ANSWER KEY

1	(b)	11	(a)	21	(c)	31	(b)	41	(a)	51	(a)	61	(d)	71	(a)
2	(b)	12	(d)	22	(c)	32	(c)	42	(a)	52	(c)	62	(b)	72	(b)
3	(a)	13	(b)	23	(d)	33	(c)	43	(c)	53	(b)	63	(c)	73	(d)
4	(c)	14	(a)	24	(d)	34	(c)	44	(b)	54	(a)	64	(c)	74	(a)
5	(a)	15	(d)	25	(d)	35	(b)	45	(a)	55	(d)	65	(a)		
6	(b)	16	(c)	26	(c)	36	(b)	46	(b)	56	(d)	66	(d)		
7	(c)	17	(a)	27	(d)	37	(d)	47	(b)	57	(a)	67	(a)		
8	(b)	18	(c)	28	(c)	38	(c)	48	(a)	58	(b)	68	(b)		
9	(d)	19	(d)	29	(c)	39	(b)	49	(b)	59	(c)	69	(b)		
10	(c)	20	(c)	30	(c)	40	(a)	50	(d)	60	(c)	70	(c)		

HINTS & SOLUTIONS

1. (b) Given that $\sin\theta$ and $\cos\theta$ are the roots of the equation $ax^2 - bx + c = 0$, so $\sin\theta + \cos\theta = \frac{b}{a}$ and

$\sin\theta\cos\theta = \frac{c}{a}$

Using the identity $(\sin\theta + \cos\theta)^2 = \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta$, we have

$$\frac{b^2}{a^2} = 1 + \frac{2c}{a} \text{ or } a^2 - b^2 + 2ac = 0$$

2. (b) Since $\tan\theta = -\frac{4}{3}$ is negative, θ lies either in second quadrant or in fourth quadrant.

Thus $\sin\theta = \frac{4}{5}$ if θ lies in the second quadrant

or $\sin\theta = -\frac{4}{5}$, if θ lies in the fourth quadrant.

3. (a) π radians $= 180°$

$1° = \frac{\pi}{180}$ radians

$\therefore\ 25° = 25 \times \frac{\pi}{180} = \frac{5\pi}{36}$

4. (c) Indeed $\sin 20° \sin 40° \sin 60° \sin 80°$.

$= \frac{\sqrt{3}}{2} \sin 20° \sin(60° - 20°) \sin(60° + 20°)$

(since $\sin 60° = \frac{\sqrt{3}}{2}$)

$= \frac{\sqrt{3}}{2} \sin 20° [\sin^2 60° - \sin^2 20°]$

$= \frac{\sqrt{3}}{2} \sin 20° \left[\frac{3}{4} - \sin^2 20°\right]$

$= \frac{\sqrt{3}}{2} \times \frac{1}{4} [3\sin 20° - 4\sin^3 20°]$

$= \frac{\sqrt{3}}{2} \times \frac{1}{4} (\sin 60°)$

$= \frac{\sqrt{3}}{2} \times \frac{1}{4} \times \frac{\sqrt{3}}{2} = \frac{3}{16}$

5. (a) We have,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{2}{(2/3)} = \frac{3}{\sin B} \Rightarrow 3 = \frac{3}{\sin B}$$

$$\Rightarrow \sin B = 1 \Rightarrow \angle B = 90°$$

6. (b) We know that the general solution of the equation
$\tan\theta = 0$ is $\theta = n\pi$, $n \in Z$. Therefore,
$\tan 2\theta = 0 \Rightarrow 2\theta = n\pi, n \in Z$

$\Rightarrow \theta = \frac{n\pi}{2}, n \in Z$

7. (c) We have,

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$

Putting $A = 45°$, we get

$$\cos 22\frac{1°}{2} = \sqrt{\frac{1+\cos 45°}{2}}$$

$$\left[\because \cos 22\frac{1°}{2} \text{ is +ve}\right]$$

$$\Rightarrow \cos 22\frac{1°}{2} = \sqrt{\frac{1+1/\sqrt{2}}{2}} = \sqrt{\frac{\sqrt{2}+1}{2\sqrt{2}}}$$

8. (b) Since A and B both lie in the IV quadrant, it follows that $\sin A$ and $\sin B$ are negative. Therefore,

$$\sin A = -\sqrt{1-\cos^2 A}$$

$$\Rightarrow \sin A = -\sqrt{1-\frac{16}{25}} = -\frac{3}{5}$$

and, $\sin B = -\sqrt{1-\cos^2 B}$

$$\Rightarrow \sin B = -\sqrt{1-\frac{144}{169}} = -\frac{5}{13}$$

Now, $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$$= \frac{4}{5}\times\frac{12}{13} - \left(\frac{-3}{5}\right)\left(\frac{-5}{13}\right) = \frac{33}{65}$$

9. (d) We have,

$$\sin A = \frac{3}{5}, \text{ where } 0 < A < \frac{\pi}{2}$$

$$\therefore \quad \cos A = \pm\sqrt{1-\sin^2 A}$$

$$\Rightarrow \quad \cos A = +\sqrt{1-\sin^2 A} = \sqrt{1-\frac{9}{25}} = \frac{4}{5}$$

In the first quadrant tangent function is positive.

$$\therefore \quad \tan A = \frac{\sin A}{\cos A} = \frac{3}{4}$$

It is given that : $\cos B = -\frac{12}{13}$ and $\pi < B < \frac{3\pi}{2}$

$$\therefore \quad \sin B = \pm\sqrt{1-\cos^2 B}$$

$$\Rightarrow \quad \sin B = -\sqrt{1-\cos^2 B}$$

[$\because$ sin is negative in the third quadrant]

$$\Rightarrow \sin B = -\sqrt{1-\left(\frac{-12}{13}\right)^2} = -\frac{5}{13}$$

In the III quadrant tangent function is positive.

$$\therefore \tan B = \frac{\sin B}{\cos B} = \frac{5}{12}$$

Now, $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$$= \frac{4}{5}\times\frac{-12}{13} - \frac{3}{5}\times\frac{-5}{13} = \frac{-33}{65}$$

11. (a) Given that $3\tan(\theta - 15°) = \tan(\theta + 15°)$ which can be rewritten as

$$\frac{\tan(\theta+15°)}{\tan(\theta-15°)} = \frac{3}{1}$$

Applying componendo and Dividendo; we get

$$\frac{\tan(\theta+15°)+\tan(\theta-15°)}{\tan(\theta+15°)-\tan(\theta-15°)} = 2$$

$$\Rightarrow \frac{\sin(\theta+15°)\cos(\theta-15°)+\sin(\theta-15°)\cos(\theta+15°)}{\sin(\theta+15°)\cos(\theta-15°)-\sin(\theta-15°)\cos(\theta+15°)} = 2$$

$$\Rightarrow \frac{\sin 2\theta}{\sin 30°} = 2 \text{ i.e., } \sin 2\theta = 1$$

giving $\theta = \frac{\pi}{4}$

12. (d) The values of θ lying between 0 and 2π and satisfying the given equations is $\theta = \frac{5\pi}{4}$.

Hence, the general value of θ satisfying the given equation is

$$\theta = 2n\pi + \frac{5\pi}{4} \Rightarrow \theta = (2n+1)\pi + \frac{\pi}{4}$$

13. (b) $\cos A = n\cos B$ and $\sin A = m\sin B$
Squaring and adding, we get
$1 = n^2\cos^2 B + m^2\sin^2 B$
$\Rightarrow\ 1 = n^2(1-\sin^2 B) + m^2\sin^2 B$
$\therefore\ (m^2-n^2)\sin^2 B = 1-n^2$

14. (a) We have.
$3A = 2A + A \Rightarrow \tan 3A = \tan(2A+A)$
$$= \frac{(\tan 2A + \tan A)}{(1-\tan 2A\tan A)}$$
$\Rightarrow \tan 3A - \tan 3A\tan 2A\tan A = \tan 2A + \tan A$
$\Rightarrow \tan 3A - \tan 2A - \tan A = \tan 3A\tan 2A\tan A$

15. (d) $\because\ -1 \le \cos\theta \le 1$
$\therefore\ -3 \le 3\cos\sqrt{3+x+x^2} \le 3$

16. (c) Let $\theta = 12°$,
Consider given expression
$$= \frac{1}{\sin 72°}\sin 12°\sin 48°\sin 54°\sin 72°$$
$$= \frac{1}{2}\frac{\sin 3(12°)\sin 54°}{\sin 72°}$$
$$= \frac{\sin 36°\sin 54°}{8\sin 36°\cos 36°} = \frac{\cos 36°}{8\cos 36°} = \frac{1}{8}$$

17. (a) Given : $\operatorname{cosec}\theta - \cot\theta = q$
We know that $1+\cot^2\theta = \operatorname{cosec}^2\theta$
$\therefore\ \sqrt{1+\cot^2\theta} - \cot\theta = q$
$\Rightarrow\ \sqrt{1+\cot^2\theta} = q + \cot\theta$
Squaring both sides, we get
$\therefore\ (1+\cot^2\theta) = (q+\cot\theta)^2$
$\Rightarrow\ 1+\cot^2\theta = q^2 + \cot^2\theta + 2q\cot\theta$
$\Rightarrow\ 1-q^2 = 2q\cot\theta$
$$\Rightarrow\ \cot\theta = \frac{1-q^2}{2q}$$

18. (c) Given : $\sin A = \sin B$ and $\cos A = \cos B$
$\Rightarrow\ \sin A = \sin(2n\pi + B)$ and
$\cos A = \cos(2n\pi + B)$
$\therefore\ A = 2n\pi + B$

19. (d) Consider
$\cos\theta\,.\cos(90-\theta) - \sin\theta\sin(90-\theta)$
$= \cos\theta\,.\,\sin\theta - \sin\theta\,.\cos\theta = 0$

20. (c) Consider $\frac{1}{2}(\sqrt{3}\sin 75° - \cos 75°)$
$$= \frac{\sqrt{3}}{2}.\sin 75° - \frac{1}{2}\cos 75°$$
$= \sin 60°.\sin 75° - \cos 60°.\cos 75°$
$= -[\cos(75°+60°)]$
$= -\cos 135°$
$= -\cos(180°-45°)$
$$= +\frac{1}{\sqrt{2}}$$

21. (c) Given : $\tan^2 A = 2\tan^2 B + 1$
$\Rightarrow\ 1+\tan^2 A = 2\tan^2 B + 1 + 1$
$\Rightarrow\ \sec^2 A = 2\sec^2 B$
$\Rightarrow\ \cos^2 B = 2\cos^2 A$
$\Rightarrow\ \cos^2 B = 1 + \cos 2A$
$\Rightarrow\ \cos^2 B - 1 = \cos 2A$
$\Rightarrow\ -\sin^2 B = \cos 2A$
$\Rightarrow\ \cos 2A + \sin^2 B = 0.$

22. (c) Consider $\left(\dfrac{\cos\left(\frac{\pi}{2}+x\right)+\sin\left(\frac{\pi}{2}+x\right)}{\cos\left(\frac{\pi}{2}-x\right)-\sin\left(\frac{\pi}{2}-x\right)}\right)^2$
$$= \left(\frac{-\sin x + \cos x}{\sin x - \cos x}\right)^2$$
$$= \left\{-\left(\frac{\sin x - \cos x}{\sin x - \cos x}\right)\right\}^2$$
$= (-1)^2 = 1$

23. (d) Tangent formula is derived as follows
$$\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)}$$
$$= \frac{\sin A\cos B + \cos A\sin B}{\cos A\cos B - \sin A\sin B}$$
$$= \frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A + \tan B}{1-\tan A\tan B}$$
Correct and proper sequential form to derive the formula is 2, 3, 1, 4.

24. (d) $\cos^3\theta + \cos^3(120+\theta) + \cos^3(\theta-120)$

Use the formulae $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\cos^3(120+\theta)$

$$= \frac{1}{4}[\cos 3(120+\theta) + 3\cos(120+\theta)]$$

And similar for $\cos^3(120-\theta)$

We get,

$$\cos^3\theta = \frac{1}{4}(\cos 3\theta + 3\cos\theta)$$

Let $x = \cos^3\theta + \cos^3(120+\theta) + \cos^3(\theta-120)$

$$\Rightarrow x = \frac{1}{4}[\{\cos 3\theta + 3\cos\theta\} + \{\cos(360^\circ+3\theta) + 3\cos(120^\circ+\theta)\} + \{\cos(3\theta-360^\circ) + 3\cos(120^\circ-\theta)\}]$$

$$= \frac{1}{4}[\cos 3\theta + 3\cos\theta + \cos 3\theta + 3\cos(120^\circ+\theta) + \cos 3\theta + 3\cos(120^\circ-\theta)]$$

$[\because \cos(-\theta) = \cos\theta, \cos(360+\theta) = \cos\theta]$

$$= \frac{1}{4}[3\cos 3\theta + 3\cos\theta + 3\cos(120^\circ+\theta) + 3\cos(120^\circ-\theta]$$

$$= \frac{1}{4}[3\cos 3\theta + 3\cos\theta + 3\{(\cos 120^\circ\cos\theta - \sin 120^\circ\sin\theta) + (\cos 120^\circ\cos\theta + \sin 120^\circ\sin\theta)]$$

$$= \frac{1}{4}[3\cos 3\theta + 3\cos\theta + 3(2\cos 120^\circ\cos\theta)\}]$$

$[\because \cos(A \pm B) = \cos A\cos B \mp \sin A\sin B]$

$$= \frac{1}{4}\left[3\cos 3\theta + 3\cos\theta - 3\times 2\times\frac{1}{2}\cos\theta\right]$$

$\left[\text{Since } \cos 120^\circ = \frac{-1}{2}\right]$

$$= \frac{1}{4}[3\cos 3\theta + 3\cos\theta - 3\cos\theta] = \frac{3}{4}\cos 3\theta$$

25. (d) As given, $\cos T = 3/5$

$$\sin^2 T = 1 - \frac{3^2}{5^2} = 1 - \frac{9}{25}$$

So, $\sin T = \frac{16}{25} = -\frac{4}{5}$

[since T is in IV quad. + ve value is ignored.]

Also, given, $\sin R = \frac{8}{17}$

$$\Rightarrow \cos^2 R = 1 - \frac{8^2}{17^2} = 1 - \frac{64}{289} = \frac{225}{289}$$

So, $\cos R = -\frac{15}{17}$

[Since R is in II quad. + ve value is ignored]

Now, $\cos(T-R) = \cos T\cos R + \sin T\sin R$

$$= \frac{3}{5}\times\frac{-15}{17} - \frac{4}{5}\times\frac{8}{17} = -\frac{45}{85} - \frac{32}{85}$$

$$= -\left[\frac{45+32}{85}\right] = -\frac{77}{85}$$

26. (c) $\cos(A+B).\cos(A-B) = (\cos A\cos B - \sin A\sin B)(\cos A\cos B + \sin A\sin B)$

$= \cos^2 A\cos^2 B - \sin^2 A\sin^2 B.$

$= \cos^2 A(1-\sin^2 B) - \sin^2 A\sin^2 B$

$= \cos^2 A - \cos^2 A\sin^2 B - \sin^2 A\sin^2 B$

$= \cos^2 A - \sin^2 B(\cos^2 A + \sin^2 A)$

$= \cos^2 A - \sin^2 B$

27. (d) The given expression is :

$\sin 20^\circ(\tan 10^\circ + \cot 10^\circ)$

$$= \sin 20^\circ\left(\tan 10^\circ + \frac{1}{\tan 10^\circ}\right)$$

$$= 2\sin 10^\circ\cos 10^\circ\left(\frac{\tan^2 10^\circ + 1}{\tan 10^\circ}\right)$$

$$= 2\sin 10^\circ\cos 10^\circ\left(\frac{\sec^2 10^\circ}{\tan 10^\circ}\right)$$

$$= 2\sin 10^\circ\cos 10^\circ\,\frac{1}{\cos^2 10^\circ.\sin 10^\circ}\cos 10^\circ$$

$= 2$

28. (c) As given $x = \cos^2\theta + \sec^2\theta$

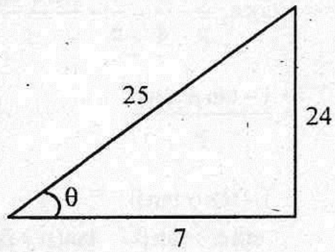
$$= \cos^2\theta + \frac{1}{\cos^2\theta}$$

$\cos^2\theta \le 1$

$$\Rightarrow \cos^2\theta + \frac{1}{\cos^2\theta} \ge 2$$

Since value of $\cos\theta$ lies between – 1 to 1

29. (c) We have, $\sin\theta = \frac{24}{25}, 0° < \theta < 90°$



$$\cos^2\theta = 1-\sin^2\theta = 1-\left(\frac{24}{25}\right)^2$$

Since lies in first quadrant $\Rightarrow \cos\theta = \frac{7}{25}$

$$\cos\theta = 1-2\sin^2\frac{\theta}{2}$$

$$2\sin^2\frac{\theta}{2} = 1-\cos\theta = 1-\frac{7}{25}$$

$$2\sin^2\frac{\theta}{2} = \frac{18}{25}$$

$$\sin^2\frac{\theta}{2} = \frac{9}{25} \Rightarrow \sin\frac{\theta}{2} = \pm\frac{3}{5}$$

$\Rightarrow \sin\frac{\theta}{2} = \frac{3}{5}$ [Negative sign discarded since θ is in first quadrant]

30. (c) $\tan\theta + \sec\theta = 4$

$$\frac{\sin\theta}{\cos\theta}+\frac{1}{\cos\theta} = 4 \Rightarrow \frac{\sin\theta+1}{\cos\theta} = 4$$

$\Rightarrow$ $\sin\theta + 1 = 4\cos\theta$

Squaring both the sides, we get

$\sin^2\theta + 1 + 2\sin\theta = 16\cos^2\theta$

$\Rightarrow$ $\sin^2\theta + 2\sin\theta + 1 = 16 - 16\sin^2\theta$

$\Rightarrow$ $17\sin^2\theta + 2\sin\theta - 15 = 0$

$\Rightarrow$ $(\sin\theta + 1)(17\sin\theta - 15) = 0$

$\Rightarrow$ $\sin\theta = -1$ (impossible) [since, $\theta \neq 90°$]

$\sin\theta = \frac{15}{17}$ (possible)

31. (b) The given expression $\sin 1950° - \cos 1950°$ can also be written as

$= \sin(10\times 180° + 150°) - \cos(10\times 180° + 150°)$

$= \sin 150° - \cos 150°$

$= \sin(90° + 60°) - \cos(90° + 60°)$

$= \cos 60° + \sin 60°$

$$= \frac{1}{2}+\frac{\sqrt{3}}{2} = \frac{\sqrt{3}+1}{2}$$

32. (c) (1) Given equation on RHS

$$(\sec\theta+\tan\theta)^2 = \left(\frac{1+\sin\theta}{\cos\theta}\right)^2 = \frac{(1+\sin\theta)^2}{\cos^2\theta}$$

$$= \frac{(1+\sin\theta)^2}{1-\sin^2\theta}$$

$$= \frac{(1+\sin\theta)^2}{(1-\sin\theta)(1+\sin\theta)} = \frac{1+\sin\theta}{1-\sin\theta} = \text{LHS.}$$

$$\Rightarrow (\sec\theta+\tan\theta)^2 = \frac{1+\sin\theta}{1-\sin\theta}$$

Statement (1) is true.

(2) LHS of given equation

$$\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta} = \sqrt{1+\tan^2\theta+1+\cot^2\theta}$$

$$\sqrt{\tan^2\theta+\cot^2\theta+2} = (\tan\theta+\cot\theta)$$

$$\sqrt{\sec^2\theta+\operatorname{cosec}^2\theta} = \tan\theta+\cot\theta$$

So, statement (2) is also correct.

Hence, both of the given statements are correct.

33. (c) Given values are $\tan A = \frac{1}{2}$ and $\tan B = \frac{1}{3}$

$$\tan(2A+B) = \frac{\tan 2A + \tan B}{1-\tan 2A\tan B}$$

$$\text{and, } \tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\frac{1}{2}}{1-\frac{1}{4}} = \frac{4}{3}$$

$$\text{So, } \tan(2A+B) = \frac{\frac{4}{3}+\frac{1}{3}}{1-\frac{4}{3}\times\frac{1}{3}} = \frac{\frac{5}{3}}{\frac{5}{9}} = 3$$

34. (c) Given that

$\tan\theta + \sec\theta = p$ ...(1)

and we know that

$\Rightarrow$ $\sec^2\theta - \tan^2\theta = (\sec\theta - \tan\theta)p$ (multiplying both the sides by $(\sec\theta - \tan\theta)$)

$\Rightarrow$ $(\sec\theta - \tan\theta)\,p = 1$

$\Rightarrow$ $\sec\theta - \tan\theta = \dfrac{1}{p}$ ...(2)

On solving equation (1) and (2), we get

$$2\sec\theta = \frac{p^2+1}{p} \Rightarrow \sec\theta = \frac{p^2+1}{2p}$$

35. (b) The given expression $\sin 10° + \sin 50° - \sin 70°$ can also be written as

$\sin 10° - (\sin 70° - \sin 50°)$

$= \sin 10° - (2\cos 60° \sin 10°)$

$= \sin 10°(2\cos 60° - 1) = \sin 10°(1-1) = 0$

36. (b) If $\tan(2A+B) = K$ then $2A + B = \tan^{-1}K$

$$\tan(2A+B) = \frac{\tan 2A + \tan B}{1 - \tan 2A \tan B}$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A} = \frac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{3}\right)^2}$$

$$= \frac{\frac{2}{3}}{1-\frac{1}{9}} = \frac{2/3}{8/9} = \frac{3}{4}$$

Hence, $\tan(2A+B) = \dfrac{\tan 2A + \tan B}{1 - \tan 2A \tan B}$

$$= \frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\cdot\frac{1}{7}} = \frac{\frac{21+4}{28}}{\frac{28-3}{28}} = 1$$

$\Rightarrow$ $\tan(2A+B) = \tan 45°$

$\Rightarrow$ $2A + B = \tan^{-1}(1) = 45°$

37. (d) Since, $p = \tan\alpha + \tan\beta$

and $q = \cot\alpha + \cot\beta$

$q = \tan\alpha + \tan\beta$

$$\Rightarrow q = \frac{1}{\tan\alpha} + \frac{1}{\tan\beta} = \frac{\tan\alpha + \tan\beta}{\tan\alpha\tan\beta}$$

$$q = \frac{p}{\tan\alpha\tan\beta}$$

Hence, $\dfrac{1}{p} - \dfrac{1}{q} = \dfrac{1}{p} - \dfrac{\tan\alpha\tan\beta}{p}$

$$= \frac{1-\tan\alpha\tan\beta}{p}$$

$$= \frac{1-\tan\alpha\tan\beta}{\tan\alpha+\tan\beta} = \frac{1}{\tan(\alpha+\beta)}$$

$= \cot(\alpha+\beta)$

38. (c) Given that $\tan\theta = m$ and $\tan 2\theta = n$

We know from fundamentals that

$$\Rightarrow \tan 3\theta = \frac{\tan\theta + \tan 2\theta}{1 - \tan\theta\tan 2\theta}$$

Since, $\tan 3\theta = \tan\theta + \tan 2\theta$.....(as given)

$$\Rightarrow \tan\theta + \tan 2\theta = \frac{\tan\theta + \tan 2\theta}{1-\tan\theta\tan 2\theta}$$

$\Rightarrow$ $(\tan\theta + \tan 2\theta)(1 - \tan\theta\tan 2\theta)$
$-(\tan\theta + \tan 2\theta) = 0$

$\Rightarrow$ $(\tan\theta + \tan 2\theta)\{1 - \tan\theta\tan 2\theta - 1\} = 0$

$\Rightarrow$ $(\tan\theta + \tan 2\theta)(\tan\theta\tan 2\theta) = 0$

$\Rightarrow$ $(m+n)(mn) = 0; \Rightarrow (m+n) = 0$

[since $m \neq 0$ and $n \neq 0$]

39. (b) As given, $\tan\alpha = \dfrac{m}{m+1}$

and $\tan\beta = \dfrac{1}{2m+1}$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

$$= \frac{\frac{m}{m+1}+\frac{1}{2m+1}}{1-\frac{m}{m+1}\times\frac{1}{2m+1}} = \frac{m(2m+1)+(m+1)}{(m+1)(2m+1)-m}$$

$$= \frac{2m^2+2m+1}{2m^2+2m+1} = 1$$

So, $\alpha + \beta = \dfrac{\pi}{4}$

0. (a) The given expression is :

$(\sec\theta - \cos\theta)(\operatorname{cosec}\theta - \sin\theta)(\cot\theta + \tan\theta)$

$$= \left(\frac{1}{\cos\theta} - \cos\theta\right)\left(\frac{1}{\sin\theta} - \sin\theta\right)\left(\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta}\right)$$

$$= \left(\frac{1-\cos^2\theta}{\cos\theta}\right)\left(\frac{1-\sin^2\theta}{\sin\theta}\right)\left(\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta}\right)$$

$$= \frac{\sin^2\theta}{\cos\theta}.\frac{\cos^2\theta}{\sin\theta} \times \frac{1}{\sin\theta.\cos\theta}$$

$$= \frac{\sin^2\theta.\cos^2\theta}{\cos^2\theta.\sin^2\theta} = 1$$

1. (a) $\cot(-870°) = -\cot(2 \times 360° + 150°)$

$= -\cot 150° = -\cot(180° - 30°)$

$= \cot 30° = \sqrt{3}.$

2. (a) Let $x = \left(\sin 22\frac{1°}{2} + \cos^2 22\frac{1°}{2}\right)^4$

$$= \left\{\left(\sin 22\frac{1°}{2} + \cos 22\frac{1}{2}°\right)^2\right\}^2$$

$$= \left(\sin^2 22\frac{1°}{2} + \cos^2 22\frac{1°}{2} + 2\sin 22\frac{1°}{2}\cos 22\frac{1°}{2}\right)^2$$

$$= (1 + \sin 45°)^2$$

$$= \left(1 + \frac{1}{\sqrt{2}}\right)^2 = \left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)^2$$

$$= \frac{2+1+2\sqrt{2}}{2} = \frac{3+2\sqrt{2}}{2}$$

3. (c) As given : $\sin 2A = \frac{4}{5}$

$$\sin 2A = \frac{2\tan A}{1+\tan^2 A}$$

$$\Rightarrow \frac{2\tan A}{1+\tan^2 A} = \frac{4}{5}$$

$\Rightarrow 10\tan A = 4 + 4\tan^2 A$

$\Rightarrow 5\tan A = 2 + 2\tan^2 A$

$\Rightarrow 2\tan^2 A - 5\tan A + 2 = 0$

$\Rightarrow 2\tan^2 A - 4\tan A - \tan A + 2 = 0$

$\Rightarrow 2\tan A(\tan A - 2) - 1(\tan A - 2) = 0$

$\Rightarrow (2\tan A - 1)(\tan A - 2) = 0$

$\Rightarrow \tan A = \frac{1}{2}$ (since $A \le \frac{\pi}{4} \Rightarrow \tan A \ne 2$)

44. (b) $\sin\frac{5\pi}{12} = \sin 75°$

$= \sin(45° + 30°)$

$= \sin 45° \cos 30° + \cos 45° \sin 35°$

$$= \frac{1}{\sqrt{2}}.\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}}.\frac{1}{2} = \frac{1}{\sqrt{2}}\left(\frac{\sqrt{3}+1}{2}\right)$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

45. (a) We work out the given statements.

1. $\sin\frac{\pi}{12} = \sin 15° = \frac{\sqrt{3}-1}{2\sqrt{2}}$

2. $\cos\frac{\pi}{12} = \cos 15° = \frac{\sqrt{3}+1}{2\sqrt{2}}$

3. $\cot\frac{\pi}{12} = \cot 15° = 2 + \sqrt{3}$

So, correct sequence is $3 > 2 > 1$.

46. (b) $\sin\left[n\pi + (-1)^n \frac{\pi}{4}\right] = (-1)^n \sin\left[(-1)^n \frac{\pi}{4}\right]$

$[\because \sin(n\pi + \theta) = (-1)^n \sin\theta]$

$$= (-1)^n (-1)^n \sin\frac{\pi}{4}$$

$\therefore \sin[(-1)^n\theta = (-1)^n \sin\theta]$

$$= (-1)^{2n} \sin\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

47. (b) Let $y = \frac{\tan x}{\tan 3x}$

$$\Rightarrow y = \frac{\tan x}{\left(\frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}\right)} = \frac{1 - 3\tan^2 x}{3 - \tan^2 x}$$

$\Rightarrow 3y - y\tan^2 x = 1 - 3\tan^2 x$

$\Rightarrow (y-3)\tan^2 x + (1-3y) = 0$

$\Rightarrow \tan^2 x = \dfrac{3y-1}{y-3}$

For tan x to be real $\dfrac{3y-1}{y-3} \geq 0$

$\Rightarrow (3y-1)(y-3) \geq 0$ and $y \neq 3$

$\Rightarrow y \leq \dfrac{1}{3}$ or $y > 3$

48. (a) Since $A + B + C = \pi$

$\therefore A + B = \pi - C \Rightarrow \tan(A+B) = \tan(\pi - C)$

$\Rightarrow \dfrac{\tan A + \tan B}{1 - \tan A \tan B} = -\tan C$

$\Rightarrow \dfrac{\tan A + \tan B}{1 - \tan A \tan B} > 0$ ...(1)

[$\because$ angle C is obtuse $\therefore \tan C < 0$]

But C is obtuse angle, so A and B will both be less than $\dfrac{\pi}{2}$

$\therefore$ Both tan A and tan B are positive.

Hence from (1),

$1 - \tan A \tan B > 0 \Rightarrow \tan A \tan B < 1$

49. (b) The given equation is $\tan x + \sec x = 2\cos x$;

$\Rightarrow \sin x + 1 = 2\cos^2 x$

$\Rightarrow \sin x + 1 = 2(1 - \sin^2 x)$;

$\Rightarrow 2\sin^2 x + \sin x - 1 = 0$;

$\Rightarrow (2\sin x - 1)(\sin x + 1) = 0$

$\Rightarrow \sin x = \dfrac{1}{2}, -1.$;

$\Rightarrow x = 30°, 150°, 270°.$

50. (d) $\pi < \alpha - \beta < 3\pi$

$\Rightarrow \dfrac{\pi}{2} < \dfrac{\alpha-\beta}{2} < \dfrac{3\pi}{2} \Rightarrow \cos\dfrac{\alpha-\beta}{2} < 0$

$\sin\alpha + \sin\beta = -\dfrac{21}{65}$

$\Rightarrow 2\sin\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2} = -\dfrac{21}{65}$ ....(1)

$\cos\alpha + \cos\beta = -\dfrac{27}{65}$

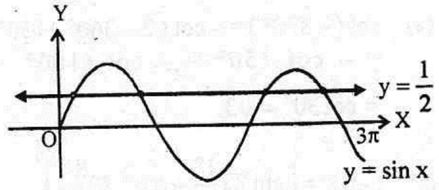
$\Rightarrow 2\cos\dfrac{\alpha+\beta}{2}\cos\dfrac{\alpha-\beta}{2} = -\dfrac{27}{65}$ ....(2)

Square and add (1) and (2)

$4\cos^2\dfrac{\alpha-\beta}{2} = \dfrac{(21)^2 + (27)^2}{(65)^2} = \dfrac{1170}{65 \times 65}$

$\therefore \cos^2\dfrac{\alpha-\beta}{2} = \dfrac{9}{130} \Rightarrow \cos\dfrac{\alpha-\beta}{2} = -\dfrac{3}{\sqrt{130}}$

51. (a)



$2\sin^2 x + 5\sin x - 3 = 0$

$\Rightarrow (\sin x + 3)(2\sin x - 1) = 0$

$\Rightarrow \sin x = \dfrac{1}{2}$ and $\sin x \neq -3$

$\therefore$ In $[0, 3\pi]$, x has 4 values.

52. (c) $\cos x + \sin x = \dfrac{1}{2} \Rightarrow 1 + \sin 2x = \dfrac{1}{4}$

$\Rightarrow \sin 2x = -\dfrac{3}{4}$, so x is obtuse and

$\dfrac{2\tan x}{1 + \tan^2 x} = -\dfrac{3}{4}$

$\Rightarrow 3\tan^2 x + 8\tan x + 3 = 0$

$\therefore \tan x = \dfrac{-8 \pm \sqrt{64-36}}{6} = -\dfrac{-4 \pm \sqrt{7}}{3}$

as $\tan x < 0$ $\therefore \tan x = \dfrac{-4-\sqrt{7}}{3}$

53. (b) We have

$\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta)$

$= -\dfrac{3}{2}$

$\Rightarrow \ 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)]+3=0$

$\Rightarrow \ 2[\cos(\beta-\gamma)+\cos(\gamma-\alpha)+\cos(\alpha-\beta)]+\sin^2\alpha+\cos^2\alpha+\sin^2\beta+\cos^2\beta+\sin^2\gamma+\cos^2\gamma=0$

$\Rightarrow \ [\sin^2\alpha+\sin^2\beta+\sin^2\gamma+2\sin\alpha\sin\beta+2\sin\beta\sin\gamma+2\sin\gamma\sin\alpha]+[\cos^2\alpha+\cos^2\beta+\cos^2\gamma+2\cos\alpha\cos\beta+2\cos\beta\cos\gamma+2\cos\gamma\cos\alpha]=0$

$\Rightarrow \ [\sin\alpha+\sin\beta+\sin\gamma]^2+(\cos\alpha+\cos\beta+\cos\gamma)^2=0$

$\Rightarrow \ \sin\alpha+\sin\beta+\sin\gamma=0$ and $\cos\alpha+\cos\beta+\cos\gamma=0$

$\therefore$ A and B both are true.

54. (a) $\cos(\alpha+\beta)=\dfrac{4}{5} \Rightarrow \tan(\alpha+\beta)=\dfrac{3}{4}$

$\sin(\alpha-\beta)=\dfrac{5}{13} \Rightarrow \tan(\alpha-\beta)=\dfrac{5}{12}$

$\tan 2\alpha=\tan[(\alpha+\beta)+(\alpha-\beta)]$

$$=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}}=\frac{56}{33}$$

55. (d) $A=\sin^2 x+\cos^4 x$

$=\sin^2 x+\cos^2 x(1-\sin^2 x)$

$=\sin^2 x+\cos^2 x-\dfrac{1}{4}(2\sin x.\cos x)^2$

$=1-\dfrac{1}{4}\sin^2(2x)$

Now $0\le\sin^2(2x)\le 1$

$\Rightarrow 0\ge-\dfrac{1}{4}\sin^2(2x)\ge-\dfrac{1}{4}$

$\Rightarrow 1\ge 1-\dfrac{1}{4}\sin^2(2x)\ge 1-\dfrac{1}{4}$

$\Rightarrow 1\ge A\ge\dfrac{3}{4}$

56. (d) $\sin 4\theta+2\sin 4\theta\cos 3\theta=0$

$\Rightarrow \sin 4\theta(1+2\cos 3\theta)=0$

$\sin 4\theta=0$ or $\cos 3\theta=-\dfrac{1}{2}$

$4\theta=n\pi;n\in I$

or $3\theta=2n\pi\pm\dfrac{2\pi}{3}, n\in I$

$\theta=\dfrac{\pi}{4},\dfrac{\pi}{2},\dfrac{3\pi}{4}$

or $\theta=\dfrac{2\pi}{9},\dfrac{8\pi}{9},\dfrac{4\pi}{9}$ $[\because \theta,\in(0,\pi)]$

57. (a) Since, $\dfrac{\alpha}{2}+\dfrac{\beta}{2}=\left(\pi-\dfrac{\gamma}{2}\right)$

$\therefore \ \tan\left(\dfrac{\alpha}{2}+\dfrac{\beta}{2}\right)=\tan\left(\pi-\dfrac{\gamma}{2}\right)$

$$\Rightarrow \frac{\tan\frac{\alpha}{2}+\tan\frac{\beta}{2}}{1-\tan\frac{\alpha}{2}\tan\frac{\beta}{2}}=-\tan\frac{\gamma}{2}$$

$\Rightarrow \tan\dfrac{\alpha}{2}+\tan\dfrac{\beta}{2}+\tan\dfrac{\gamma}{2}=\tan\dfrac{\alpha}{2}\tan\dfrac{\beta}{2}\tan\dfrac{\gamma}{2}$

58. (b) Since, $\sin\theta=\dfrac{1}{2}$ and $\cos\phi=\dfrac{1}{3}$

$\Rightarrow \theta=\dfrac{\pi}{6}$ and $0<\left(\cos\phi=\dfrac{1}{3}\right)<\dfrac{1}{2}$ $\left(\because 0<\dfrac{1}{3}<\dfrac{1}{2}\right)$

$\Rightarrow \theta=\dfrac{\pi}{6}$ and $\dfrac{\pi}{3}<\phi<\dfrac{\pi}{2}$

$\Rightarrow \dfrac{\pi}{2}<\theta+\phi<\dfrac{2\pi}{3}$ $\therefore \ \theta\in\left(\dfrac{\pi}{2},\dfrac{2\pi}{3}\right)$

59. (c) Let $f(x) = \tan x - x$

We know that, for $0 < x < \frac{\pi}{2}$

$\Rightarrow \tan x > x$

$\therefore$ $f(x) = \tan x - x$ has no root in $\left(0, \frac{\pi}{2}\right)$

For $\frac{\pi}{2} < x < \pi$, $\tan x$ is negative

$f(x) = \tan x - x < 0$

$\therefore$ $f(x) = 0$ has no root in $\left(\frac{\pi}{2}, \pi\right)$

For $\frac{3\pi}{2} < x < 2\pi$, $\tan x$ is negative

$f(x) = \tan x - x < 0$

So, $f(x) = 0$ has no root in $\left(\frac{3\pi}{2}, 2\pi\right)$

We have, $f(x) = 0 - \pi < 0$

66. (d) Given, $\cos^2\theta + \sin\theta + 1 = 0$

$(1 - \sin^2\theta) + \sin\theta + 1 = 0$

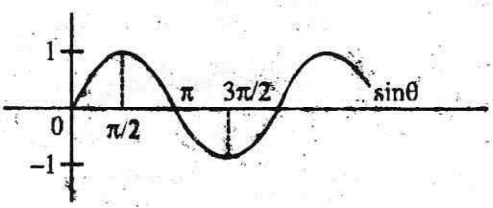
$\Rightarrow (1 - \sin\theta)(1 + \sin\theta) + (\sin\theta + 1) = 0$

$\Rightarrow (1 + \sin\theta)(1 - \sin\theta + 1) = 0$

$\Rightarrow \sin\theta = -1$ and $\sin\theta = 2$,

Which is not possible because

$-1 \leq \sin\theta \leq 1$



$\therefore$ $\sin\theta = \frac{3\pi}{2}$

$\therefore$ $\frac{3\pi}{2} \in \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right)$

67. (a) Consider the equation

$\tan x + \tan 2x + \tan x . \tan 2x = 1$

$\Rightarrow \tan x + \tan 2x = 1 - \tan x \tan 2x$

$\Rightarrow \tan(x + 2x)[1 - \tan x \tan 2x]$

$= 1 - \tan x \tan 2x$

$\Rightarrow \tan 3x = \frac{1 - \tan x \tan 2x}{1 - \tan x \tan 2x}$

$\left(\because \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}\right)$

$\Rightarrow \tan 3x = 1 = \tan\frac{\pi}{4}$

$\Rightarrow 3x = n\pi + \frac{\pi}{4}, n \in z$

$\Rightarrow x = \frac{n\pi}{3} + \frac{\pi}{12}, n \in z$

Hence, solution of the given equation will be

$x = \frac{n\pi}{3} + \frac{\pi}{12}$

68. (b) Given : $x + \frac{1}{x} = 2\cos\theta$ ...(1)

Cubic both sides in eq^n (1) we get

$x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 8\cos^3\theta$

$\Rightarrow x^3 + \frac{1}{x^3} + 3(2\cos\theta) = 8\cos^3\theta$

$\Rightarrow x^3 + \frac{1}{x^3} = 8\cos^3\theta - 6\cos\theta$

$\Rightarrow x^3 + \frac{1}{x^3} = 2(4\cos^3\theta - 3\cos\theta) = 2\cos 3\theta$

69. (b) Consider $\cos\frac{\pi}{3}.\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}$

Let $A = \frac{\pi}{7}$ then

$\cos\frac{\pi}{3}.\cos\frac{\pi}{7}.\cos\frac{2\pi}{7}.\cos\frac{4\pi}{7}$

$= \frac{1}{2}\cos A \cos 2A \cos 4A$

$= \frac{1}{2}\cos A \cos 2A \cos 2^2 A$

$= \frac{1}{2}.\frac{\sin 2^3 A}{2^3 \sin A} = \frac{1}{2}\frac{\sin 8A}{8 \sin A}$

$\left(\text{using } \cos A \cos 2A \cos 2^2 A \ldots\ldots\ldots \cos 2^{n-1} A = \dfrac{\sin 2^n A}{2^n \sin A}\right)$

$$= \frac{1}{16}\frac{\sin(7A+A)}{\sin A} = \frac{1}{16}\frac{\sin(\pi + A)}{\sin A}$$

$$= -\frac{\sin A}{16 \sin A} = -\frac{1}{16}.$$

70. (c) Given $\cos x = \sqrt{3}(1 - \sin x)$

$\Rightarrow \quad \cos x = \sqrt{3} - \sqrt{3}\sin x$

$\Rightarrow \quad \sqrt{3}\sin x + \cos x = \sqrt{3}$

Divide this equation by $\sqrt{a^2 + b^2}$

i. e., $\sqrt{(\sqrt{3})^2 + (1)^2} = 2$

we get $\dfrac{\sqrt{3}}{2}\sin x + \dfrac{1}{2}\cos x = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \cos\dfrac{\pi}{6}.\sin x + \sin\dfrac{\pi}{6}.\cos x = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \sin\left(x + \dfrac{\pi}{6}\right) = \dfrac{\sqrt{3}}{2}$

$\Rightarrow \quad \sin\left(x + \dfrac{\pi}{6}\right) = \sin\dfrac{\pi}{3} \qquad \ldots(i)$

we know that if $\sin\theta = \sin\alpha$

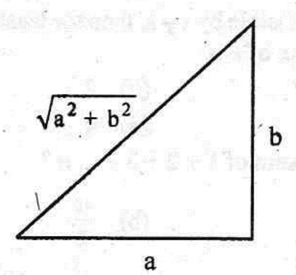
$\Rightarrow \quad \theta = n\pi + (-1)^n \alpha$

$\therefore$ Equation (i) becomes

$$x + \frac{\pi}{6} = n\pi + (-1)^n.\frac{\pi}{3}$$

$\Rightarrow \quad x = n\pi + (-1)^n.\dfrac{\pi}{3} - \dfrac{\pi}{6}$

71. (a) Given $\tan x = \dfrac{b}{a}$



$\Rightarrow \quad \sin x = \dfrac{b}{\sqrt{a^2+b^2}}$ and $\cos x = \dfrac{a}{\sqrt{a^2+b^2}}$

Thus, $a\cos 2x + b\sin 2x$

$= a(\cos^2 x - \sin^2 x) + b(2\sin x.\cos x)$

$$= a\left[\frac{a^2 - b^2}{a^2 + b^2}\right] + 2b\left[\frac{ab}{a^2+b^2}\right] = \frac{a^3 + ab^2}{a^2+b^2}$$

$$= \frac{a(a^2+b^2)}{(a^2+b^2)} = a.$$

72. (b) Let $A = \tan 20° + 2\tan 50° - \tan 70°$

$$A = \frac{\sin 20°}{\cos 20°} - \frac{\sin 70°}{\cos 70°} + \frac{2\sin 50°}{\cos 50°}$$

$$= \frac{\sin 20°}{\cos 20°} - \frac{\sin(90° - 20°)}{\cos(90° - 20°)} + \frac{2\sin(90° - 40°)}{\cos(90° - 40°)}$$

$$= \frac{\sin 20°}{\cos 20°} - \frac{\cos 20°}{\sin 20°} + \frac{2\cos 40°}{\sin 40°}$$

$$= \frac{\sin^2 20° - \cos^2 20°}{\sin 20°.\cos 20°} + \frac{2\cos 40°}{\sin 40°}$$

$$= \frac{-\cos 40° \times 2}{2\sin 20°.\cos 20°} + \frac{2\cos 40°}{\sin 40°}$$

$$= \frac{-2\cos 40°}{\sin 40°} + \frac{2\cos 40°}{\sin 40°} = 0.$$

73. (d) Given, $\sin 2x + 2\sin x + 2\cos x + 1 = 0$

$\Rightarrow 1 + \sin 2x + 2(\sin x + \cos x) = 0$

$\Rightarrow (\sin x + \cos x)^2 + 2(\sin x + \cos x) = 0$

$\Rightarrow (\sin x + \cos x)(\sin x + \cos x + 2) = 0$

$\therefore \sin x + \cos x = 0$ or $\sin x + \cos x = -2$

But, $\sin x + \cos x = -2$ is inadmissible.

Since, $|\sin x| \le 1, |\cos x| \le 1$

$\therefore \sin x + \cos x = 0 \Rightarrow \sin\left(x + \dfrac{\pi}{4}\right) = 0$

$\Rightarrow x + \dfrac{\pi}{4} = n\pi \Rightarrow x = n\pi - \dfrac{\pi}{4}$

74. (a) From $\sin x + \sin^2 x = 1$, we get $\sin x = \cos^2 x$ ...(1)

Now the given expression is equal to

$\cos^6 x(\cos^6 x + 3\cos^4 x + 3\cos^2 x + 1) - 1$

$= \cos^6 x(\cos^2 x + 1)^3 - 1 = \sin^3 x(\sin x + 1)^3 - 1$ [From (1)]

$= (\sin^2 x + \sin x)^3 - 1 = 1 - 1 = 0$