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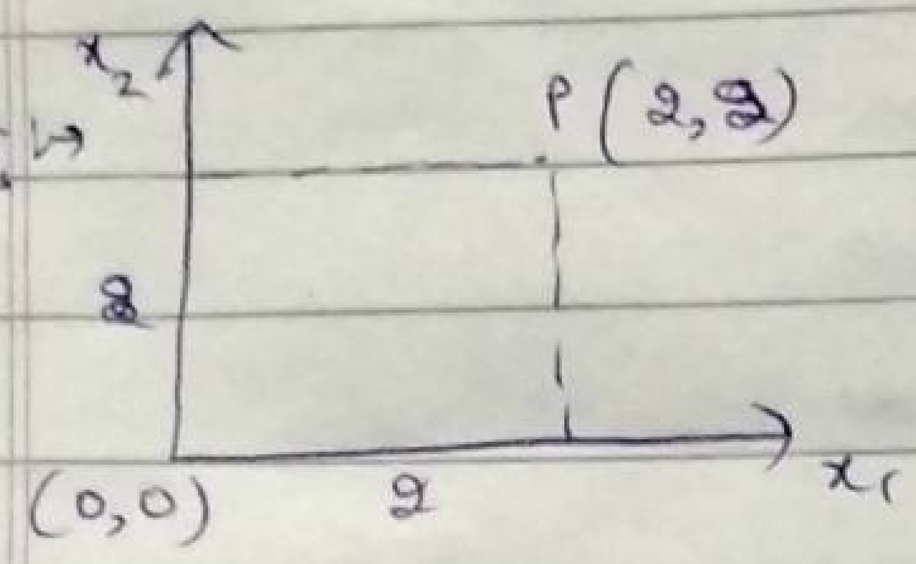
Linear Algebra

1. Point / Vector :-

2D point :-

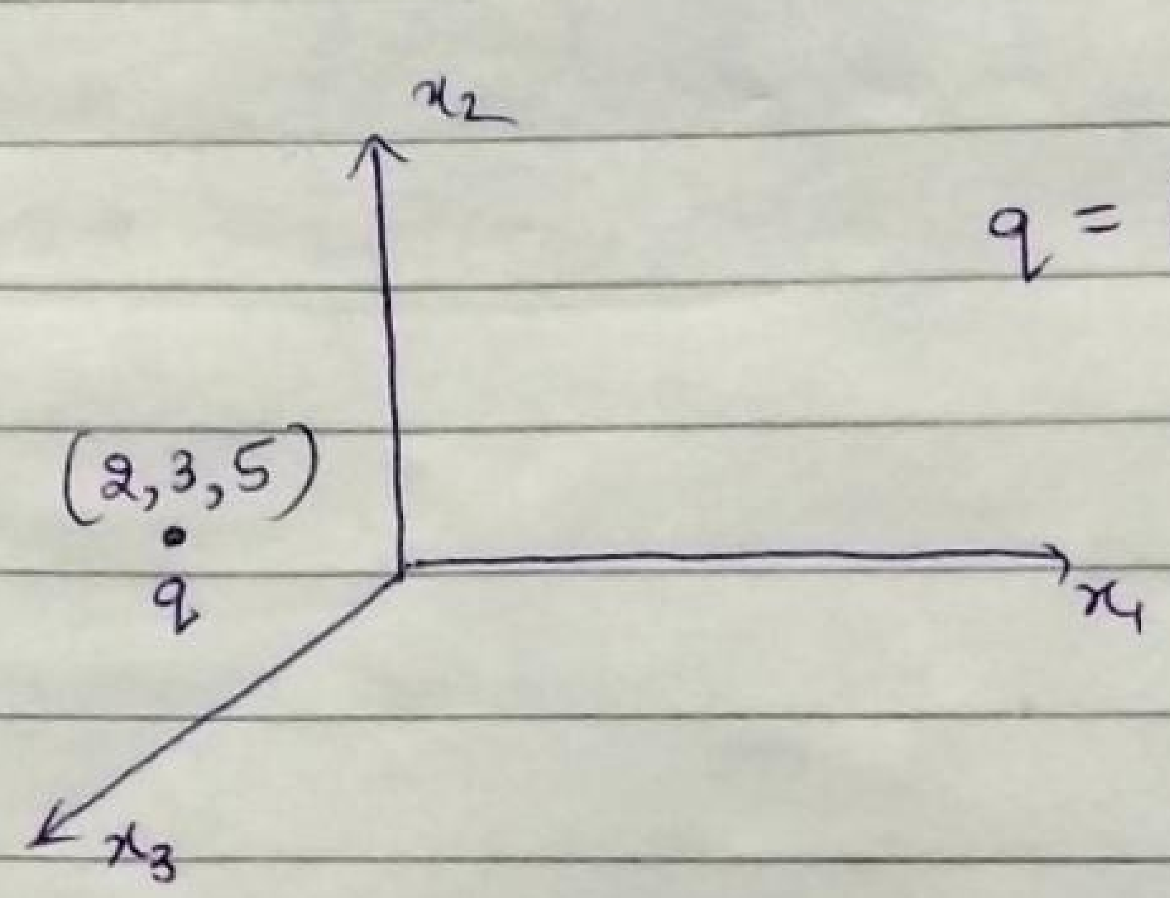


$P = [2, 3]$

$\hookrightarrow x_2$ component

$\hookrightarrow x_1$ component

3D- point :-



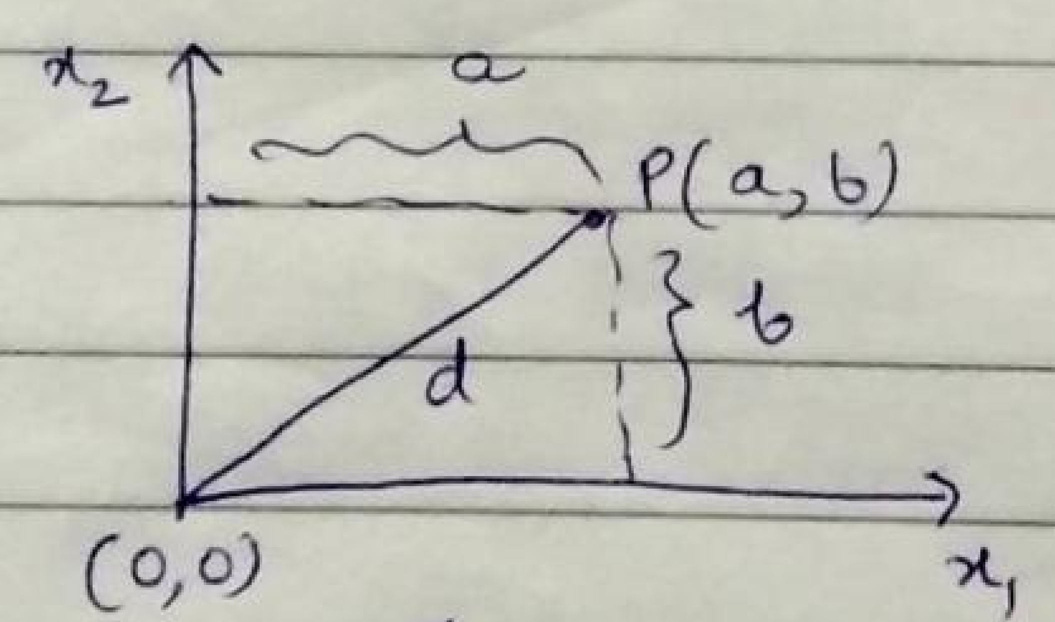
$q = [2, 3, 5]$

nd point :- $x = [2, 3, 4, 1, 5 \cdots \sim]$

So, points are represented in form of a vector.

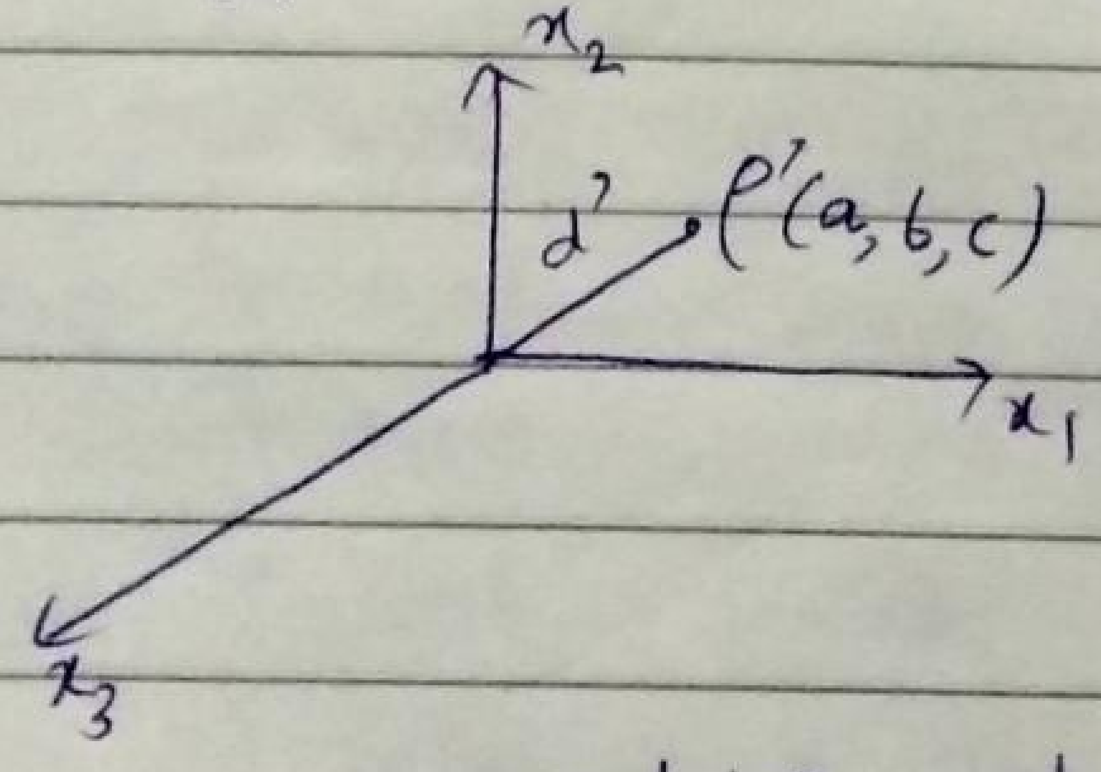
* Distance of a point from origin :-

2D :-



$d = \sqrt{a^2 + b^2}$ (pythagoras)

3D :-

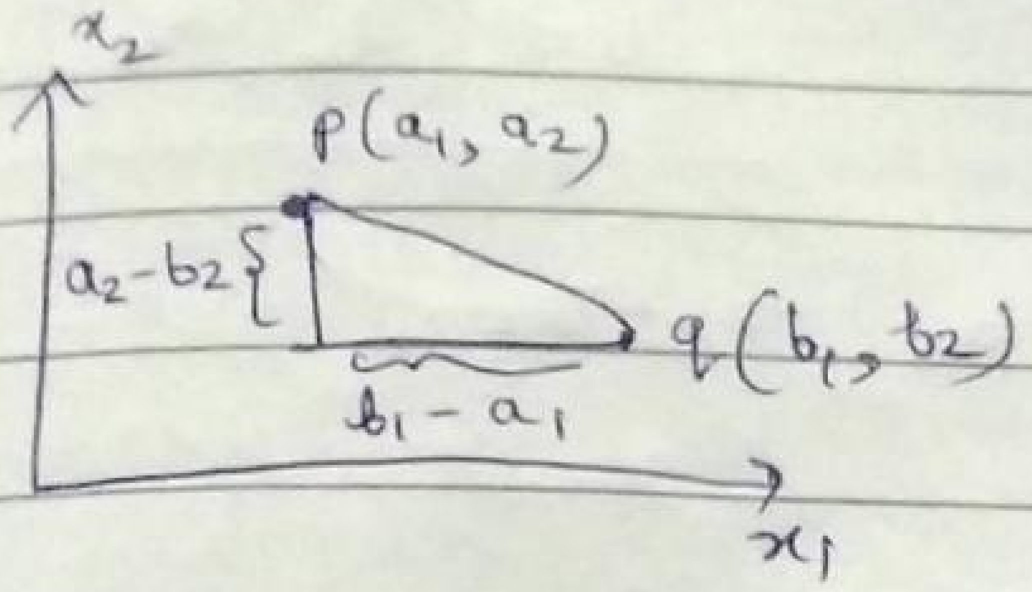


$d' = \sqrt{a^2 + b^2 + c^2}$

nd :- $d = \sqrt{a_1^2 + a_2^2 + a_3^2 + \cdots + a_n^2}$

* Distance b/w two points :-

2D :-



$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2}$$

3D :-

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

nD :-

$$d = \sqrt{(a_1 - b_1)^2 + (a_2 - b_2)^2 + \cdots + (a_n - b_n)^2}$$
$$= \text{sqrt}\left(\sum_{i=1}^{n} (a_i - b_i)^2\right)$$

* column vector :-

$$\underset{n\times 1}{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}_{n\times 1}$$

$n \times 1$ → columns, ↓ rows

* Row vector :-

$$A = [a_1, a_2, a_3, \cdots a_n]$$

$(1 \times n)$ → row, columns

* matrix :-

$A_{m\times n}$

$$\begin{array}{c} \\ 1 \\ 2 \\ \vdots \\ m \end{array} \begin{array}{c} 1 \quad 2 \quad 3 \quad \cdots \quad n \\ \left[\begin{array}{ccccc} & & & & \\ & & & & \\ & & & & \\ & & & & \end{array} \right]_{m \times n} \end{array}$$

array of arrays.

* Addition of vectors :→ $a = [a_1, a_2, \ldots, a_n]$
$b = [b_1, b_2, \ldots, b_n]$

$$c = a + b = [a_1 + b_1, a_2 + b_2, \ldots, a_n + b_n]$$

* Multiplication of vectors :→

If someone said there is a vector a_n then by default its a column vector.

$$a \cdot b = |a||b| \cos\theta$$
$$|a| = \sqrt{a_1^2 + a_2^2} \;,\quad |b| = \sqrt{b_1^2 + b_2^2}$$

$$a \cdot b = a_1 b_1 + a_2 b_2 = |a||b| \cos\theta$$
$$\Rightarrow \theta = \cos^{-1}\left\{\frac{a_1 b_1 + a_2 b_2}{|a||b|}\right\}$$

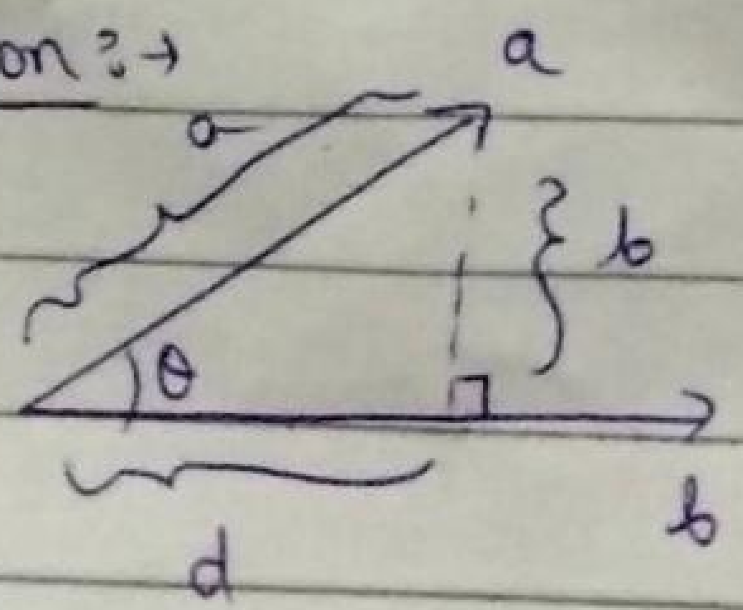
⇒ if $a \cdot b = 0$, then the two vectors are perpendicular to each other.

⇒ $a \cdot a = a_1 a_1 + a_2 a_2 + \cdots + a_n a_n$
$= a_1^2 + a_2^2 + \cdots + a_n^2$
$= |a|^2$

↓

distance of a from origin.

* Projection :→

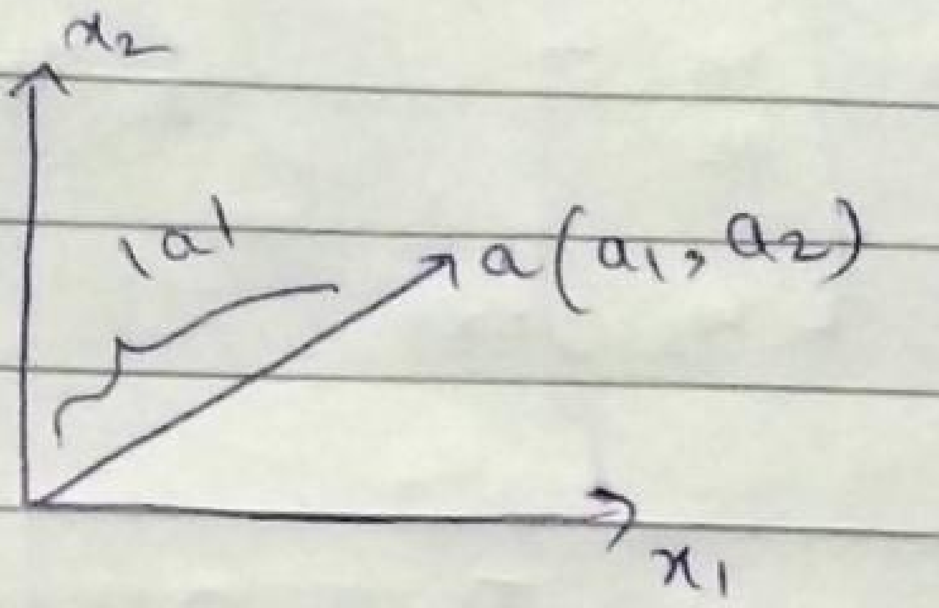


Projection of a on b
$$d = |a| \cos\theta$$

$$a \cdot b = \sum_{i=1}^{n} a_i b_i = |a|\,|b| \cos\theta$$

$$\boxed{d = \frac{a \cdot b}{|b|}} = \frac{|a|\,\cancel{|b|}\cos\theta}{\cancel{|b|}}$$

* Unit vector :→



$$\hat{a} = \frac{a}{|a|}$$

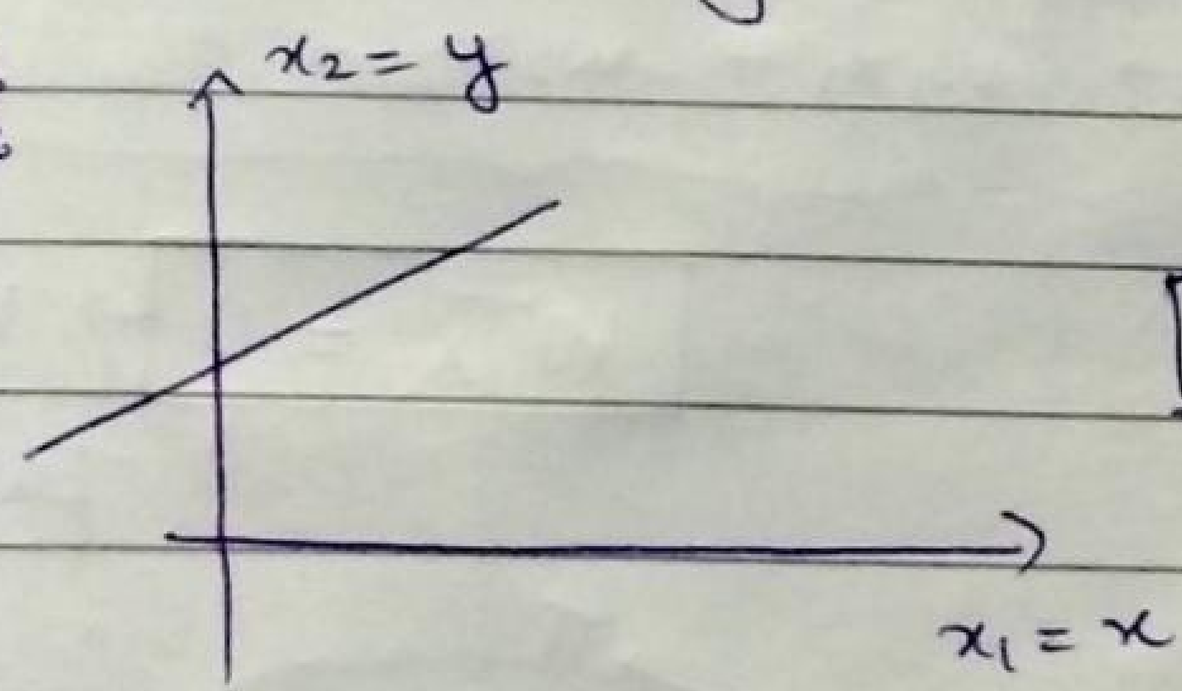
$\hat{a}$ → same direction as a

$|\hat{a}| = 1$

* Line :→ $ax + by + c = 0$ → general form.

&

2D :→

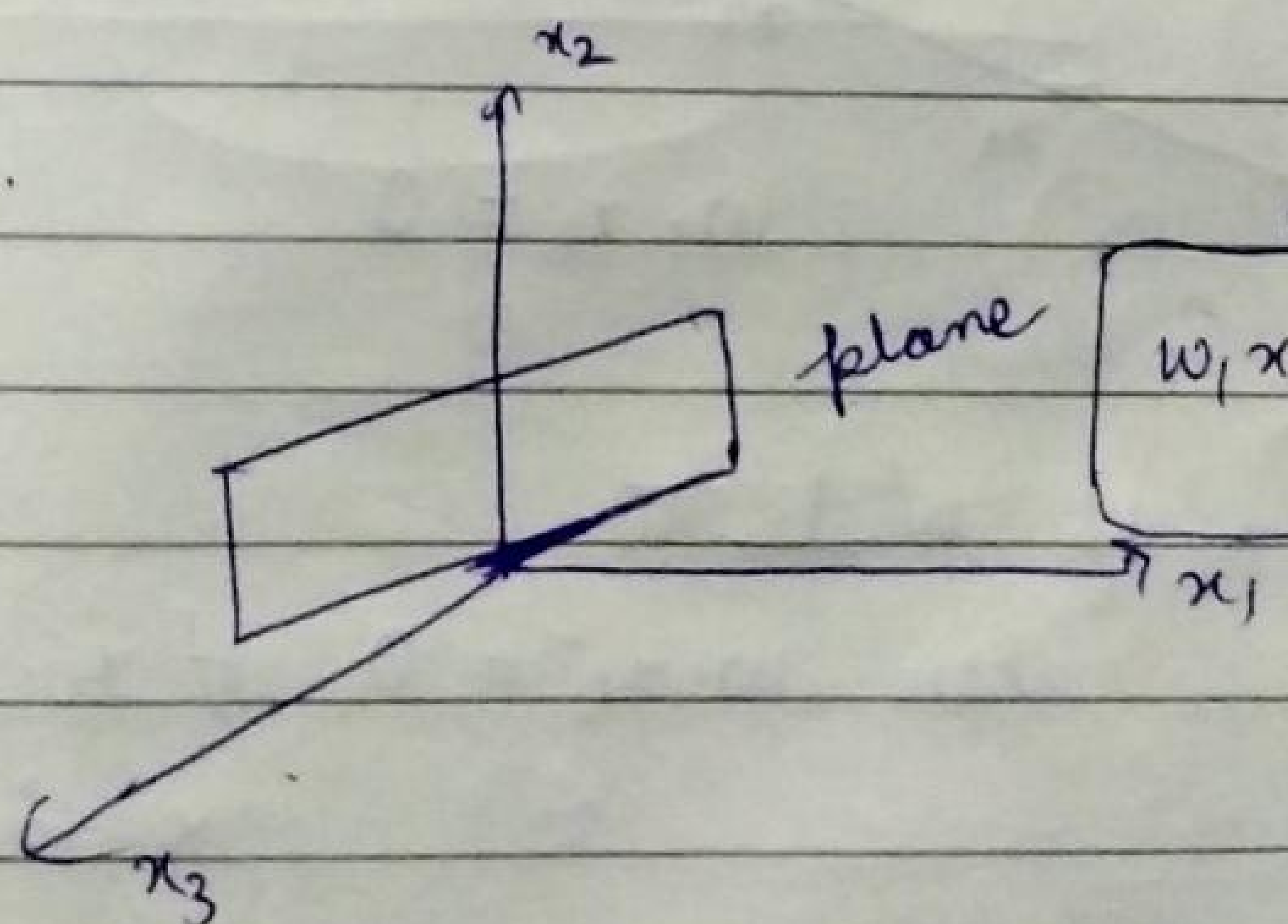


$ax_1 + bx_2 + c = 0$

$\boxed{w_1x_1 + w_2x_2 + w_0 = 0}$

↓ 2D

3D :→ Plane.



$ax + by + cz + d = 0$

$\boxed{w_1x_1 + w_2x_2 + w_3x_3 + w_0 = 0}$

↓

plane (3D)

nD :→ Hyperplane :→ $w_0 + w_1x_1 + w_2x_2 + \cdots + w_nx_n = 0$

Summation notation ← $\boxed{w_0 + \sum_{i=1}^{n} w_ix_i = 0}$

row vector

vector notation :→ $w_0 + \underbrace{[w_1, w_2, \cdots w_n]}_{W_{1\times n}} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}}_{x_{n\times 1}} = 0$

column vector

$W_{n\times 1} = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_3 \end{bmatrix}_{n\times 1}$, $x_{n\times 1} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}_{n\times 1}$

$\boxed{\pi_n = w_0 + w^T x = 0}$ → hyperplane

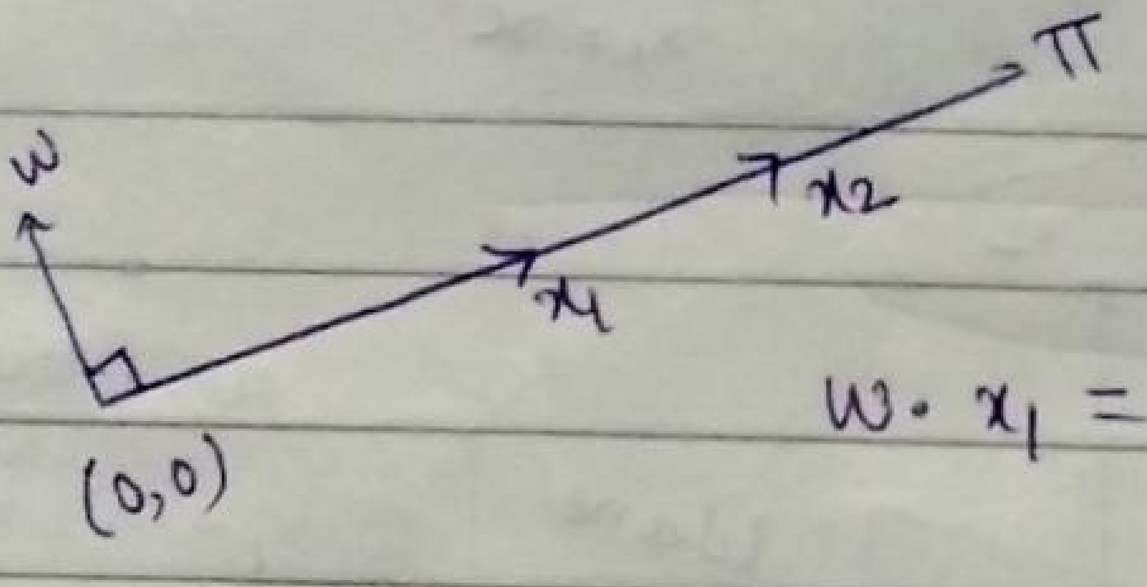
line passing through origin :→

line :→ 2D :→ $w_1x_1 + w_2x_2 = 0$

plane :→ 3D :→ $w_1x_1 + w_2x_2 + w_3x_3 = 0$

hyperplane :→ nD :→ $\underbrace{w_1x_1 + w_2x_2 + \cdots + w_nx_n = 0}$

$\boxed{W^T X = 0}$ → hyperplane passing through origin



$w \cdot x_1 = 0$

If $w \perp \pi$

then $w \cdot x_i = 0 \quad \forall x_i \in \pi$

, x is any point on plane.

$\hat{w} = \dfrac{w}{|w|}$

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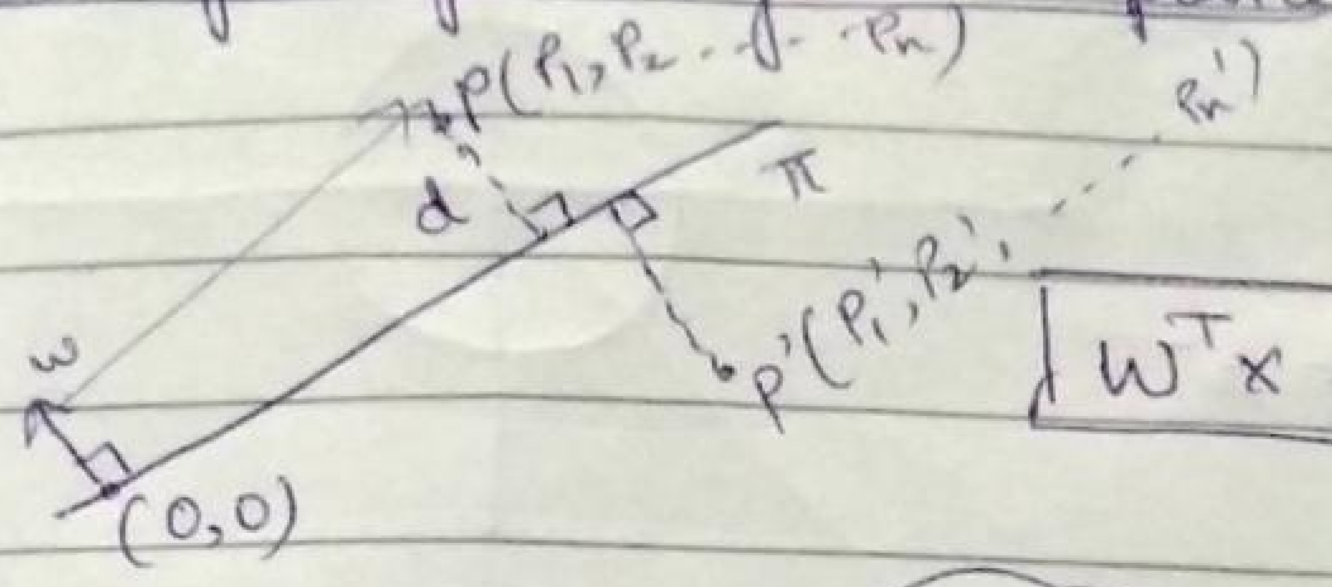
* Distance of a point from a plane :→



$\boxed{W^T x = 0}$ → Equation of plane

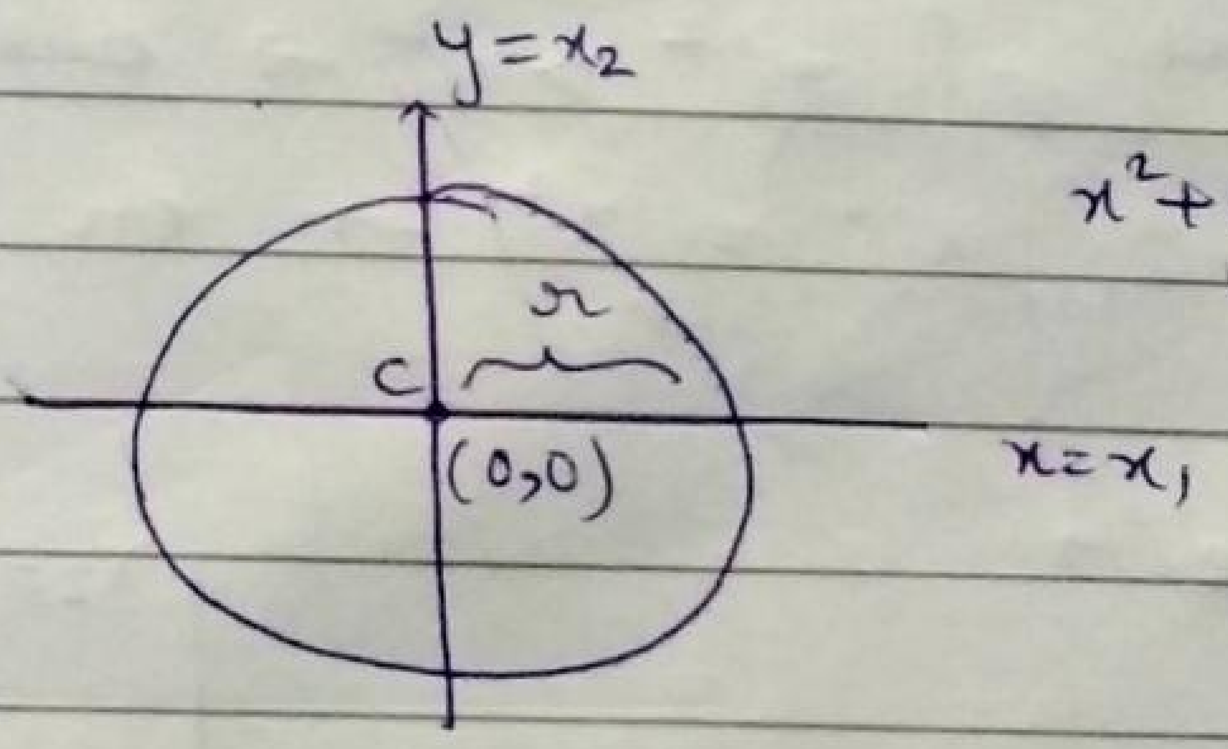
$W_{n \times 1}$ $\quad$ $P_{n \times 1}$

+ve $\boxed{d = \dfrac{W^T P}{|W|}}$ $\quad$ $\boxed{d' = \dfrac{W^T P'}{|W|}}$ −ve

Dividing a space into two spaces by a plane. those parts are called half spaces.

* Circle :→

2D :→



$x^2 + y^2 = r^2$

$C : (h, k) \rightarrow \boxed{(x-h)^2 + (y-k)^2 = r^2}$

$P(x_1, x_2)$ → How to know its position based on circle?

if $x_1^2 + x_2^2 < r^2$ = → p lies inside

if $x_1^2 + x_2^2 > r^2$ = p lies outside

if $x_1^2 + x_2^2 = r^2$ = p lies on circle.

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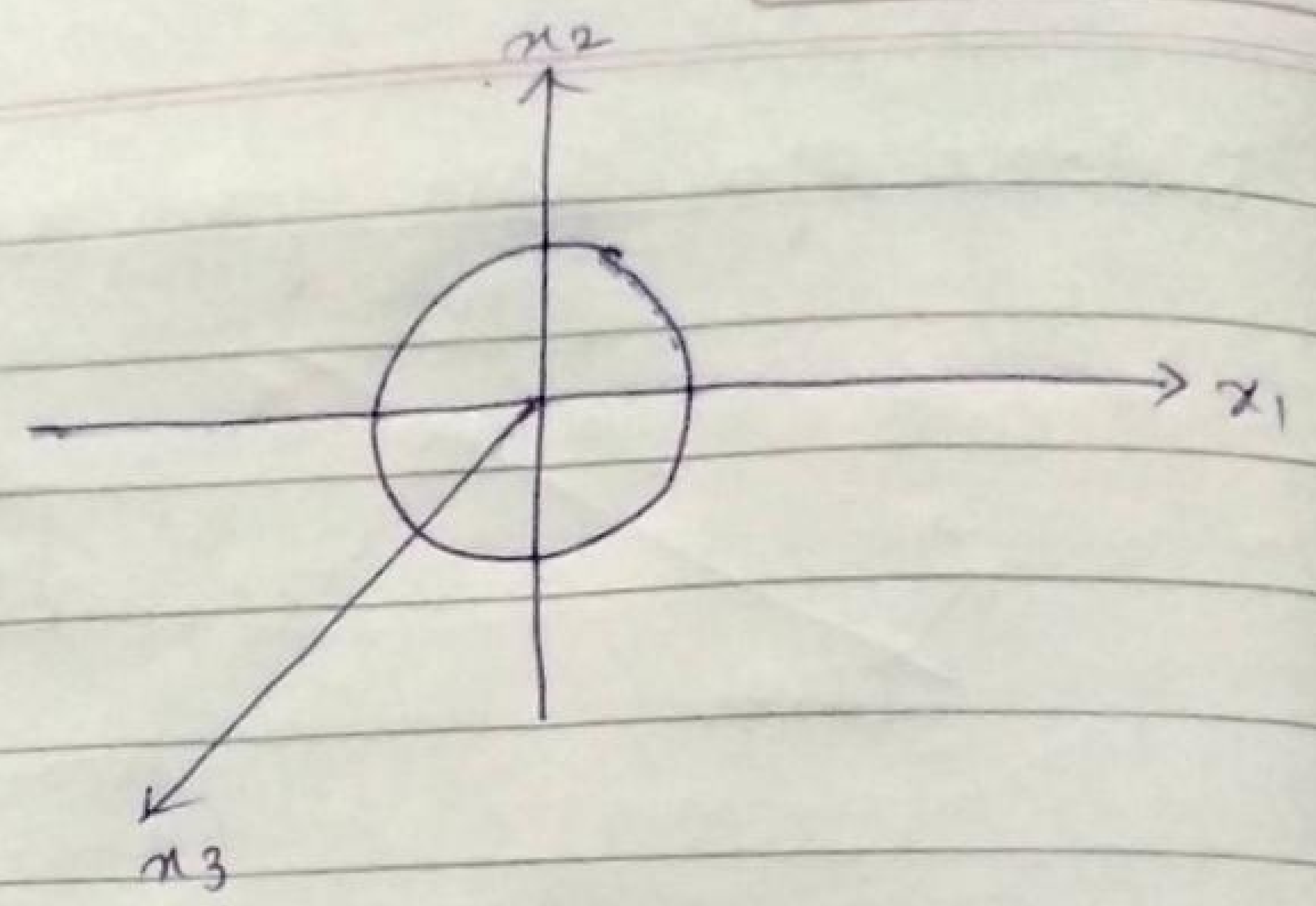
3D :→ Sphere



$x_1^2 + x_2^2 + x_3^2 = r^2$

nD :→ $x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2 = r^2$

hyper-sphere :→ $\boxed{\sum_{i=1}^{n} x_i^2 = r^2}$

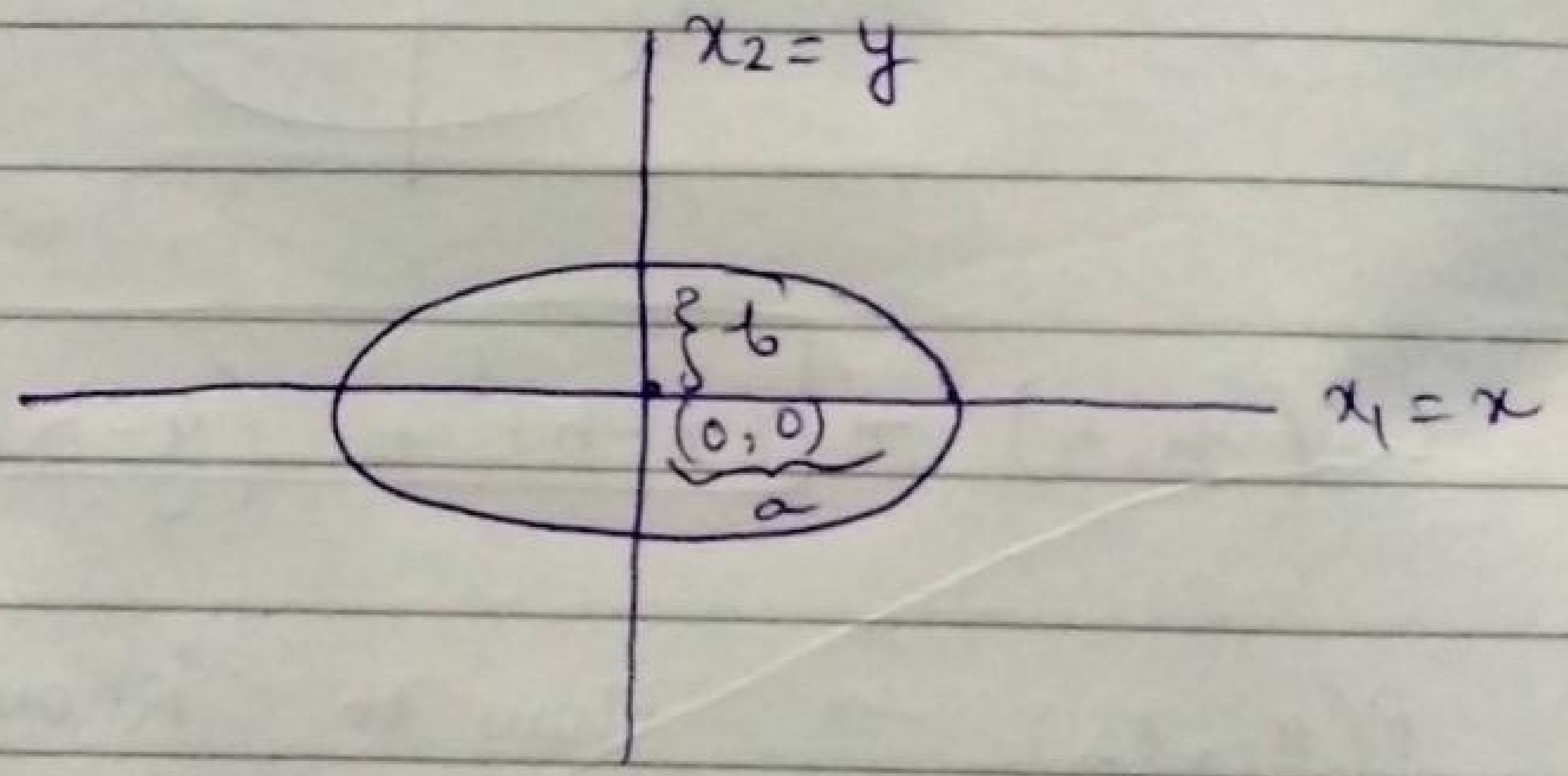
$\sum_{i=1}^{n} x_i^2 < r^2$ → inside hyper sphere

$\sum_{i=1}^{n} x_i^2 > r^2$ → outside hypersphere

$\sum_{i=1}^{n} x_i^2 = r^2$ → on circle.

* Ellipse :→

2D :→



$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

if $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} < 1$ → p lies inside ellipse

if $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} > 1$ → outside

if $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$ → on ellipse

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3D :→ Ellipsoid :→ $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$

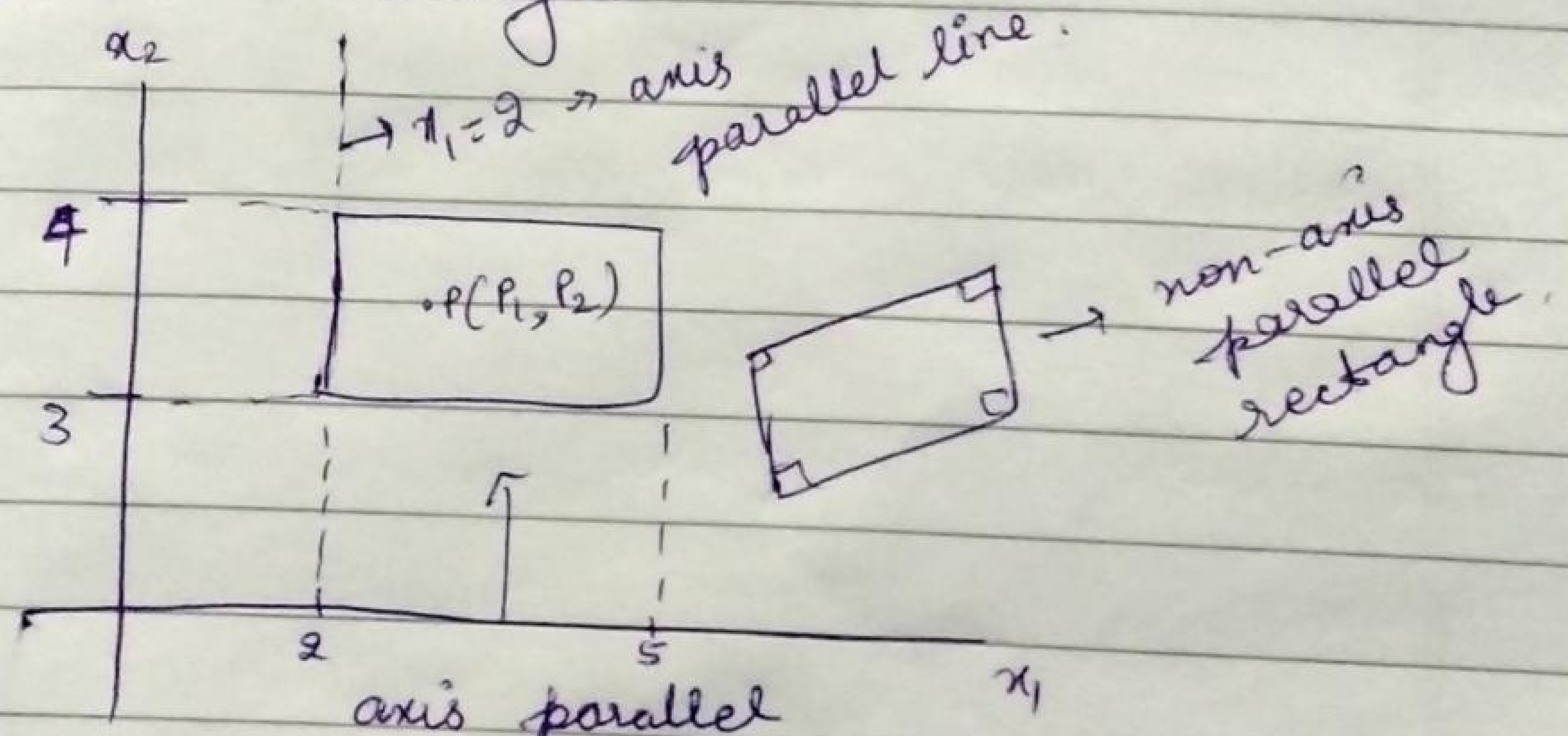
Same inside, outside and on properties, like 2D

nD :→ $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} + \cdots + \frac{x_n^2}{z^2} = 1$

same inside, outside & on properties like 2D.

* Squares and Rectangles :→

2D :→



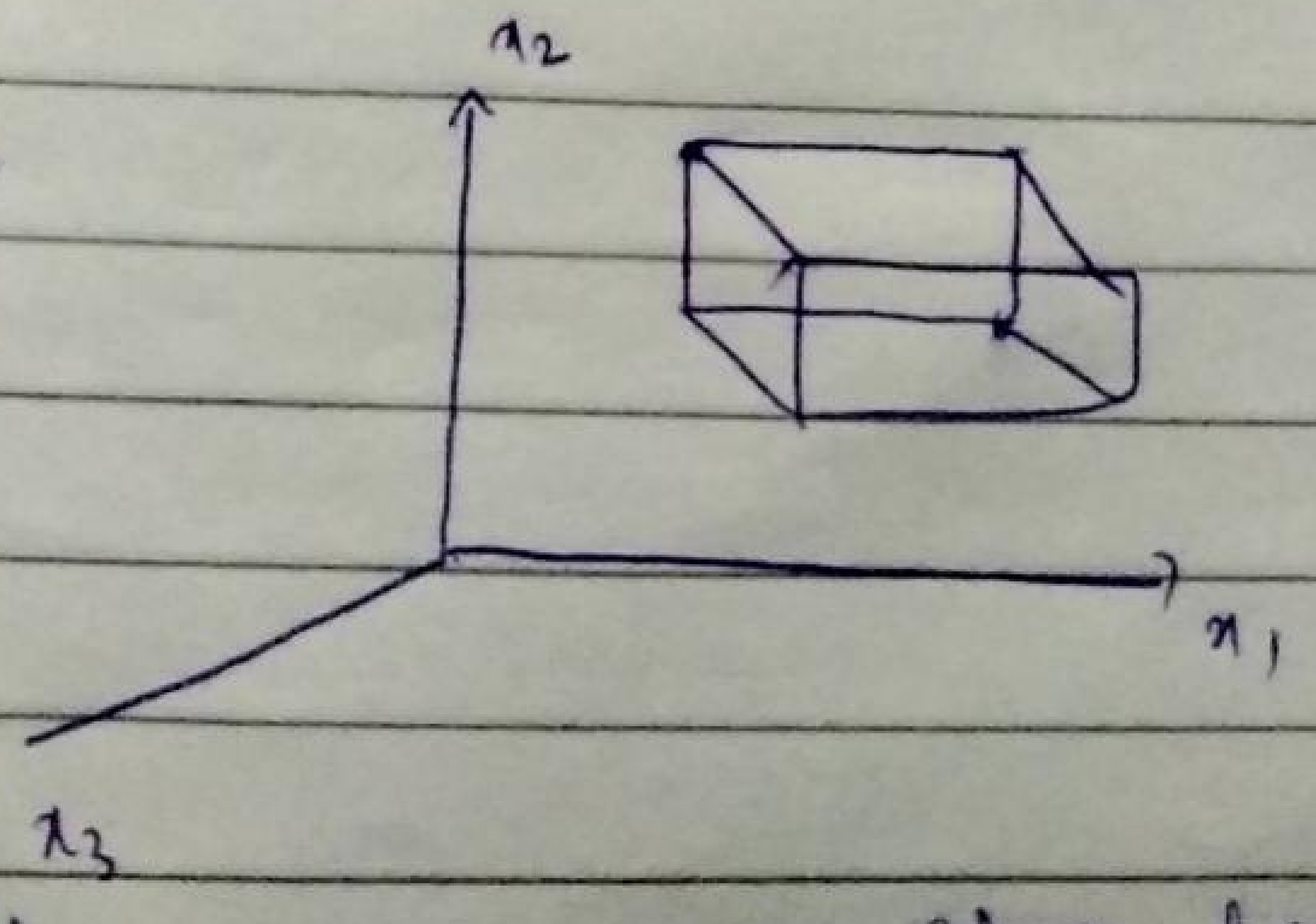
axis parallel rectangle

If $P_1 < 5$ && $P_1 > 2$

if $P_2 > 3$ && $P_2 < 4$

then P lies inside this rectangle.

3D :→ Cuboid :→



same idea here too

nD → Hyper-cuboid :→ same idea here too.