```
In [1]:
         import numpy as np
         import pandas as pd
         import scipy.stats as stats
         import matplotlib.pyplot as plt
         from sklearn.datasets import load_boston
         import seaborn as sns
         from sklearn.model selection import train_test_split
         from statsmodels.stats.outliers influence import variance inflation factor
         from sklearn.linear model import Ridge, Lasso, RidgeCV, LassoCV, ElasticNet, E
         lasticNetCV, LinearRegression
         import statsmodels.formula.api as smf
         from sklearn.metrics import mean squared error, r2 score
In [2]: boston = load boston()
         bos = pd.DataFrame(boston.data, columns=boston.feature names)
In [3]:
         bos.head()
Out[3]:
                          INDUS CHAS
                                                                         TAX PTRATIO
               CRIM
                      ΖN
                                         NOX
                                                RM
                                                     AGE
                                                             DIS
                                                                  RAD
                                                                                           B LST.
          0.00632
                     18.0
                            2.31
                                    0.0
                                        0.538
                                               6.575
                                                     65.2
                                                           4.0900
                                                                   1.0
                                                                        296.0
                                                                                       396.90
                                                                                                4.
                                                                                  15.3
            0.02731
                      0.0
                            7.07
                                    0.0
                                        0.469
                                               6.421
                                                     78.9
                                                           4.9671
                                                                   2.0
                                                                        242.0
                                                                                  17.8
                                                                                       396.90
                                                                                                9.
            0.02729
                            7.07
                                                                        242.0
                      0.0
                                    0.0 0.469
                                               7.185
                                                     61.1
                                                           4.9671
                                                                   2.0
                                                                                  17.8
                                                                                       392.83
                                                                                                4.
             0.03237
                                                                        222.0
                      0.0
                            2.18
                                    0.0
                                        0.458
                                               6.998
                                                     45.8
                                                           6.0622
                                                                   3.0
                                                                                  18.7
                                                                                       394.63
                                                                                                2.
             0.06905
                      0.0
                            2.18
                                    0.0 0.458 7.147
                                                     54.2
                                                           6.0622
                                                                   3.0
                                                                        222.0
                                                                                  18.7
                                                                                       396.90
                                                                                                5.
In [4]:
         print(boston.keys())
         dict_keys(['data', 'target', 'feature_names', 'DESCR', 'filename'])
         bos['MEDV'] = boston.target
In [5]:
In [6]:
         bos.head()
Out[6]:
               CRIM
                      ZN INDUS CHAS
                                         NOX
                                                RM AGE
                                                             DIS RAD
                                                                         TAX PTRATIO
                                                                                           B LST.
            0.00632
                     18.0
                            2.31
                                        0.538
                                               6.575
                                                     65.2
                                                           4.0900
                                                                        296.0
                                                                                       396.90
                                                                                                4.
                                    0.0
                                                                   1.0
                                                                                  15.3
            0.02731
                            7.07
                      0.0
                                    0.0 0.469
                                               6.421
                                                     78.9
                                                           4.9671
                                                                        242.0
                                                                                       396.90
                                                                                                9.
                                                                   2.0
                                                                                  17.8
                            7.07
            0.02729
                      0.0
                                    0.0
                                        0.469
                                               7.185
                                                     61.1
                                                           4.9671
                                                                   2.0
                                                                        242.0
                                                                                  17.8
                                                                                       392.83
                                                                                                4.
            0.03237
                      0.0
                            2.18
                                    0.0
                                        0.458
                                               6.998
                                                     45.8
                                                           6.0622
                                                                   3.0
                                                                        222.0
                                                                                  18.7
                                                                                       394.63
                                                                                                2.
             0.06905
                                    0.0 0.458 7.147
                                                                   3.0 222.0
                      0.0
                            2.18
                                                     54.2 6.0622
                                                                                  18.7
                                                                                       396.90
                                                                                                5.
```

```
In [7]: bos.isnull().sum()
Out[7]: CRIM
                      0
         \mathsf{ZN}
                      0
                      0
         INDUS
         CHAS
                      0
         NOX
                      0
                      0
         RM
         AGE
                      0
         DIS
                      0
         RAD
                      0
                      0
         TAX
         PTRATIO
                      0
         LSTAT
                      0
         MEDV
                      0
         dtype: int64
```

There are no missing values

In [8]: print(boston.DESCR)

.. _boston_dataset:

Boston house prices dataset

Data Set Characteristics:

:Number of Instances: 506

:Number of Attributes: 13 numeric/categorical predictive. Median Value (a ttribute 14) is usually the target.

:Attribute Information (in order):

- CRIM per capita crime rate by town

- ZN proportion of residential land zoned for lots over 25,000

sq.ft.

- INDUS proportion of non-retail business acres per town

- CHAS Charles River dummy variable (= 1 if tract bounds river; 0 otherwise)

- NOX nitric oxides concentration (parts per 10 million)

RM average number of rooms per dwelling

- AGE proportion of owner-occupied units built prior to 1940

- DIS weighted distances to five Boston employment centres

- RAD index of accessibility to radial highways

- TAX full-value property-tax rate per \$10,000

- PTRATIO pupil-teacher ratio by town

- B 1000(Bk - 0.63)^2 where Bk is the proportion of blacks by

town

- LSTAT % lower status of the population

- MEDV Median value of owner-occupied homes in \$1000's

:Missing Attribute Values: None

:Creator: Harrison, D. and Rubinfeld, D.L.

This is a copy of UCI ML housing dataset.

https://archive.ics.uci.edu/ml/machine-learning-databases/housing/

This dataset was taken from the StatLib library which is maintained at Carneg ie Mellon University.

The Boston house-price data of Harrison, D. and Rubinfeld, D.L. 'Hedonic prices and the demand for clean air', J. Environ. Economics & Management, vol.5, 81-102, 1978. Used in Belsley, Kuh & Welsch, 'Regression diagnostics ...', Wiley, 1980. N.B. Various transformations are used in the table on pages 244-261 of the latter.

The Boston house-price data has been used in many machine learning papers that address regression problems.

- .. topic:: References
- Belsley, Kuh & Welsch, 'Regression diagnostics: Identifying Influential Data and Sources of Collinearity', Wiley, 1980. 244-261.
 - Quinlan, R. (1993). Combining Instance-Based and Model-Based Learning. In

Proceedings on the Tenth International Conference of Machine Learning, 236-24 3, University of Massachusetts, Amherst. Morgan Kaufmann.

In [9]: bos.describe()

Out[9]:

	CRIM	ZN	INDUS	CHAS	NOX	RM	AGE	
count	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506.000000	506
mean	3.613524	11.363636	11.136779	0.069170	0.554695	6.284634	68.574901	3
std	8.601545	23.322453	6.860353	0.253994	0.115878	0.702617	28.148861	2
min	0.006320	0.000000	0.460000	0.000000	0.385000	3.561000	2.900000	1
25%	0.082045	0.000000	5.190000	0.000000	0.449000	5.885500	45.025000	2
50%	0.256510	0.000000	9.690000	0.000000	0.538000	6.208500	77.500000	3
75%	3.677083	12.500000	18.100000	0.000000	0.624000	6.623500	94.075000	5
max	88.976200	100.000000	27.740000	1.000000	0.871000	8.780000	100.000000	12
4								•

In [10]: bos.info()

<class 'pandas.core.frame.DataFrame'>
RangeIndex: 506 entries, 0 to 505
Data columns (total 14 columns):

#	Column	Non-Null Count	, Dtype
0	CRIM	506 non-null	float64
1	ZN	506 non-null	float64
2	INDUS	506 non-null	float64
3	CHAS	506 non-null	float64
4	NOX	506 non-null	float64
5	RM	506 non-null	float64
6	AGE	506 non-null	float64
7	DIS	506 non-null	float64
8	RAD	506 non-null	float64
9	TAX	506 non-null	float64
10	PTRATIO	506 non-null	float64
11	В	506 non-null	float64
12	LSTAT	506 non-null	float64
13	MEDV	506 non-null	float64

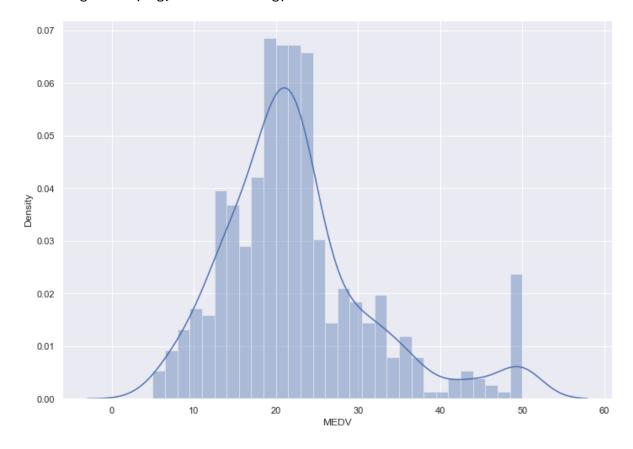
dtypes: float64(14)
memory usage: 55.5 KB

```
In [11]:
         # visualize the relationship between the features and the response using scatt
         erplots
         fig, axs = plt.subplots(4, 4, figsize=(24, 24))
         # unpack all the axes subplots
         axe = axs.ravel()
         for i in range(len(bos.drop(columns='MEDV').columns)):
             bos.plot(kind='scatter', x=bos.columns[i], y='MEDV', ax=axe[i])
             plt.xlabel(bos.columns[i])
                                                  0.2
                                                                      0.2
```

```
In [12]: sns.set(rc={'figure.figsize':(11.7,8.27)})
    sns.distplot(bos['MEDV'], bins=30)
    plt.show()
```

C:\Users\Urvi\AppData\Roaming\Python\Python37\site-packages\seaborn\distribut ions.py:2551: FutureWarning: `distplot` is a deprecated function and will be removed in a future version. Please adapt your code to use either `displot` (a figure-level function with similar flexibility) or `histplot` (an axes-lev el function for histograms).

warnings.warn(msg, FutureWarning)



```
In [13]: correlation_matrix = bos.corr().round(2)
# annot = True to print the values inside the square
sns.heatmap(data=correlation_matrix, annot=True)
```

Out[13]: <AxesSubplot:>



As seen in correlation matrix:

- LSTAT, PTRATIO, RM are highly correlated with MEDV
- · TAX and RAD are highly correlated
- INDUS, NOX and AGE are high correlated with DIS
- INDUS is highly correlated with ZN, TAX, NOX

```
In [14]: X = bos.drop(columns='MEDV')
y = bos.MEDV

In [15]: X.shape[1]

Out[15]: 13

In [16]: ## Checking vif
vif = pd.DataFrame()
vif["VIF"] = [variance_inflation_factor(X.values, i) for i in range(X.shape[1 ])]
vif["Features"] = X.columns
```

In [17]: vif

Out[17]:

	VIF	Features
0	2.100373	CRIM
1	2.844013	ZN
2	14.485758	INDUS
3	1.152952	CHAS
4	73.894947	NOX
5	77.948283	RM
6	21.386850	AGE
7	14.699652	DIS
8	15.167725	RAD
9	61.227274	TAX
10	85.029547	PTRATIO
11	20.104943	В
12	11.102025	LSTAT

Observations in VIF:

• INDUS, NOX, RM, AGE, DIS, RAD, TAX, PTRATIO, B, LSTAT have vif > 10

```
In [18]: ## Dropping TAX, as tax and rad had high correlation
    X = bos.drop(columns=['MEDV', 'TAX'])
    y = bos.MEDV
    ## Checking vif
    vif = pd.DataFrame()
    vif["VIF"] = [variance_inflation_factor(X.values, i) for i in range(X.shape[1])]
    vif["Features"] = X.columns
    vif
```

Out[18]:

	VIF	Features
0	2.100323	CRIM
1	2.697230	ZN
2	11.743319	INDUS
3	1.136630	CHAS
4	71.972959	NOX
5	77.946536	RM
6	21.377489	AGE
7	14.641579	DIS
8	5.599479	RAD
9	82.355181	PTRATIO
10	20.104332	В
11	11.098281	LSTAT

```
In [19]: ## Dropping NOX, as Nox and Dis, indus had high correlation
X = bos.drop(columns=['MEDV', 'TAX', 'NOX'])
y = bos.MEDV
## Checking vif
vif = pd.DataFrame()
vif["VIF"] = [variance_inflation_factor(X.values, i) for i in range(X.shape[1])]
vif["Features"] = X.columns
vif
```

Out[19]:

1/15/2021

	VIF	Features
0	2.098376	CRIM
1	2.696984	ZN
2	9.930116	INDUS
3	1.134868	CHAS
4	58.059674	RM
5	19.826168	AGE
6	14.485117	DIS
7	5.405429	RAD
8	82.299026	PTRATIO
9	19.872129	В
10	10.116939	LSTAT

```
In [20]: ## Dropping NOX, as Nox and Dis, indus had high correlation
X = bos.drop(columns=['MEDV', 'TAX', 'NOX', 'B', 'AGE', 'DIS'])
y = bos.MEDV
## Checking vif
vif = pd.DataFrame()
vif["VIF"] = [variance_inflation_factor(X.values, i) for i in range(X.shape[1])]
vif["Features"] = X.columns
vif
```

Out[20]:

	VIF	Features
0	2.048566	CRIM
1	1.801368	ZN
2	8.164547	INDUS
3	1.123261	CHAS
4	43.767132	RM
5	4.867376	RAD
6	57.716508	PTRATIO
7	8.061584	LSTAT

```
In [21]:
         x train,x test,y train,y test = train test split(X, y, test size = 0.25, rando
          m state=355)
In [22]: regression = LinearRegression()
          regression.fit(x_train, y_train)
Out[22]: LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None, normalize=Fals
         e)
In [23]: regression.score(x_train, y_train)
Out[23]: 0.6852743389124581
In [24]: # Let's create a function to create adjusted R-Squared
          def adj r2(x,y):
              r2 = regression.score(x,y)
              n = x.shape[0]
              p = x.shape[1]
              adjusted_r2 = 1-(1-r2)*(n-1)/(n-p-1)
              return adjusted_r2
In [25]: | adj_r2(x_train,y_train)
Out[25]: 0.6784694597538086
         lm = smf.ols(formula='MEDV ~ CRIM + ZN + INDUS + CHAS + RM + RAD + PTRATIO + L
In [26]:
          STAT', data=bos).fit()
          lm.conf_int()
Out[26]:
                          0
                                   1
          Intercept 10.619480 27.102404
             CRIM
                  -0.157797
                            -0.019711
               ΖN
                  -0.029286
                             0.018326
            INDUS
                   -0.137108
                             0.061141
             CHAS
                   1.523579
                             5.150357
               RM
                    3.549548
                             5.242013
              RAD -0.012834
                             0.147742
          PTRATIO -1.179944
                            -0.668984
            LSTAT -0.647857
                            -0.452509
```

In [27]:

Out[27]:

lm.summary() **OLS Regression Results** Dep. Variable: **MEDV** R-squared: 0.692 Model: OLS Adj. R-squared: 0.687 Method: Least Squares F-statistic: 139.5 Fri, 15 Jan 2021 Prob (F-statistic): 7.94e-122 Log-Likelihood: Time: 20:49:17 -1542.4 No. Observations: 506 AIC: 3103. **Df Residuals:** 497 BIC: 3141. **Df Model:** 8 **Covariance Type:** nonrobust std err P>|t| 0.975] coef [0.025 Intercept 18.8609 4.195 4.496 0.000 10.619 27.102 CRIM -0.0888 0.035 -2.526 0.012 -0.158 -0.020 ΖN -0.0055 0.012 -0.452 0.651 -0.029 0.018 **INDUS** -0.0380 0.050 -0.753 0.452 -0.137 0.061 **CHAS** 3.3370 0.923 3.615 0.000 1.524 5.150 RM4.3958 0.431 10.206 0.000 3.550 5.242 0.0675 **RAD** 0.041 1.651 0.099 -0.013 0.148 **PTRATIO** -0.9245 0.000 0.130 -7.110 -1.180 -0.669 **LSTAT** -0.5502 0.050 -11.067 0.000 -0.648 -0.453 **Omnibus:** 184.679 **Durbin-Watson:** 0.978 Prob(Omnibus): 0.000 Jarque-Bera (JB): 851.037 Skew: 1.567 Prob(JB): 1.58e-185

Warnings:

Kurtosis:

8.527

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

579.

As p values for ZN and INDUS > 0.05, we fail to reject null hypothesis. Hence, there is no correlation between MEDV and ZN, INDUS. So, we drop them

Cond. No.

```
In [28]: lm = smf.ols(formula='MEDV ~ CRIM + CHAS + RM + RAD + PTRATIO + LSTAT', data=b
os).fit()
lm.summary()
```

Out[28]:

OLS Regression Results

Dep. Variable:	MEDV	R-squared:	0.691
Model:	OLS	Adj. R-squared:	0.688
Method:	Least Squares	F-statistic:	186.4
Date:	Fri, 15 Jan 2021	Prob (F-statistic):	5.75e-124
Time:	20:49:17	Log-Likelihood:	-1542.7
No. Observations:	506	AIC:	3099.
Df Residuals:	499	BIC:	3129.
Df Model:	6		

Covariance Type: nonrobust

	coef	std err	t	P> t	[0.025	0.975]
Intercept	18.2899	4.086	4.476	0.000	10.262	26.318
CRIM	-0.0885	0.035	-2.527	0.012	-0.157	-0.020
CHAS	3.2827	0.912	3.601	0.000	1.492	5.074
RM	4.4174	0.428	10.316	0.000	3.576	5.259
RAD	0.0567	0.038	1.497	0.135	-0.018	0.131
PTRATIO	-0.9159	0.126	-7.274	0.000	-1.163	-0.668
LSTAT	-0.5583	0.047	-11.950	0.000	-0.650	-0.467

 Omnibus:
 182.840
 Durbin-Watson:
 0.979

 Prob(Omnibus):
 0.000
 Jarque-Bera (JB):
 833.465

 Skew:
 1.553
 Prob(JB):
 1.04e-181

 Kurtosis:
 8.467
 Cond. No.
 478.

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

As p values for RAD > 0.05, we fail to reject null hypothesis. Hence, there is no correlation between MEDV and RAD. So, we drop them

```
In [29]: lm = smf.ols(formula='MEDV ~ CRIM + CHAS + RM + PTRATIO + LSTAT', data=bos).fi
t()
lm.summary()
```

Out[29]:

OLS Regression Results

Covariance Type:

Dep. Variable: **MEDV** R-squared: 0.690 Model: OLS Adj. R-squared: 0.687 Method: Least Squares F-statistic: 222.7 **Date:** Fri, 15 Jan 2021 Prob (F-statistic): 1.11e-124 Log-Likelihood: Time: 20:49:17 -1543.9 No. Observations: AIC: 3100. 506 **Df Residuals:** 500 BIC: 3125. **Df Model:** 5

coef std err t P>|t| [0.025 0.975] 16.5736 4.221 0.000 8.859 Intercept 3.927 24.288 **CRIM** -0.0625 0.030 -2.052 0.041 -0.122 -0.003 **CHAS** 3.3879 0.910 3.723 0.000 1.600 5.176 RM 4.5262 0.423 10.713 0.000 3.696 5.356 **PTRATIO** -0.8489 0.118 -7.204 0.000 -1.080 -0.617 **LSTAT** -0.5397 0.045 -11.970 0.000 -0.628 -0.451

nonrobust

 Omnibus:
 198.984
 Durbin-Watson:
 0.968

 Prob(Omnibus):
 0.000
 Jarque-Bera (JB):
 1006.250

 Skew:
 1.670
 Prob(JB):
 3.13e-219

 Kurtosis:
 9.048
 Cond. No.
 417.

Warnings:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

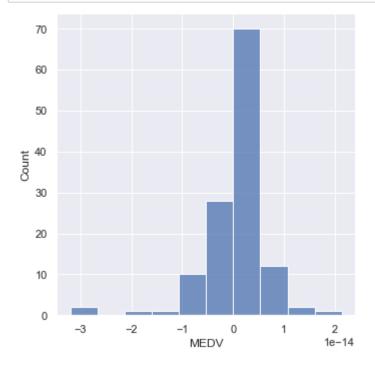
```
In [31]: X = bos.drop(columns=['ZN', 'INDUS', 'NOX', 'AGE', 'DIS', 'RAD', 'TAX', 'B'])
         x_train,x_test,y_train,y_test = train_test_split(X, y, test_size = 0.25, rando
         m state=355)
         regression = LinearRegression()
         regression.fit(x train, y train)
         regression.score(x_test,y_test)
Out[31]: 1.0
In [32]: | adj_r2(x_test,y_test)
Out[32]: 1.0
In [33]: # Lasso Regularization
         # LassoCV will return best alpha and coefficients after performing 10 cross va
         lidations
         lasscv = LassoCV(alphas = None,cv = 10, max iter = 100000, normalize = True)
         lasscv.fit(x train, y train)
Out[33]: LassoCV(alphas=None, copy_X=True, cv=10, eps=0.001, fit_intercept=True,
                 max_iter=100000, n_alphas=100, n_jobs=None, normalize=True,
                 positive=False, precompute='auto', random state=None,
                 selection='cyclic', tol=0.0001, verbose=False)
In [34]: # best alpha parameter
         alpha = lasscv.alpha
         alpha
Out[34]: 0.00046719468953559777
In [35]: #now that we have best parameter, let's use Lasso regression and see how well
          our data has fitted before
         lasso reg = Lasso(alpha)
         lasso_reg.fit(x_train, y_train)
Out[35]: Lasso(alpha=0.00046719468953559777, copy X=True, fit intercept=True,
               max iter=1000, normalize=False, positive=False, precompute=False,
               random_state=None, selection='cyclic', tol=0.0001, warm_start=False)
In [36]: | lasso_reg.score(x_test, y_test)
Out[36]: 0.999999999681303
```

```
In [37]: # Using Ridge regression model
         # RidgeCV will return best alpha and coefficients after performing 10 cross va
         lidations.
         # We will pass an array of random numbers for ridgeCV to select best alpha fro
         m them
         alphas = np.random.uniform(low=0, high=10, size=(50,))
         ridgecv = RidgeCV(alphas = alphas,cv=10,normalize = True)
         ridgecv.fit(x_train, y_train)
Out[37]: RidgeCV(alphas=array([3.28930915, 7.89545669, 1.6628005, 6.70285091, 2.74752
         509,
                7.14862378, 4.11504209, 8.03652793, 6.05656481, 0.33067785,
                3.69672012, 7.11045599, 2.15930961, 7.92899791, 4.52094033,
                8.66256149, 7.82366878, 3.48542093, 3.9133991, 1.3756095,
                4.62807315, 2.88971323, 0.44889159, 8.11791991, 5.09407137,
                7.30088967, 5.44378713, 4.94060507, 6.42312384, 0.8735612,
                7.10870453, 7.49889476, 6.7303034, 1.54226575, 7.58070917,
                3.83952096, 6.47679764, 2.56581659, 3.02340132, 7.05430819,
                5.50345811, 8.08686833, 9.2267722, 5.65233148, 4.02877869,
                8.39449767, 3.76151812, 6.30254226, 4.0210066, 7.70982734]),
                 cv=10, fit intercept=True, gcv mode=None, normalize=True, scoring=Non
         e,
                 store_cv_values=False)
In [38]: ridgecv.alpha
Out[38]: 0.33067784763377017
In [39]: ridge model = Ridge(alpha=ridgecv.alpha )
         ridge_model.fit(x_train, y_train)
Out[39]: Ridge(alpha=0.33067784763377017, copy_X=True, fit_intercept=True, max_iter=No
         ne,
               normalize=False, random state=None, solver='auto', tol=0.001)
In [40]: ridge model.score(x test, y test)
Out[40]: 0.999999996706648
```

As Lasso, Ridge and Linear Regression Test score is almost same, model is not overfitted.

```
In [42]: elasticCV.alpha
Out[42]: 0.1654493254711398
In [43]: elasticCV.l1 ratio
Out[43]: 0.5
In [44]: elasticnet reg = ElasticNet(alpha=elasticCV.alpha , 11 ratio=0.5)
         elasticnet reg.fit(x train, y train)
Out[44]: ElasticNet(alpha=0.1654493254711398, copy X=True, fit intercept=True,
                   11 ratio=0.5, max iter=1000, normalize=False, positive=False,
                   precompute=False, random_state=None, selection='cyclic', tol=0.000
         1,
                   warm start=False)
In [45]: elasticnet reg.score(x test, y test)
Out[45]: 0.9999959500003314
In [46]: # model evaluation for training set
         y train predict = regression.predict(x train)
         rmse = (np.sqrt(mean_squared_error(y_train, y_train_predict)))
         r2 = r2_score(y_train, y train predict)
         print("The model performance for training set")
         print("-----")
         print('RMSE is {}'.format(rmse))
         print('R2 score is {}'.format(r2))
         print("\n")
         # model evaluation for testing set
         y_test_predict = regression.predict(x_test)
         rmse = (np.sqrt(mean_squared_error(y_test, y_test_predict)))
         r2 = r2_score(y_test, y_test_predict)
         print("The model performance for testing set")
         print("-----")
         print('RMSE is {}'.format(rmse))
         print('R2 score is {}'.format(r2))
         The model performance for training set
         RMSE is 5.620667389044694e-15
         R2 score is 1.0
         The model performance for testing set
         RMSE is 6.3428626160647376e-15
         R2 score is 1.0
```

```
In [62]: predicted = regression.predict(x_test)
    residuals = y_test-predicted
    sns.set(rc={'figure.figsize':(11,8)})
    sns.displot(residuals, bins=10)
    plt.show()
```



```
In [64]: # mean of residuals is almost 0
  residuals.mean()
```

Out[64]: -6.224242468764657e-16

```
In [65]: ## Hence all the assumptions are satisfied
```

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```
In [ ]:
```