Set Operations: Union and Intersection Cheat Sheet

Definitions

- **Union**: $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$
- **Intersection**: $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Key Properties

General Laws

Property	Union	Intersection
Commutative	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Associative	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Idempotent	$A \cup A = A$	$A \cap A = A$
Identity	$A \cup \emptyset = A$	$A \cap U = A$ (U = universal set)
Domination	$A \cup U = U$	$A \cap \emptyset = \emptyset$

Distributive Laws

1. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
2. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Absorption Laws

- $A \cup (A \cap B) = A$
- $A \cap (A \cup B) = A$

Proof Strategies

Equality Proofs

To prove $X = Y$:

1. Show $X \subseteq Y$
   (Assume $x \in X$, prove $x \in Y$)
2. Show $Y \subseteq X$
   (Assume $y \in Y$, prove $y \in X$)

Subset Relations

To prove $A \subseteq B$:
Assume $a \in A$, then show $a \in B$.

Proof Examples

1. Distributive Law Proof: $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

> **Proof**
> **(i) Show $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$**
> Let $x \in A \cup (B \cap C)$.
> Then $x \in A$ **or** $x \in B \cap C$.
>
> - If $x \in A$, then $x \in A \cup B$ and $x \in A \cup C$, so $x \in (A \cup B) \cap (A \cup C)$.
> - If $x \in B \cap C$, then $x \in B$ and $x \in C$. Thus $x \in A \cup B$ and $x \in A \cup C$, so $x \in (A \cup B) \cap (A \cup C)$.
>
> **(ii) Show $(A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$**
> Let $x \in (A \cup B) \cap (A \cup C)$.
> Then $x \in A \cup B$ **and** $x \in A \cup C$.
>
> - If $x \in A$, then $x \in A \cup (B \cap C)$.
> - If $x \notin A$, then $x \in B$ (from $x \in A \cup B$) and $x \in C$ (from $x \in A \cup C$). Thus $x \in B \cap C$, so $x \in A \cup (B \cap C)$.

> $\therefore$ Equality holds by double inclusion. ■

2. Absorption Law Proof: $A \cap (A \cup B) = A$

> **Proof**
> **(i) Show $A \cap (A \cup B) \subseteq A$**
> Let $x \in A \cap (A \cup B)$.
> Then $x \in A$ **and** $x \in A \cup B$. Thus $x \in A$.
>
> **(ii) Show $A \subseteq A \cap (A \cup B)$**
> Let $x \in A$.
> Then $x \in A \cup B$ (since $A \subseteq A \cup B$), so $x \in A \cap (A \cup B)$.
>
> $\therefore$ Equality holds by double inclusion. ■

De Morgan's Laws (Sets)

- $(A \cup B)^c = A^c \cap B^c$
- $(A \cap B)^c = A^c \cup B^c$

Tips for Proofs

- **Membership Tables**: Verify identities by enumerating cases.
- **Venn Diagrams**: Visualize relationships (but not formal proofs).
- **Logical Equivalence**: Convert $\cup$ to $\vee$ and $\cap$ to $\wedge$ in logic statements.