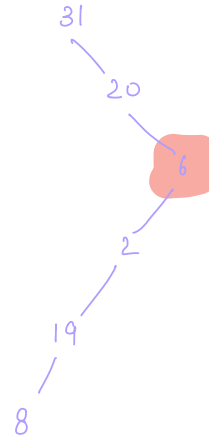
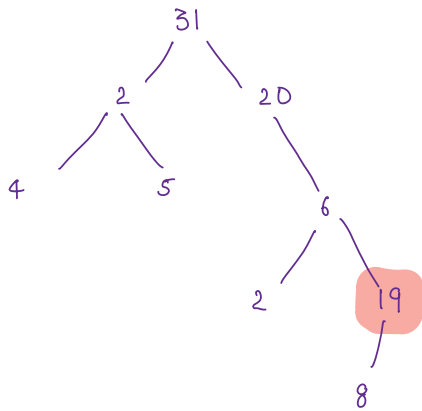


Lecture ÷ Trees-4

Agenda

- Morris inorder traversal
- kth smallest element in BST
- LCA in BST
- LCA in BT.

Morris Inorder traversal [Hard]

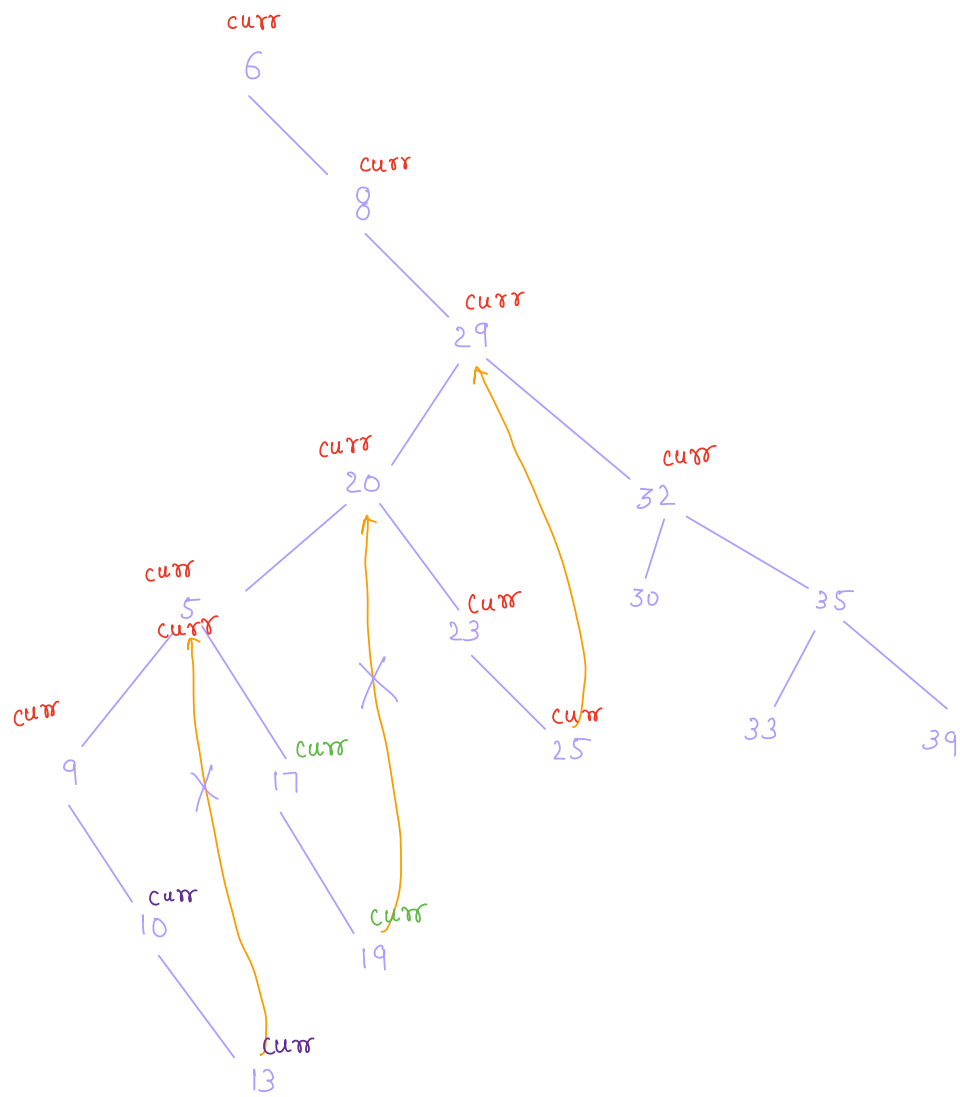


Last el of inorder: 19

claim: In a B-T inorder traversal —

Last node = extreme right of root.

Inorder	Recursively:	TC: $O(n)$ SC: $O(h)$
	Iterative	TC: $O(n)$ SC: $O(h)$
	Morris	TC: $O(n)$ SC: $O(1)$



Print: 6 8 9 10 13 5 17 19 20

 23 25 29

TC: $O(n)$

SC: $O(1)$

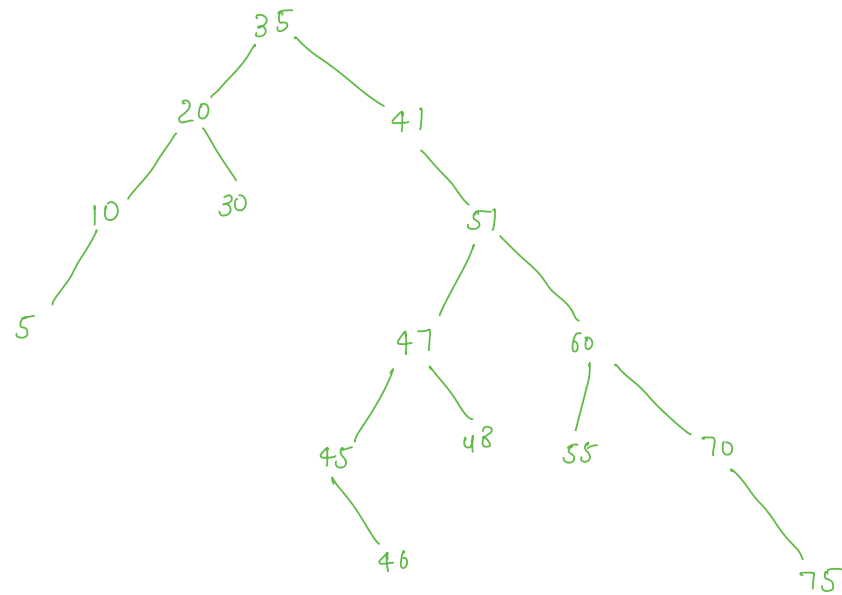
```

void morrisInorder(Node root) {
    Node curr = root;
    while( curr != null) {
        if( curr.left == null) {
            print(curr.data);
            curr = curr.right;
        } else {
            Node last = curr.left;
            while( last.right != null &&
                last.right != curr) {
                last = last.right;
            }
            if( last.right == null) {
                last.right = curr;
                curr = curr.left;
            } else {
                last.right = null;
                print(curr.data);
                curr = curr.right;
            }
        }
    }
}

```

last.right = null
 last.right != null

Q42 kth smallest el in a BST.



$k=1 \rightarrow 5$

$k=2 \rightarrow 10$

$k=3 \rightarrow 20$

$k=4 \rightarrow 30$

$k=5 \rightarrow 35$

$k=6 \rightarrow 41$

$k=7 \rightarrow 45$

Idea1: Inorder of BST = sorted array.

find kth el of sorted array.

TC: $O(n)$

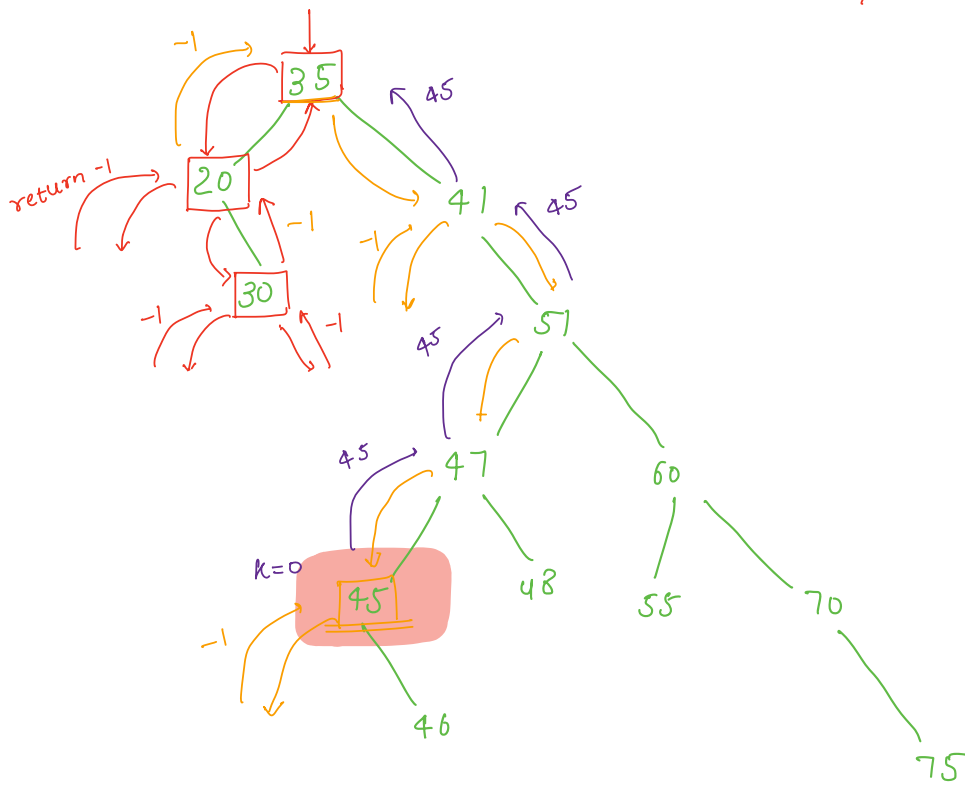
SC: $O(n)$

Idea 2:

TC: $O(\ln)$

$$Sc: 0(h)$$

$k = \cancel{5} \ \cancel{4} \ \cancel{3} \ \cancel{2} \ \cancel{1} \ 0$



```

int k;

int kthsmallest(Node root) {
    if (root == null) {
        return -1;
    }

    int left = kthsmallest(root.left);

    if (left != -1) {
        return left;
    }

    k--;
    if (k == 0) {
        return root.data;
    }

    return kthsmallest(root.right);
}

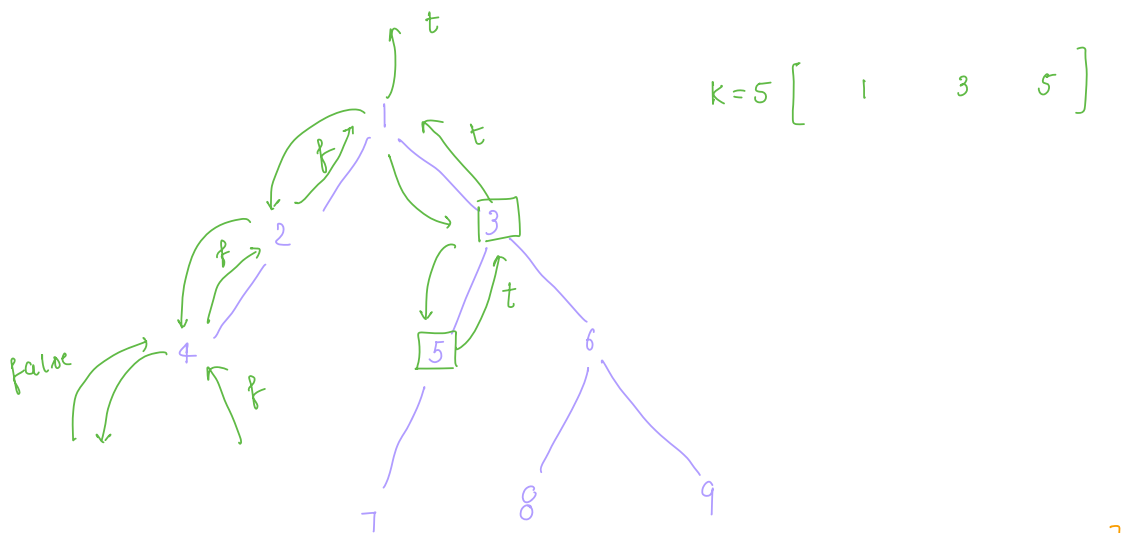
```

TC: $O(n)$

SC: $O(h)$

Break: 8:18 - 8:30 AM

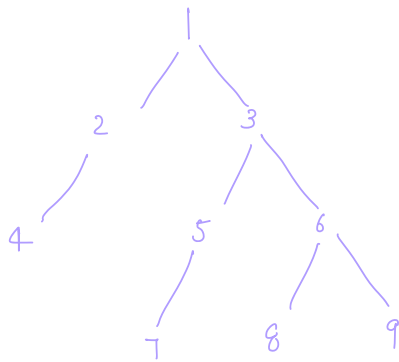
Qn Root to Node Path



```
List<Integer> list;
boolean search(Node root, int k) {
    if (root == null) {
        return false;
    }
    if (root.data == k) {
        list.add(root.data);
        return true;
    }
    boolean left = search(root.left, k);
    if (left == true) {
        list.add(root.data);
        return true;
    }
    boolean right = search(root.right, k);
    if (right == true) {
        list.add(root.data);
        return true;
    }
    return false;
}
```

[5 3 1]

Lowest common Ancestor



$$\text{lca}(7, 9) = 3$$

$$\text{lca}(5, 8) = 3$$

$$\text{lca}(4, 8) = 1$$

$$\text{lca}(3, 8) = 3$$

Root to node path:

$$7: [1, 3, 5, 7] \rightarrow \text{ans} = 3$$

$$8: [1, 3, 6, 8]$$

$$5: [1, 3, 5]$$

$$9: [1, 3, 6, 9] \rightarrow \text{ans} = 3$$

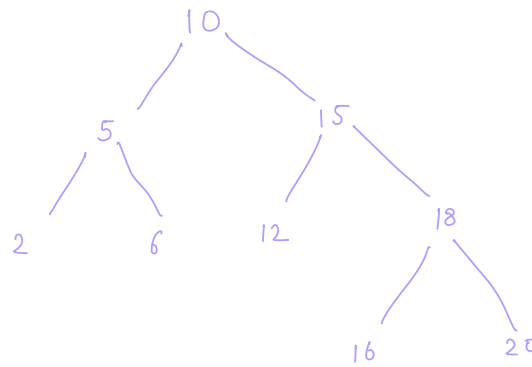
$$6: [1, 3, 6]$$

$$8: [1, 3, 6, 8] \rightarrow \text{ans} = 6$$

TC: $O(n)$

SC: $O(h)$

LCA of BST ★★★



$$\text{lca}(12, 20) = 15$$

Logic: $x = 2$ and $y = 6$ [LCA is in left subtree]

$x = 16$ and $y = 20$ [LCA " " right " "]

$x = 2$ and $y = 16$ [LCA == root]
Left subtree Right subtree

```
Node lcaBST(Node root, x, y) {
    curr = root;
    while(curr != null) {
        // x & y are in LST
        if(curr.data > x && curr.data > y) {
            curr = curr.left;
        } else if(curr.data < x && curr.data < y) {
            curr = curr.right;
        } else {
            return curr;
        }
    }
}
```

TC: $O(\text{height})$

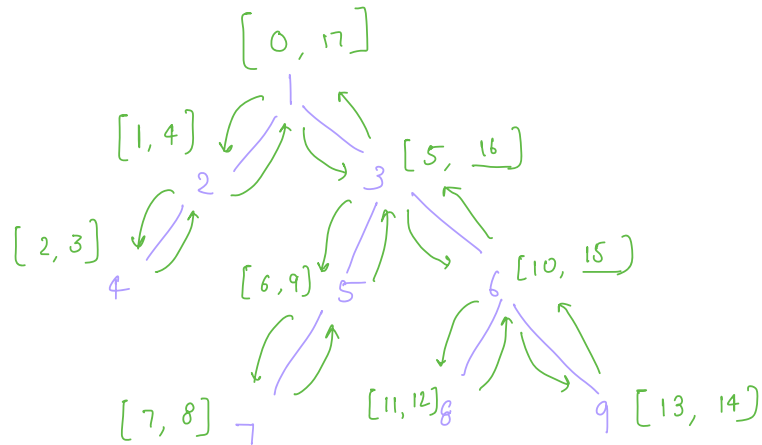
SC: $O(1)$

In time and out time

time at which
you first come
at a node

time at which
you go from
that node

$t=0$



Inmap < Integer, Node >

0 : 1
1 : 2
2 : 4
5 : 3
6 : 5
7 : 7
11 : 8
13 : 9
10 : 6

outmap

17 : 0
4 : 2
.....
}

```
Map<Integer, Node> inMap;
```

```
Map<Integer, Node> outMap;
```

```
t=0;
```

```
void traversal(Node root) {
```

```
    if (root == null) {
```

```
        return;
```

```
    }
```

```
    inMap.put(t, root);
```

```
    t++;
```

```
    traversal(root.left);
```

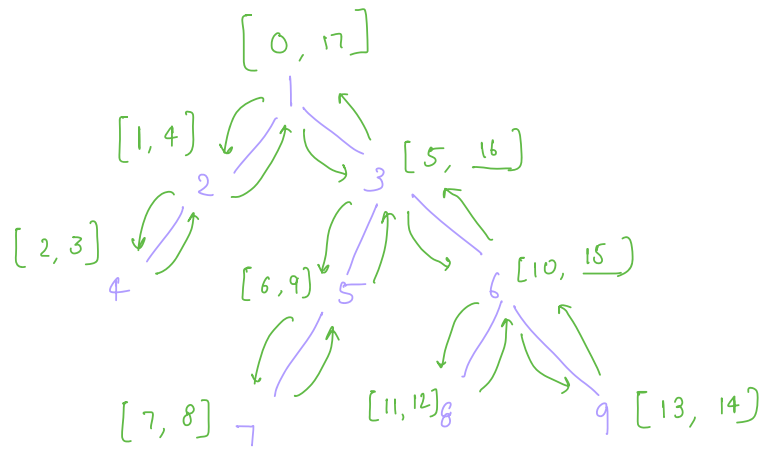
```
    traversal(root.right);
```

```
    outMap (t, root);
```

```
    t++;
```

```
}
```

Qu Given a queries, find LCA of all queries.



1. $\left. \begin{array}{l} \text{in}(x) < \text{in}(y) \\ \text{out}(x) > \text{out}(y) \end{array} \right\} \rightarrow x \text{ is ancestor of } y$

$x = 5 \text{ and } y = 7 \left[\text{anc} = 5 \right]$

$\text{in}(5) = 6 \quad \text{in}(7) = 7 \quad \text{in}(x) < \text{in}(y)$

$\text{out}(5) = 9 \quad \text{out}(7) = 8 \quad \text{out}(x) > \text{out}(y)$

2. $\left. \begin{array}{l} \text{in}(y) < \text{in}(x) \\ \text{out}(y) > \text{out}(x) \end{array} \right\} \rightarrow y \text{ is ancestor of } x$

Algo: if (x is ancestor of y) { return x; }

optional

↓
in(x) < in(y) && out(x) > out(y)

if (y is ancestor of x) { return y; }

in(y) < in(x) && out(y) > out(x)

curr = root;

while (curr != null) {

h/w { if (curr.left is ancestor of x &
in(curr.left) < in(x) &&
out(" ") > out(x)
curr.left " " " y) {
curr = curr.left;

h/w } else if (curr.right is ancestor of x &&
curr.right " " " y) {
curr = curr.right;

h/w } else {
return curr;
}
}

TC: $O(\underline{a * h}) + O(n)$

SC: $O(n)$