

**Team Selection Test for the 15<sup>th</sup> European Girls' Math Olympiad**

**United States of America**

**Day I**

**Thursday, December 11, 2025**

*Time limit:* 4.5 hours. If you need to add page headers after the time limit, you must do so under proctor supervision. Proctors may not answer clarification questions.

You may keep the problems, but you cannot discuss them publicly until they are posted by staff online.

**EGMO TST 1.** Let  $n$  be a positive integer. Prove that one can paint the non-zero coefficients of the polynomial

$$f(x_1, x_2, \dots, x_n) = \prod_{k=0}^n (x_1 + x_2 + \dots + x_n - k)$$

with  $2^n - 1$  colors such that the coefficients of each color have sum 0, and each color is used at least once.

**EGMO TST 2.** Let  $p$  be a prime and let  $a$  and  $b$  be positive integers less than  $p$ . Show that

$$\sum_{k=1}^b (-1)^{\lfloor (a-1)k/p \rfloor + \lfloor ak/p \rfloor} \geq 0.$$

**EGMO TST 3.** Let  $S$  be a subset of  $\mathbb{R}^2$  such that any triangle of area 1 contains at least 1 point of  $S$  in its strict interior. Prove that for any positive integer  $n$  and real number  $\varepsilon > 0$ , there exists a triangle with area at most  $\varepsilon$  containing at least  $n$  points of  $S$  in its strict interior.

Team Selection Test for the 15<sup>th</sup> European Girls' Math Olympiad

United States of America

Day II

Thursday, January 8, 2026

*Time limit:* 4.5 hours. If you need to add page headers after the time limit, you must do so under proctor supervision. Proctors may not answer clarification questions.

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**EGMO TST 4.** For a positive integer  $n$ , let  $c_n$  be the smallest possible value of  $\max(a, b)$  over all pairs of distinct positive integers  $a$  and  $b$  such that

$$\operatorname{lcm}(a, b) + k = \operatorname{lcm}(a + k, b + k)$$

for all  $0 \leq k \leq n$ . Find all positive integers  $n$  for which  $c_n = c_{n+1}$ .

**EGMO TST 5.** Let  $n$  be a positive integer. In the infinite lattice  $\mathbb{Z}^2$ ,  $n$  points are colored red while the rest are colored blue. Each red point is labeled with the distance to the nearest blue point in the same row or column. Find the smallest real number  $\alpha$  for which the sum of all labels does not exceed  $100n^\alpha$ , independent of  $n$  and the placement of the red points.

(Note: A *row* is the set of points with a given  $y$ -coordinate, and a *column* is the set of points with a given  $x$ -coordinate.)

**EGMO TST 6.** Let  $ABC$  be an acute scalene triangle with circumcircle  $\Gamma$ , and let  $M$  the midpoint of  $BC$ . Let  $\omega$  be the circumcircle of triangle formed by  $BC$  and the two common external tangents of the circumcircles  $ABM$  and  $ACM$ . Prove that the internal bisector of  $\angle BAC$  and the perpendicular bisector of  $AM$  intersect on the radical axis of  $\omega$  and  $\Gamma$ .