Team Selection Test for the 67^{th} International Mathematical Olympiad

United States of America

Day I

Thursday, December 11, 2025

Time limit: 4.5 hours. If you need to add page headers after the time limit, you must do so under proctor supervision. Proctors may not answer clarification questions.

You may keep the problems, but you cannot discuss them publicly until they are posted by staff online.

IMO TST 1. Let n be a positive integer. Prove that one can paint the non-zero coefficients of the polynomial

$$f(x_1, x_2, \ldots, x_n) = \prod_{k=0}^{n} (x_1 + x_2 + \cdots + x_n - k)$$

with $2^n - 1$ colors such that the coefficients of each color have sum 0, and each color is used at least once.

IMO TST 2. Let p be a prime and let a and b be positive integers less than p. Show that

$$\sum_{k=1}^{b} (-1)^{\lfloor (a-1)k/p \rfloor + \lfloor ak/p \rfloor} \geq 0.$$

IMO TST 3. Prove that for any subset S of $\mathbb{R}^2$, there exists a (not necessarily axis-aligned) rectangle of area 1 that contains either 0 or more than 2025 points in its strict interior.

Team Selection Test for the 67$^{\text{th}}$ International Mathematical Olympiad

United States of America

Day II

Thursday, January 8, 2026

Time limit: 4.5 hours. If you need to add page headers after the time limit, you must do so under proctor supervision. Proctors may not answer clarification questions.

You may keep the problems, but you cannot discuss them publicly until they are posted by staff online.

IMO TST 4. Let n be a positive integer. In the infinite lattice $\mathbb{Z}^2$, n points are colored red while the rest are colored blue. Each red point is labeled with the distance to the nearest blue point in the same row or column. Find the smallest real number α for which the sum of all labels does not exceed $100n^\alpha$, independent of n and the placement of the red points.

(Note: A *row* is the set of points with a given y-coordinate, and a *column* is the set of points with a given x-coordinate.)

IMO TST 5. Let ABC be an acute scalene triangle with circumcircle Γ, and let M the midpoint of BC. Let ω be the circumcircle of triangle formed by BC and the two common external tangents of the circumcircles ABM and ACM. Prove that the internal bisector of $\angle BAC$ and the perpendicular bisector of AM intersect on the radical axis of ω and Γ.

IMO TST 6. A positive integer is called *chaotic* if it can be expressed as $a^3 + b^3 + abc$ for positive integers $a \geq b \geq c$. Show that there is no infinite increasing arithmetic progression consisting of only chaotic positive integers.