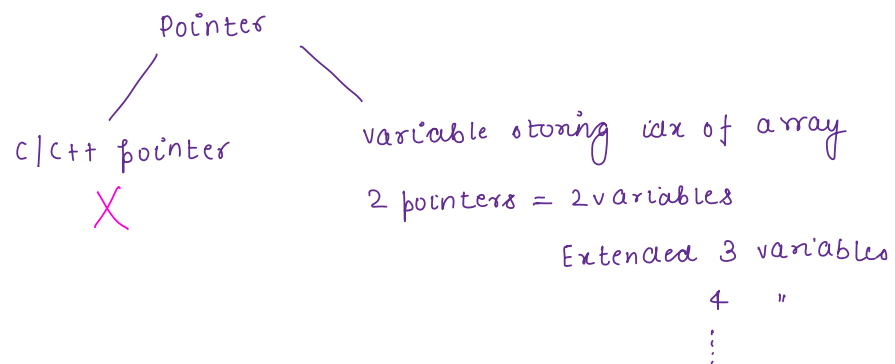


Lecture ÷ Two pointers

Agenda

- 2 pointers ✓
- 4 interview problems.

Class starts at 7:07 AM



Qul Given $arr[n]$ distinct sorted elements, check if there exists a pair (i, j) such that $arr[i] + arr[j] = k$ and $i \neq j$.

Ex. $arr[] = [3, 7, 8, 11, 14, 19, 20]$, $k = 25$
 $arr[3] + arr[4] = 25$, return true.

Ideas:

1. > Go to all pairs, calculate the sum and cmp with k

```
for (i=0; i<n; i++) {  
    for (j=i+1; j<n; j++) {  
        if (arr[i] + arr[j] == k) {  
            return true;  
        }  
    }  
}
```

TC: $O(n^2)$
SC: $O(1)$

2. > Optimisation :- Hashmap

TC: $O(n)$
SC: $O(n)$

3> Sorted array = Binary search

arr[] = $\begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 7 & 8 & 11 & 14 & 19 & 20 \end{bmatrix}$, $k=25$
 ↑
 i

Idea:

```
boolean pair(int[] arr, int k) {  
    n = arr.length;  
    for (i=0; i<n; i++) { —  $O(n)$   
        int a = arr[i];  
        int b = k-a;  
        // Apply b-search from [i+1 ---- n-1]  
        s = i+1, e = n-1;  
        while (s <= e) {  
            mid = s +  $\left(\frac{e-s}{2}\right)$ ;  
            if (arr[mid] == b) {  
                return true;  
            }  
            if (arr[mid] < b) {  
                s = mid+1;  
            } else {  
                e = mid-1;  
            }  
        }  
    }  
    return false;  
}
```

TC: $O(n \log n)$
SC: $O(1)$

a	b	search from
3	$25-3=22$	from [1 ---- n-1]
7	$25-7=18$	[2 ---- 6]
8	$25-8=17$	[3 ---- 6]
11	$25-11=14$	[4 ---- 6] ↑ return true;

$O(\log n)$

4>

Two pointer?

arr[11] = $\left[\begin{array}{cccccccccc} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ -3 & 0 & 1 & 3 & 6 & 8 & 11 & 14 & 18 & 25 \end{array} \right]$, $x = 17$

$\begin{array}{ccccccccc} \uparrow & \uparrow & \uparrow & \uparrow & & \uparrow & \uparrow & \uparrow \\ p1 & p1 & p1 & p1 & & p2 & p2 & p2 \end{array}$

p1	p2	arr[p1] + arr[p2]
0	9	$-3 + 25 = 22 > 17$
0	8	$-3 + 18 = 15 < 17$
1	8	$0 + 18 = 18 > 17$
1	7	$0 + 14 = 14 < 17$
2	7	$1 + 14 = 15 < 17$
3	7	$3 + 14 = 17 == 17$ (return true);

Code:

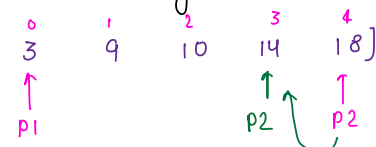
```
boolean checksum( int[] arr, int k) {  
    n = arr.length;  
    p1 = 0;  
    p2 = n-1;  
    while( p1 < p2 ) {  
        if (arr[p1] + arr[p2] == k) {  
            return true;  
        }  
        if (arr[p1] + arr[p2] < k) {  
            // inc p1  
            p1++;  
        } else {  
            // dec p2  
            p2--;  
        }  
    }  
    return false;  
}
```

TC: $O(n)$

SC: $O(1)$

Q why does this logic works?

$arr[] = [3, 9, 10, 14, 18]$ $K=19$



$p1$	$p2$	$arr[p1] + arr[p2]$
0	4 [fix]	$3 + 18 = 21 > 19$
1	-----	$9 + 18 = 27 > 19$
2	-----	$10 + 18 = 28 > 19$
3	-----	$14 + 18 = 32 > 19$

Obs: can I get 18 as one of the element?

↳ No [discard it]

Q2 Given $arr[n]$ sorted elements, check if there exists a pair (i, j) such that $arr[j] - arr[i] = k$ and $i \neq j$

and ~~$k > 0$~~ k can be anything.

Ex: $arr[] = [-3, 0, 1, 3, 6, 8, 11, 14, 21, 25]$, $k = 5$.

$arr[5] - arr[3] = 5$ return true;

$arr[4] - arr[2] = 5$

Case 1:

	P1	P2	$arr[P2] - arr[P1]$	
	0	9	$25 - (-3) = 28 > 5$	[wrong]
Move P2 →	0	8	$21 - (-3) = 24 > 5$	diff is $> k$ [Ambiguity]
Move P1 →	1	9	$25 - 0 = 25 > 5$	

Case 2:

	P1	P2	$arr[P2] - arr[P1]$	
	$(\frac{n}{2} + 1)$	$(\frac{n}{2} + 2)$		[wrong]
	5	6	$11 - 8 = 3 < 5$	
move P2 →	5	7	$14 - 8 = 6 > 5$	diff is $< k$ [Ambiguity]
move P1 →	4	6	$11 - 3 = 8 > 5$	

$$\text{arr}[] = \begin{bmatrix} -3 & 0 & 1 & 3 & 6 & 8 & 11 & 14 & 21 & 25 \end{bmatrix}, k=5$$

\uparrow \uparrow
 p_1 p_2

Case 3:

p_1	p_2	$\text{arr}[p_2] - \text{arr}[p_1]$	[correct]
0	1	$0 - (-3) = 3 < 5$	$[p_2++]$
0	2	$1 - (-3) = 4 < 5$	$[p_2++]$
0	3	$3 - (-3) = 6 > 5$	p_1++
1	3	$3 - 0 = 3 < 5$	p_2++
1	4	$6 - 0 = 6 > 5$	p_1++
2	4	$6 - 1 = 5 == 5$	return true;

$$\text{arr}[] = \begin{bmatrix} -3 & 0 & 1 & 3 & 6 & 8 & 11 & 14 & 21 & 25 \end{bmatrix}, k=5$$

\uparrow \uparrow \uparrow
 p_1, p_2 p_1 p_2

Case 4:

p_1	p_2	$\text{arr}[p_2] - \text{arr}[p_1]$	[correct]
8	9	$25 - 21 = 4 < 5$	p_1--
7	9	$25 - 14 = 11 > 5$	p_2--
7	8	$21 - 14 = 7 > 5$	p_2--
7	7		p_1-- — return false [wrong]
6	7	$14 - 11 = 3 < 5$	p_1--
5	7	$14 - 8 = 6 > 5$	p_2--

⋮
⋮
⋮
do on

Code:

```
boolean diff(int[] arr, int k) {  
    n = arr.length;  
    if (k < 0) { k = k * -1};  
    // case 3: p1 = 0, p2 = 1.  
  
    p1 = 0, p2 = 1  
    while ( p2 < n. ) {  
        if ( arr[p2] - arr[p1] == k) {  
            return true;  
        }  
        if ( arr[p2] - arr[p1] > k) {  
            p1++;  
            if ( p1 == p2) {  
                p2++;  
            }  
        } else {  
            p2++;  
        }  
    }  
    return false;  
}
```

conclusion

① $arr[3] = [\overset{0}{4} \quad \overset{1}{10} \quad \overset{2}{13}]$, $k = 8$

Case3:	p1	p2	$arr[p2] - arr[p1]$
	0	1	$10 - 4 = 6 < 8$ $p2++$
	0	2	$13 - 4 = 9 > 8$ $p1++$
	1	2	$13 - 10 = 3 < 8$ $p2++$
	1	3	→ out of bound [stop]

② $arr[] = [\overset{0}{4} \quad \overset{1}{10} \quad \overset{2}{13} \quad \overset{3}{23}]$, $k = 0$

[$p1 == p2$]

Try out yourself

<u>case3:</u>	p1	p2	$arr[p2] - arr[p1]$
	0	1	$10 - 4 = 6 > 0$ $p1++$
	1	1	if ($p1 == p2$) ; $p2++$
	1	2	$13 - 10 = 3 > 0$ $p1++$
	2	2	if ($p1 == p2$) $p2++$
	2	3	$23 - 13 = 10 > 0$; $p1++$
	3	3	if ($p1 == p2$) $p2++$
	3	4	→ stop

* If $k < 0$?

$$\text{arr}[3] = [\overset{0}{4} \quad \overset{1}{10} \quad \overset{2}{13}] , k = -3$$

$$\text{Pair: } k = -3 ; \quad \text{arr}[1] - \text{arr}[2]$$

$$k = 3 ; \quad \text{arr}[2] - \text{arr}[1]$$

obs: if $\text{arr}[i] - \text{arr}[j] == k \iff \text{arr}[j] - \text{arr}[i] == -k$

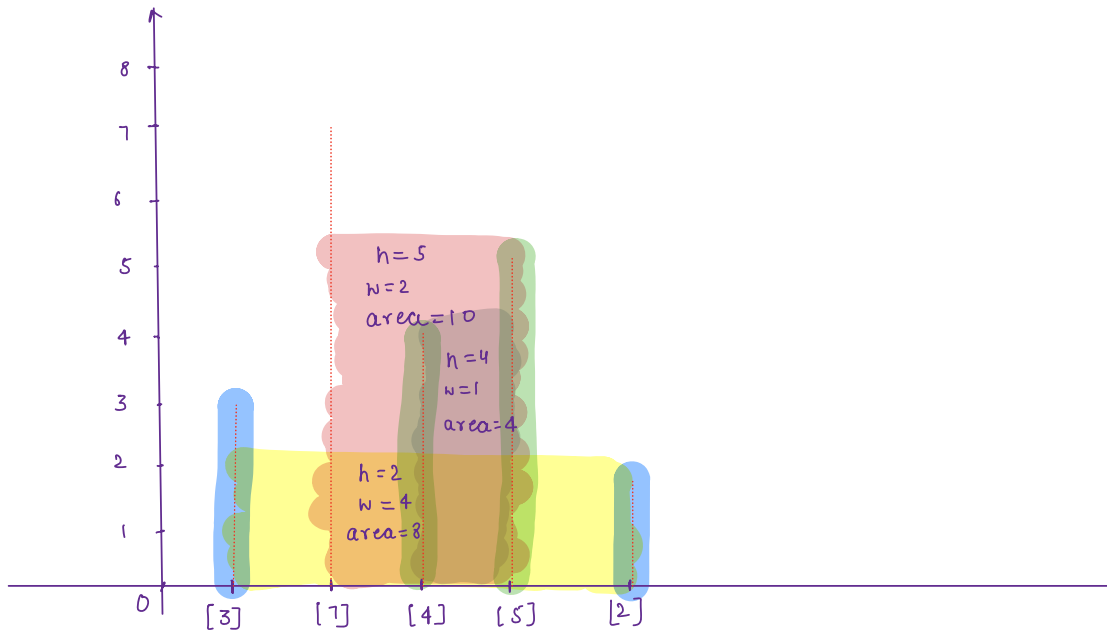
↑
Pair with k exists, $-k$ will also exist.

Break:- 8:38 AM

Ques: Given $arr[n]$ elements, $arr[i]$ represents height of each wall.
find max water accumulated b/w any 2 walls?

Note: Between any two walls, width = 1.

Example: $arr[5] = [3, 1, 4, 5, 2]$, $ans = 10$.



Idea1: Go to all pairs. —
calculate area and compare with max area encountered

TC: $O(n^2)$

SC: $O(1)$

Idea2: calculate area b/w max and s-max wall. [wrong]

$arr: [3, 2, 5, 6, 1, 2, 3, 4, 1, 2]$

↑ ↑ ↑

area: $5 * 1 = 5$

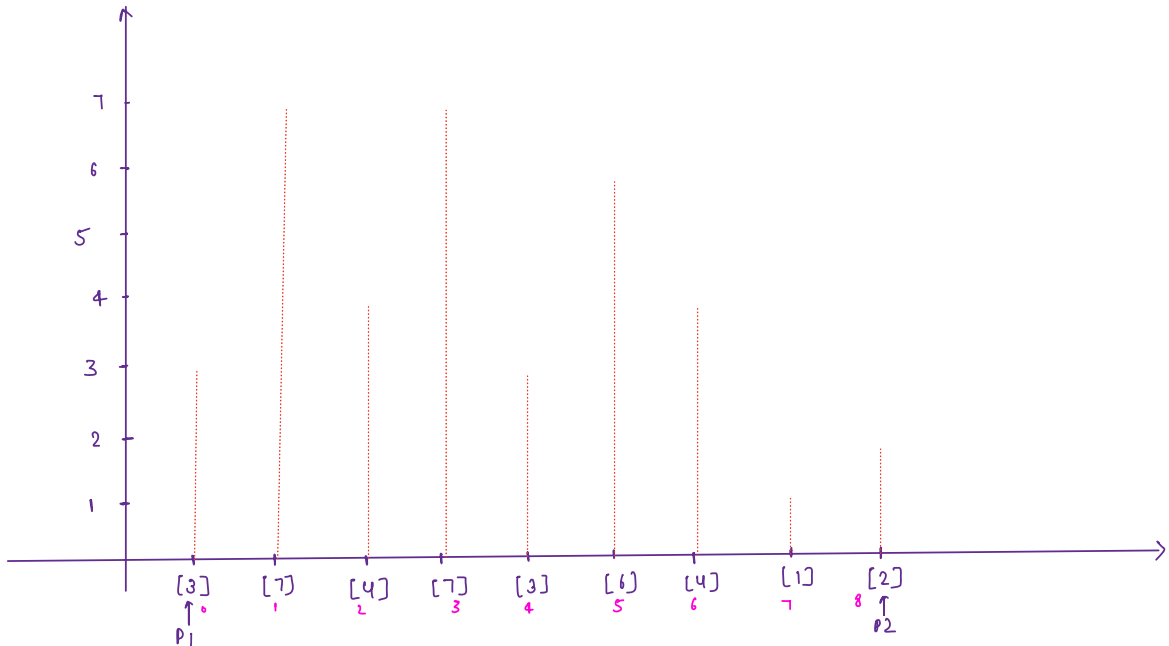
area: $2 * 9 = 18$

Idea3 Using 2 pointer?

arr[10] = [3⁰ 7¹ 4² 7³ 3⁴ 6⁵ 4⁶ 1⁷ 2⁸]

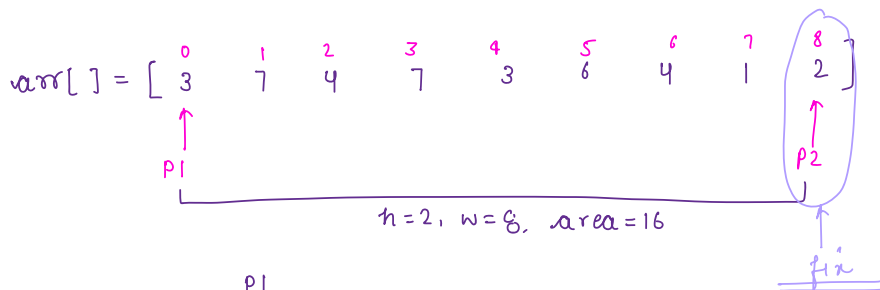
↑
p1

↑
p2



p1	p2	$h = \min(arr[p1], arr[p2])$	$w = p2 - p1$	area	move	final ans
0	8	$h = \min(3, 2) = 2$	$8 - 0 = 8$	$2 * 8 = 16$	p2--	16
0	7	$h = \min(3, 1) = 1$	$7 - 0 = 7$	$1 * 7 = 7$	p2--	16
0	6	$h = \min(3, 4) = 3$	$6 - 0 = 6$	$3 * 6 = 18$	p1++	18
1	6	$h = \min(7, 4) = 4$	$6 - 1 = 5$	$4 * 5 = 20$	p2--	20
1	5	$h = \min(7, 6) = 6$	$5 - 1 = 4$	$6 * 4 = 24$	p2--	24
1	4	$h = \min(7, 3) = 3$	$4 - 1 = 3$	$3 * 3 = 9$	p2--	24
1	3	$h = \min(7, 7) = 7$	$3 - 1 = 2$	$7 * 2 = 14$	p1++	24
2	3	$h = \min(4, 7) = 4$	$3 - 2 = 1$	$4 * 1 = 4$	p1++	24
3	3	→ stop				

* Which pointer to move any why?
 ↳ Pointer with min height.



P1	P2	
0	8	→ area=16
1	8	→ area=2 * 7 = 14
2	8	→ area=2 * 6 = 12
3	8	→ area=2 * 5 = 10
4	8	→ area=2 * 4 = 8
5	8	→ area=2 * 3 = 6
6	8	→ area=2 * 2 = 4
7	8	→ area=1 * 1 = 1.

area ↓

Discard P2.

code:

```
int maxWaterAccumulated(int[] arr) {  
    ans = 0;  
    n = arr.length;  
    p1 = 0, p2 = n-1;  
    while (p1 < p2) {  
        if (arr[p1] <= arr[p2]) {  
            ans = max(ans, arr[p1] * (p2 - p1));  
            p1++;  
        } else {  
            ans = max(ans, arr[p2] * (p2 - p1));  
            p2--;  
        }  
    }  
    return ans;  
}
```

TC: $O(n)$

SC: $O(1)$

Q4: Given 3 sorted arrays $A[]$, $B[]$, $C[]$ of size n .

find min value of $\max(A[i], B[j], C[k]) - \min(A[i], B[j], C[k])$

Example: $A[4] = [\overset{0}{3} \quad \overset{1}{14} \quad \overset{2}{16} \quad \overset{3}{23}] \Rightarrow \text{ans} = 3$

$B[4] = [\overset{0}{-6} \quad \overset{1}{23} \quad \overset{2}{24} \quad \overset{3}{30}]$

$C[4] = [\overset{0}{-15} \quad \overset{1}{15} \quad \overset{2}{26} \quad \overset{3}{31}]$

i	j	k	$\max(A[i], B[j], C[k])$	$\min(A[i], B[j], C[k])$	value
0	0	0	$\max(3, -6, -15) = 3$	$\min(3, -6, -15) = -15$	$3 - (-15) = 18$
1	1	1	$\max(14, 23, 15) = 23$	$\min(14, 23, 15) = 14$	$23 - 14 = 9$
3	3	3	$\max(23, 30, 31) = 31$	$\min(23, 30, 31) = 23$	$31 - 23 = 8$
3	1	2	$\max(23, 23, 26) = 26$	$\min(23, 23, 26) = 23$	$26 - 23 = 3$

Idea: consider all triplets —

find ans and compare with min ans encountered

TC: $O(n^3)$

SC: $O(1)$

Idea 2:

$$A[4] = \begin{bmatrix} 3 & 14 & 16 & 23 \end{bmatrix}$$

$$B[4] = \begin{bmatrix} -6 & 23 & 24 & 30 \end{bmatrix}$$

$$C[4] = \begin{bmatrix} -15 & 15 & 26 & 31 \end{bmatrix}$$

p1	p2	p3	$\max(A[p1], B[p2], C[p3])$	$\min(A[p1], B[p2], C[p3])$	ans
0	0	0	$\max(3, -6, -15) = 3$	$\min(3, -6, -15) = -15$ Increase p3; p3++	$3 - (-15) = 18$
0	0	1	$\max(3, -6, 15) = 15$	$\min(3, -6, 15) = -6$ incr p2; p2++	$15 - (-6) = 21$
0	1	1	$\max(3, 23, 15) = 23$	$\min(3, 23, 15) = 3$ p1++	$23 - 3 = 20$
1	1	1	$\max(14, 23, 15) = 23$	$\min(14, 23, 15) = 14$ p1++	$23 - 14 = 9$
2	1	1	$\max(16, 23, 15) = 23$	$\min = 15$; p3++	$23 - 15 = 8$
2	1	2	$\max(16, 23, 26) = 26$	$\min = 16$ p1++	$26 - 16 = 10$
⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮

TC: $O(n)$
SC: $O(1)$

Code: Try out yourself.

*

$$A = \begin{pmatrix} 3 & 14 & 16 & 23 \end{pmatrix}$$

⁰ ¹ ² ³
_{P1}

$$B = \begin{pmatrix} -6 & 23 & 24 & 30 \end{pmatrix}$$

_{P2}

$$C = \begin{pmatrix} -15 & 15 & 26 & 31 \end{pmatrix}$$

_{P3}

P1	P2	P3	max	min	ans
0	0	0	3	-15	$3 - (-15) = 18$
1	1		23	-15	$23 - (-15) = 38$
1	2		24	-15	$24 - (-15) = 39$
2	2		24	-15	$24 - (-15) = 39$

for P3, I can never get any min ans. so discard it
P3++

Thankyou 😊