Exercises: Modular arithmetic part 2

1. Solve the following congruences with the smallest positive solution.

   (a) $207 + 712 \mod 5$;

   $207 + 712 \equiv 2 + 2 \equiv 4 \mod 5$.

   Note, here we have used $207 \equiv 2 \mod 5$ and $712 \equiv 2 \mod 5$. We then substitute the large values for their smaller congruent.

   (b) $63 + 97 \mod 25$;

   $63 + 97 \equiv 13 + (-3) \equiv 10 \mod 25$.

   Note, here we have used $63 \equiv 13 \mod 25$ and $97 \equiv -3 \mod 25$. Alternatively, you could have used $97 \equiv 22 \mod 25$ and got the same answer, but sometimes a small negative value can be nicer.

   (c) $211 + 50 + 5 \mod 19$;

   $211 + 50 + 5 \equiv 2 + 12 + 5 \equiv 19 \equiv 0 \mod 19$.

   Note, Although the proposition describes adding two values, this can be extend to three or more values.

2. Solve the following congruences with the smallest positive solution.

   (a) $(21)(51) \mod 20$;

   $(21)(51) \equiv (1)(11) \equiv 11 \mod 20$.

   Note, here we have used $21 \equiv 1 \mod 20$ and $51 \equiv 11 \mod 20$. We then substitute the large values for their smaller congruent.

   (b) $(72)(130) \mod 35$;

   $(72)(130) \equiv (2)(25) \equiv 50 \equiv 15 \mod 35$.

   Note, here we have used $72 \equiv 2 \mod 35$ and $130 \equiv 25 \mod 35$.

   (c) $3713^5 \mod 1237$;

   $3713^5 \equiv 2^5 \equiv 32 \mod 1237$.

Note, here we have used $3713 \equiv 2 \mod 1237$. Since $3713^5 = 3713 \times 3713 \times 3713 \times 3713 \times 3713 \equiv 2 \times 2 \times 2 \times 2 \times 2 \mod 1237$ we can simply replace 3713^5 with 2^5 in the congruence.