Exercises: Common divisors and Eulcid's algorithm

1. List the divisors of 48 and 56. What is their greatest common divisor?

   The divisors of 48 are $\pm 1, \pm 2, \pm, 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 16, \pm 24, \pm 48$.

   The divisors of 56 are $\pm 1, \pm 2, \pm, 4, \pm 7, \pm 8, \pm 14, \pm 28, \pm 56$.

   The greatest common divisor is 8.

2. (a) Write 56 in the form $56 = q(48) + r$ where q and r are quotient and remainder.

   $56 = (1)48 + 8$.

   (b) Consequently, write the greatest common divisor in the form $s(48) + t(56)$.

   $8 = 56 - 48$.

3. (a) Use Euclid's Algorithm to find the greatest common divisor of 216 and 196.

$$\begin{aligned} 216 &= (1)(196) + 20 \\ 196 &= (9)(20) + 16 \\ 20 &= (1)(16) + 4 \\ 16 &= (4)(4) + 0 \end{aligned}$$

   Therefore, the greatest common divisor is 4.

   (b) Use Euclid's Algorithm to find the greatest common divisor of 357 and 798.

$$\begin{aligned} 798 &= (2)(357) + 84 \\ 357 &= (4)(84) + 21 \\ 84 &= (4)(21) + 0 \end{aligned}$$

   Therefore, the greatest common divisor is 21.

(c) Use Euclid's Algorithm to show 795 and 539 are coprime.

$$\begin{aligned} 795 &= (1)(539) + 256 \\ 539 &= (2)(256) + 27 \\ 256 &= (9)(27) + 13 \\ 27 &= (2)(13) + 1 \\ 13 &= (13)(1) + 0 \end{aligned}$$

Therefore, the greatest common divisor is 1 and 795 and 539 are coprime.

4. Write each greatest common divisor above in terms of the two integers.

(a)

$$\begin{aligned} 4 &= (1)(20) - (1)(16) \\ &= (1)(20) - (1)(196 - (9)(20) = (10)(20) - (1)(196) \\ &= (10)(216 - 196) - (1)(196) = (10)(216) - (11)(196) \end{aligned}$$

(b)

$$\begin{aligned} 21 &= (1)(357) - (4)(84) \\ &= (1)(357) - (4)(798 - (2)(357) = (9)(357) - (4)(798) \end{aligned}$$

(c)

$$\begin{aligned} 1 &= (1)(27) - (2)(13) \\ &= (1)(27) - (2)(256 - (9)(27)) = (19)(27) - (2)(256) \\ &= (19)(539 - (2)(256)) - (2)(256) = (19)(539) - (40)(256) \\ &= (19)(539) - (40)(795 - (1)539) = (59)(539) - (40)(795) \end{aligned}$$