## Exercises: Enigma

1. The German Navy Enigma was exactly the same as the army's machine except they had three extra rotors. How many ways are there to pick 3 rotors from a choice of 8?

$$8 \times 7 \times 6 = 336$$

2. In 1942 the German U-Boats added a fourth rotor to the machine, which was not a choice and was not interchangable with the other navy rotors. How many ciphers did the U-Boat Enigma Machine have?

$$\frac{336 \times 26! \times 26^4}{2^{10}10!6!} \approx 2.3 \times 10^{22}$$

3. Here is a complete table of the ciphers performed by each rotor, reflector, and variants.

INPUT	A	В	С	D	Е	F	G	Η	Ι	J	K	L	Μ	N	О	Ρ	Q	R	$\mathbf{S}$	Т	U	V	W	X	Y	$\mathbf{Z}$
Rotor I	Е	K	Μ	F	L	G	D	Q	V	Z	N	Т	О	W	Y	Н	X	U	S	Р	A	Ι	В	R	С	J
Rotor II	A	J	D	K	$\mathbf{S}$	I	R	U	X	В	L	Η	W	Τ	M	$\mathbf{C}$	Q	G	$\mathbf{Z}$	N	Ρ	Y	F	V	Ο	$\mathbf{E}$
Rotor III	В	D	F	Н	J	L	$\mathbf{C}$	Ρ	R	Т	X	V	Z	N	Y	$\mathbf{E}$	Ι	W	G	A	K	Μ	U	$\mathbf{S}$	Q	Ο
Rotor IV	E	$\mathbf{S}$	Ο	V	Ρ	$\mathbf{Z}$	J	A	Y	Q	U	I	R	Н	X	$\mathbf{L}$	N	F	Τ	G	K	D	$\mathbf{C}$	M	W	В
Rotor V	V	$\mathbf{Z}$	В	R	G	I	T	Y	U	Р	$\mathbf{S}$	D	N	Н	L	X	A	W	Μ	J	Q	Ο	F	$\mathbf{E}$	$\mathbf{C}$	K
Navy Rotor VI	J	Ρ	G	V	Ο	U	M	F	Y	Q	В	$\mathbf{E}$	N	Η	Z	R	D	K	A	S	X	L	Ι	$\mathbf{C}$	Τ	W
Navy Rotor VII	N	$\mathbf{Z}$	J	Η	G	R	$\mathbf{C}$	X	Μ	Y	$\mathbf{S}$	W	В	Ο	U	F	A	I	V	L	Ρ	$\mathbf{E}$	K	Q	D	Τ
Navy Rotor VIII	F	K	Q	Η	Т	L	X	Ο	$\mathbf{C}$	В	J	$\mathbf{S}$	Р	D	Z	R	A	M	$\mathbf{E}$	W	N	I	U	Y	G	V
U-Boat Beta rotor	L	$\mathbf{E}$	Y	J	V	$\mathbf{C}$	N	Ι	X	W	Р	В	Q	Μ	D	R	Τ	A	K	Z	G	F	U	Н	Ο	$\mathbf{S}$
U-Boat Gamma rotor	F	$\mathbf{S}$	Ο	K	A	N	U	$\mathbf{E}$	R	Η	Μ	В	Τ	I	Y	$\mathbf{C}$	W	L	Q	Р	$\mathbf{Z}$	X	V	G	J	D
reflector B	Y	R	U	Η	Q	$\mathbf{S}$	L	D	Р	X	N	G	Ο	K	M	I	$\mathbf{E}$	В	$\mathbf{F}$	Z	$\mathbf{C}$	W	V	J	A	Τ
reflector C	F	V	Ρ	J	Ι	A	Ο	Y	Е	D	R	Z	X	W	G	С	Τ	K	U	Q	$\mathbf{S}$	В	N	M	Η	L

These ciphers are when the ring setting is A, and when the rotor is also at position A.

Kickover points for rotors I-V these are at position R, F, W, K, and A, respectively. For rotors VI-VII these are at A and at N.

(a) We are given the following Enigma settings:

Rotors: III, II, I (placed in the machine from left to right);

Ring Setting: AAA; Rotor Position: AAA; Reflector B;

Plugboard:

$$(AU)(BE)(CJ)(DO)(FT)(GP)(HZ)(IW)(KN)(LS)(M)(Q)(R)(V)(X)(Y).$$

What does an input of G become?

$$G \mapsto P \mapsto H \mapsto U \mapsto K \mapsto N \mapsto N \mapsto T \mapsto L \mapsto S$$