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Routing a Heterogeneous Fleet of Vehicles

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Abstract

In the well-known Vehicle Routing Problem (VRP), a set of identical vehicles, based at a central depot, is to be optimally routed to supply customers with known demands subject to vehicle capacity constraints.

An important variant of the VRP arises when a fleet of vehicles characterized by different capacities and costs, is available for distribution activities. The problem is known as the Mixed Fleet VRP or as the Heterogeneous Fleet VRP.

This paper gives an overview of approaches from the literature to solve heterogeneous VRPs. In particular, we classify the different variants described in the literature and, as no exact algorithm has been presented for any variants of heterogeneous VRP, we review the lower bounds and the heuristic algorithms proposed. Computational results, comparing the performance of the different heuristic algorithms on benchmark instances, are also discussed.

1 Introduction

The Vehicle Routing Problem (VRP) is one of the most studied combinatorial optimization problems and is concerned with the optimal design of routes to be used by a fleet of vehicles to serve a set of customers. Since it was first proposed by Dantzig and Ramser (1959), hundreds of papers were devoted to the exact and approximate solution of the many variants of this problem, as the Capacitated VRP (CVRP), in which a homogeneous fleet of vehicles is available and the only considered constraint is the vehicle capacity, or the VRP with Time Windows (VRPTW), in which customers may be served within a specified time interval and the schedule of the vehicle trips should be determined as well.

More recently, a greater attention has been devoted to more complex variants of the VRP, sometimes named “rich” VRPs, that are closer to the practical distribution problems that VRP models. In particular, these variants are characterized by multiple depots, multiple trips to be performed by the vehicles, multiple vehicle types, or other operational issues as loading constraints. Trying to systematize such a huge literature is a challenging and useful activity that has attracted considerable efforts in the scientific community. Among the various surveys on VRP we mention the book by Toth and Vigo (2002) and the more recent update by Cordeau et al. (2006), whereas specific surveys on rich VRPs may be found in Bräysy et al. (2002).

This paper considers an important variant of the VRP in which a fleet of vehicles characterized by different capacities and costs, is available for the distribution activities. The problem is known as the Mixed Fleet VRP or as the Heterogeneous Fleet VRP, and was first considered in a structured way in Golden et al. (1984). We mainly concentrate on the basic problems including capacity constraints only, which received greater attention in the literature. Moreover, we briefly review the approaches recently proposed to include time windows constraints. Case-studies and applications related to the solution of Heterogeneous VRPs can be found in Semet and Taillard (1993), Rochat and Semet (1994), Brandão and Mercer (1997), Prins (2002), Wu et al. (2005) and Tavakkoli-Moghaddam et al. (2006). In addition, Engevall et al. (2004) use a game-theoretic approach to model, as a Vehicle Routing game, the problem of allocating to the customers the cost of the heterogeneous fleet.

The paper is organized as follows. The next section introduces the notation used throughout the paper and describes the variants of heterogeneous VRPs with capacity constraints studied in the literature. Section 3 presents the main integer programming formulations and discusses lower bounding approaches. Finally, Section 4 reviews heuristics and metaheuristics and reports on their performances.

2 Notation and problem variants

A directed graph $G = (V, A)$ is given, where $V = \{0, 1, \ldots, n\}$ is the set of $n + 1$ nodes and A is the set of arcs. Node 0 represents the depot, while the remaining node set $V' = V \setminus \{0\}$ corresponds to the n customers.

Each customer $i \in V'$ requires a supply of q_i units from the depot (we assume $q_0 = 0$). A heterogeneous fleet of vehicles is stationed at the depot and is used to supply the customers. The vehicle fleet is composed by m different vehicle types, with $M = \{1, \ldots, m\}$. For each type $k \in M$, m_k vehicles are available at the depot, each having a *capacity* equal to Q_k. Each vehicle type is also associated with a *fixed cost*, equal to F_k modelling, e.g., rental or capital amortization costs. In addition, each arc $(i, j) \in A$ and each vehicle type $k \in M$ has a non-negative *routing cost* c_{ij}^k.

A *route* is defined as the pair (R, k), where $R = (i_1, i_2, \ldots, i_{|R|})$, with $i_1 = i_{|R|} = 0$ and $\{i_2, \ldots, i_{|R|-1}\} \subseteq V'$, is a simple circuit in G containing the depot, and k is the type of vehicle associated with the route. In the following, R will be used to refer both to the visiting sequence and to the set of customers (including the depot) of the route. A route (R, k) is *feasible* if the total demand of the customers visited by the route does not exceed the vehicle capacity Q_k (i.e., $\sum_{h=2}^{|R|-1} q_{i_h} \leqslant Q_k$). The cost of a route corresponds to the sum of the costs of the arcs forming the route, plus the fixed cost of the vehicle associated with it (i.e., $F_k + \sum_{h=1}^{|R|-1} c_{i_h i_{h+1}}^k$).

The most general version of the Heterogeneous VRP consists of designing a set of feasible routes with minimum total cost, and such that:

i) each customer is visited by exactly one route;

ii) the number of routes performed by vehicles of type $k \in M$ is not greater than m_k.

Two versions of the problem naturally arise: the *symmetric* one, when $c_{ij}^k = c_{ji}^k$, for every pair i, j of customers and for each vehicle type $k \in M$, and the *asymmetric* version, otherwise. In addition, several variants of these general problems were presented in the literature, depending on the available fleet, and the type of considered costs. In particular, the following problem characteristics were modified:

i) the vehicle fleet is composed by an *unlimited* number of vehicles for each type, i.e., $m_k = +\infty$, $\forall k \in M$;

ii) the fixed costs of the vehicles are *not considered*, i.e., $F_k = 0$, $\forall k \in M$;

iii) the routing costs are *vehicle-independent*, i.e., $c_{ij}^{k_1} = c_{ij}^{k_2} = c_{ij}$, $\forall k_1, k_2 \in M$, $k_1 \neq k_2$, and $\forall (i, j) \in A$.

Table 1: Problem variants presented in the literature

Problem Name	**Fleet Size**	**Fixed Costs**	**Routing Costs**	**References**
HVRPFD	*Limited*	*Considered*	*Dependent*	Li et al. (2006)
HVRPD	*Limited*	*Not considered*	*Dependent*	Taillard (1999), Gendreau et al. (1999), Prins (2002), Tarantilis et al. (2003), Tarantilis et al. (2004), Li et al. (2006)
FSMFD	*Unlimited*	*Considered*	*Dependent*	Ferland and Michelon (1988), Teodorovic et al. (1995), Choi and Tcha (2006)
FSMD	*Unlimited*	*Not considered*	*Dependent*	Taillard (1999), Gendreau et al. (1999), Wassan and Osman (2002), Choi and Tcha (2006)
FSMF	*Unlimited*	*Considered*	*Independent*	Gheysens et al. (1984), Golden et al. (1984), Gheysens et al. (1986), Desrochers and Verhoog (1991), Salhi and Rand (1993), Osman and Salhi (1996), Taillard (1999), Ochi et al. (1998a), Ochi et al. (1998b), Gendreau et al. (1999), Liu and Shen (1999), Wassan and Osman (2002), Dullaert et al. (2002), Renaud and Boctor (2002), Choi and Tcha (2006), Yaman (2006), Dell'Amico et al. (2006)

Table 1 summarizes the different problem variants that were actually considered in the literature, together with the corresponding references. The different problem variants have been referred in the literature using different names, although there is a certain homogeneity toward calling Heterogenous VRPs the variants with limited number of vehicles, and Fleet Size and Mix those with unlimited ones. Therefore, we adopted a unified naming convention, that uses two acronyms (HVRP and FSM) and adds them two letters indicating whether fixed or routing costs are considered: we used "F" for fixed costs and "D" for vehicle dependent routing costs, respectively. Thus, in this paper we will refer to the problem variants as follows (see Table 1):

(a) Heterogeneous VRP with Fixed Costs and Vehicle Dependent Routing Costs (HVRPFD);

(b) Heterogeneous VRP with Vehicle Dependent Routing Costs (HVRPD);

(c) Fleet Size and Mix VRP with Fixed Costs and Vehicle Dependent Routing Costs (FSMFD);

(d) Fleet Size and Mix VRP with Vehicle Dependent Routing Costs (FSMD);

(e) Fleet Size and Mix VRP with Fixed Costs (FSMF).

The variants with time windows are denoted by adding TW to the acronym of the specific problem.

For FSMF, a usual assumption on the vehicle types in M imposes that they are *undominated*, i.e., ordered so that $Q_1 < Q_2 < \ldots < Q_m$ and $F_1 < F_2 < \ldots < F_m$.

All the problems described above are $\mathcal{NP}$-hard as they are natural generalizations of the Travelling Salesman Problem (TSP).

3 Mathematical formulations and lower bounds

In this section, we describe some of the mathematical formulations and lower bounds presented in the literature for heterogeneous vehicle routing problems. As far as we are aware, no exact algorithm has ever been developed for any of the different versions of heterogeneous vehicle routing problems described in the previous section.

Most integer programming formulations of the basic VRP use binary variables as vehicle flow variables to indicate if a vehicle travels between two customers in the optimal solution. In this way, decision variables combine assignment constraints, modelling vehicle routes, with commodity flow constraints, modelling movements of goods. Formulations of this type were first proposed by Garvin et al. (1957) to model an oil delivery problem and later extended by Gavish and Graves (1982).

Gheysens et al. (1984) formulate the FSMF using three-index binary variables x_{ij}^k as vehicle flow variables that take value 1 if a vehicle of type k travels directly from customer i to customer j, and 0 otherwise. In addition, flow variables y_{ij} specify the quantity of goods that a vehicle carries when it leaves customer i to service customer j. The formulation, below written for HVRPFD which is the most general variant, is as follows:

$$(F1) \quad z(F1) = Min \sum_{k\in M} F_k \sum_{j\in V'} x_{0j}^k + \sum_{k\in M} \sum_{\substack{i,j\in V\\ i\neq j}} c_{ij} x_{ij}^k \tag{1}$$

$$s.t. \sum_{k\in M} \sum_{i\in V} x_{ij}^k = 1, \qquad \forall j \in V' \tag{2}$$

$$\sum_{i\in V} x_{ip}^k - \sum_{j\in V} x_{pj}^k = 0, \qquad \forall p \in V', \forall k \in M \tag{3}$$

$$\sum_{j\in V'} x_{0j}^k \leqslant m_k, \qquad \forall k \in M \tag{4}$$

$$\sum_{i\in V} y_{ij} - \sum_{i\in V} y_{ji} = q_j, \qquad \forall j \in V' \tag{5}$$

$$q_j x_{ij}^k \leqslant y_{ij} \leqslant (Q_k - q_i) x_{ij}^k, \qquad \forall i, j \in V, i \neq j, \forall k \in M \tag{6}$$

$$y_{ij} \geqslant 0, \qquad \forall i, j \in V, i \neq j \tag{7}$$

$$x_{ij}^k \in \{0, 1\}, \ \ \forall i, j \in V, i \neq j, \forall k \in M \tag{8}$$

In the above formulation, constraints (2) and (3) ensure that a customer is visited exactly once and that if a vehicle visits a customer, it must also depart from it. The maximum number of vehicles available for each vehicle type is imposed by constraints (4). Constraints (5) are the commodity flow constraints: they specify that the difference between the quantity of goods a vehicle carries before and after visiting a customer is equal to the demand of that customer. Finally, constraints (6) ensure that the vehicle capacity is never exceeded.

Golden et al. (1984) proposed a formulation for the FSMF similar to formulation $F1$ where the capacity and subtour elimination constraints are modelled with an extension of the Miller-Tucker-Zemlin (MTZ) inequalities for the TSP (see Miller et al. (1960)). Other mathematical formulations for FSMF were presented by Yaman (2006), who described six different formulations: the first four based on the use of MTZ inequalities to model subtour elimination and the last two based on flow variables.

Another important type of formulation for Heterogeneous VRPs can be obtained by extending the Set Partitioning (SP) model of the VRP that was originally proposed by Balinski and Quandt (1964) and associates a binary variable with each feasible route. The formulation, again written for HVRPFD, can be described as follows.

Let $\mathscr{R}_k$ be the index set of all feasible routes for a vehicle of type $k \in M$. Each route $\ell \in \mathscr{R}_k$ has an associated cost $d_{\ell k}$. Let $\mathscr{B}_{ik} \subset \mathscr{R}_k$ be the index subset of the routes for a vehicle of type k covering customer $i \in V'$. In the following we will use R_ℓ to indicate the subset of vertices (i.e., $R_\ell = \{0, i_1, i_2, \ldots, i_h\}$) visited by route ℓ.

Let $\xi_{\ell k}$ be a binary variable that is equal to 1 if and only if route $\ell \in \mathscr{R}_k$ belongs to the optimal solution.

The SP model is as follows:

$$(F2) \quad z(F2) = \min \sum_{k \in M} \sum_{\ell \in \mathscr{R}_k} d_{\ell k} \xi_{\ell k} \tag{9}$$

$$s.t. \ \sum_{k \in M} \sum_{\ell \in \mathscr{B}_{ik}} \xi_{\ell k} = 1, \quad \forall i \in V' \tag{10}$$

$$\sum_{\ell \in \mathscr{R}_k} \xi_{\ell k} \leqslant m_k, \quad \forall k \in M \tag{11}$$

$$\xi_{\ell k} \in \{0,1\}, \quad \forall \ell \in \mathscr{R}_k, \forall k \in M. \tag{12}$$

Constraints (10) specify that each customer $i \in V'$ must be covered exactly by one route and constraints (11) require that at most m_k routes are selected for a vehicle of type $k \in M$.

Note that in the case of FSMF, each route $\ell_1 \in \mathscr{R}_{k_1}$ is *dominated* by another route $\ell_2 \in \mathscr{R}_{k_2}$, if $k_1 > k_2$ and $R_{\ell_1} = R_{\ell_2}$. This happens since for FSMF we have an unlimited number of vehicles of type k_2 and $d_{\ell_2 k_2} < d_{\ell_1 k_1}$. Thus, sets $\{\mathscr{R}_k\}$ can be redefined as the sets of *undominated* feasible routes.

Mathematical formulations for the time windows variants of the problem were described in Ferland and Michelon (1988), Dell'Amico et al. (2006) and Bräysy et al. (2006).

3.1 Lower bounds

Lower bounds for FSMF were proposed by Golden et al. (1984), Yaman (2006) and Choi and Tcha (2006). These latter authors also described lower bounds for FSMFD and FSMD. In this section, we present the lower bound of Golden et al. (1984), and we briefly examine those proposed by Choi and Tcha (2006) and by Yaman (2006).

Let us consider the FSMF problem and suppose (without loss of generality) that the customers are numbered according to decreasing distance from the depot (i.e., $c_{01} \leqslant c_{02} \leqslant \ldots \leqslant c_{0n}$). Given a route (R, k), the *pivot* of a route is defined as the vertex $i^* \in R$ such that $c_{0i^*} = \max_{j \in R \setminus \{0\}} \{c_{0j}\}$ (i.e., i^* is the customer of the route located farthest from the depot). In those cases where more than one vertex produces the maximum of the expression, we call *pivot* of route (R, k) the vertex having the smallest index.

Using the definition of a pivot, the set of routes $\mathscr{R}_k$ can be partitioned as $\mathscr{R}_{1k} \cup \mathscr{R}_{2k} \cup \ldots \cup \mathscr{R}_{nk}$, where $\mathscr{R}_{ik}$ is the index set of all routes having as a pivot the customer $i \in V'$ and using a vehicle of type $k \in M$. Let us denote the cost of a route $\ell \in \mathscr{R}_{ik}$ as $d^i_{\ell k}$. Moreover, let $\mathscr{B}_{jik} \subset \mathscr{R}_{ik}$ be the index subset of the routes for a vehicle of type k, for the pivot i and covering customer $j \in V'$. Finally, let $\xi^i_{\ell k}$ be a binary variable that is equal to 1 if and only if route $\ell \in \mathscr{R}_{ik}$ belongs to the optimal solution. Starting from $F2$, the FSMF can be formulated as follows:

$$(F3) \quad z(F3) = \min \sum_{k \in M} \sum_{i \in V'} \sum_{\ell \in \mathscr{R}_{ik}} d^i_{\ell k} \xi^i_{\ell k} \tag{13}$$

$$s.t. \sum_{k \in M} \sum_{i \in V'} \sum_{\ell \in \mathscr{B}_{jik}} \xi^i_{\ell k} = 1, \quad \forall j \in V' \tag{14}$$

$$\xi^i_{\ell k} \in \{0, 1\}, \quad \forall \ell \in \mathscr{R}_{ik}, \forall k \in M, \forall i \in V'. \tag{15}$$

Note that in the case of FSMF, constraints (11) of formulation $F2$ are redundant as $m_k = +\infty$, $\forall k \in M$.

If the cost matrix $\{c_{ij}\}$ is symmetric and satisfies the triangle inequality, a lower bound to FSMF can be obtained from formulation $F3$ by computing the cost $d^i_{\ell k}$ as $d^i_{\ell k} = 2c_{0i} + F_k$, i.e., by approximating the route cost with the radial component associated with the pivot of the route, plus the fixed cost of the vehicle assigned to it.

The above observation leads to the following relaxation of formulation $F3$. Let ξ_{ik} be a binary variable which is equal to 1 if and only if a route for the pivot i, using a vehicle of type k, is in the solution, and 0 otherwise. In addition, let x_{ijk}, with $j \geqslant i$, be a binary variable which is equal to 1 if and only if customer j is served by a route having pivot i and vehicle type k. Then, the optimal solution of the following mixed integer programming problem gives a valid lower bound to FSMF:

$$(LB1) \quad z(LB1) = \min \sum_{k \in M} \sum_{i \in V'} (2c_{0i} + F_k) \xi_{ik} \tag{16}$$

$$s.t. \sum_{k \in M} \sum_{i \in V'} x_{ijk} = 1, \quad \forall j \in V' \tag{17}$$

$$\sum_{j \in V'} q_j x_{ijk} \leqslant Q_k \xi_{ik}, \quad \forall i \in V', \forall k \in M \tag{18}$$

$$x_{ijk} \in \{0, 1\}, \quad \forall i, j \in V', j \geqslant i, \forall k \in M \tag{19}$$

$$\xi_{ik} \in \{0, 1\}, \quad \forall i \in V', \forall k \in M. \tag{20}$$

Constraints (17) state that each customer must be assigned to a pivot, while constraints (18) impose the vehicle capacities. Note that the definition of variables $\{x_{ijk}\}$ implies $\xi_{1k} = 1$, for some $k \in M$.

A relaxation of lower bound $LB1$ can be obtained if the integrality constraints (19) are relaxed and variables $\{\xi_{ik}\}$ are assumed to be general integer (i.e., a customer can be a pivot of more than one route). In addition, let s_k be the sum of demands of customers for which vehicle type k is the smallest one that can service the demand, and define $s_{m+1} = 0$. Then, the optimal solution of the following mixed integer programming problem gives a valid lower bound to FSMF:

$$(LB2) \quad z(LB2) = \min \sum_{k \in M} \sum_{i \in V'} (2c_{0i} + F_k)\xi_{ik} \tag{21}$$

$$s.t. \ (17), (18) \text{ and} \tag{22}$$

$$\sum_{j \in V'} q_j x_{ijk} \geqslant s_k - \sum_{h=k+1}^{m} (\sum_{j \in V'} q_j x_{ijh} - s_h), \quad \forall k \in M \tag{23}$$

$$0 \leqslant x_{ijk} \leqslant 1, \quad \forall i, j \in V', j \geqslant i, \forall k \in M \tag{24}$$

$$\xi_{ik} \geqslant 0 \text{ integer }, \quad \forall i \in V', \forall k \in M. \tag{25}$$

Constraints (23) impose that the sum of the customer demands served by all the vehicles of type $k \in M$ must be greater than or equal to s_k, minus the demand which can be served by all vehicles having capacities greater than Q_k. Note that the demand of a customer can be split among the pivots which are selected in the solution of lower bound $LB2$.

Lower bound $z(LB2)$ can be efficiently computed by using the procedure proposed by Golden et al. (1984) which can be described as follows. Let $D_{tot} = \sum_{i \in V'} q_i$ be the total customer demand, and let $\hat{G} = (\hat{V}, \hat{A})$ be a directed graph where $\hat{V} = \{0, 1, \ldots, D_{tot}\}$. Associate with each vertex $q \in \hat{V} \setminus \{D_{tot}\}$ the cost $C(q) = 2c_{0h}$, where the index h is such that the following inequalities are satisfied:

$$\sum_{j=0}^{h-1} q_j \leqslant q < \sum_{j=0}^{h} q_i. \tag{26}$$

The set $\hat{A}$ of arcs of graph $\hat{G}$ is composed by the following arcs: for each vehicle of type $k \in M$, there is an arc from node $q \in \hat{V}$ to $\min\{D_{tot}, q + Q_k\}$ with cost equal to $C(q) + F_k$ if and only if $q \geqslant \sum_{h=k+1}^{m+1} s_h$. Note that in graph $\hat{G}$, the first s_m vertices represent the demands that require the largest vehicle type. On the other hand, the first q_1 vertices represent the demand of the farthest customer.

The cost of the shortest path in graph $\hat{G}$ from vertex 0 to vertex D_{tot} gives lower bound $z(LB2)$. Note that, each arc used in the shortest path corresponds to a pivot i and to a vehicle type k, i.e., to a variable ξ_{ik} of formulation $LB2$.

Yaman (2006) proposed several lower bounds based on cutting-plane techniques used to strengthen the LP relaxation of six mathematical formulations of FSMF. Also in Yaman (2006), a comparison among the LP relaxation of the different mathematical formulations is reported. The following families of valid inequalities were considered to improve the lower bounds given by the different LP relaxations: *covering type inequalities*, *subtour elimination inequalities*, *generalized large multistar inequalities* and valid inequalities based on the lifting of the MTZ constraints.

Choi and Tcha (2006) proposed lower bounds for FSMFD, FSMD and FSMF, based on the set partitioning formulation $F2$, which were computed using a column generation technique. More precisely, the lower bounds were derived from the relaxation of the partitioning formulation $F2$ into a covering formulation, where the SP columns correspond to the set of q-routes (see Christofides et al. (1981)), where a q-route is a (not necessarily simple) circuit covering the depot and a subset of customers, whose total demand is equal to q.

The computational testing for FSMF, is generally performed by using a set of 20 symmetric instances proposed by Golden et al. (1984) (in the following, referred to as the G20 instances), that are extensions to FSMF of classical VRP test instances. In addition, some authors considered just the 12 such instances that are defined by using Euclidean distances (referred to as the G12 instances).

Table 2 reports a comparison on the quality of the lower bounds obtained for the FSMF by Golden et al. (1984), Yaman (2006) and Choi and Tcha (2006). The table reports the lower bounds on the G12 set, which were used in both Yaman (2006), and Choi and Tcha (2006). In particular, in order to make a fair comparison among the different lower bounds, we computed the lower bound of Golden et al. (1984) using real-valued distance data, and the lower bound values produced by the best formulation proposed by Yaman (2006). In the table, columns labelled LB report the lower bound values, while columns labelled $\%LB$ report the percentage ratio of lower bounds computed with respect to the best upper bound known in the literature, reported in column UB (i.e., $\%LB = 100\ LB/UB$).

Table 2 shows that the best lower bound is, on average, that computed by Choi and Tcha (2006). On the set of instances, the lower bound of Yaman (2006) dominates the lower bound of Golden et al. (1984) and it is not dominated by the lower bound of Choi and Tcha (2006) (see instances 4E, 6E, 14E and 19E). Furthermore, the lower bound of Golden et al. (1984) is not dominated by the lower bound of Choi and Tcha (2006) (see instance 14E).

Table 2: Lower bounds for FSMF

Problem	n	UB	Golden et al. (1984) LB	$\%LB$	Yaman (2006) LB	$\%LB$	Choi and Tcha (2006) LB	$\%LB$
3E	20	961.03	791.91	82.40	931.05	96.88	951.61	99.02
4E	20	6437.33	6265.26	97.33	6387.76	99.23	6369.15	98.94
5E	20	1007.05	876.23	87.01	957.70	95.10	988.01	98.11
6E	20	6516.00	6385.47	98.00	6466.94	99.25	6451.62	99.01
13E	50	2406.36	2118.49	88.04	2365.78	98.31	2392.77	99.44
14E	50	9119.03	8873.58	97.31	8943.94	98.08	8748.57	95.94
15E	50	2586.37	2327.46	89.99	2503.61	96.80	2544.84	98.39
16E	50	2720.43	2440.41	89.71	2650.76	97.44	2685.92	98.73
17E	75	1744.83	1380.03	79.09	1689.93	96.85	1709.85	98.00
18E	75	2371.49	2001.71	84.41	2276.31	95.99	2342.84	98.79
19E	100	8659.74	8290.01	95.73	8574.33	99.01	8431.87	97.37
20E	100	4039.49	3607.86	89.31	3931.79	97.33	3995.16	98.90
		Avg.		89.86		97.52		98.39

4 Heuristic algorithms

Due to the intrinsic difficulty of this family of routing problems, all solution approaches presented so far in the literature are heuristic algorithms. In addition, they generally are adaptations or extensions of the methods proposed in the last decades for the basic VRP variants, as the VRP and the VRP with Time Windows.

In this section, we briefly review the main contributions to the approximate solution of Heterogeneous VRPs. We separately examine traditional construction heuristics and metaheuristics and we provide, where available, information about their computational performance. To this end, for each presented approach, we report the average percentage gaps of the published heuristic solution values with respect to the current best known ones, as well as the average computing time required, expressed in seconds of various CPUs. Moreover, the detailed published results are summarized in Tables 3 to 6.

The detailed results for the instances proposed by Golden et al. (1984) are reported in two tables: the first one, Table 3, collects all the results obtained by considering integer-valued distances, whereas Table 4 accounts for results obtained with real-valued distances.

As to the variants with vehicle-dependent routing costs, namely HVRPD and FSMD, the computational testing is generally performed by using an adaptation of eight instances of the G12 set, as proposed by Taillard (1999), referred to as the T8 instance set. The detailed results for instance set T8 can be found in Tables 5 and

6.

Tables 3 to 6 report for each instance, the problem name, the number of customers n, the number of vehicle types m and the value of the best solution found in the literature. The last two lines of each table report the average percentage gap and the number of the best known solutions found by the corresponding heuristic, respectively. Given the value z of a heuristic solution and the best upper bound known z_{best} for the corresponding instance, the percentage gap is computed as $100(z - z_{best})/z_{best}$.

The data of all instances and more details on the published results can be found at `http://or.ingce.unibo.it/research/hvrp`.

4.1 Construction heuristics

The first comprehensive study of Heterogeneous VRPs, and in particular devoted to FSMF, is due to Golden et al. (1984) that presented the formulation and the lower bounding procedure described in Section 3. Moreover, Golden et al. (1984) proposed constructive heuristics that adapted to FSMF the savings and giant-tour based approaches for VRP (see Clarke and Wright (1964) and Beasley (1983), respectively). As to savings approaches, four different expressions were proposed to incorporate the heterogeneous fleet concept in the savings computation and enhance the algorithm performance. As reported by Golden et al. (1984), the direct adaptation of the Clarke and Wright (1964) algorithm (CW) produces solutions with average percentage gap equal to 14.31% with respect to the current best known integer solutions for G20 set. Combined Savings (CS) include in the savings formula the variation of the fixed costs associated with the route merging: the resulting average improvement on CW is 4.25%. Optimistic and Realistic Opportunity Savings (denoted as OOS and ROS, respectively) add to CS two different terms, that favor the opportunity of having residual capacity on the vehicle used to service the merged routes. In this case, the improvement on CW is 1.15% and 5.75%, respectively. Finally, ROS-γ adds to ROS the *route shape parameter* proposed by Gaskell (1967), and Yellow (1970). This latter approach is used in a multi-start fashion, by considering 31 different values of γ parameter between 0 and 3. The resulting percentage gap with respect to the best known solution values is equal to 3.79%, corresponding to an improvement on CW as high as 8.18% (see Table 3).

Desrochers and Verhoog (1991) further extended savings-based approaches to FSMF by adopting the matching-based savings heuristic proposed by Altinkemer and Gavish (1987) for the VRP. The basic savings expression considers the cost difference of the TSPs associated with the routes involved in the current merging, rather than the simpler classical one. Various extensions of the savings formula, similar to those proposed by Golden et al. (1984), are considered. At each iteration, the pair of routes to be merged is chosen as that corresponding to the largest savings in the solution of a matching problem over the current savings matrix. As reported in Table 3, the two proposed savings expressions, called ROM-ρ and ROM-γ, produced solutions with an average percentage gap equal to 1.70% and 2.01% on the G20 instances, respectively, corresponding to improvements on the CW results by 10.05% and 9.80%, respectively.

The giant-tour based approaches proposed by Golden et al. (1984) are two-phase algorithms in which first a TSP over all the nodes is, heuristically, solved so as to obtain an uncapacitated tour. Such tour, in the second phase is partitioned into the final capacitated set of routes. Two different ways of defining the initial giant tour were adopted, namely with and without the depot in the tour. In this latter case, which on average produced slightly better results, the partitioning is obtained by solving a suitably defined shortest path problem. Also in this case a multi-start framework is obtained by applying the partitioning step to different initial tours. The best obtained solutions were refined by using 2-opt and OR-opt procedures (see Lin (1965) and Or (1976)). As shown in Table 3, this approach, called MGT+Or-opt, proved able to produce solutions within 1.21% from the best integer known ones on the G20 test instances.

A different giant-tour based approach for FSMF was introduced by Gheysens et al. (1983), in which a penalty function that allows for a limited capacity violation in the tour partitioning step is used, so as to favour the presence in the routes of some empty space that may be possibly exploited in the refining step. This method has been tested on the 16 smallest instances of the G20 set allowing for an average percentage gap of 1.35% (see column "Penalty" of Table 3). More recently, Teodorovic et al. (1995) used a giant-tour approach to solve the stochastic version of the HFVRP, where customer's demand may vary stochastically and the initial tour is obtained through the Bartholdi and Platzman (1982) spacefilling curves heuristic for the TSP.

Gheysens et al. (1984) and Gheysens et al. (1986) developed an extension to FSMF of the well-known Fisher and Jaikumar (1981) algorithm for VRP, where the initial fleet is determined through the lower bounding procedure of Golden et al.

(1984). Gheysens et al. (1984) report an average percentage gap of 0.68% on the 15 smallest instances of the G20 set (see column "LB(5)+VRP" of Table 3).

Ferland and Michelon (1988) introduced three different heuristic methods to solve the FSMFD with Time Windows. The first one directly uses the three-index mathematical formulation of the problem and simplifies it by discretizing the time windows, so as to obtain a possibly solvable integer problem. The two remaining heuristics are constructive approaches in which the solution is obtained by iteratively assigning customers to the routes through the solution of either a matching or a transportation problem. No computational testing of the proposed methods was reported by the authors.

Salhi and Rand (1993) described a heuristic for the FSMF that starts from a solution obtained by heuristically solving a VRP with a single vehicle capacity, selected among the available ones. This starting solution is then iteratively improved by several procedures that attempt, in turn, to change the vehicle type assigned to each route, merging or removing routes and moving customers from one route to another. The average percentage gap of this method is equal to 0.59%, whereas the average computing time of a VAX 8700 computer is equal to 2 seconds. Osman and Salhi (1996) extended the heuristic proposed by Salhi and Rand (1993) by (i) enlarging the neighborhood size allowing moves which can lead to a utilization of a larger-sized vehicle and (ii) using a multi-start technique to restart the heuristic with the best solution found at the end of the previous iteration. Osman and Salhi (1996) used real-valued distances hence their results cannot be compared directly with those obtained by Salhi and Rand (1993). The average percentage gap is equal to 0.90% (see column "MRPERT" of Table 4) and the average computing time is equal to 5.65 seconds on a VAX 4500 computer.

Taillard (1999) proposed a heuristic column generation method for FSMF, FSMD and HVRPD. In this approach a large set of routes is initially obtained by solving homogeneous fleet VRPs for each vehicle type. Then, the final set of routes is selected by solving a set partitioning problem to ensure that each customer is served by exactly one route. This method, tested for the FSMF on the 8 largest instances in G20, produces very good results: in fact the average percentage gap is equal to 0.14% and the average computing time over five runs is 2648 seconds on a Sun Sparc work station (50 MHz). The results obtained with the T8 set for HVRPD, show an average percentage gap of 2.00%, and an average computing time around 2000 seconds on the same machine. For FSMD, the percentage gap is equal to

0.92%, requiring about the same amount of time. A similar approach is used by Renaud and Boctor (2002) to solve the FSMF, where the set of routes is obtained by using problem-specific extensions of the sweep algorithm for the VRP (see Gillett and Miller (1974)). The average percentage gap of the best solutions found out of several runs on set G20 is equal to 0.47%.

More recently, Choi and Tcha (2006) proposed a heuristic approach based on a column generation technique, to derive high quality heuristic solutions for FSMD and FSMF. More precisely, they (i) computed a lower bound to FSMD and to FSMF as the optimal solution cost of the LP relaxation of the covering to partition relaxation of formulation $F2$ and (ii) solved, using a branch-and-bound algorithm, a restricted Set Partitioning problem obtained by limiting the set of all feasible routes to the set of routes generated by the column generation algorithm in computing the lower bound. The FSMD method has been tested on T8 instances obtaining percentage gaps of 0.15% over the best known solutions, requiring on average 81 seconds on a Pentium IV 2.6GHz processor. With respect to the FSMF, this method produces on G12 instances an average percentage gap equal to 0.004%, with an average computing time of 150 seconds on the same machine.

We conclude this section by examining the two construction heuristics that were developed for the time windows variant of FSMF, denoted as FSMFTW. Liu and Shen (1999) proposed a two-phase algorithm in which an initial solution is obtained through a savings algorithm that evaluates the insertion of complete routes in all possible insertion places of the other routes, and also takes into account the vehicle scheduling component associated with the time windows. In the second phase, an improvement procedure is then applied to several best fleet solutions found during the first stage: intra-route customer shifting and inter-route customer exchanges are performed. Computational results were performed on a set of 168 test instances, hereafter called LS168, derived from the Solomon (1987) VRPTW test set. The proposed algorithm was also used to solve the G20 FSMF instances and obtained an average percentage gap equal to 0.96%.

Dullaert et al. (2002) extended to FSMFTW the sequential insertion algorithm proposed by Solomon (1987) for the VRPTW. In particular, the adopted insertion criterion combines standard insertion cost evaluations used in the VRPTW with a new term that incorporates the Golden et al. (1984) modified saving expressions. Dullaert et al. (2002) reported in their tables only the component of the objective function relative to the schedule times. Thus, their results cannot be compared with the results of the heuristics reporting also the fixed cost component.

4.2 Metaheuristics

Following the evolution of the general VRP literature, since the last decade of XX century, metaheuristic approaches started to be applied to the solution of heterogeneous VRP as well.

One of the first such algorithms is the genetic approach proposed by Ochi et al. (1998a) for FSMF that creates an initial population by means of a sweep-based heuristic. The same algorithm is tested in a parallel framework in Ochi et al. (1998b), but both papers do not report details of the computational testing.

Tabu search approaches for this problem family were developed by Osman and Salhi (1996), Gendreau et al. (1999), and Wassan and Osman (2002). All these algorithms were extensions to FSMF and to HVRPD of approaches already proposed for the VRP, using the problem-specific feasibility check and objective function evaluation. In particular, Osman and Salhi (1996) used a 1-interchange neighborhood together with a simple tabu list mechanism, whereas Wassan and Osman (2002) mixes several effective strategies to improve the overall quality of the algorithm: reactive and variable-neighborhood search mechanisms, based on λ-interchange neighborhoods are combined with efficient data management techniques for handling tabu lists and hashing functions. Finally, the tabu search of Gendreau et al. (1999) embeds a classical algorithm based on the GENIUS neighborhoods with the adaptive memory mechanism of Rochat and Taillard (1995). The Osman and Salhi (1996) algorithm performance on the G20 instances shows an average percentage gap of about 0.68% (see column "TSVFM" of Table 4) with respect to the best known solutions. Gendreau et al. (1999) algorithm was tested on G12 test instances, obtaining an average percentage gap of 0.24%, with an average computing time of 765 seconds on a Sun Sparcstation 10. The algorithm was also tested on the T8 instances, obtaining an average percentage gap of 0.21% within an average computing time of 1151 seconds. The Wassan and Osman (2002) tabu search produces good results: on G20 test bed the average percentage gap is 0.14% and on T8 this gap is 0.47%, but the average computing time increases to 1215 seconds and to 2098 seconds, respectively, using a Sun Sparc server 1000.

Tarantilis et al. (2003) and later Tarantilis et al. (2004) developed two list-based threshold accepting metaheuristic for the HVRPD: both methods start with an initial solution generated by a constructive heuristic. Then, in the threshold based phase, neighboring solutions with respect to the current one are generated by implementing randomly selected moves. The moves are accepted according to the evaluation of their threshold values, which are compared to a list storing the M best threshold values found during the search. Different ways of updating the threshold list are considered in the two papers. Tarantilis et al. (2003) method has been tested on the T8 test instances and produces results on average 1.85% (see column “LBTA” of Table 5) far from the best known solutions within, on average, 223 seconds on a Pentium III, 550 MHz Pc. The results of Tarantilis et al. (2004) are slightly better, obtaining on the T8 instances a gap of 1.68% within an average computing time of 607 seconds on a Pentium II/400 Pc. In addition, when initialized with high-quality solutions as those obtained by Taillard (1999) column generation approach, these algorithms performance is enhanced allowing for an average percentage gap equal to 0.08% (see column “LBTA1” of Table 5).

Li et al. (2006) considered a similar approach, based on a record-to-record algorithm: a deterministic variant of the simulated annealing metaheuristic. This method has been tested on T8 instances and on five large-scale instances with 200 to 360 customers from Golden et al. (1998). On T8 problems they obtained an average percentage gap of 1.09% and an average computing time of 286 seconds on an Athlon 1 GHz Pc.

Dell'Amico et al. (2006) proposed a ruin-and-recreate approach (see Schrimpf et al. (2000)) for FSMFTW. In particular, a parallel insertion procedure is used both to obtain the initial solution and to possibly complete partial ones that are produced during the ruin step. This step is performed by selecting a target route to be ruined according to a criterion which is inspired by those adopted in bin packing solution metaheuristics. The proposed approach outperformed both the Liu and Shen (1999) and Dullaert et al. (2002) algorithms on the LS168 test instances.

Finally, Bräysy et al. (2006) proposed a new deterministic annealing metaheuristic for FSMFTW. The metaheuristic is based on three phases: (i) initial solutions are generated by means of a savings-based heuristic combining diversification strategies with learning mechanisms, (ii) an attempt is made to reduce the number of routes in the initial solution with a new local search procedure and (iii) the solution from the second phase is further improved by a set of four local search operators that are embedded in a deterministic annealing framework to guide the improvement process. The computational experiments on LS168 benchmark instances show that the suggested method outperforms the previously published results and improved almost all the best known solutions.

Table 3: Comparison of the best known results for FSMF with integer distances data

				Golden et al. (1984)		Gheysens et al. (1984)		Desrochers and Verhoog (1991)		Salhi and Rand (1993)
Problem	n	m	Best	ROS-γ	MGT+Or-opt	LB(5)+VRP	Penalty	ROM-ρ	ROM-γ	
1S	12	3	602	618	622	618	634	606	**602**	614
2S	12	3	722	768	**722**	**722**	**722**	730	**722**	**722**
3E	20	5	966	1009	**966**	968	**966**	990	980	1003
4E	20	3	6447	6937	6930	6451	6473	6547	6891	**6447**
5E	20	5	1013	1048	**1013**	1030	1023	1040	1032	1015
6E	20	3	6516	6522	6974	6518	6953	6517	6517	**6516**
7S	30	5	7354	7452	7389	**7354**	7372	7421	7444	7402
8S	30	4	2362	2468	2367	**2362**	2370	2387	2389	2367
9S	30	5	2209	2266	2220	2261	2226	2231	2231	**2209**
10S	30	4	2370	2424	**2370**	2388	2371	2393	2387	2377
11S	30	4	4763	4953	**4763**	4788	4805	4862	4911	4819
12S	30	6	4092	4221	4136	4133	4248	4254	4248	**4092**
13E	50	6	2437	2566	2438	–	**2437**	2525	2508	2493
14E	50	3	9132	9178	**9132**	9156	**9132**	9155	9196	9153
15E	50	3	2621	2763	2640	**2621**	2640	2622	2642	2623
16E	50	3	2765	2894	2822	–	–	2809	2868	**2765**
17E	75	4	1767	1958	1783	–	–	1877	1877	**1767**
18E	75	6	2432	2520	**2432**	–	–	2489	2489	2439
19E	100	3	8700	8741	8721	–	–	**8700**	**8700**	8751
20E	100	3	4187	4293	4195	–	–	4248	4280	**4187**
Average %				3.79	1.21	0.68	1.35	1.70	2.01	0.59
N. of best sol.				0	7	4	4	1	3	8

Table 4: Comparison of the best known results for FSMF with real distances data

Problem	n	m	Best	Osman and Salhi (1996) MRPERT	Osman and Salhi (1996) TSVFM	Taillard (1999)	Gendreau at al. (1999)	Liu and Shen (1999)	Renaud and Boctor (2002)	Wassan and Osman (2002)	Choi and Tcha (2006)
1S	12	3	602.00	606.00	**602.00**	–	–	**602.00**	**602.00**	**602.00**	–
2S	12	3	722.00	**722.00**	**722.00**	–	–	**722.00**	**722.00**	**722.00**	–
3E	20	5	961.03	971.95	971.24	**961.03**	**961.03**	972.04	963.61	**961.03**	**961.03**
4E	20	3	6437.33	6447.80	6445.10	**6437.33**	**6437.33**	6444.72	**6437.33**	**6437.33**	**6437.33**
5E	20	5	1007.05	1015.13	1009.15	1008.59	**1007.05**	1014.05	1007.96	**1007.05**	**1007.05**
6E	20	3	6516.00	6516.56	6516.56	6516.47	6516.46	**6516.00**	6537.74	6516.47	6516.47
7S	30	5	7273.00	7377.00	7310.00	–	–	7313.33	7346.00	**7273.00**	–
8S	30	4	2346.00	2352.00	2348.00	–	–	2347.00	2347.00	**2346.00**	–
9S	30	5	2209.00	**2209.00**	**2209.00**	–	–	2214.96	2211.00	**2209.00**	–
10S	30	4	2355.00	2377.00	2363.00	–	–	2368.00	2362.00	**2355.00**	–
11S	30	4	4755.00	4787.00	**4755.00**	–	–	4777.77	4761.00	**4755.00**	–
12S	30	6	4087.00	4092.00	4092.00	–	–	4101.00	4092.00	**4087.00**	–
13E	50	6	2406.36	2462.01	2471.07	2413.78	2408.41	2465.03	2406.43	2422.10	**2406.36**
14E	50	3	9119.03	9141.69	9125.65	**9119.03**	**9119.03**	9132.00	9122.01	9119.86	**9119.03**
15E	50	3	2586.37	2600.31	2606.72	**2586.37**	**2586.37**	2608.00	2618.03	**2586.37**	**2586.37**
16E	50	3	2720.43	2745.04	2745.01	2741.50	2741.50	2808.96	2761.96	2730.08	**2720.43**
17E	75	4	1744.83	1766.81	1762.05	1747.24	1749.50	1806.05	1757.21	1755.10	**1744.83**
18E	75	6	2371.49	2439.40	2412.56	2373.63	2381.43	2415.94	2413.39	2385.52	**2371.49**
19E	100	3	8659.74	8704.20	8685.71	8661.81	8675.16	8684.00	8687.31	**8659.74**	8664.29
20E	100	3	4039.49	4166.03	4188.73	4047.55	4086.76	4148.04	4094.54	4061.64	**4039.49**
Average %				0.90	0.68	0.14	0.24	0.96	0.47	0.14	0.004
N. of best sol.				2	4	4	5	3	3	13	10

Table 5: Comparison of the best known results for HVRPD with real distances data

				Taillard (1999)	Tarantilis et al. (2003)		Tarantilis et al. (2004)	Li et al. (2006)
Problem	n	m	Best		LBTA	LBTA1		
13E	50	6	1491.86	1518.05	1519.96	**1491.86**	1519.96	1517.84
14E	50	3	603.38	615.64	612.51	**603.38**	611.39	607.53
15E	50	3	999.82	1016.86	1017.94	**999.82**	1015.29	1015.29
16E	50	3	1131.00	1154.05	1148.19	**1131.00**	1145.52	1144.94
17E	75	4	1038.60	1071.79	1071.67	**1038.60**	1071.01	1061.96
18E	75	6	1820.18	1870.16	1852.13	**1820.18**	1846.35	1823.58
19E	100	3	1108.24	1117.51	1125.64	**1108.24**	1123.83	1120.34
20E	100	3	1534.17	1559.77	1558.56	1544.54	1556.35	**1534.17**
Average %				2.00	1.85	0.08	1.68	1.09
N. of best sol.				0	0	7	0	1

Table 6: Comparison of the best known results for FSMD with real distances data

				Taillard (1999)	Gendreau et al.(1999)	Wassan and Osman (2002)	Choi and Tcha (2006)
Problem	n	m	Best				
13E	50	6	1491.86	1494.58	**1491.86**	1499.69	**1491.86**
14E	50	3	603.21	**603.21**	**603.21**	608.57	**603.21**
15E	50	3	999.82	1007.35	**999.82**	**999.82**	**999.82**
16E	50	3	1131.00	1144.39	1136.63	**1131.00**	**1131.00**
17E	75	4	1031.00	1044.93	**1031.00**	1047.74	1038.60
18E	75	6	1801.40	1831.24	**1801.40**	1814.11	**1801.40**
19E	100	3	1100.56	1110.96	1105.44	**1100.56**	1105.44
20E	100	3	1530.16	1550.36	1541.18	**1530.16**	1530.43
Average %				0.92	0.21	0.47	0.15
N. of best sol.				1	5	4	5

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