Department of Astronautics

U.S. Air Force Academy, CO

Technical Report Writing Guide

May 2018

Introduction:

Good technical writing skills are critical to your success in the Air Force and the engineering community. A good technical report is objective, clear, concise, and convincing. This guide outlines the Department of Astronautics' requirements for a full-length report and a short summary report. It follows national standards set by the American Institute of Aeronautics & Astronautics [1] and the science & engineering community in general. (See references at end of this guide for more information.) A checklist for a clear report [2] is given in Appendix A and an example full-length report is included (Appendix B) to help you. The full-length report contains many of the components you will use when writing a technical report.

Responsibilities & Ethics:

- When writing a technical report, you are responsible for presenting your findings as honestly, accurately, and objectively as possible regardless of the consequences. Do not let your biases influence your paper.
- If this is an individual effort report, you are responsible for accomplishing your own work. Do not copy or use someone else's report, seeking help from someone else other than a current instructor in that course is not allowed. If you use a statement or conclusion from a textbook or journal article, be sure to give proper credit by including a reference section. In addition, include the DF documentation statement in accordance with FOI 36-173.

    Examples:
    "Documentation: None" – if you received no help.
    "Documentation: Dr. Einstein clarified that the satellite controller design was required for the pitch axis only."

  Bottom line, perform and present your own work!
- If this is a group effort report, you are also responsible for contributing equally to the work associated with the project and the writing of the report. Do not sit back and wait for someone in your group to tell you what to do. Take a pro-active role as a team player.

Active vs. Passive Voice:

Use active voice where possible to create simpler, more concise sentences [3]

Example:

Passive: The satellite's pitch angle was controlled using magnetorquers
Active: Magnetorquers controlled the satellite's pitch angle (Preferred)

First Person vs. Third Person:

It is generally more accepted in technical writing to use third person.

Example:
First Person: I fractured the rocket motor casing under 1200 PSI of pressure.
Third Person: The rocket motor casing fractured under 1200 PSI of pressure. (Preferred)

Full-Length Report Outline:

- **Cover Sheet**

  - Includes project title, name(s), section, instructor's name, and date

- **Abstract**

  - A brief summary of the paper
    - State the objectives & scope of the investigation
    - Describe the methods used (no equations)
    - Summarize your results (in words, no numbers)
    - State your principle conclusions
  - Maximum of 200 words
  - No acronyms or abbreviations
  - Written in past tense

- **Nomenclature**

  - List all symbols in alphabetical order – English group first, then Greek, and finally numbers

- **Introduction Material**

  - Objectives
    - Provide a clear and concise statement describing the purpose of the report. For example, "The objective of this project is to determine the relationship between range and launch angle of a small projectile with a constant launch speed." This should come directly from the lab/project requirements.

  - Approach
    - Briefly describe the process used to satisfy your objectives and outline the flow of ideas presented in the report. Do <u>not</u> discuss the details of your findings in this section.

  - Objectives and approach are written in present tense

- **Theory –** In this section, you should present the detailed information required to understand the approach and accomplish the objectives. This is where you should discuss the details of your design. Discuss in text form with embedded equations.

    - Assumptions
        - Describe all assumptions and approximations. If you have none, state so. Explain why the assumptions are valid and highlight any limitations.

    - Mathematical Techniques
        - Describe the theory including all mathematical laws and formulas used. Note where assumptions are used. Use mathematical equations and <u>not</u> computer code. Point out any references, texts, or publications used.
        - This section should be in chronological order.
        - Be precise and give enough details so a competent reader can repeat your work.
        - Do not put predictions or results in this section.

    - The Theory section is written in present tense

- **Theoretical Predictions**

    - Present the results of your theoretical developments logically and neatly with enough narrative to allow your instructor to understand your presentation. Give bottom-line predictions, both tabular and graphic, while referring the reader to an appendix for all detailed calculations. <u>Remember</u>: You are <u>predicting</u> what will take place during the testing phase of the project, if any. At this point you do not know what the actual results will be. Be sure to include all units.

    - Written in past tense

- **Experimental Results**

    - Present the actual computer-calculated and/or experimental results logically and neatly. Use tables and/or figures along with enough narrative to help the reader understand the presentation. It is easier to communicate the meaning of results when you present them in this form. If your results are in the form of computer output, do not simply stick in pages of output in this section. Instead you should build tables with sections of your computer output. Make them look neat and professional. The reader should only have to go to the Appendix to get the details behind your results, not the results themselves. Be sure to include all units.

    - Written in present tense

- **Discussion**

    - This section is the heart of your report. It gives you the opportunity to tie together all aspects of the project and demonstrate to the reader that you have a thorough working knowledge of the concepts presented previously. You should discuss your results in a concise and logical fashion to convince the reader that your solution satisfies the stated objectives of the project. If experimental results are presented, compare them to predicted results and discuss any differences. Often, a table showing both predicted and actual results, and the percent error between them, is useful in your discussion. Describe potentially significant sources of error as well as inappropriate assumptions and approximations that might account for these differences. Don't simply list sources of error, but also try to quantify their effects. Remember, this is the section where you demonstrate that you really understand what you've done throughout the project.

    - Written in present tense

- **Conclusions and Recommendations**

    - Give a concise summary (objective, approach, the most important numerical results) of the entire project. If appropriate, discuss ways to improve your results and identify other problems to which your solution can be applied. Finish this section by highlighting the important conclusions that can be drawn from your results. Do not simply say, "I conclude that this computer program works." A reader should be able to read the introduction and this section and know exactly what happened.

    - Written in present tense

<< The abstract can be single-spaced and all other sections can be double-spaced >>

- **Appendices**

    - An appendix includes supplementary material used to reinforce the report’s findings. Begin each appendix with a narrative explaining the process and purpose. Each Appendix should be labeled (i.e. Appendix A, etc.), have a title, and stand alone. Use only those appendices that are applicable. Examples include:
        - Computer programs
        - Sample runs
        - Calculations

Short Summary Report Outline:

Writing a short summary report can be more difficult than a full-length report. You have to say everything in a more condensed format. You have to be very succinct. The descriptions of these sections are the same as in the full-length report.

- **Cover Page**
- **Introduction**
  - Objectives
  - Approach
- **Main Body**
  - Assumptions
  - Brief Math Technique
  - Theoretical Predictions
  - Experimental Results
- **Discussion/Conclusions/Recommendations**
- **Appendices**

Report Format:

- All full-length reports should be bound (not stapled).
    - Can be a 3-ring binder
    - Can be plastic comb bound – the cadet library will do this for free
- Short summary reports must be stapled as a minimum.

Page:

- Use 1" margins on all sides
- 10-point font size
- Times New Roman font
- Page numbers at bottom (centered). Cover page is not numbered.

Numbers:

- Be precise and use units
- Put a space between numbers and units (i.e. 10 km)

Vectors:

- Either boldface variables representing a vector or put a bar over the variable (i.e. **R** or $\bar{R}$)
    - Note: Most refereed journals require a vector to be boldface

Abbreviations:

- Never use abbreviations in a report title and almost never in an abstract.
- Abbreviate a term if it is used more than just a few times.

Equations:

- Equations should be on a line of their own (not included within a paragraph) and numbered. Define symbols immediately following the equation unless they are already defined in the Nomenclature [4].

Example:

$$a_{drag} = -\frac{1}{2}\rho\left(\frac{C_d A}{m}\right)V^2 \qquad (1)$$

where a is acceleration due to drag, ρ is the air density (kg/m^3), C_d is the coefficient of drag on the satellite, A is the area of the satellite susceptible to drag (m^2), m is the mass of the satellite (kg) and V is the velocity of the satellite (m/s).

Tables:

- Tables should be numbered and labeled at the top of the table [5].
- Include units for all variables/numbers.
- The entire table, including number/label, should appear on one page. Do not split between pages.
- A table should not appear until it is referenced in the report.
- If at all possible, configure them as ‘portrait’, not ‘landscape’.

Example:

TABLE 3: Measurement Characteristics of Thule Ground Station

Measurement	**Biases**	**Standard Deviation, σ**
ρ (km)	0.0708	0.026
Az (deg)	0.0013	0.026
El (deg)	0.0075	0.022

Graphs/Figures:

- Should be numbered and labeled at the bottom of the graph/figure [5]
- Axes should be labeled with units.
- If multiple lines appear, they should each be labeled.
- The entire graph/figure, including number/label, should appear on one page. Do not split between pages.
- A graph/figure should not appear until it is referenced in the report.
- If at all possible, configure them as 'portrait', not 'landscape'.

Example:

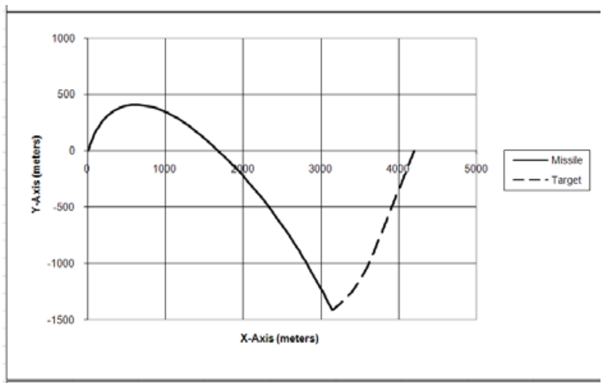

Figure 1. AIM-9 vs. Target Aircraft Intercept Engagement, XY Plane

<u>Tables vs. Graphs:</u> If the data shows pronounced trends, making an interesting visual, use graphs. If you're presenting just numbers with no exciting trends, use tables.

References:

Indicate a reference by placing the reference number with square brackets (i.e. [3]) at the end of the sentence or grouping of words you are referencing. References are placed at the end of the report in the order they appear in the text and should be numbered accordingly.

Type of Reference	**Example**
Book	Bate, R. R., Mueller, D. D., and White, J. E. *Fundamentals of Astrodynamics*, Dover Publications, New York, 1971, pp. 419-422.
Article	Vergez, P., Sauter, L., and Dahlke, S., "An Improved Kalman Filter for Satellite Orbit Predictions," *Journal of the Astronautical Sciences*, Vol 52, No. 3, July-Sept 2004, pp. 359-380.
Personal Communication	Hashida, Y., Private Communication, Surrey Space Centre, Guildford, UK, May 2007
Class Handout	Johnson, A. "Analysis of the Feasibility of Demonstrating Pulsed Plasma Thrusters on FalconSat-3." Department of Astronautics, USAF Academy, CO, 2004.
Internet	Motorola, "Power PC", http://www.mot.com/SPS/PowerPC, 17 Mar 2008.

References

[1] AIAA, "Manuscript Style & Format", http://www.writetrack.net/aiaa/documents/Manustyleformat.pdf, July 2008.

[2] Borowick, J. N., *How to Write a Lab Report*, Prentice Hall, Inc., Upper Saddle River, New Jersey, 2000.

[3] Day, R. A., *How to Write & Publish a Scientific Paper*, The Oryx Press, Phoenix, Arizona, 1998.

[4] Hancock, E., *Mastering the Craft of Science Writing*, The Johns Hopkins University Press, Baltimore Maryland, 2003.

[5] Barrass, R., *Scientists Must Write, A Guide to Better Writing for Scientists, Engineers and Students*, Routledge Taylor & Francis Group, UK, 2002.

Appendix A

Checklist for Principles of Clear Lab Report Writing [2]

Keeping a Natural Writing Style

Structure

1. Are lists with headings used?
2. Are visuals included when they would be helpful to the reader?

Word Use

1. Are your words concise?
2. Are your ideas stated directly?
3. Have you avoided pompous language?
4. Is your information stated positively rather than negatively?
5. Is communication of ideas your priority?
6. Are key nouns and verbs repeated when necessary?

Communicating Effectively

Contents

1. Did you build headings and information on ideas and concepts previously presented?
2. Did you state your purpose clearly?
3. Did you include only relevant information?
4. Were the abilities and limitations of your readers considered?

Word Use

1. Is the style and tone appropriate for this type of report?
2. Are ambiguous and vague words avoided?
3. Are clichés and slang avoided?
4. Are words with only one meaning included?
5. Are abbreviations avoided unless commonly abbreviated?

Writing Convincingly

Sentences

1. Do sentences begin with the central idea or concept?
2. Are sentences short enough for readers to understand easily?
3. Do sentences have strong beginnings?

Excessive Word Use

1. Have unnecessary words been eliminated?
2. Have redundant words and phrases been eliminated?

Appropriate Word Use

1. Are strong conjunctions used to interrelate thoughts?
2. Are overly descriptive words avoided?

Tense

Is the present tense used for all items discussed in the report, the past tense for all actions completed before the conception of the report, and the future tense for all future or hypothetical events?

Appendix B

Sample Technical Paper

Orbital Prediction – Kepler's Method

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C2C John Smith

Maj. Knowsall
AstroEngr 310, M2A
30 Aug 2017

Documentation: Referenced our notes from class and used some figures from presentations available on the K:drive. Received EI from Maj. Knowsall a couple of times to clarify assumptions and report format because she knows a lot! Many of the figures were copied from the text "Understanding Space"

Abstract

Predicting spacecraft orbits is critical to ensuring they are where they need to be to accomplish their mission. It also serves to prevent collisions with other objects in space and evaluate the health of the spacecraft. The prediction process is very complex, requiring numerical integration techniques to accurately access the spacecraft's location. However, there is a technique which can be accomplished analytically under the restricted 2-body equations of motion (EOM) assumptions, which basically states that the spacecraft orbit's orientation and size with respect to an inertial reference frame is constant. If the spacecraft is in a circular orbit, prediction is easy due to the spacecraft's constant rotational speed. For an elliptical orbit, the spacecraft is changing speed all the time (depending on how close it is to the Earth), and therefore, predicting the orbit is not so straight forward. The technique presented in this paper to predict the spacecraft's location in an elliptical orbit (under the 2-body EOM) is Kepler's method. Given the current spacecraft location (the 6 Classical Orbital Elements), a time of flight, and the accuracy needed; the future spacecraft location can be calculated analytically. The technique was applied to an elliptical orbit example. The answers were then validated by comparing them to a numerical simulation of the same orbit, using the same 2-body assumptions. For the orbit selected and the time of flight, Kepler's method produced a highly accurate answer for the future spacecraft's location.

Nomenclature

a	= semi-major axis (km)
COEs	= Classical Orbital Elements
e	= eccentricity
E	= eccentric anomaly (deg)
EOM	= Equations of Motion
i	= inclination (deg)
IJK	= Geocentric Equatorial reference frame (inertial)
k	= orbital revolutions
M	= mean anomaly (deg)
n	= mean motion (km/sec)
TOF	= time of flight (sec)
tol	= accuracy tolerance (rad)
Ω	= right ascension of the ascending node (deg)
ω	= argument of perigee (deg)
υ	= true anomaly (deg)
μ	= Earth's gravitational constant (km^3/sec^2)

Objective

The objective of this paper is to predict the future location of a spacecraft in an elliptical orbit given the spacecraft's current location, a time of flight, and the accuracy of that prediction. This will be done using Kepler's method under the restricted 2-body EOM assumptions. In addition, the solution is validated against a robust numerical simulation of the 2-body EOM. The 6 Classical Orbital Elements (COEs) are used to completely describe the spacecraft's orbit and location around the Earth. They are listed in Table 1 and shown with respect to the Earth in Figures 1 and 2, [1].

Table 1: The 6 Classical Orbital Elements

COE	Description	Units
a	Size	Km
e	Shape	
i	Tilt of the orbital plane with respect to the Earth's equatorial plane	Deg
Ω	Angle from the vernal equinox to the ascending node	Deg
ω	Angle from the ascending node to perigee of the orbit	Deg
υ	Angle from perigee to the spacecraft's position in orbit	Deg

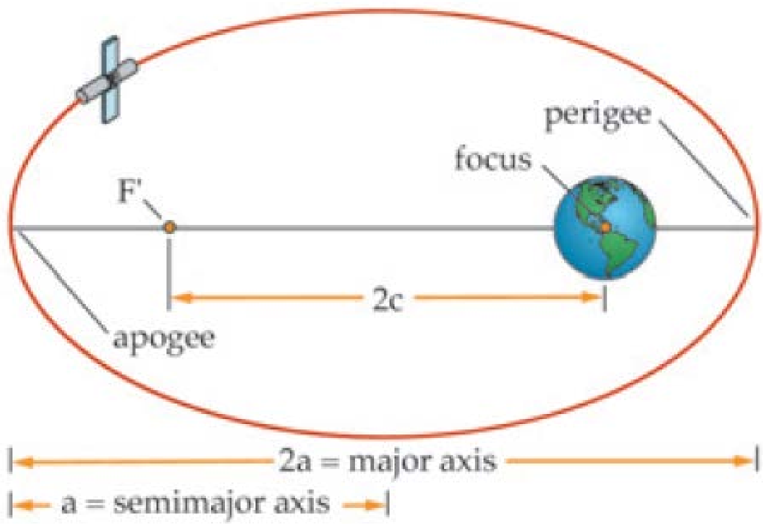

Figure 1: Size and shape of a spacecraft orbit around the Earth

where

c = distance between foci of the ellipse

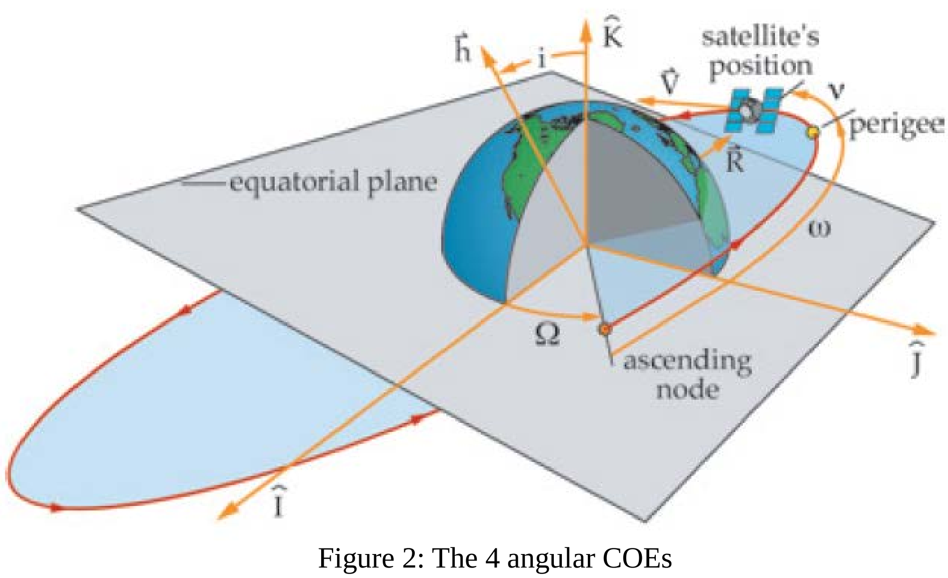

Figure 2: The 4 angular COEs

where

IJK = Geocentric Equatorial inertial coordinate frame, where $\hat{I}$ points to the vernal equinox.
$\bar{R}$ = Spacecraft's position vector in the IJK frame (km)
$\bar{V}$ = Spacecraft's velocity vector in the IJK frame (km/sec)
Ascending node = node where the spacecraft passes from south to north through the equator

Approach

Given the assumptions that the spacecraft's orbit is constant in size, shape, and orientation with respect to the Earth (the 2-Body assumptions), all of the COEs remain constant except true anomaly, υ. Therefore,

$$a_f = a_i \tag{1}$$
$$e_f = e_i \tag{2}$$
$$i_f = i_i \tag{3}$$
$$\Omega_f = \Omega_i \tag{4}$$
$$\omega_f = \omega_i \tag{5}$$

where $(\ )_i$ denotes initial values and $(\ )_f$ final values.

True anomaly will not only change, it will change at a different rate along the elliptical trajectory. Kepler's method is used because it relates motion along an ellipse to the average motion along a circle. This involves placing a circle over the ellipse and calculating the average motion of a moving object on the ellipse over one revolution.

To validate Kepler's method, a numerical simulation of the 2-Body EOM with a Runge-Kutta 4 algorithm was used to provide an accurate numerical solution to the same problem. The simulation converts the initial COEs to their respective initial position and velocity vectors in the IJK frame. The simulation then propagates the position and velocity vectors to the specified final time. Finally, the future COEs are calculated from the future position and velocity vectors. These results were compared to the analytic results using Kepler's method.

Assumptions

The assumptions used are for the restricted 2-body EOM:

- The spacecraft in orbit is not subject to forces due to drag, thrust, gravity from any other object other than the Earth, or any other forces like solar radiation, etc.
- Earth's mass is much greater than the mass of the spacecraft
- Earth is a perfect sphere with uniform density
- The spacecraft's mass is constant
- The IJK coordinate frame is sufficiently inertial

Mathematical Technique

Given the initial COEs (a_i, e_i, i_i, Ω_i, ω_i, and υ_i), TOF, and accuracy tolerance (tol), the initial eccentric anomaly, E_i, is calculated first. Figure 3 shows the relationship of E_i to υ_i.

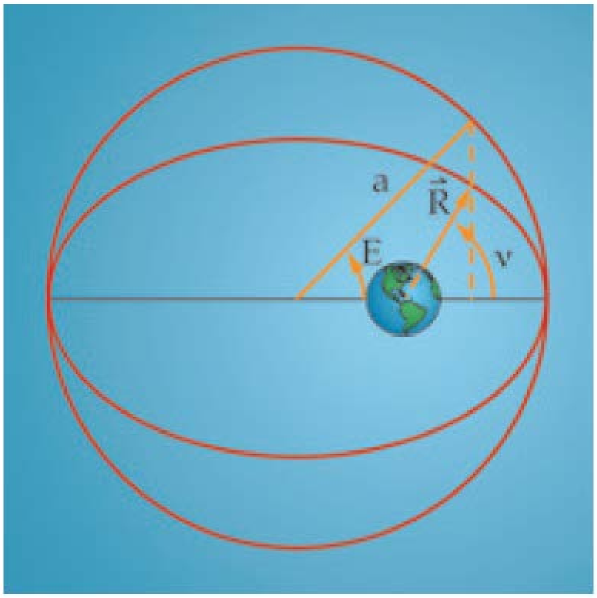

Figure 3: Eccentric anomaly vs true anomaly

$$\cos(E_i) = \frac{e + \cos(\upsilon_i)}{1 + e\cos(\upsilon_i)} \tag{6}$$

where

e = eccentricity (since it is a constant, the subscripts are dropped)

E_i has 2 possible solutions (E_i or 360^o-E_i). From Figure 3, if υ_i is less than 180^o, E_i less than 180^o. Therefore, to determine which value of E_i is correct, use the half-plane check:

$$\text{If } \upsilon_i > 180^o,\ E_i = 360^o - E_i \tag{7}$$

Since the rate of change of υ changes over the orbit, the same is true for the rate of change of E. Kepler then defined the average motion of E over one revolution, and called this Mean Anomaly, M. There is no geometrical representation of M. It's the average motion of E over one revolution. The equation for M_i is

$$M_i = E_i - e \sin(E_i) \tag{8}$$

Note, because of the nature of the equation, E_i must be in radians, which results in M_i in radians. Given TOF, the future Mean Anomaly, M_f can be calculated using

$$M_f = M_i + n(TOF) \tag{9}$$

where

TOF = time of flight (sec)
n = spacecraft's mean motion (rad/sec), and comes from the equation

$$n = \sqrt{\frac{\mu}{a^3}} \tag{10}$$

where

μ = Earth's gravitational constant (km^3/sec^2)
a = semi-major axis (km)

It is possible that the given TOF be greater than the spacecraft's orbital period, and will cause the spacecraft to go through several full revolutions before it reaches its final position. If the spacecraft makes k full revolutions, subtract the full revolutions using

$$M_f = M_f - 2k\pi \tag{11}$$

Given M_f (rad) solve for E_f using Equation 8.

$$E_f = M_f + e \sin(E_f) \tag{12}$$

E_f cannot be solved for directly because it is imbedded inside the sin function. It must be calculated indirectly through the following iterative approach:

1. Guess a value of E_f ($E_f = M_f$)
2. Plug this value of E_f into the right side of Equation 12 and solve for a new E_f
3. Now plug this new value of E_f into the right side of Equation 12 and solve again
4. Repeat this process until the changes in E_f are less than a specified accuracy tolerance (tol)

Convert E_f to degrees and solve for final true anomaly, υ_f

$$\cos(\nu_f) = \frac{cos(E_f) - e}{1 - ecos(E_f)} \tag{13}$$

There are 2 possible solutions to υ_f, so apply the half-plane check:

$$\text{If } E_f > 180^o,\ \upsilon_f = 360^o - \upsilon_f \tag{13}$$

For the numerical integration routine, the 2-body EOM was used, Equation 14.

$$\ddot{R} + \frac{\mu}{R^3} = \bar{0} \tag{14}$$

where

R = Radius vector from the center of the Earth to the spacecraft (km)

Theoretical Predictions

Kepler's method was applied to the following example:

a_i = 10,000 km
e_i = 0.1
i_i = 45°
Ω_i = 0°
ω_i = 90°
υ_i = 170°
TOF = 2 hrs
tol = 0.001 rad

This represents an elliptical orbit. The final values of a, e, i, Ω, and ω are the same as the initial values, under the restricted 2-body EOM assumptions.

Given the semi-major axis, a, for the orbit; the orbital period is 2.76 hrs using

$$Period = 2\pi\sqrt{\frac{a^3}{\mu}} \tag{15}$$

The spacecraft is almost at apogee. With a TOF of 2 hrs, the spacecraft should make 1 complete revolution and then be less than half way into the next orbit. i.e. the final true anomaly, υ_f, should be less than 180°.

Experimental Results

As mentioned, all of the COEs remain constant except true anomaly. Using Kepler's method, the following results are obtained (Table 2). The calculations are in Appendix A.

Table 2: Analytic Solutions from Kepler's Method

Variable	Values
E_i	168.95° / 2.9487 rad
M_i	167.88° / 2.9296 rad
n	6.3134837×10^{-4} rad/sec
M_f	68.33° / 1.1925 rad
E_f	73.83° / 1.2885 rad
υ_f	79.418° / 1.3861 rad

The numerical integration routine is then run using the same initial conditions, Appendix B. Results are compared to Kepler's method in Table 3.

Table 3: COE Analytic Solutions vs Numerical Solutions

Variable	Kepler's Solution	Numerical Solution	% Error
a_f	10000 km	10000 km	2.2028×10^{-13}
e_f	0.1	0.1	1.4228×10^{-11}
i_f	45°	45°	2.8272×10^{-16}
Ω_f	0°	0°	0
ω_f	90°	90°	2.8933×10^{-9}
υ_f	79.418°	79.395°	2.897×10^{-4}

Discussion

The objective of this effort was to show that Kepler's method is a valid tool for performing analytic calculations to predict a spacecraft's orbit location under the 2-body EOM assumptions. The results show this clearly. The calculation of final true anomaly (Table 2) was in line with the predicted results – the spacecraft ended up in the first half plane with a value less than 180°. It was assumed that all of the COEs except true anomaly would remain constant. When these results are compared to the simulated results, all of the COEs except true anomaly remained within 10^{-9} of their original value, thus validating this assumption. The true anomaly values differed by 10^{-4}. From the results in Appendix A, it took 3 iterations to achieve an accuracy of 0.001. This is the time consuming part of Kepler's method.

Conclusions and Recommendations

It is shown in this report that Kepler's method is a viable tool for analytically predicting orbiting spacecraft with the 2-body EOM assumptions. It could be used to provide a first cut at a solution. The COEs that are assumed to be constant are very close to being true. The comparison of final true anomaly from Kepler's method to the numerical method shows that Kepler's method is very accurate within the 2-body EOM assumptions. To improve the accuracy, the tolerance could be tightened. Instead of 0.001, make it much smaller. The trade-off is that there will be more iteration calculations of final eccentric anomaly (seen in Appendix A), and therefore it will take longer to find the final value. It must be noted that the COEs were not propagated very far from the initial values. The accuracy of future true anomaly will degrade as the TOF is increased. If more accuracy is need by eliminating some or all of the 2-body EOM assumptions, then the recommendation is to use a numerical simulation which takes into account many of the forces a spacecraft faces.

Appendix A

Analytic Calculations of True Anomaly using Kepler's Method

Given the following initial conditions and accuracy tolerance, Kepler's method is used to propagate the spacecraft's location in orbit given the 2-body EOM assumptions.

a_i = 10,000 km
e_i = 0.1
i_i = 45°
Ω_i = 0°
ω_i = 90°
υ_i = 170°
TOF = 2 hrs
tol = 0.001 rad

First, solve for initial Eccentric Anomaly using Equation 6.

$$E_i = cos^{-1}\left(\frac{0.1 + \cos(170^o)}{1 + 0.1cos(170^o)}\right) = 168.95^o \; or \; 191.05^o \tag{A-1}$$

Since $\upsilon_i < 180^o$, $E_i = 168.95^o$

Now, solve for initial Mean Anomaly using Equation 8.

$$M_i = 2.949 \text{ rad} - 0.1 \sin(168.95^o) = 2.93 \text{ rad} \tag{A-2}$$

where 2.949 is E_i in radians.

Mean motion is the next required variable, Equation 10.

$$n = \sqrt{\frac{398600.5}{10000^3}} = 6.3135X10^{-4} \; rad/sec \tag{A-3}$$

With mean motion, future Mean Anomaly is derived from Equation 9.

$$M_f = 2.93 \text{ rad} + 6.3135X10^{-4} \text{ (7200 sec)} = 7.4757 \text{ rad} \tag{A-4}$$

Since this is greater than 2π (meaning that the spacecraft has passed one revolution), apply Equation 11 where k = 1.

$$M_f = 7.4757 \text{ rad} - 2\pi = 1.1925 \text{ rad} \tag{A-5}$$

To solve for future Eccentric Anomaly, Equation 12 is used. Start by setting the initial guess of E_f as M_f. Now plug this value into Equation 12.

$$E_f = 1.1925 \text{ rad} + 0.1 \sin(1.1925*180/\pi) = 1.2854 \text{ rad} \tag{A-6}$$

Now substitute this value of Ef back into Equation 12 to obtain a better E_f. Repeat until E_f changes by less than the specified accuracy tolerance, tol.

$$E_f = 1.1925 \text{ rad} + 0.1 \sin(1.2854*180/\pi) = 1.2885 \text{ rad} \quad \text{(A-7)}$$

$$E_f = 1.1925 \text{ rad} + 0.1 \sin(1.2885*180/\pi) = 1.2885 \text{ rad} \quad \text{(A-8)}$$

Therefore,

$$E_f = 1.2885 \text{ rad} = 73.828^o \quad \text{(A-9)}$$

Finally, solve for future True Anomaly with Equation 13.

$$\nu_f = cos^{-1}\left(\frac{\cos(73.828^o) - 0.1}{1 - 0.1\cos(73.828^o)}\right) = 79.418^o \; or \; 280.512^o \quad \text{(A-10)}$$

Since $E_f < 180^o$, $\boldsymbol{\nu_f = 79.418^o}$

Appendix B

Numerical Solutions of COEs using RK4 Simulation

The numerical simulation was given the same initial COEs that were used in Kepler's method. The initial position and velocity vectors were calculated from these COEs. Using these initial conditions, Equation 14 was numerically integrated forward in time by 2 hours, providing the future position and velocity vectors. The future COEs were then calculated from the position and velocity vectors.

```
            Initial Orbital Elements:
a      =  10000.000000 km
e      =      0.100000
Inc    =     45.000000 degs
RAAN   =      0.000000 degs
Arpg   =     90.000000 degs
Nuo    =    170.000000 degs
M      =      0.000000 degs

Uo     =           NaN degs
lo     =           NaN degs
CapPi  =           NaN degs

Orbital Orbital Elements 2 hrs later:
a      =  10000.000000 km
e      =      0.100000
Inc    =     45.000000 degs
RAAN   =      0.000000 degs
Arpg   =     90.000000 degs
Nuo    =     79.394551 degs
M      =     68.302533 degs

Uo     =      0.000000 degs
lo     =           NaN degs
CapPi  =           NaN degs

R =   -9555.031284 I   1265.096155 J   1265.096155 K
Mag =    9721.088386(Km)
V =      -1.802346 I     -4.410152 J     -4.410152 K
Mag =       6.492098(Km/sec)

            Error in Orbital Orbital Elements:

delta_a      =  2.202796e-09 km
delta_e      =  1.422848e-12
delta_Inc    = -1.272222e-14 degs
delta_RAAN   =  0.000000e+00 degs
delta_Arpg   =  2.893300e-09 degs
delta_Nuo    = -9.060545e+01 degs
delta_M      =  6.830253e+01 degs

delta_Uo     =           NaN degs
delta_lo     =           NaN degs
delta_CapPi  =           NaN degs
```

References

[1] Sellers, Jerry, *Understanding Space, An Introduction to Astronautics,* McGraw-Hill Higher Education, New York, 2005