

5. **Student Walking Distance.** This criterion balances the total distance that students walk from home to and from their bus stops against route length.

Routing Constraints

As well as having objectives to evaluate the routes, several different constraints may be imposed on the school bus routes from board-specific policies and from the capacity of a school bus. These constraints include:

1. an upper bound on the number of students on each route (bus capacity);
2. an upper bound on the length (or travel time) on each route; and,
3. an upper bound on the total travel time (both pickup and delivery) that students can travel.

However, due to the high population density of students in urban areas, urban school bus routes tend to be relatively short because bus capacity is reached after a small number of bus stops are visited. Consequently, the length and total travel time constraints discussed above are not binding and are, therefore, dropped from the problem formulation.

In measuring a bus's load early primary students are only weighted 2/3 as much as older students to reflect the fact that students in the early primary grades are placed three to a seat while older (and larger) students are placed two to a seat on the school bus.

Additionally, in the model that follows, it is assumed that schools are serviced by a homogeneous fleet of buses so that each bus has the same capacity.

Problem Formulation

This section gives an integer programming formulation of the USBRP for urban areas as described previously. First, consider the following definitions:

Sets

S	$=$	set of cardinality 1 representing the school
B	$=$	set of all potential bus stop sites
I	$=$	$S \cup B$, all potential routing points
J	$=$	set of all students
K	$=$	the set of school buses

Parameters

$$W = \text{school bus capacity}$$

$$c_{ij} = \begin{cases} \text{walking distance from home of student } j \in J \text{ to routing point } i \in I\text{; or,} \\ \text{distance along street network between routing points } i \in I \text{ and } j \in I \end{cases}$$

$$S_j = \text{maximum walking distance for student } j \in J \text{ to a bus stop}$$

$$v_j = \text{load of student } j \in J \text{ (} 2/3 \text{ if student } j \text{ is in early primary grades and 1 otherwise)}$$

$$n = \text{the number of routing points, equivalent to } |I|$$

Note that the distance parameter, c_{ij}, has two cases considered in its definition. The first case refers to the walking distance between a student's residence and a potential bus stop location or the school. The second case refers to the distance along the street network between two potential bus stop locations or a potential bus stop location and the school.

Variables

$$z_{ij} = \begin{cases} 1 & \text{if student } j \in J \text{ is assigned to bus stop } i \in B \\ 0 & \text{otherwise} \end{cases}$$

$$u_i = \begin{cases} 1 & \text{if a bus stop is located at site } i \in B \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ik} = \begin{cases} 1 & \text{if routing point } i \in I \text{ is serviced by vehicle } k \in K \\ 0 & \text{otherwise} \end{cases}$$

$$x_{ijk} = \begin{cases} 1 & \text{if point } i \in I \text{ immediately precedes point } j \in I \text{ on route } k \in K \\ 0 & \text{otherwise} \end{cases}$$

Measures

$$
\begin{aligned}
f_1 &= \sum_{i,j \in I; k \in K} c_{ij} x_{ijk} && \text{Total Bus Route Length} \\
f_2 &= \sum_{i \in B; j \in J} c_{ij} z_{ij} && \text{Student Walking Distance} \\
f_3 &= \sum_{k \in K} \left(\sum_{i \in B; j \in J} v_j y_{ik} z_{ij} \right)^2 && \text{Load Balancing} \\
f_4 &= \sum_{k \in K} \left(\sum_{i,j \in I} c_{ij} x_{ijk} - \frac{\sum_{ijk} c_{ij} x_{ijk}}{|K|} \right)^2 && \text{Length Balancing}
\end{aligned}
$$

Program

Given these definitions it is possible to formulate the following multiobjective nonlinear integer program describing the USBRP.

$$
\begin{array}{llllll}
\text{Minimize} & (f_1, f_2, f_3, f_4) & & & & \\
\text{subject to} & c_{ij} z_{ij} & \leq & S_j & i \in B; j \in J & (1) \\
& z_{ij} & \leq & u_i & i \in B; j \in J & (2) \\
& \sum_{i \in B} z_{ij} & = & 1 & j \in J & (3) \\
& \sum_{i \in B} \left(y_{ik} \sum_{j \in J} v_j z_{ij} \right) & \leq & W & k \in K & (4) \\
& \sum_{k \in K} y_{ik} & = & \begin{cases} |K| & i \in S \\ u_i & i \in B \end{cases} & & (5) \\
& \sum_{i \in I} x_{ijk} & = & y_{jk} & j \in I; k \in K & (6) \\
& \sum_{j \in I} x_{ijk} & = & y_{ik} & i \in I; k \in K & (7) \\
& \sum_{\ell \in B} f_{i\ell k} - \sum_{\ell \in B} f_{\ell i k} & = & \sum_{j \in B} x_{ijk} & i \in B; k \in K & (8) \\
& f_{ijk} & \leq & n x_{ijk} & i, j \in I; k \in K & (9) \\
& f_{ijk} & \geq & 0 & i, j \in I; k \in K & (10) \\
& u_i & \in & \{0, 1\} & i \in B & (11) \\
& x_{ijk} & \in & \{0, 1\} & i, j \in I; k \in K & (12) \\
& y_{ik} & \in & \{0, 1\} & i \in I; k \in K & (13) \\
& z_{ij} & \in & \{0, 1\} & i \in B; j \in J & (14)
\end{array}
$$

Constraints (1) ensure that the students do not have to walk further than the maximum walking distance set by the school board to their bus stop. Constraints (2) guarantee that students are only assigned to bus stops that are in use. Constraints (3) force each eligible student to be assigned to a bus stop. Constraints (4) are the vehicle capacity constraint. Note that these constraints are nonlinear since both y_{ik} and z_{ij} are variables. Constraints (5) ensure that each bus stop to which eligible students are assigned is only included on one route and that the school is on all routes. Constraints (6) and (7) specify that a bus that visits a bus stop also leaves the bus stop and that it only visits the bus stops on its own route. Constraints (8), (9) and (10) prevent the formation of subtours using flow variables (f_{ijk}). Constraints (11) through (14) are the integrality restrictions.

The mathematical formulation presented above cannot be used directly to solve the problem for two reasons. First, the problem formulation cannot be solved directly in a reasonable amount of time since the LRP is a *NP*-hard problem (Laporte, 1988) and there is no known polynomial time algorithm for the solution of *NP*-hard problems (Johnson and Papdimitrou, 1985). Second, the formulation generates a problem with a a very large number of variables and constraints. For example, to specify a sample SBRP with 20 potential bus stop sites, 100 students, and 3 school bus routes would require over 3400 variables and over 6800 constraints.

Hence, the USBRP cannot be solved in a reasonable amount of time using integer programming techniques and this formulation. Therefore, a heuristic approach must be adopted in order to generate solutions to the USBRP. We first briefly review heuristic techniques that have been developed for school bus routing. We then introduce a new solution methodology based on the formulation presented above which provides solutions to the USBRP under conditions of both equity and efficiency.

Heuristic Solution Methods for the School Bus Routing Problem

In an early paper on school bus routing, Bodin and Berman (1979) concentrated on solving the USBRP. In their method, a student was manually assigned to the nearest "ministop" or potential bus stop location (the terms here are used interchangeably). From these ministops, the set of actual bus stops was manually selected by the analyst who then assigned each student to the nearest actual bus stop from their ministop. The main convenience of the ministop concept was that it significantly reduced the amount of data needed; only a network with the ministops as nodes was required, not an entire street network. The actual bus stops were then combined into a large tour which was then partitioned into a series of routes.

In Dulac, Ferland and Fogues (1980) students were located on the street segment on which they lived and were then subsequently assigned to an incident street intersection. A subset of the incident nodes was selected as potential bus stop locations. Next, the potential bus stop with the largest number of students within walking distance was selected and all students within walking distance were assigned to this stop. This procedure was repeated until all students were assigned to a bus stop. Then the VRP was solved for the selected bus stops using either the Clarke and Wright (1964) or the Insertion procedure (Rosenkrantz, Sterns and Lewis, 1977). A deficiency in this method is due to the fact that students were allocated to bus stops before routing decisions were made. This resulted in a solution that could have more bus routes than necessary.

The deficiency with the previous method was addressed by Chapleau, Ferland, and Rousseau (1985). In their approach, the incident nodes were first grouped into clusters using a metric that estimated the route detour caused by adding an incident node to the cluster. Then, bus stops were selected for each cluster by considering the maximum number of students in that cluster within walking distance of a potential bus stop. After the bus stops were selected for each cluster, the stops were then routed by a heuristic traveling salesman problem algorithm.

Unfortunately, the current solution methods outlined above cannot deal with the multiobjective formulation of the USBRP presented above. However, the "cluster-first/route-second" decomposition of the USBRP and the clustering metric developed by Chapleau et al. are incorporated in the multiobjective method presented here. The next section outlines this new solution methodology that explicitly takes the multiobjective nature of the problem into account.

A Heuristic Solution Method for the Multiobjective USBRP

In order to develop a solution heuristic for the USBRP, it is useful to consider the problem as consisting of the following three related subproblems:

1. a location problem for determining school bus stops;

2. an allocation problem for assigning students to these bus stops; and,

3. a routing problem for generating the routes that visit the bus stops.

Both locational and routing decisions are made in identifying the bus routes. As mentioned above, a problem with these characteristics is known as a Location-Routing Problem (LRP) (Laporte, 1988). In the literature, two main heuristic strategies have been proposed for LRPs which differ in the order in which the three subproblems are solved.

The first heuristic is known as the Location-Allocation-Routing (LAR) strategy. Dulac et al. (1980) describe a school bus routing algorithm that uses this approach. Within the confines of a maximum walking distance constraint from a student's home to his/her bus stop, the Dulac et al (1980) algorithm determines a set of bus stops for a school and assigns all eligible students to these stops. Subsequently, a standard Vehicle Routing Problem (VRP) heuristic is applied to the selected bus stops in order to generate the school bus routes.

With this approach, the selection of the bus stops and the assignment of the students to these stops is undertaken without accounting for the effect of these decisions on the routing subproblem and capacity constraints. This can create several difficulties. For example, since vehicle capacity constraints are ignored in the location-allocation phase of this approach there is a tendency to generate an excessive number of routes whenever several bus stops have a relatively large number of students assigned to them. The task of balancing the number of students per bus can also be complicated by this approach.

The second heuristic is known as the Allocation-Routing-Location (ARL) strategy. In this approach, the students are first allocated into clusters so that each cluster can be serviced by a single school bus route. Then, for each cluster, the school bus stops are selected and a school bus route that visits the selected stops is generated. Chapleau et al (1985) use this approach in their school bus routing algorithm.

There are several advantages the second heuristic has over the LAR strategy. First, effective load-balancing can be accomplished during the allocation phase by assigning students to the clusters individually or in small numbers. Second, it is possible to keep the number of school bus routes to a minimum because these two objectives are independent of the location of the bus stops and of the route generated to service these stops. However, the route length-balancing objective cannot be explicitly controlled since each route is generated after the allocation phase and is dependent on the dispersion of students within the individual clusters.

Despite this drawback, the second heuristic is adopted for the improved solution methodology proposed here. The implementation of this methodology involves two algorithms which explicitly consider the multiobjective nature of the USBRP. The first is a districting algorithm and the second is a routing algorithm. These algorithms are discussed below.

The Districting Algorithm

The purpose of the districting algorithm is to group the students into clusters, each of which can be serviced by a unique school bus route. Due to school bus capacity constraints the minimum number of clusters, M_K, that must be generated for a given problem is:

$$M_K = \left\lceil \frac{\text{number of students}}{\text{bus capacity}} \right\rceil$$

where $\lceil a \rceil$ is the smallest integer greater than or equal to a. To generate the clusters, we define a multiobjective Vehicle Routing Problem (VRP) and use a heuristic method to generate solutions to this problem. This VRP uses the number of students assigned to each node as the demand that has to be serviced and the depot is defined as the school since all routes begin and end at the school. The solution of this VRP assigns each node to a particular route and this assignment is used as the basis for defining the student clusters. Thus the districting algorithm is transformed into an algorithm for the VRP. Note that whereas the LAR strategy outlined in the introduction used the solution of the VRP to produce the routing of the actual school bus stops, the VRP discussed in this section is formulated to identify the student clusters. To emphasize this point, this particular VRP formulation is referred to henceforth as the clustering-VRP formulation.

Not all objectives of the USBRP can be dealt with during the districting algorithm since the location of the school bus stops, the assignment of students to these stops, and the school bus routes are not yet defined. The two USBRP objectives that can be explicitly considered during the districting phase are minimizing the number of routes and load-balancing. This is a consequence of the fact that these objectives are independent of the actual school bus routes selected and only depend on the clusters chosen.

Although the length-balancing USBRP objective is not examined in this approach (due to the ordering of the phases), two additional objectives are included in the districting algorithm in an attempt to fulfill several other USBRP goals. The first additional objective is to minimize the total length of the clustering-VRP routes. Since the nearest street intersections (or nodes) to the students' residences are used as the demand points, since these nodes tend to be a superset of the actual bus stops, and since these nodes are used on an actual street network this objective provides a reasonable surrogate for the length of the actual school bus routes. The other clustering-VRP objective involves the compactness measure of Chapleay at al. (1985) which is defined and discussed in Appendix A. This measure ensures that the clusters chosen are compact and oriented in elliptical clusters around the school and helps to compensate for cases where the clustering-VRP route length objective produces clusters which translate into ineffective school bus routes.

In the past, clustering-VRP formulations have been avoided for the following reasons:

1. Large problems could have over 500 nodes to service and heuristics used for the VRP proved to be very slow in generating solutions to such problems.

2. These same heuristics were designed to minimize the total route distance and not the number of routes. Moreover, they could not deal with multiple objectives.

Bowerman et al. (1993) proposed a new heuristic for solving the VRP that addresses these concerns. This heuristic, termed the Space Filling Curve with Optimal Partitioning heuristic (SFC OP), is very fast; it handles larger problems particularly well when compared to other VRP heuristics; it can consider multiple objectives; and, it can generate the minimum possible number of routes. The heuristic operates by ordering all the demand points, in this case the street intersections to which students are assigned, into one long tour using a SFC heuristic (Bartholdi and Platzman, 1988). This tour is then partitioned into the minimum number of routes using dynamic programming techniques. The objective function for the tour partitioning is formulated to consider the multiple objectives of the districting algorithm.

Recall that the districting algorithm has four objectives: minimizing the number of routes, minimizing the length of the routes, load-balancing, and maximizing the compactness of the routes. The goal of minimizing the number of routes is the dominant objective. This goal is met by formulating the SFC OP partitioning heuristic so that the number of routes is the dominant objective, *i.e.,* a solution with fewer routes is always favoured over a solution with more. The three remaining objectives, involving clustering-VRP route length, load balancing, and compactness, are combined into a single overall objective using the weighting method (Szidarovszky, Gerson and Duckstein, 1986) by forming a weighted sum of these three objectives. These weights are assigned to each objective by a decision maker, typically the school board transportation planner, according to her or his opinion of the relative importance of each objective. The weights can be chosen arbitrarily, however it is important to note that small changes in the weights can yield much larger changes in the values of the objective functions in the solution since a different configuration can result. If a decision maker assigned the following weights to the relative importance of each objective: w_a for route length, w_b for compactness, and w_3 for load balancing, then this overall objective function might be written as:

$$\hat{f} = w_a f_a + w_b f_b + w_3 f_3. \tag{16}$$

where f_a is the clustering-VRP route length, f_b is the compactness measure of the solution, and f_3 is load balancing objective as defined on page 10. Note that both f_a and f_b are not objectives defined in the formulation but are surrogate objectives as discussed above. However, (16) would yield poor results since the three objectives have vastly different ranges. This difficulty can be overcome by standardizing the objectives so that each has an equivalent scale. This is accomplished by generating the solution to the clustering-VRP for each of the individual objectives (i.e., obtain the best possible value for each individual objective, denoted by f_i^* for $i = a, b, 3$), and by obtaining an average value for each objective (denoted by $\overline{f_i}$ for $i = a, b, 3$). The SFC OP heuristic generates intermediate solution values by chosing different starting points; these values are used to calculate the average value for each objective. Using these values the overall weighted objective function is defined as:

$$f = w_a \left(\frac{f_a - f_a^*}{\overline{f_a} - f_a^*} \right) + w_b \left(\frac{f_b - f_b^*}{\overline{f_b} - f_b^*} \right) + w_3 \left(\frac{f_3 - f_3^*}{\overline{f_3} - f_3^*} \right) \tag{17}$$

This results in each standardized objective having values of 0 as its minimum and 1 as its average. Any solution that minimizes (17) is Pareto Optimal as long as there are no zero weights assigned by the decision maker (Szidarovszky, Gerson and Duckstein, 1986).

Decision makers can examine the effects of different parameter weightings to find a desired set of school bus routes. For example, if attempting to load balance introduces large or unwieldy routes then the emphasis placed on load balancing can be reduced. Similarly, if the clustering-VRP route length criterion does not result in sensible clusters then the weighting of the compactness measure can be increased to force the clusters to become more compact.

The solution to the clustering-VRP that is generated by applying the SFC OP heuristic to the objective function specified in (17) results in each student node being assigned to a specific cluster (i.e., all student nodes on a single clustering-VRP route are assigned to the same cluster). For each of these clusters a bus stop routing algorithm is then executed to select a subset of the potential bus stop sites, to assign students to the selected bus stops, and to generate a route that visits these bus stops. This bus stop routing algorithm is discussed in the next section.

The Bus Stop Routing Algorithm

Whereas the districting algorithm assigns each student node to a particular cluster, the bus stop routing algorithm generates a specific school bus route that visits a subset of the potential bus stop sites so that every student on the route is within walking distance of a bus stop. Since we are only generating a single route at a time with this algorithm, we can no longer balance factors between the routes. However, the two criteria which are of interest in this phase are the total length of the bus routes and the total student walking distance to their bus stops.

The solution procedure for this phase is a combination of the COVTOUR heuristic of Current and Schilling (1989) and the insertion heuristic of Yurtsever (1988). Here again we use the weighting method to form a single objective function. If both criteria are measure in the same units, for example kilometres, then there is no need to standardize these measures as there was in the districting algorithm. The overall objective function can be obtained using the weighting method (Szidarovszky, Gerson and Duckstein, 1986) by adding the weighted objectives. If w_1 and w_2 are the relative weights assigned to the school bus route length criteria and to the student walking distance criteria respectively, f_1 is the school bus route length, and f_2 is the total student walking distance, then the overall objective for the bus stop routing algorithm would be written as:

$$f = w_1 f_1 + w_2 f_2.$$

Note that w_1/w_2 measures the importance of reducing bus distance compared to student walking distance. For example, if $w_1/w_2 = 20$ then increasing total school bus route length by 1 km is equivalent to reducing total student walking distance by 20 km. Thus, the importance placed on student walking distance can be varied so that increasing the length of the school bus routes produces a significant reduction in the average distance the students walk to their bus stop.

The algorithm can be described by the following three steps:

1. Find P covering sets of bus stops.

2. For each set of bus stops, generate a school bus route on this set of stops.

3. Find the solution from the previous step that has the least total weighted distance and add bus stops to reduce this total distance.

Note that the first two steps of this algorithm are the same as the COVTOUR heuristic of Current and Schilling (1989). Each of these three steps is described in more detail below.

Find P covering sets of bus stops

This step of the algorithm finds a set of bus stops for each route so that every student in a cluster is assigned to a bus stop within the maximum walking distance from their homes. This method is based on the Kolesar-Walker heuristic (1974) for solving the unicost set covering problem. The procedure is described by the following algorithm:

Algorithm 1

1. Repeat the following procedure P times, where P is the number of potential bus stop sites. Let p represent the current iteration number.

2. Mark all students for this route as being unassigned to any bus stop.

3. Select the bus stop that has the p^{th} most students within maximum walking distance of it and assign all these students to this bus stop.

4. Select the remaining bus stop with the largest number of unassigned students within maximum walking distance of it and assign these students to this bus stop. Repeat this step until all students on the route have been assigned to a bus stop.

5. Execute the routing procedure, '**Find a tour of the bus stops**,' discussed in the next section on the selected bus stop sites to obtain the total weighted distance of the corresponding route.

From the P different routes generated by this algorithm select the route that has the least total weighted distance and use this route as the basis for performing the bus stop insertion heuristic.

Find a tour of the bus stops

Given a subset of the potential bus stop sites (i.e., the stops selected in each of the P repetitions of Algorithm 1), this procedure finds a route that visits these bus stops. There are three steps to this procedure.

Algorithm 2

1. Since the Kolesar-Walker heuristic may not have assigned students to the nearest *selected* bus stop, reassign students to the nearest selected bus stop and calculate the total student walking distance, denoted by f_2.

2. Perform a 2-opt procedure (Lin, 1965) on the selected bus stops and the school node to form a tour that visits the selected bus stops and school. The 2-opt heuristic was chosen since the number of bus stops is small, typically less than 10, and this heuristic is computationally efficient and gives excellent results. Calculate the total length of this route, denoted by f_1.

3. Calculate the total weighted distance associated with this route, $w_1 f_1 + w_2 f_2$.

The algorithm in the previous section selects the route that has the least total weighted distance and performs the bus stop insertion heuristic described in the next section to reduce the total weighted distance.

Bus stop insertion heuristic

Whenever $w_2 \neq 0$, this procedure attempts to decrease the total weighted distance obtained with Algorithm 2 by adding bus stops to the route. This is possible since a sufficient decrease in the student walking distance can still reduce the total weighted distance even if the length of the route increases. The algorithm is based on the insertion heuristic of Yurtsever (1988) and is described as follows:

Algorithm 3

1. Pick one of the currently unselected bus stops and add this stop to the route (by running Algorithm 2 on the new set of selected bus stops).

2. Calculate the amount of walking distance saved, ∂f_2, by adding this stop to the route. Note that ∂f_2 represents the reduction in total student walking distance.

3. Calculate the route distance increase, ∂f_1, caused by adding this stop to the route.

4. After performing steps 1 through 3 on all unselected bus stop sites select the unselected bus stop that gives the best improvement to the route using one of the following selection rules:

   Maximum Improvement Select the stop that decreases the objective function the most, i.e., has the largest negative value of $w_1 \partial f_1 - w_2 \partial f_2$.

   Best Ratio Select the stop that yields the maximum savings in student walking distance when route distance is increased by one, i.e., maximizes $\dfrac{w_2 \partial f_2}{w_1 \partial f_1}$.

5. If no bus stop improves the route or if there are no more potential bus stop sites to add, then stop. Otherwise add the selected bus stop to the route and return to step 1.

Yurtsever (1988) noted that the best ratio selection rule gave the best results in empirical tests.

In summary, the algorithm for the USBRP is divided into two phases. The first phase groups the students into clusters that can be serviced by a single school bus route. This grouping is accomplished by posing the problem in a multiobjective VRP formulation known as the clustering-VRP, and generating a solution using the SFC OP VRP heuristic reported in Bowerman el al (1993). Then, in the second phase, the bus stop routing algorithm is executed on each of the resulting clusters.

The next section discusses the results of empirical tests of this method on sample data from Wellington Country, Ontario using various parameter weightings.

Results

Empirical tests were conducted on this solution methodology using sample data from John McCrae Public School located in the south end of the City of Guelph, Ontario. This school is administered by the Wellington County Board of Education (WCBE), which provided the data on student grades and residential locations. In the WCBE, school bus services are provided to students who live outside a specified walking distance from their school. Students who live closer than this distance are expected to walk to their schools and bus transportation is not provided. These maximum walking distance limits vary by grade level. The actual limits used by the WCBE are 1.6 km for students in Kindergarten to Grade 2, 3.2 km for students in Grade 3 to Grade 8, and 4.8 km for students in secondary school. For these tests, forty-four street intersection nodes were chosen from the street network to serve as potential school bus sites.

Figure 1 is a map of the area surrounding the school. Included on this map is the spatial distribution of the students used in the test problem, the location of the school, the potential bus stop sites, and the pattern of the street network. The street network used for testing was undirected.

The solution method was implemented as part of a a series of four programs written in the 'C' language called within the procedure interface of the TransCAD Geographical Information System (Caliper Corporation, 1990). The four modules that were implemented are: the student management module,

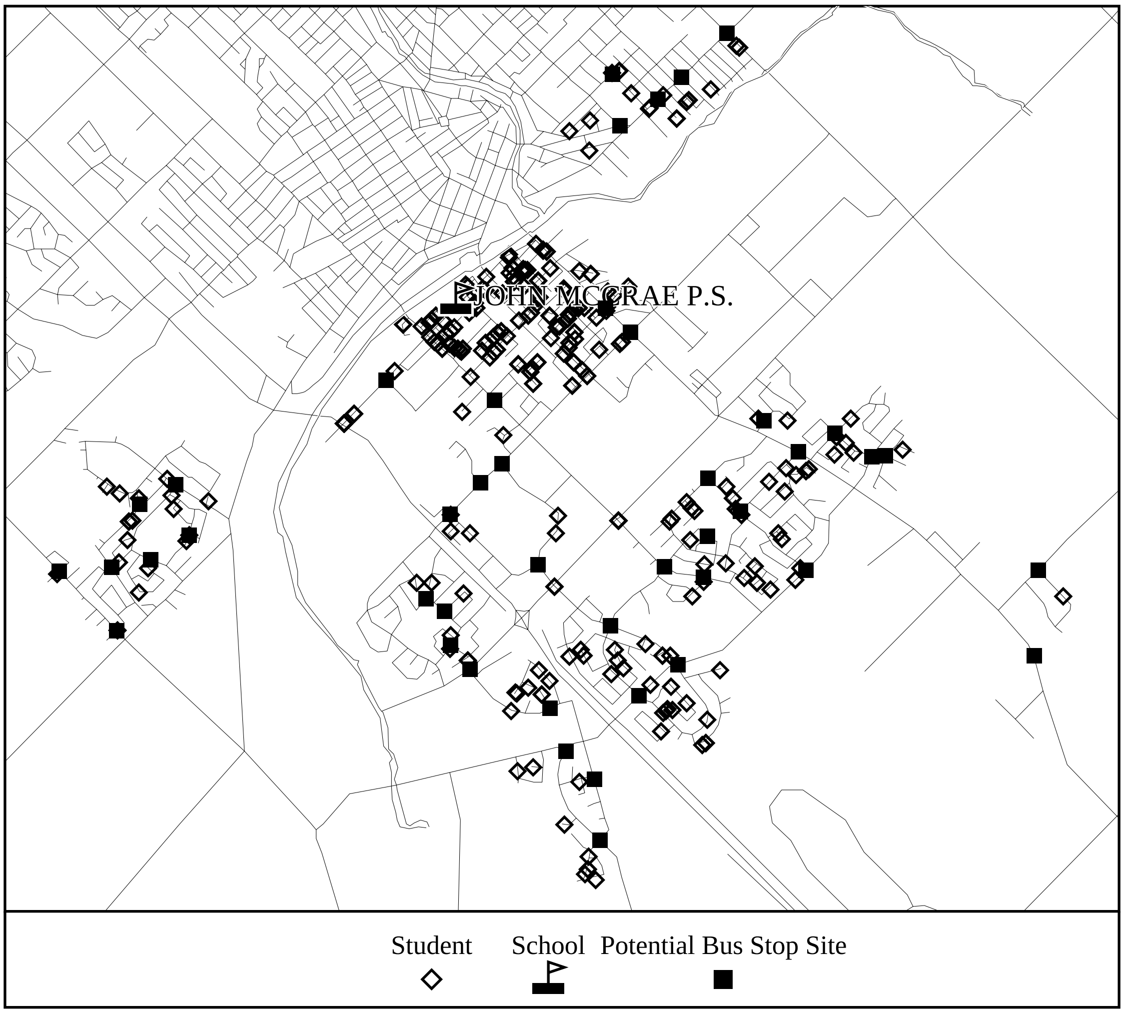

Figure 1: The spatial distribution of students and potential bus stop sites for the test problem.

the attendance area and node assignment module, the distance table creation module, and the school bus routing module. The modules used TransCAD to provide a user friendly interface to the programs and to manage the student and street data. Additionally, the db_VISTA III database management system (Raima Corporation, 1990) was used to access the various TransCAD databases. db_VISTA was chosen since it is the database management system that TransCAD uses to organize its databases.

For this problem the total number of students eligible for school bus transportation is 138. With the school bus capacity set to 50 at least three school bus routes are required to satisfy the demand. Also, the routes will be perfectly load-balanced if each route carried 46 students. The school bus routing algorithm outlined above was tested by varying the clustering-VRP objective weights and by varying the two bus stop routing algorithm weights. These tests are described in detail below.

Effects of Varying Clustering Weights

The school bus routing algorithm was executed on the sample problem using five different configurations of the three clustering-VRP weights. In reality, these various weights would be set by the school board transportation planner according to the relative importance that he or she attaches to each criterion. The first three configurations result in only one criterion (the one having weight 1.0) being used during the districting phase. The fourth configuration weights all three criteria equally. The final configuration was chosen to be, in the authors' opinion, a typical weighting that a school board transportation planner might choose. This weighting is a compromise between reducing the school bus route length to be nearly minimal while still incorporating equity goals of length balancing in the design of the school bus routes. In each case, student walking distance was not considered.

As can be seen in Table 1, where the results of these tests are summarized, the effect of changing the weights of the various clustering-VRP criteria had the expected result on the school bus routes and student clusters generated by the school bus routing algorithm. In particular, the more a particular criterion is emphasized (by increasing its relative weight), the lower the value of that criterion in the final solution. The school bus routes that resulted from Run 5 are shown in Figure 2. It is interesting to note that even though the length-balancing objective is not considered by this algorithm, the difference in the length of the routes was small; the routes varied from 10 km to 14 km. In a typical situation these routes would take between 15 and 20 minutes to traverse from the school to all the bus stops and back to the school excluding the time required to perform the bus stops.

Effects of the Student Walking Distance Weight

The effects of the various weighting criteria on the districting process were examined in the previous section. Additional tests were conducted to determine the effect of the student walking distance weight on the overall routing process. The larger the value of this weight, the smaller the reduction in total student walking distance has to be in order to add a new bus stop to a route. For example, if the bus distance weight is 1.0 and the walking distance weight is 0.01 then in order to add 10 metres to the overall length of a route, there must be a reduction of 1000 metres in student walking distance. Thus as the weight applied to the student walking distance criterion increases with respect to the weight applied to the bus distance criterion, the total bus route length would be expected to increase while the average student walking distance would decrease.

The school bus route generation program was executed with the student walking distance criterion weight varying from 0 to 0.25. The bus distance criterion was given a weight of 1.0. Again, the school board transportation planner would fix the value according to some school board policy or vary the value to ensure that the average student walking distance is below some distance

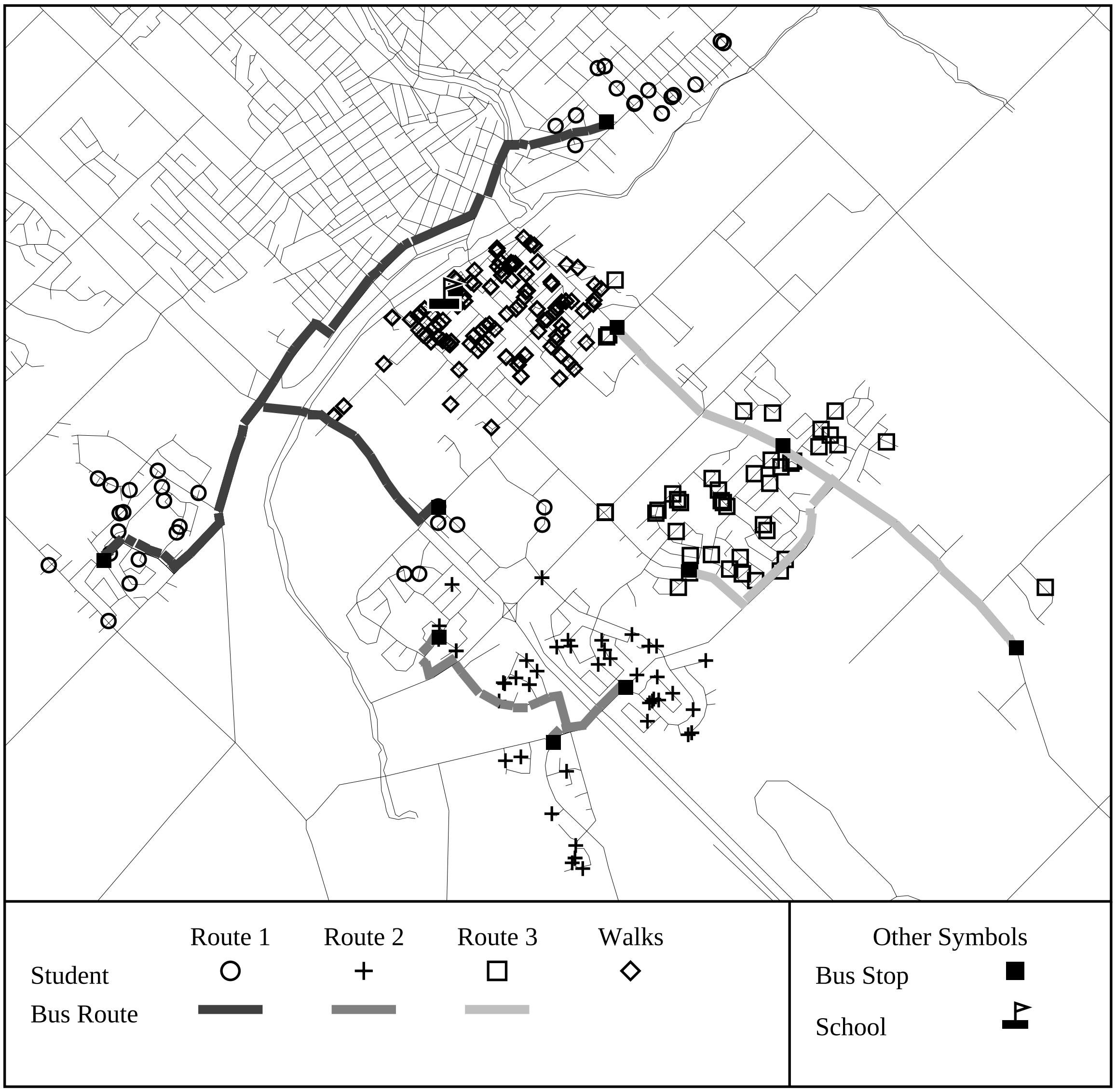

Figure 2: School bus routes and student assignments generated with clustering-VRP weights set to 0.7 route length, 0.2 load balancing, and 0.1 compactness.

Note: each school bus route starts and ends at the school and that the sections of the school bus route going from the school to the first bus stop and from the last bus stop to the school have been omitted for the sake of clarity.

standard. The total bus route distance and average student walking distance for these runs are recorded in Table 2 and shown graphically in Figure 3. For all of these runs the clustering-VRP route length, load balancing and compactness criterion weights were fixed at the same values as used for Run 5 in the previous section, namely 0.7, 0.2, and 0.1, respectively.

The results from these runs are consistent with expectations. When the weight associated with student walking distance was increased the total bus route length increased and the average student walking distance to the bus stop decreased. However, it is perhaps surprising to note that with the weighting parameter set to 0.1 the total bus route distance increased by approximately 250 m yet the average student walking distance decreased by a full 174 m. Since this corresponds to 138 students, the total student walking distance was reduced by over 24 km.

Conclusion

This paper has discussed the characteristics of a school bus routing problem for urban areas. The mathematical formulation developed for the USBRP required both equity and efficiency considerations, given the multiobjective nature of the service being delivered.

The USBRP was modeled as a multiobjective variant of the Location-Routing Problem (LRP). The USBRP contains three inter-related tasks: the selection of the school bus stops, the assignment of students to these school bus stops, and the generation of school bus routes to server the stops. Additionally, the objectives were to minimize the bus route distance, to balance student walking distance with bus route distance, and to perform both load-balancing and length-balancing between the routes.

Two algorithms were developed for generating solutions to the USBRP that incorporate these equity considerations in the assignment of students to bus

	Weights			Results				
	VRP	Load		Ext.	Load	Comp.	Bus	Avg
Run	Route	Bal.	Comp.	(km)	Bal. (N^2)	(km^2)	Len (km)	Walk (m)
1	1.0	0.0	0.0	9.983	56	662.8	35.944	592.0
2	0.0	1.0	0.0	14.783	0	1053.7	39.713	598.8
3	0.0	0.0	1.0	16.560	96	640.7	41.280	547.1
4	1.0	1.0	1.0	12.407	8	703.1	38.264	589.8
5	0.7	0.2	0.1	11.175	24	647.5	35.945	589.8

Table 1: The effects of varying the clustering parameters.

Notes: In this table VRP Route, Load Bal. and Comp. refer to the weights assigned to the clustering-VRP route length objective, load balancing objective, and compactness objective, respectively. (The value of the load balancing objective is the variance of the load values, i.e., it is the sum of the squares of the load on each bus route minus the average load on all three bus routes.) Ext. is the extension or the total additional distance added to the initial clustering-VRP tour that visits all student nodes by breaking this tour into routes. Bus Len is the total length of the school bus routes, and Avg Walk is the average distance the students walk to their bus stop.

Student Walking Distance Weight	Total Bus Distance	Average Student Walking Distance
0.00	35.945	589.8
0.01	35.949	435.2
0.05	36.002	430.3
0.10	36.199	415.8
0.15	36.403	404.5
0.20	37.354	368.2
0.25	37.869	351.9

Table 2: Total bus route distance and average student walking distance for varying values of the student walking distance weight parameter

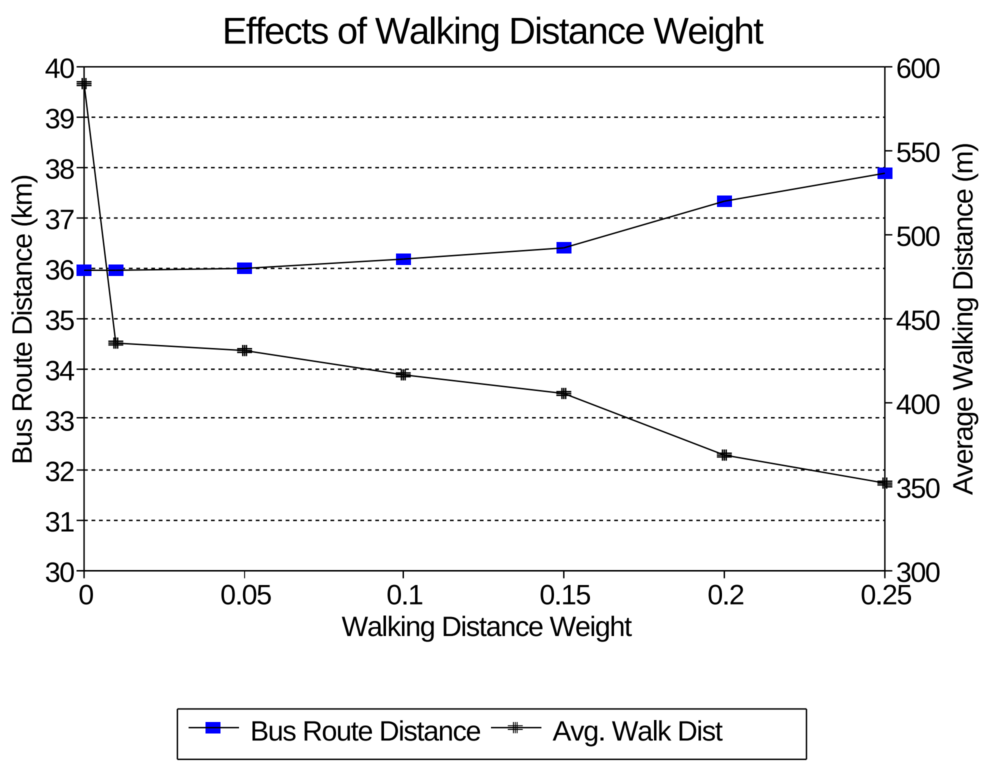

Figure 3: Effects of varying walking distance weight

routes and the subsequent generation of the routes. Due to the arrangement of the algorithms in the solution methods, it was not possible to explicitly consider length balancing between routes. We consider this to be a noncritical issue in the USBRP since, in virtually all cases tested, the routes were relatively short and the difference in their lengths was small.

The user-defined weighting measures introduced in both the districting and routing stages allow school board transportation planners to emphasize the various considerations as they see fit. The larger the weight placed on a criterion the lower the value of that criterion in the final solution. School bus routes were generated for a variety of weighting factors. The following cases were considered: each clustering criterion being the only factor weighted, each clustering criterion being equally weighted, and what was felt to be a "typical" weighting that a school board transportation planner would use. For the final clustering, the value of the student walking distance weight was varied between runs.

We feel that the solution method developed in this paper has several unique features. It is the only method for school bus routing, that the authors are aware of, which examines the problem from a multiobjective viewpoint for considering such things as student walking distance and load-balancing and allows the transportation planner to explicitly weigh these considerations. Although this method was designed for jurisdictions in Ontario, Canada, we feel that it is easily applicable to other jurisdictions. It may be necessary to change the student eligibility requirements or to add additional objectives to either algorithm in order to suit the requirements of other jurisdictions. In addition, although the method was tested on an undirected street network, we believe that the method would function equally well on street networks with more realistic traffic constraints.

By explicitly examining these considerations decision makers can generate a set of school bus routes that are both economically efficient and politically acceptable to the public.

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Appendix A – Definition of Compactness Measure

The districting algorithm discussed in this paper includes a compactness measure as one of the objectives. This compactness measure, which was developed by Chapleau et al. (1985), is described in this section.

Assume that each student is assigned to a node on the street network. Let the distance between two given points x and y be defined as:

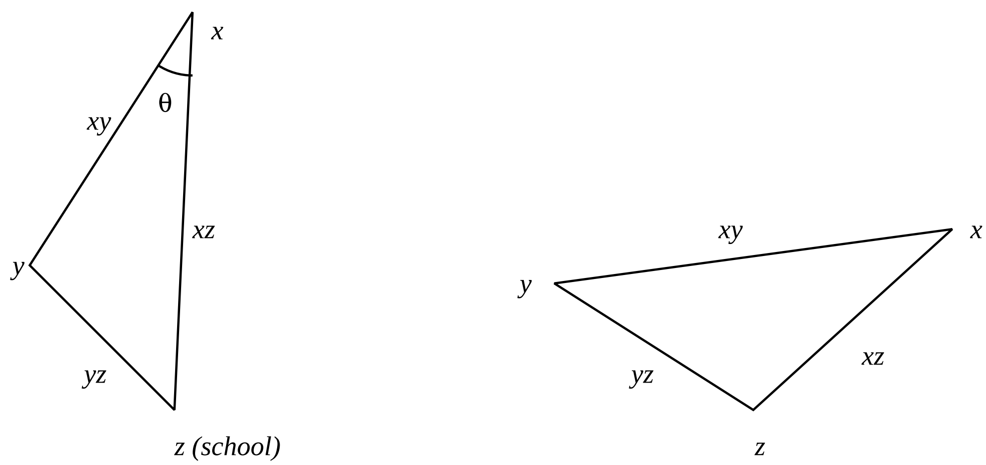
$$\mathrm{cfrdis}(x,y) = \begin{cases} xy^2 \left(1 + \lambda \left[1 - \dfrac{(xy^2 - yz^2 + xz^2)}{2\, xy\, xz}\right]\right) & \text{if } xy \leq yz \\[2ex] xy^2 \left(1 + 2\lambda \dfrac{yz}{xy}\right) & \text{if } xy > yz \end{cases}$$

where z is the school location, and ab is the shortest distance between a and b in the network. Note that x is assumed to be further from z than y (i.e., $xy > xz$). The parameter λ is a penalty parameter that affects the geometry of the clusters. Different cluster shapes are generated by different values of λ. Chapleau et al. (1985) state that good results that involved elliptical clusters centered around the school were obtained for a school bus routing problem using $\lambda = 1/2$.

This definition of distance measures the penalty of including node y in a cluster by measuring the potential detour in the route necessary to include that node in the cluster. If straight line distances are used between points, instead of the shortest distances along the network, then the distance definition for the case $xy \leq yz$ would become:

$$\mathrm{cfrdis}(x,y) = xy^2\,(1 + \lambda \sin\theta)$$

where θ is defined as in Figure 4. Thus if x, y, and z are collinear then $\sin\theta = 0$ and $\mathrm{cfrdis}(x,y)$ is simply the square of the straight line distance. As θ increases so does $\sin\theta$ and this results in a larger penalty that reflects the detour incurred



Case 1: xy ≤ yz

Case 2: xy > yz

Figure 4: CFR Distance definitions

by adding this node to the cluster. If the school is in between nodes x and y $(xy > yz)$ then the second definition of cfrdis is used. This measure increases the penalty as the ratio yz/xy increases.

Using this distance measure, the compactness of a cluster, k, can be computed as:

$$\mathrm{Comp}_k = \sum_{m \in M_k} \mathrm{cfrdis}(m, m_k^*), \;\; 1 \leq k \leq K;$$

where M_k is the set of student nodes included in cluster k, and m_k^* is the centroid of the cluster given by:

$$m_k^* = \arg\min_{n \in M_k} \left\{ \sum_{m \in M_k} \mathrm{cfrdis}(m, n) \right\}.$$