

Lecture ÷ Greedy Algorithms (Approaches)

Agenda

- Introduction
- Toys
- Distribute candies
- finish max jobs.

Qul There is a limited time sale going on:

$A[i]$ = sale end time of i th toy.

$B[i]$ = beauty of i th toy.

Time starts from $t=0$, and it takes 1 unit of time

to buy each toy and toy can be bought only if

$T < A[i]$.

Buy toys such that beauty is maximised. [Medium-Hard]

Eg:

sale end $A[] =$

0	1	2	3	4
3	1	3	2	3

 toy beauty

beauty $B[] =$

6	5	3	1	9
↑	↑	↑	↑	
6	5	3	1	

 $t = \emptyset \times \times \times 3$

toy	beauty
0	6
2	3
4	9
	<u>18</u>

Idea: Buy most expensive toy first [Wrong]

$A[] =$

0	1	2	3	4
3	1	3	2	3

 toy beauty

$B[] =$

6	5	3	1	9
---	---	---	---	---

 $t = \emptyset \times \times 3$

toy	beauty
4	9
0	6
2	3
	<u>18</u>

Wrong ans

contradiction

$A[] =$

0	1
1	2

$B[] =$

3	1500
---	------

$t = \emptyset 1$

0	- 3
1	- 1500
	<u>1503</u>

What about this?

A[] = ⁰3 ¹1 ²3 ³2 ⁴3

B[] = 6 5 3 1 9

t = ~~0~~ / ~~1~~ / ~~2~~ 3

toy
1
4
0

beauty
5
9
6
20

Idea: Buy everything → asc order of time

A[] = ⁰1 ¹3 ²3 ³3 ⁴5 ⁵5 ⁶5 ⁷8

B[] = 5 2 7 1 4 3 8 1
↑ ↑ ↑ ↑ ↑ ↑ ↑

t = ~~0~~ / ~~1~~ / ~~2~~ / ~~3~~ / ~~4~~ / ~~5~~ / ~~4~~ / ~~5~~ 6

Min heap

5 2 7
4 3 8
1

ans = 5 + 7 + 4 + 3 + 8 + 1 = 28 Ans

Correcting an incorrect item bought in past

↳ Min heap

A[] = ⁰3 ¹3 ²3 ³3
B[] = [4 8 12 16]

t = ~~0~~ / ~~1~~ / ~~2~~ 3

toy beauty
0 4
2 12
1 8

Correct :- 16 , 12 , 8

Code:

```
class Pair {  
    int e;  
    int beauty;  
  
    Pair (x, y) {  
        e = x;  
        beauty = y;  
    }  
}
```

```
int maximiseBeauty (A[], B[]) {  
    n = A.length;
```

// Sort in asc order of time

```
Pair[] arr = new Pair[n];
```

$O(n)$ — for($i=0$; $i < n$; $i++$) {

```
    arr[i] = new Pair(A[i], B[i]);
```

```
}
```

$O(n \log n)$ — Arrays.sort(arr, (u, v) \rightarrow u.e - v.e));

```
PriorityQueue<Integer> pq = new PriorityQueue<>();
```

```
int t = 0;
```

$O(n \log n)$ — for($i=0$; $i < n$; $i++$) {

```
    int e = arr[i].e;
```

```
    int b = arr[i].beauty;
```

```

    if( t < e) {
        pq.add(b);

        t++;
    } else {
        if( b <= pq.peek()) {
            continue;
        } else {
            pq.poll(); —  $O(\log n)$ 
            pq.add(b);
        }
    }

    int ans = 0;
    while( ! pq.isEmpty()) { —  $(n \log n)$ 
        ans += pq.poll();
    }

    return ans;
}

```

TC: $O(n \log n)$

SC: $O(n)$

Ques There are n students with marks.

Teacher has to distribute candies such that -

a) Every student should have atleast one candy.

b) student with more marks than neighbours should have more candies.

find min no. of candies to distribute [medium]

$$A[] = \begin{bmatrix} 1 & 5 & 2 & 1 \\ 1 & \cancel{1} & \cancel{1} & 1 \\ & 2 & 2 & \\ & 3 & & \end{bmatrix} \quad \text{ans} = 1 + 3 + 2 + 1 = 7$$

$$A[] = \begin{bmatrix} 8 & 10 & 6 & 2 \\ 1 & \cancel{1} & \cancel{1} & 1 \\ & 2 & 2 & \\ & 3 & & \end{bmatrix} \quad \text{ans} = 1 + 3 + 2 + 1 = 7 \text{ Ans}$$

$$A[] = \begin{bmatrix} 4 & 4 & 4 & 4 & 4 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \quad \text{ans} = 5.$$

$$A[] = \begin{bmatrix} 1 & 6 & 3 & 1 & 10 & 12 & 20 & 5 & 2 \\ 1 & \cancel{1} & \cancel{1} & 1 & \cancel{1} & \cancel{1} & \cancel{1} & \cancel{1} & 1 \\ & 2 & 2 & & 2 & 3 & 4 & 2 & \\ & 3 & & & & & & & \end{bmatrix}$$
$$\text{ans} = 1 + 3 + 2 + 1 + 2 + 3 + 4 + 2 + 1 = 19 \text{ Ans}$$

Logic:

$A[] = \begin{bmatrix} 1 & 6 & 3 & 1 & 10 & 12 & 20 & 5 & 2 \\ 1 & \cancel{2} & 1 & 1 & \cancel{2} & \cancel{3} & \cancel{4} & 1 & 1 \end{bmatrix}$

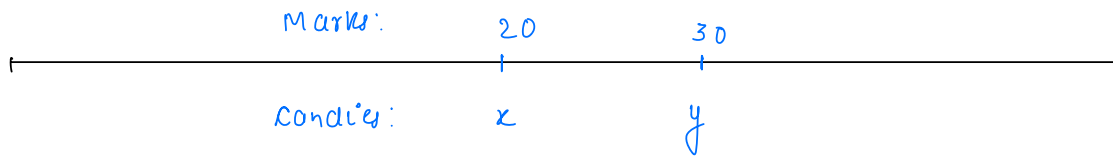
$L \rightarrow R$

$\begin{bmatrix} 1 & \cancel{2} & \cancel{1} & 1 & 2 & 3 & 4 & \cancel{1} & 1 \end{bmatrix}$

$R \rightarrow L$

$\begin{bmatrix} 1 & 3 & 2 & 1 & 2 & 3 & 4 & 2 & 1 \end{bmatrix}$

Why this works?



$L \rightarrow R: \quad x < y$

$R \rightarrow L \quad x < y + \underline{\underline{\text{delta}}}$

Code:

```
int minimiseCondiar ( A[] ) {  
    n = A.length;  
    int[] res = new int[n];  
  
    for (i=0; i<n; i++) {  
        res[i] = 1;  
    }  
  
    L → R  
  
    for (i=1; i<n; i++) {  
        if ( A[i] > A[i-1] ) {  
            res[i] = res[i-1] + 1;  
        }  
    }  
  
    R → L  
  
    for (i=n-2; i>=0; i--) {  
        if ( A[i] > A[i+1] ) {  
            if ( res[i] <= res[i+1] ) {  
                res[i] = res[i+1] + 1;  
            }  
        }  
    }  
  
    int ans = 0;  
    for ( int el: res ) {  
        ans += el;  
    }  
    return ans;  
}
```

TC: $O(n)$
SC: $O(n)$

Break: 8:17 - 8:28 AM

Ques: Given n jobs with their start and end time.
find **max jobs** that can be completed if only
one job can be done at a time. [medium]

1
9 am ————— 11 am

2 ✓
2 pm ————— 4 pm

3
7 pm ————— 9 pm

4 ✓
10 am ————— 1 pm

5 ✓
4 pm ————— 6 pm

6 ✓
7 pm ————— 8 pm

7
10 am ————— 3 pm

8 ✓
8 pm ————— 10 pm

ans = 5

ip format

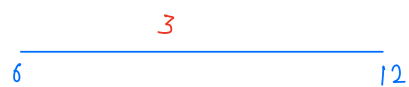
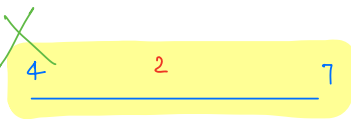
$s[] = [\overset{0}{1} \quad \overset{1}{5} \quad \overset{2}{8} \quad \overset{3}{7} \quad \overset{4}{12} \quad \overset{5}{13}]$

$e[] = [2 \quad 10 \quad 10 \quad 11 \quad 10 \quad 19]$

Greedy ideas

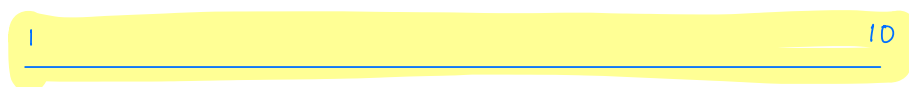
1. Shortest duration
2. Shortest start time
3. ' end time

1.) Shortest duration. ~~X~~

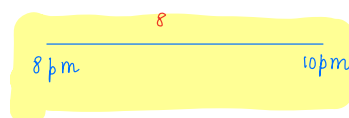
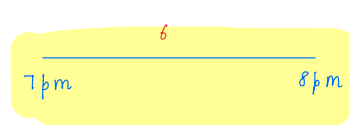
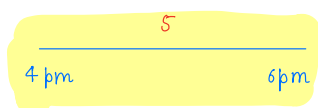
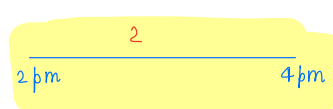
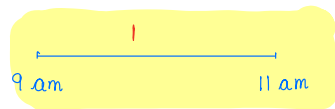


ans = 1 [wrong]

2.) Early start time ~~X~~



3.) Early end time ÷ (start time + duration) less



ans = 5

$s[] = [\overset{0}{1} \quad \overset{1}{5} \quad \overset{2}{8} \quad \overset{3}{7} \quad \overset{4}{12} \quad \overset{5}{13}]$

$e[] = [2 \quad 10 \quad 10 \quad 11 \quad 13 \quad 19]$

last job ended = ~~2~~ ~~10~~ ~~10~~ ~~11~~ ~~13~~ 19

ans = ~~1~~ ~~2~~ ~~3~~ 4

Code:

```
class Pair {
    int s;
    int e;
    Pair(x, y) {
        s = x;
        e = y;
    }
}
```

```
int maximizeJobs(s[], e[]) {
```

```
    Pair[] arr = new Pair[n];
```

$O(N)$ — for ($i=0$; $i < n$; $i++$) {

```
        arr[i] = new Pair(s[i], e[i]);
```

```
    }
```

$O(N \log N)$ — Arrays.sort(arr, (u, v) \rightarrow u.e - v.e));

ans = 1;

lastJobEnded = arr[0].e;

$O(n)$ — for (i=1; i<n; i++) {

pair p = arr[i];

if (p.s >= lastJobEnded) {

ans += 1;

lastJobEnded = p.e;

}

}

return ans;

}

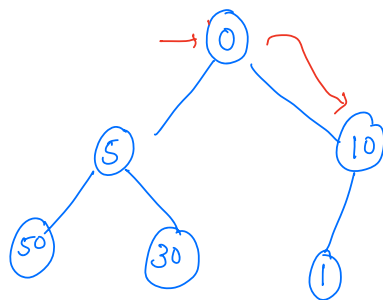
TC: $O(n \log n)$

SC: $O(n)$

Greedy: मलमल

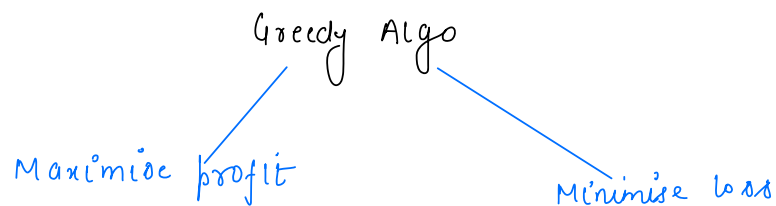
→ Approach to solve optimisation problems
by making greedy / optimal choices

Ex:



Max sum from root to
leaf

Greedy does not work.



Thankyou 😊