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First-ordinary Differential Equations

Example (3) :- Solve the differential equation $\frac{dy}{dx} = \frac{6x^2-2x+1}{\cos y + e^y}$

1. Seperate variable :- $(\cos y + e^y)dy = (6x^2-2x+1)dx$
2. integrate :- $\int(\cos y + e^y)dy = \int(6x^2-2x+1)dx$

$\Rightarrow \sin y + e^y = 2x^3 - x^2 + x + c$

general solution :- $\sin y + e^y = 2x^3 - x^2 + x + c$

Example (4): Solve the differential equation $3e^x \tan y \, dx + (1-e^x)\sec^2 y \, dy = 0$

1. Seperate variable :- $(1-e^x)\sec^2 y \, dy = -3e^x \tan y \, dx$

$\frac{(1-e^x)\sec^2 y \, dy}{(1-e^x)\tan y} = \frac{3e^x \tan y \, dx}{(1-e^x)\tan y}$

$\Rightarrow \frac{\sec^2 y}{\tan y} dy = \frac{3e^x}{(e^x-1)} dx$

2. integrate :- $\int \frac{\sec^2 y}{\tan y} dy = 3\int \frac{e^x}{e^x-1} dx$

$\ln|\tan y| = 3\ln|e^x-1| + \ln|c|$

$\ln|\tan y| = \ln|(e^x-1)^3| + \ln|c|$

$\ln|\tan y| = \ln|C(e^x-1)^3|$

$e^{\ln|\tan y|} = e^{\ln|C(e^x-1)^3|}$

general solution :- $|\tan y| = |C(e^x-1)^3|$

Example (6): Solve the intial-value Problem (IVP) :- $y' = 2x\cos(y)$, $y(0) = \frac{\pi}{4}$

1. Seperate variable :- $\frac{dy}{\cos(y)} = 2x\,dx$
2. integrate :- $\int \frac{dy}{\cos^2(y)} = \int 2x\,dx$

simplify:-

$\int \sec^2(y)\,dy = \int 2x\,dx$

$\Rightarrow \tan(y) = x^2 + c$

general solution :- $\tan(y) = x^2 + c$

$y(0) = \frac{\pi}{4}$

$\tan(\frac{\pi}{4}) = 0^2 + c$

$1 = c$

Particular solution:- $\tan(y) = x^2 + 1$

Equations Reducible to Separable

Example (9): Solve the differential equation :- $y' = (8x+2y-1)^2$

Let $z = 8x+2y-1$

$\frac{dz}{dx} = 8 + 2y' \Rightarrow \frac{dy}{dz} = \frac{1}{2}\left(\frac{dz}{dx} - 8\right)$

Substitute :- $\frac{1}{2}\left(\frac{dz}{dx} - 8\right) = z^2 \Rightarrow \frac{dz}{dx} = 2z^2 + 8$

Seperate the variables :- $\frac{dz}{2z^2+8} = dx$

integrate :- $\frac{1}{2}\int \frac{1}{z^2+4}\,dz = \int dx \Rightarrow \frac{1}{4}\tan^{-1}\left(\frac{z}{2}\right) = x + c_1$

$\Leftrightarrow \tan^{-1}\left(\frac{z}{2}\right) = 4x + c$

general solution :- $\tan^{-1}\left(\frac{8x+2y-1}{2}\right) = 4x + c$

Homogeneous Equations

Example (11): Solve the differential equation :- $(x^3 + y^2\sqrt{y^2+x^2})\,dx - xy\sqrt{x^2+y^2}\,dy = 0$

$M(x,y) = x^3 + y^2\sqrt{y^2+x^2}$, $N(x,y) = -xy\sqrt{x^2+y^2}$

degree (n) = 3

$xy\sqrt{x^2+y^2}\,dy = (x^3 + y^2\sqrt{y^2+x^2})\,dx$

$\frac{dy}{dx} = \frac{x^3 + y^2\sqrt{y^2+x^2}}{xy\sqrt{x^2+y^2}}$

put $\frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{x^3 + v^2x^2\sqrt{v^2x^2+x^2}}{xvx\sqrt{x^2+v^2x^2}}$

$= \frac{1+v^2\sqrt{v^2+1}}{v\sqrt{v^2+1}} = \frac{1}{v\sqrt{v^2+1}} + v$

seperat the variables :- $x\frac{dv}{dx} = \frac{1}{v\sqrt{v^2+1}}$

$\Rightarrow v\sqrt{v^2+1}\,dv = \frac{1}{x}\,dx$

integrate :- $\frac{1}{2}\int 2v\sqrt{v^2+1}\,dv = \int \frac{1}{x}\,dx$

$= \frac{1}{2}\,\frac{(v^2+1)^{3/2}}{3/2} = \ln|x| + \ln|c|$

$= \frac{1}{3}(v^2+1)^{3/2} = \ln|cx|$ $\quad v = \frac{y}{x}$

general solution :- $\left[\left(\frac{y}{x}\right)^2 + 1\right]^{3/2} = 3\ln|cx|$

Example (13): Solve the intial-value Problem (IVP):- $y' = \sec\left(\frac{y}{x}\right) + \frac{y}{x}$, $y(1) = \frac{\pi}{2}$

Put $\frac{y}{x} = v \Rightarrow y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Substitute:- $v + x\frac{dv}{dx} = \sec v + v$

Separate the variables:- $x\frac{dv}{dx} = \sec v \Rightarrow \frac{dv}{\sec v} = \frac{dx}{x}$
$= \cos v \, dv = \frac{1}{x} \, dx$

integrate:- $\int \cos v \, dv = \int \frac{1}{x} \, dx$
$= \sin v = \ln|x| + c$
put $v = \frac{x}{y}$

general solution :- $\sin\left(\frac{y}{x}\right) = \ln|x| + c$
$y(1) = \frac{\pi}{2}$, $x = 1$

$\sin\left(\frac{\pi/2}{1}\right) = \ln|1| + c \Rightarrow 1 = 0 + c \Rightarrow c = 1$

Particular Solution :- $\sin\left(\frac{y}{x}\right) = \ln|x| + 1$

Equations Reducible to Either Homogeneous or Seperable