Calculating Option Greeks using Black-Scholes Model

Overview

This notebook provides a comprehensive implementation of the Black-Scholes option pricing model, demonstrating the calculation of key option Greeks. Option Greeks are sensitivity measurements that describe how the price of an option changes in response to various factors.

Key Components

- Option Pricing
- Delta Calculation
- Gamma Calculation
- Vega Calculation
- Theta Calculation
- Rho Calculation

Required Libraries

In [1]:
```
import numpy as np
from scipy.stats import norm
import matplotlib.pyplot as plt

plt.style.use('ggplot')
```

Model Parameters

In [2]:
```
r = 0.055           # Risk-free interest rate
S = 600             # Current stock price
K = 600             # Strike price
T = 365 / 365       # Time to expiration (in years)
sigma = 0.1625      # Volatility

K_range = np.linspace(400, 800, 500)  # Strike prices from 400 to 800
```

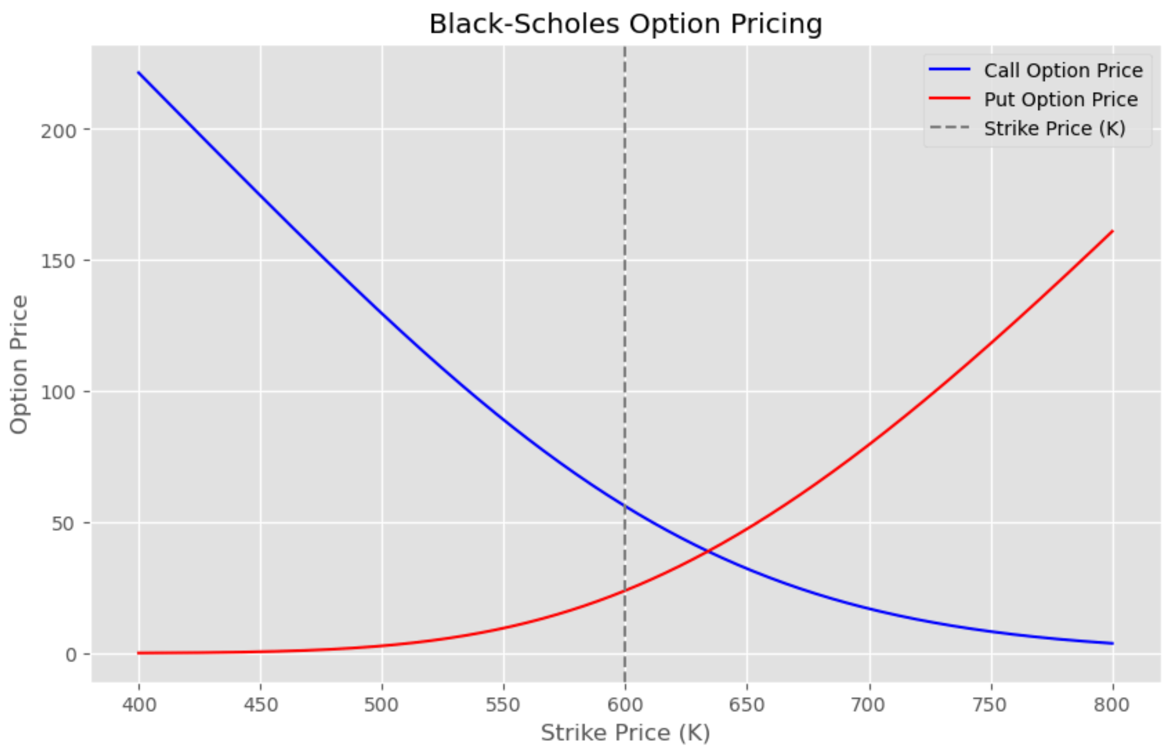
Black-Scholes Option Pricing Function

In [3]:
```
def blackScholes(r, S, K, T, sigma, type="c"):
    """
    Calculate the theoretical price of a call or put option using Black-S

    Parameters:
    - r: Risk-free interest rate
```

```python
    - S: Current stock price
    - K: Strike price
    - T: Time to expiration (in years)
    - sigma: Volatility of the underlying asset
    - type: Option type - 'c' for Call, 'p' for Put

    Returns:
    - Theoretical option price
    """
    # Calculate d1 and d2 parameters
    d1 = (np.log(S / K) + (r + sigma**2 / 2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)

    try:
        if type == "c":
            # Call option pricing formula
            price = S * norm.cdf(d1, 0, 1) - K * np.exp(-r * T) * norm.cd
        elif type == "p":
            # Put option pricing formula
            price = K * np.exp(-r * T) * norm.cdf(-d2, 0, 1) - S * norm.c
        return price
    except:
        print("Please confirm option type, either 'c' for Call or 'p' for
```

In [4]:

```python
# Calculate call and put option prices
call_prices = [blackScholes(r, S, K, T, sigma, type="c") for K in K_range
put_prices = [blackScholes(r, S, K, T, sigma, type="p") for K in K_range]

# Plot the results
plt.figure(figsize=(10, 6))
plt.plot(K_range, call_prices, label="Call Option Price", color="blue")
plt.plot(K_range, put_prices, label="Put Option Price", color="red")
plt.axvline(x=K, color="gray", linestyle="--", label="Strike Price (K)")
plt.title("Black-Scholes Option Pricing")
plt.xlabel("Strike Price (K)")
plt.ylabel("Option Price")
plt.legend()
plt.show()
```



Option Greeks Calculation Functions

Delta

Delta measures the rate of change of the theoretical option value with respect to changes in the underlying asset's price.

$$\Delta = \frac{\partial V}{\partial S}$$

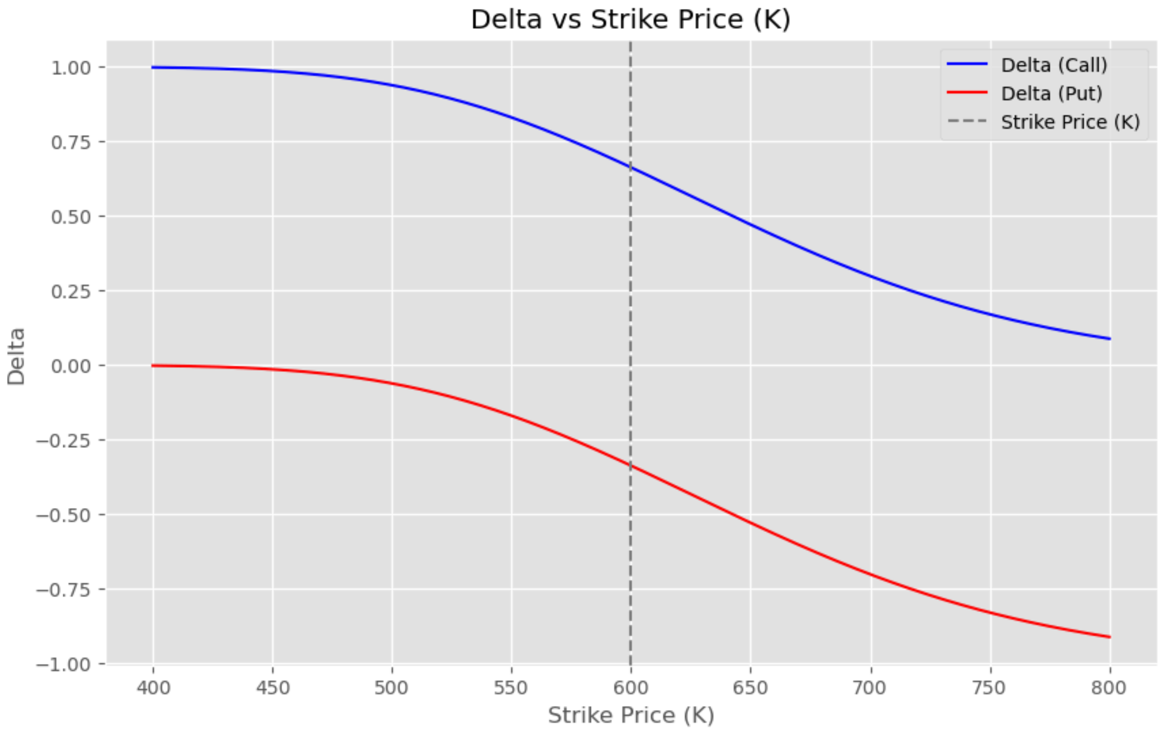
$$\Delta_{call} = \Phi(d1)$$

$$\Delta_{put} = -\Phi(-d1)$$

In [5]:
```
def delta_calc(r, S, K, T, sigma, type="c"):
    """
    Calculate Delta: Rate of change of option price with respect to under

    Delta represents the hedge ratio or the equivalent stock position of
    - For calls: Ranges from 0 to 1
    - For puts: Ranges from -1 to 0

    Parameters same as Black-Scholes function

    Returns:
    - Delta value
    """
    d1 = (np.log(S / K) + (r + sigma**2 / 2) * T) / (sigma * np.sqrt(T))
    try:
        if type == "c":
            delta_calc = norm.cdf(d1, 0, 1)
        elif type == "p":
            delta_calc = -norm.cdf(-d1, 0, 1)
```

```
        return delta_calc
    except:
        print("Please confirm option type, either 'c' for Call or 'p' for
```

In [6]:
```
# Calculate Delta values for calls and puts
call_deltas = [delta_calc(r, S, K, T, sigma, type="c") for K in K_range]
put_deltas = [delta_calc(r, S, K, T, sigma, type="p") for K in K_range]

# Plot the results
plt.figure(figsize=(10, 6))
plt.plot(K_range, call_deltas, label="Delta (Call)", color="blue")
plt.plot(K_range, put_deltas, label="Delta (Put)", color="red")
plt.axvline(x=K, color="gray", linestyle="--", label="Strike Price (K)")
plt.title("Delta vs Strike Price (K)")
plt.xlabel("Strike Price (K)")
plt.ylabel("Delta")
plt.legend()
plt.show()
```



Gamma

Gamma measures the rate of change in the delta with respect to changes in the underlying price.

$$\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 V}{\partial S^2}$$

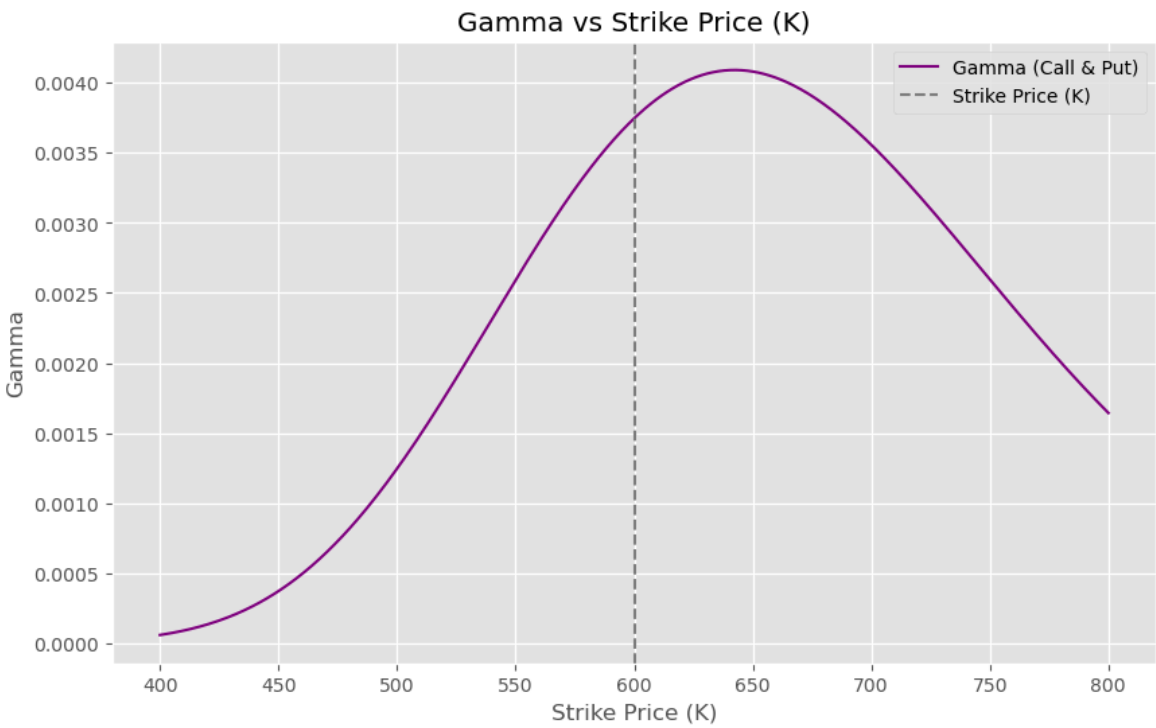
$$\Gamma = \frac{\phi(d1)}{S\sigma\sqrt{\tau}}$$

In [7]:
```
def gamma_calc(r, S, K, T, sigma, type="c"):
    """
    Calculate Gamma: Rate of change of Delta with respect to underlying a
```

```
    Gamma measures the curvature of the option price's relationship to th
    - Highest near the money
    - Symmetric for calls and puts

    Parameters same as Black-Scholes function

    Returns:
    - Gamma value
    """
    d1 = (np.log(S / K) + (r + sigma**2 / 2) * T) / (sigma * np.sqrt(T))
    try:
        gamma_calc = norm.pdf(d1, 0, 1) / (S * sigma * np.sqrt(T))
        return gamma_calc
    except:
        print("Please confirm option type, either 'c' for Call or 'p' for
```

In [8]:

```
# Calculate Gamma values for the stock price range
gamma_values = [gamma_calc(r, S, K, T, sigma) for K in K_range]

# Plot the results
plt.figure(figsize=(10, 6))
plt.plot(K_range, gamma_values, label="Gamma (Call & Put)", color="purple
plt.axvline(x=K, color="gray", linestyle="--", label="Strike Price (K)")
plt.title("Gamma vs Strike Price (K)")
plt.xlabel("Strike Price (K)")
plt.ylabel("Gamma")
plt.legend()
plt.show()
```



Vega

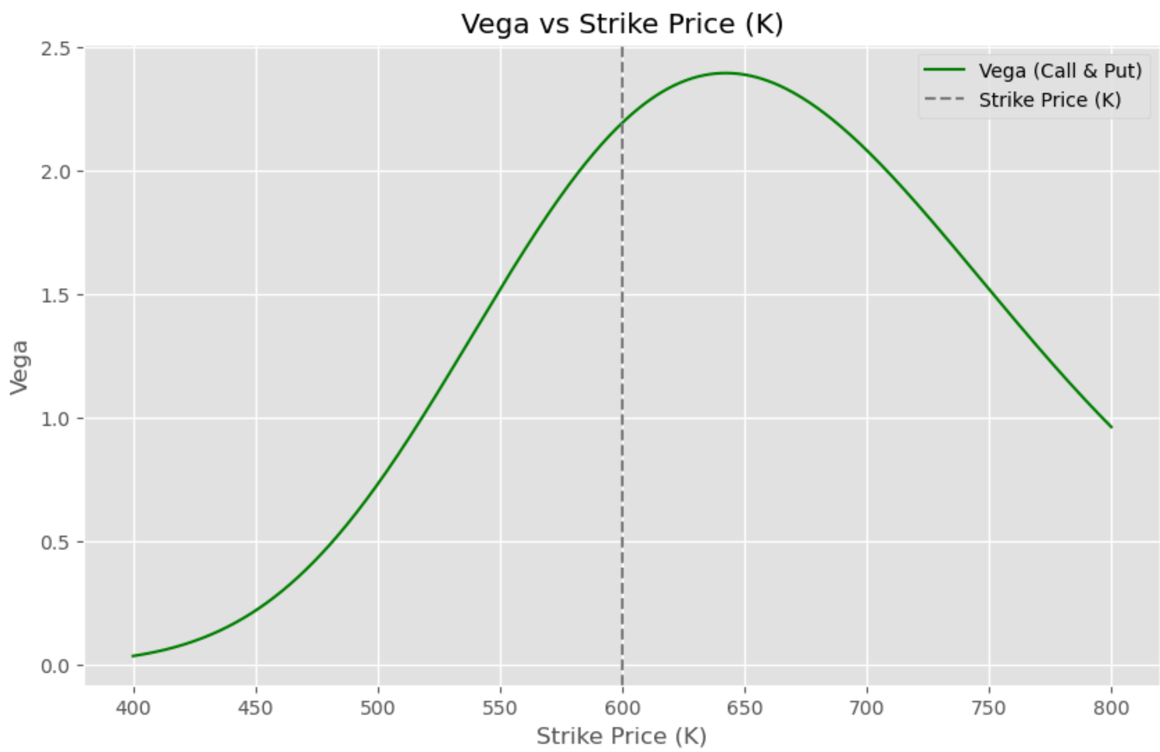
Vega measures sensitivity to volatility. Vega is the derivative of the option value with respect to the volatility of the underlying asset.

$$\upsilon = \frac{\partial V}{\partial \sigma}$$

$$\upsilon = S\phi(d1)\sqrt{\tau}$$

In [9]:
```
def vega_calc(r, S, K, T, sigma, type="c"):
    """
    Calculate Vega: Sensitivity of option price to volatility changes

    Vega measures how much an option's price changes with volatility
    - Highest for at-the-money options
    - Multiplied by 0.01 to represent percentage point change

    Parameters same as Black-Scholes function

    Returns:
    - Vega value
    """
    d1 = (np.log(S / K) + (r + sigma**2 / 2) * T) / (sigma * np.sqrt(T))
    try:
        vega_calc = S * norm.pdf(d1, 0, 1) * np.sqrt(T)
        return vega_calc * 0.01  # Convert to percentage points
    except:
        print("Please confirm option type, either 'c' for Call or 'p' for
```

In [10]:
```
# Calculate Vega values for the stock price range
vega_values = [vega_calc(r, S, K, T, sigma) for K in K_range]

# Plot the results
plt.figure(figsize=(10, 6))
plt.plot(K_range, vega_values, label="Vega (Call & Put)", color="green")
plt.axvline(x=K, color="gray", linestyle="--", label="Strike Price (K)")
plt.title("Vega vs Strike Price (K)")
plt.xlabel("Strike Price (K)")
plt.ylabel("Vega")
plt.legend()
plt.show()
```



Theta

Theta measures the sensitivity of the value of the derivative to the passage of time - time decay.

$$\Theta = -\frac{\partial V}{\partial \tau}$$

$$\Theta_{call} = -\frac{S\phi(d1)\sigma}{2\tau} - rK\exp{(-rT)}\Phi(d2)$$

$$\Theta_{put} = -\frac{S\phi(d1)\sigma}{2\tau} + rK\exp{(-rT)}\Phi(-d2)$$

In [11]:
```
def theta_calc(r, S, K, T, sigma, type="c"):
    """
    Calculate Theta: Rate of time decay of the option

    Theta measures how much value an option loses as time passes
    - Typically negative (option loses value as expiration approaches)
    - Divided by 365 to get daily time decay

    Parameters same as Black-Scholes function

    Returns:
    - Theta value (daily time decay)
    """
    d1 = (np.log(S / K) + (r + sigma**2 / 2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)
    try:
        if type == "c":
            theta_calc = -S * norm.pdf(d1, 0, 1) * sigma / (2 * np.sqrt(T
        elif type == "p":
            theta_calc = -S * norm.pdf(d1, 0, 1) * sigma / (2 * np.sqrt(T
        return theta_calc / 365  # Daily time decay
    except:
        print("Please confirm option type, either 'c' for Call or 'p' for
```

In [12]:
```
# Calculate Theta values for the strike price range
theta_values_call = [theta_calc(r, S, K, T, sigma, type="c") for K in K_r
theta_values_put = [theta_calc(r, S, K, T, sigma, type="p") for K in K_ra

# Plot the results
plt.figure(figsize=(10, 6))
plt.plot(K_range, theta_values_call, label="Theta (Call)", color="blue")
plt.plot(K_range, theta_values_put, label="Theta (Put)", color="red")
plt.axvline(x=S, color="gray", linestyle="--", label="Stock Price (S)")
plt.title("Theta vs Strike Price (K)")
plt.xlabel("Strike Price (K)")
plt.ylabel("Theta")
plt.legend()
plt.show()
```

Rho

Rho measures the sensitivity to the interest rate.

$$\rho = \frac{\partial V}{\partial r}$$

$$\rho_{call} = K\tau \exp{(-rT)} \Phi(d2)$$

$$\rho_{put} = -K\tau \exp{(-rT)} \Phi(-d2)$$

In [13]:
```
def rho_calc(r, S, K, T, sigma, type="c"):
    """
    Calculate Rho: Sensitivity of option price to interest rate changes

    Rho measures how much an option's price changes with interest rates
    - Multiplied by 0.01 to represent percentage point change

    Parameters same as Black-Scholes function

    Returns:
    - Rho value
    """
    d1 = (np.log(S / K) + (r + sigma**2 / 2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)
    try:
        if type == "c":
            rho_calc = K * T * np.exp(-r * T) * norm.cdf(d2, 0, 1)
        elif type == "p":
            rho_calc = -K * T * np.exp(-r * T) * norm.cdf(-d2, 0, 1)
        return rho_calc * 0.01  # Convert to percentage points
    except:
        print("Please confirm option type, either 'c' for Call or 'p' for
```

In [14]:
```python
# Calculate Rho values for the strike price range
rho_values_call = [rho_calc(r, S, K, T, sigma, type="c") for K in K_range
rho_values_put = [rho_calc(r, S, K, T, sigma, type="p") for K in K_range]

# Plot the results
plt.figure(figsize=(10, 6))
plt.plot(K_range, rho_values_call, label="Rho (Call)", color="blue")
plt.plot(K_range, rho_values_put, label="Rho (Put)", color="red")
plt.axvline(x=S, color="gray", linestyle="--", label="Stock Price (S)")
plt.title("Rho vs Strike Price (K)")
plt.xlabel("Strike Price (K)")
plt.ylabel("Rho")
plt.legend()
plt.show()
```

Comprehensive Option Greeks Calculation

In [15]:
```python
# Option type selection
option_type = "p"  # Put option in this example

# Calculate and print all Greeks
print("Option Type: ", "Put" if option_type == "p" else "Call")
print("      Price: ", round(blackScholes(r, S, K, T, sigma, option_type)
print("      Delta: ", round(delta_calc(r, S, K, T, sigma, option_type),
print("      Gamma: ", round(gamma_calc(r, S, K, T, sigma, option_type),
print("      Vega : ", round(vega_calc(r, S, K, T, sigma, option_type), 3
print("      Theta: ", round(theta_calc(r, S, K, T, sigma, option_type),
print("      Rho  : ", round(rho_calc(r, S, K, T, sigma, option_type), 3)
```

```
Option Type:  Put
      Price:  23.9
      Delta:  -0.337
      Gamma:  0.004
      Vega :  2.192
      Theta:  -0.015
      Rho  :  -2.263
```

Additional Notes

- This implementation uses the standard Black-Scholes model assumptions
- Assumes European-style options
- Does not account for dividends
- Requires further validation for real-world trading

References

- Black, F., & Scholes, M. (1973). The Pricing of Options and Corporate Liabilities
- Hull, J. C. (2017). Options, Futures, and Other Derivatives