2019

Child Robot Interaction System

A CROWDSOURCING COMPETITION

UMAIR SARWAR, ANA VILLANUEA

Scenario and Game Theory Contest Descriptions

Scenario:

Current technology is almost entirely targeted towards evaluating human robot interactions in different environments. However, there is an emerging trend towards introducing the new generations growing up to the concept of robots. The main difference between children and adults is that children have a different, immature cognitive development. Pretend play and anthropomorphisation are relevant to the ability of children to engage with robots and treat them as life-like agents. While a lot of these robots already exist in the market, particularly catered towards STEM education and teaching programming, there is almost nothing that caters to using the senses of touch and smell via Haptic Feedback. Making use of these senses can integrate a whole new experience in robots, that can make the experience of interaction more real for the children as well as engage the children who are visually and auditory impaired. A stakeholder/sponsor is interested in developing this Haptic technology and integrating it into children robots that would increase the experience for the children and cater to a larger set of children. Cost of descaling Haptic feedback , feedback accuracy and integration are key to the user experience as a higher efficiency in all of these means that the learning experience is enhanced.

In this project, the objective is to set up and model the open crowdsourcing contest that the sponsor is conducting to develop such integrated robots and technology. The effect of the seeding amount on the possible solutions will also be considered in this decision scenario. This will be like a startup seeding done in cells where the initial project development, ideation and prototyping will be assessed to make a final decision and allocate the funds.

Contest:

The stakeholder calls on proposals from startups or non-profit organizations to develop the technology to procure robots especially targeted towards children STEM to children. After several rounds to eliminate many applicants, the stakeholder has decided to go with two startups that promise to deliver more effective, accessible, and cheap haptic feedback to children, while teaching them about STEM principles. The seeding will go to the startup that is capable of delivering scalable and deployable robotics that includes a haptic feedback system.

In this case, both startups will maximize payout, which equal the prize money and discounts the money that will go towards research and development. We are assuming that both companies are aware and have access to cutting edge technology to achieve their objectives. Both firms are also encouraged to look outside of the company and bring in researchers and partners from universities or research laboratories as needed, in order to provide new solutions and a testing bed of the technology.

This type of contest provides us with the perfect scenario to model mathematically, while including a simultaneous game, in which each startup is not aware of the competitor's solution (and partners brought into the project) until the game is over. Thus, we are deciding on a discrete number of participants (2 startups), a final event (final decision of the stakeholder), and the outcome (seed money).

STRATEGIC DECISIONS:

DECISION MAKERS:

1. The primary decision makers in this are the startups and nonprofits that are participating in the contest. They have to make various decisions ranging from investments, ideation, prototyping, etc.
2. The secondary decision maker is the sponsor (joint team from NSF and Department of Education) who are running the seeding contest and deciding who will get the seed money and how much.

DECISION MAKER'S OBJECTIVES:

1. The main objective of every organization bidding for the seed money is to maximize the money allocated to them for development and mass production. That means a higher monetary payoff for everyone involved, more budget to spend and develop the technology. They also have to be careful of the amount they are spending initially on the technology development as it can be sunk cost if they don't end up getting the funding. They also have to maximize their probability of winning the contest and the seeding, so they have to present the best possible solution while minimizing cost.
2. The objectives that the sponsor would be looking for is the quality and cost of the robotic integration presented and developed. Since these are children robots, children have a short span with the toy so the solutions can be too expensive that parents can't afford, yet they have to achieve their goal of inducing STEM skills via integrating Haptic feedback to the children. The quality will be assessed using the cost and the compactness of the Haptic feedback system generated.

VARIABLES AND CONSTRAINTS:

The variables to consider for the startups and on profits are very basic: cost of investment that will go into developing the technology. In the final stage, both the startups would be looking to spend a minimum amount yet present the best solution they can so if they don't win, money doesn't get wasted. They also have to be able to present how they will reduce the cost of the system itself and be able to sell the best, yet cheapest Haptic solution.

The strategic variables for the sponsor include which firm wins the seeding amount based on the solution they present and how good it is. The sponsor is a secondary decision maker. The decision making relevant to game theory is applicable primarily to the two startups that are competing for the seeding amount. This money and what the sponsor are offering is just being used as a benchmark of how much the firms would be willing to spend and how they manage the budget allocations. Hence, the seeding amount is important to the primary decision makers in the long run and is being used in the game theory modelling.

INTERDEPENDENCIES AMONG DECISIONS:

The game is played simultaneously by two startups, in which each startup is not aware of the competitor's technological solution and available seed money until the final decision is made (i.e., until the deadline of the proposal). Thus, each startup is kept in the dark as to what the other is doing in terms of innovation and allocation of resources. We are assuming that each startup is its own rational decision maker and that each have available cutting-edge technology or at least access to research partners to bring into the company for this last round to make their proposal more competitive.

In this contest, interdependency is framed relative to the prize money. In the case of the startups, the startup that demonstrates the most compact robotic haptics system will the winner of the seed money. In the case of a tie or negligible difference between the startups, we will consider the level of engagement and learning obtained from their students. Thus, the showcase of prolonged user studies in nurseries or classrooms will also be a determining factor in deciding which startup will obtain the seed money.

To determine how much research and development will be invested towards developing a compact robotic haptics system we will have to make several assumptions, based on available haptics system being deployed into robotics and what the latest compact technology coming out (primarily from research labs) in renowned human-computer interaction conferences. Then, we will provide estimates on how much money would it cost to develop such technology based on past investments into children robotics for STEM education. As we move forward into modeling these investments, we will provide 2 sources from which we choose our estimates: (a) money invested into STEM education, (b) money invested into haptic robotics systems. To account for the technology that the startups have to develop, we consider a combination of both areas for investment (i.e., STEM education and haptic robotics). The compactness of the system will be modeled against the seed money to produce a predictive relationship between the seed money that the startups will invest towards technological innovation (R&D) and the improvement in compactness of their systems based on what is available in the market. Figure are provided in Appendix A of the plots generated from research data. We also assume that a market research was previously done by each startup and that both have access to this data when deciding how to invest the seed money and propose their solutions to the proposal.

MATHEMATICAL MODELS

VARIABLES OF STRATEGY SPACE

Strategic variables for the two startups: investment cost from the seed money (C). Every startup decides how much money from the seed money will go into R&D.

Strategic variables for the stakeholder: Prize seed money(Π). The stakeholder decides how much the prize seed money will be offered so that the startups come up with the best technology or make strategic alliances.

OBJECTIVES OF DECISION MAKERS IN TERMS OF STRATEGIC VARIABLES

Design outcome (q): The criteria to select the haptic feedback device will be with respect to compactness (c) of the system, while enabling 3DOF rotation and translation of its joints. By these standards we decide that the function of design solution (q) is determined from the increase in the compactness of a fully rotational and translational system:

$$q(C) = c(C) = \alpha * \ln(C) + \beta,$$

where α = 3.1932 and β = 1.3326.

We used *Haptipedia,* the first haptic database of prototype and commercial haptic systems to estimate the relationship between the translational size and rotational degrees of freedom (compactness ratio) and the investment in millions. In our case, we use the compactness ratio and number of motors to estimate the relationship between compactness and scaled investment in millions of dollars, as described in Appendix A.

We calculated the equation for the design solution by finding an equation based on the trendline of the graph from compactness ratio and investment in millions. The stakeholder will select the startup winner of the contest based on increased compactness (c) of the haptics system. In this scenario, the strategy space for investment of the prize seed money is as follows: because we are expecting each startup to invest in terms of improving compactness, the strategy space for this cost needs to reach a compromise value such that the increase in compactness is ensured and delivered. This means that the increase in the compactness needs to be a positive value--greater than zero--to be competitive. It follows that since we need to model the compactness as a value greater than zero, we have the following equations:

$$\alpha * \ln(C) + \beta \geq 0$$

and

$$C \geq e^{-\beta/\alpha}$$

With these parameters, we can determine an upper bound and lower bound. In this scenario, we know that the startups will have to invest in R&D an amount lower than the prize seed money. This will be our upper bound: $C \leq \Pi$, where C is the cost that each will incur with R&D and Π is the prize seed money given to each startup. The startups cannot spend more money in R&D than the seed money they receive, because if they do so, then the participation would prove non profitable for them. Thus, our strategy space falls within the following bounds: $[e^{-\beta/\alpha}, \Pi]$. We determine that $\alpha = 3.1932$ and $\beta = 1.3326$, and $e^{-\beta/\alpha} \approx 0.659$

OBJECTIVE FUNCTIONS/EXPECTED PAYOFFS (Π_i) BY EACH STARTUP

Each startup will aim to optimize its objective function based on the stakeholder's decision to evaluate the startup technological design solution. The function will be highly dependent on the design quality of the startup and the expected solution by the opponent startup. In the case of the first startup, the objective function will aim to optimize the payoff as illustrated in:

$$\Pi_1(C_1,C_2) = \Pi * Pr(q_1 * C_1 > q_2 * C_2) - C_1,$$

where Π is the seed money prize to be given by the stakeholder in the contest outcome, $Pr$is the probability of the startup of winning the contest, and C_1,$C_2$are the strategic variables (R&D investment costs) for startups 1 and 2, respectively.

In the case of the second startup, the objective function will aim to optimize the payoff as illustrated in:

$$\Pi_2(C_1,C_2) = \Pi * Pr(q_2 * C_2 > q_1 * C_1) - C_2$$

For these scenarios, the probability of winning for each startup can be modeled using a contest success function as determined by the following expressions:

$$Pr(q_i > q_j) = \frac{q_i}{q_i + q_j} = \frac{\alpha * ln(C_i) + \beta}{\alpha * (ln(C_i) + ln(C_j)) + 2\beta},$$

With this optimization process, which serves to increase the design quality of the solution and the probability of winning the seed money prize. A binary approach to the two available probabilities for each startup can be treated as follows: winning can be modeled as a continuous function instead of a 1 value if $q_i > q_j$ or a 0 value. To validate these modeled function, we have to review the assumptions we have about the contest. For example, we know that each startup is blind to what each competitor is choosing for a design solution and to the amount of money that each will invest in R&D for the solution. With these parameters, we need to model that the probability of one startup of winning the seed money is actually a function with respect of what the other startup is proposing as a design solution. This parameter of design solution quality is decisive in predicting which startup will win the seed money from the stakeholder. Other parameters can be considered negligible or not important to be modeled in order to predict the winning startup. Thus, our parameters in terms of the design solution quality is linearly dependent on the probability of winning, since we can arrive to this value by predicting this linear relationship from our design solution quality, q. We can model the probability of one startup of winning the seed prize money with the following equation:

$$Pr(q_i > q_j) = \frac{f(q_i)}{f(q_i) + f(q_j)},$$

in which f is a function of q, the design quality solution, with respect to the probability of a startup winning the seed money prize. In our scenario, given the linear relationships between quality of design solution and the probability of winning, we can simplify the function equals the quality of design, thus $f(q) = q$. We make this assumption of only prioritizing quality of design to determine the probability of a startup winning, because the other determining factor--levels of children's engagement with the haptics robot--is only used in case of a tie between both startups, which would be highly unlikely and would require a large investment by the startups to prototype and test among children subjects. Such an extra investment falls beyond the scope of the proposal each startup is prepared to present to the stakeholders. Any other factors, biases, preferences, observations to be made or presented by the stakeholders cannot be anticipated, thus cannot be taken into consideration in terms of our mathematical modeling. We can assume that any external criteria for choosing a design made by the stakeholders can only be speculated by the startups, if taken into consideration at all.

BEST REPLY CORRESPONDENCES

Best response for a firm is the strategy that is best for them in all cases, leading to both the competitors reaching a Nash equilibrium. In the case of our example, the best response function will be of the form invest an amount in the R&D and prototype development which is for sure less than the maximum payout and high enough to win a good chunk of the distribution. The startups can maximize their payout by putting in a lot of work into the system and development, which in turn ensures highest chances of winning the see money. The risk however is that what happens if the startups' solution is not the best and they lost money. So it is a tradeoff and with the goal to maximize their overall payouts, the best reply correspondence will be driven. Since we already know that the maximum and minimum of the investment they can make is the same for both, we can start by deriving one best response function and assume that others will be identical to it and symmetric functions.

With firm 1, the BR_1can be derived with the basis of the argument as below. Our goal is to maximize the profit so:

$$Q_1^* = \frac{argmax}{Q_1}(Q_1,Q_2)$$
$$\pi_1\left(C_1, C_{2(fixed)}\right) = \Gamma \Pr\left(q_1 * C_1 > q_2 * C_2\right) - C_1$$
$$\frac{\partial \pi_1}{\partial C_1} = 0$$
$$\frac{\partial \pi_1}{\partial C_1} = \pi \frac{\partial}{\partial C_1}\Pr(q_1 C1 > q_2 C2) - 1$$
$$C_1 = BR_1(C_2)$$

Using the *diff* and optimization toolbox in Matlab, this analytical model of the equation was solved in numerical terms by putting in values. The next steps were followed with the assumptions that both the derived functions will be symmetric.

$$\boldsymbol{BR_1}(\boldsymbol{C_2}) = \boldsymbol{BR_2}(\boldsymbol{C_1})$$

As it can be observed from the differential equation above, the difference between the two best response functions is similar with the probabilities just switching around and since they are dependent on each other, that makes the function identical. However, all the difference and change in response shape comes from how the payout (*Π*)reacts and changes the response and slopes of the best response functions. To help analyze the effect of the probable seed money being provided by the investor, different payouts ranging from 50k to 200k were evaluated over 25k increments and analyzed in Matlab. The figure below shows the best response function of player 1 with regard to the strategy of costing and investment of player 2.

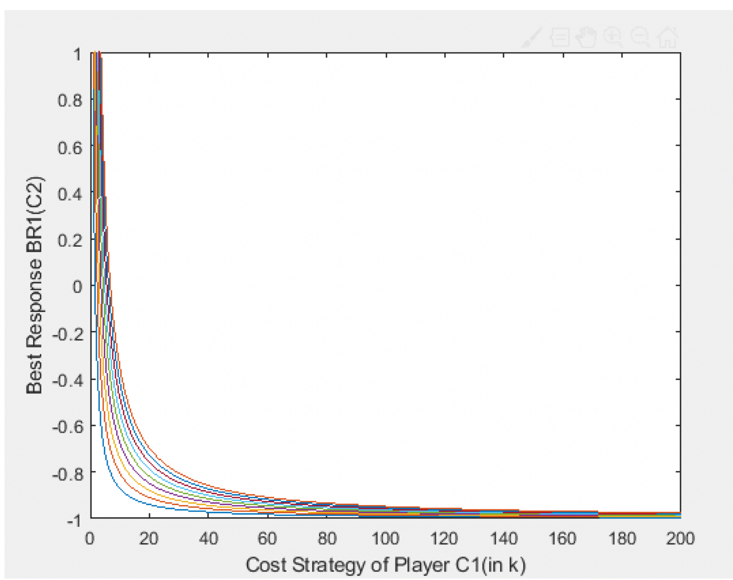


Figure 1.Best Reply for 1 wrt to 2

The graph in Figure 1 can be read in terms of increasing direction following the direction of the arrow shown. It intuitively makes sense as well as when the amount of cost investment from player 2 increases on the x axis, the best response output for player 1 also increases and hence will in turn increase outcomes and improve the Nash equilibria which will be discussed later on.

This representation of outcome of BR of player 1 with regards to player 2 has also been discussed and evaluated through the use of a contour to see the general effect. Figure 2 below shows the contour plot for such a case and you can see that with increasing payout moving to the right, the general area of efficient and profitable investment increases as the area on the contour graph tends to become bigger with expected payout increase. Hence, companies can invest more to be able to maintain a chance to win.

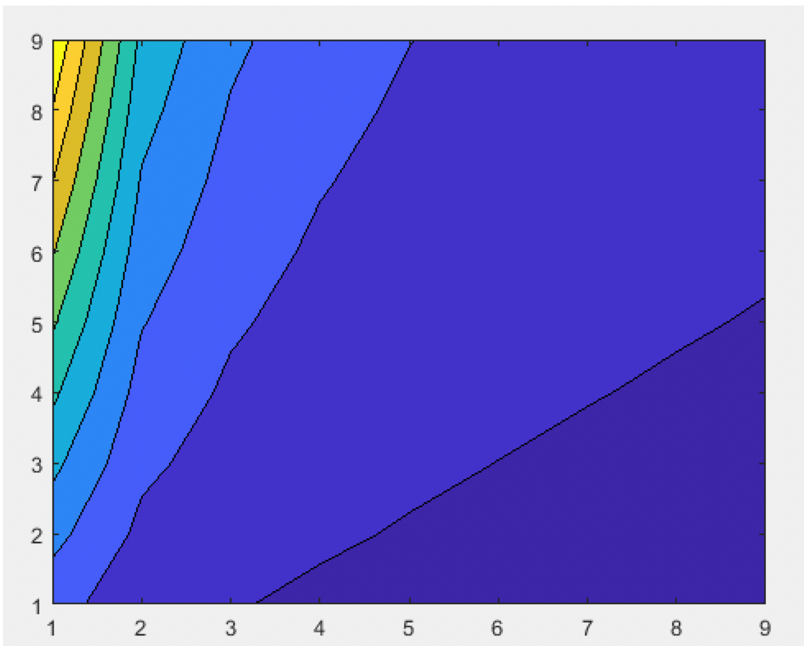


Figure 2. Contour Graph for Best Reply of 1 wrt to 2

Nash Equilibrium

The best response functions that were generated above can be used to get the Nash equilibrium of the two startups competing for the seed money. As shown by the two plots above that the amount of seed money has a great impact on the expected investment of the company. Higher the payout set by the stakeholders--NSF in our case--the more the startups would be willing to invest to bid for that number and the Nash equilibrium can be used to obtain these payouts at each prize money that the startup would aim for. Plotting the best response functions of the two startups together would help obtain the Nash equilibrium which would be the point of intersection of the two curves with each other. For showing how the Nash equilibrium adjusts value with payout, Nash equilibrium was found at five randomly chosen increments of the payout increase and can be seen in the images presented below.

As for the approach, as both the best response functions were symmetric,

$$BR_1(C_2) = BR_2(C_1)$$

And

$$C_1^* = BR_1(C_2) \& \; C_2^* = BR_2(C_1)$$

So we can say ,

$$C_1^* = C_2^*$$

Using the above logic, the best response function and cost of investment by firm one was applicable and made to be in terms of the investment of itself to simplify the equation. Using the simplified relations and the same procedure of numerical values in matlab, the list of values was plotted against each other over the maximum payoff they can obtain set to 200k. The graphs below from Figure 3 to Fig 6 show the Nash equilibria in such cases of four payouts that were investigated.

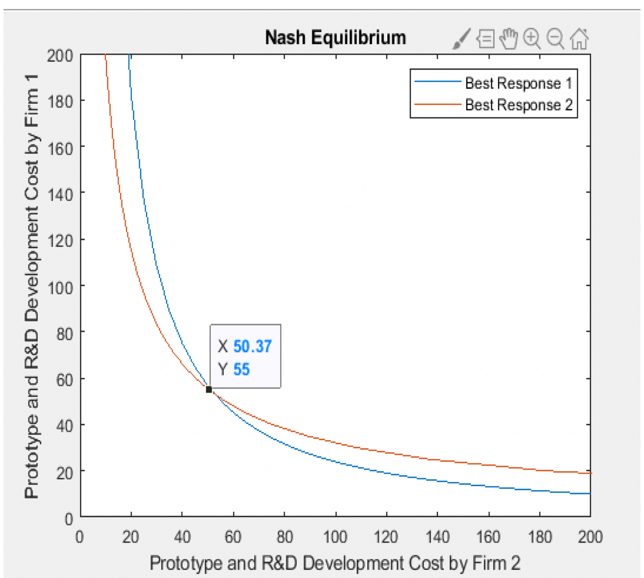


Figure 3.Nash Equilirbium for 50k payout

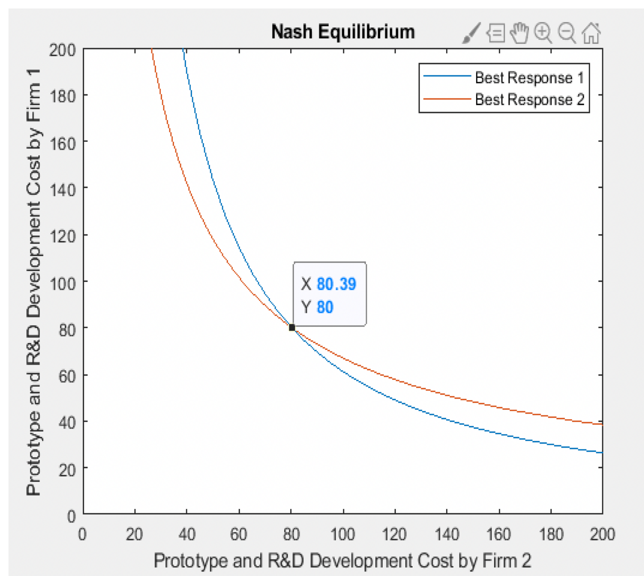
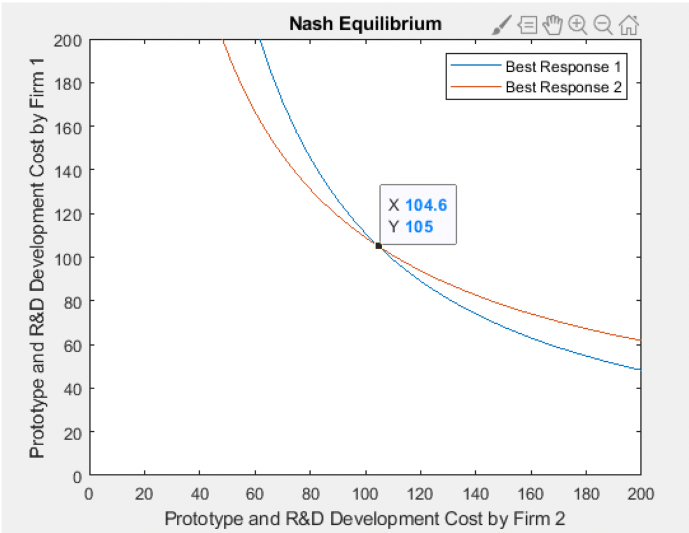


Figure 4.Nash Equilbirum for 100k Payout



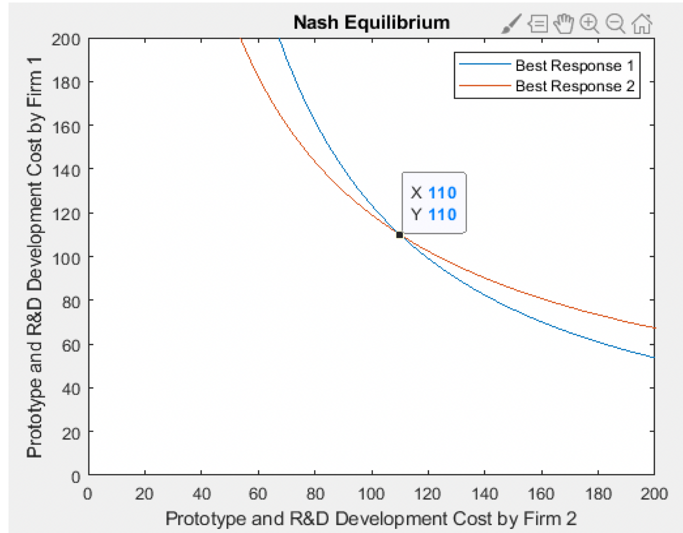


Figure 5.Nash equilibrium for 175k Payout

Figure 6.Nash Equilibrium for 200k Payout

As can be seen from the Nash equilibria in the images above, considering a varied prize out rate, the value of the Nash equilibrium changes. The higher the seed money on offer by the stakeholder, there is more incentive for the company to invest in development. Seeing and analyzing the 200k highest payout, the maximum value they can invest for is 200k and that logically makes sense as the value can be split amongst the two so they might want to try just another extra 10k to get a 55:45 share and gain an upper hand of some sort but in the case that they do not win, they do not want to have a higher loss. Hence, the best outcome for them becomes $C_1^* = C_2^* = 110k$ when the payout is about 200k.

Seeing the varied and increasing Nash equilibrium outcome from min to max payout, the results were also plotted to see the effect. As can be seen from the graph below, as you increase the payout amount, the value of the outcome of the equilibria increases. The increase with a linearly best fit equation with the equation y=0.3635x+40.544(Fig 7.). With the positive gradient of the equation, you can suggest that the higher the payout, the more there is incentive for the companies to invest in the development of the prototype.

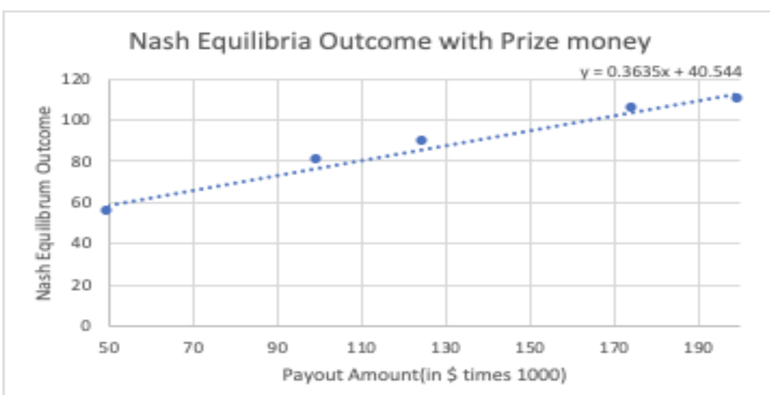


Figure 7.Effect of Payout

Nash Equilibria vs. Pareto Optimal Solutions

The pareto-optimal solutions we calculated demonstrate that there are not any available solutions that are good for one startup while having negative connotations for the other. The way we have positioned the game means that the startups are antagonistic towards each other because it is a zero-sum game. Thus, if one startup wins the money prize, then the other loses all the possible investment. However, in terms of other possible solutions, in which the prize money can be split or the resources shared among the startups, we can come up for several different interpretations of the Pareto-optimal solutions. These different scenarios we can obtain are as follows:

1. *Sharing:* This scenario would be complicated to model, because it would signify an arrangement between the startups. Presumably, the stakeholder (NSF) has no stipulations that forbid the two startups from essentially merging but may still require both startups from submitting a proposal each. This means that each startup may present somewhat similar reports, while prioritizing the one startup where the main technological improvement came from. Thus, the reports would signify a 100%-win certainty, but the legalities of IPs being split would have to be discussed between the startups.
2. *Splitting:* Splitting the money would only be possible if the prize money were larger than 200k, because we know that under any prize the startups mean to invest less than 50k. Thus, provided that the startups still mean to make a profit, then each startup would mean to profit at least 50k. However, the more money spent, then better technology can be generated by the startups. And we can observe this by the fact that at a 200k prize, then the startups mean to spend upwards of 100k in the technology.
3. *Zero-sum game:* This scenario means that each startup is playing for winner takes all. The risk is greater in that if a startup gives more money and wins, then the other startup loses all the investment from the seed money. However, this scenario may be preferable for the startups, if each has already spent some year(s) coming up with their technology, thus their $C>0$, and they would like the investment money to move towards the production phase.

   Although this is the most likely scenario, it is difficult to model Nash Equilibria vs. the Pareto Optimal, because each startup is capable of investing equal capital to the R&D phase, at least as far as they know. In our case, Pareto Optimal solutions present too many opportunities of investment as compared to the Nash equilibria point, which is essentially the same amount for both startups. Although in real life circumstances, a tie in performance or compactness would not be very likely, in our modeling, we can assume that one startup proposes accurate force feedback or some other metric to sway the stakeholder in their direction.

The Pareto Optimal payoff are based on the weight parameters for each startup are calculated by a single expected investment function, and the way by which each weight is assigned to a startup will be calculated by the stakeholder, who possibly assigns this contest to a director. This NSF director can potentially analyze what will be the weight for each startup $w1$ and $w2$. The NSF director will be the best to distribute these weights, because he or she the best understanding of what type of technology the NSF is looking for, as well as the education impact it will have, and the educational centers it can be deployed to. Simultaneously, the director will procure that the prize money also benefits the agenda of the NSF.

Thus, the prize seed money for each startup would the following:

$$\pi_{pareto}(C_1, C_2, w_1, w_2) = \mathrm{w}_1(\Pi \Pr(q_1(C_1) > q_2(C_2)) - C_1) + \mathrm{w}_2(\Pi \Pr(q_2(C_2) > q_1(C_1)) - C_2)$$

Then, our Pareto Optimal payoff can be calculated with the following:

$$C_{1,pareto}(w_1, w_2), C_{2,pareto}(w_1, w_2) = \underset{C_1, C_2}{\operatorname{argmax}} \, \pi_{pareto}(C_1, C_2, w_1, w_2)$$

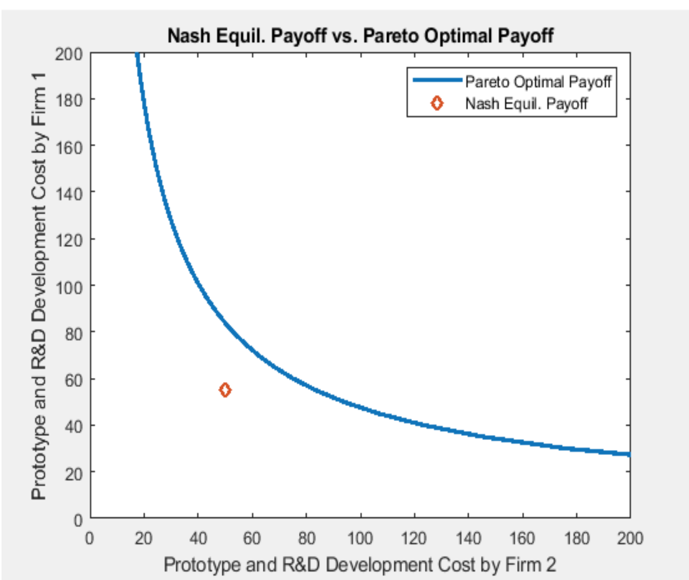


Figure 8.Pareto Optimal for 50k Payout

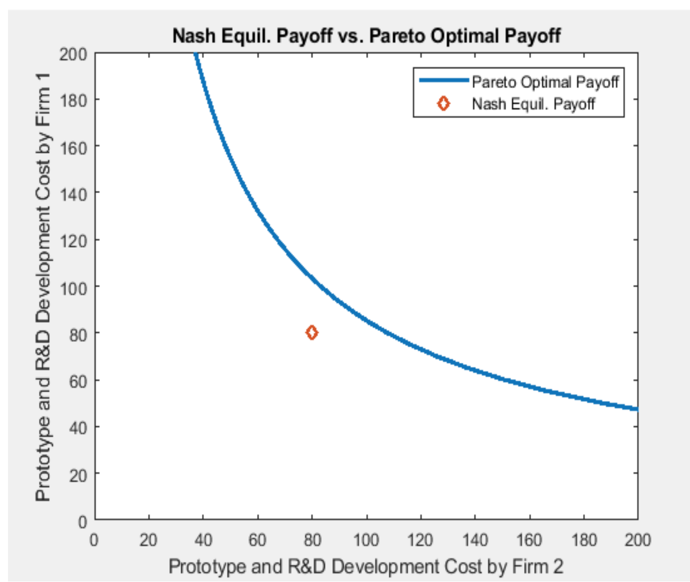
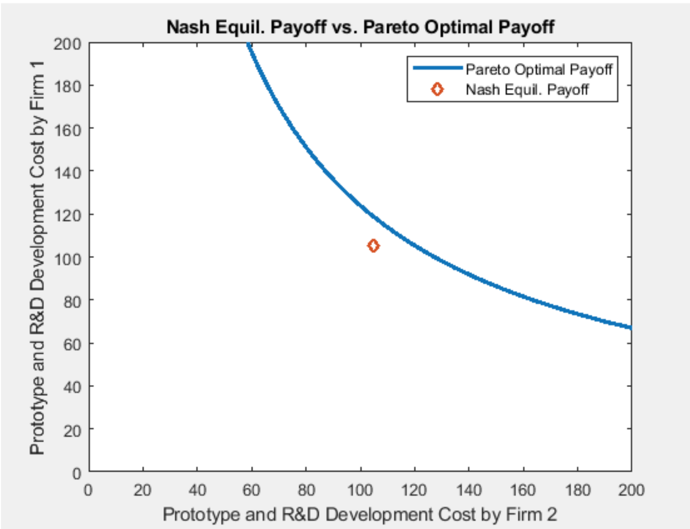


Figure 9.Pareto Optimal for 100k payout



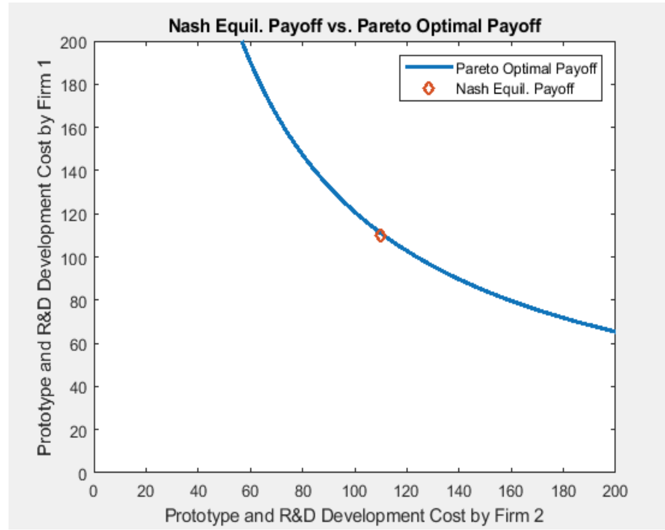


Figure 10.Pareto Optimal for 175k Payout

Figure 11.Pareto Optimal for 200k Payout

The Pareto Optimal payoff compared to Nash equilibria for every prize money are shown from Figure 8 to Fig 11. We observe that when $w1 = w2 = 0.5$, this means then that both startups are valued the same, thus the Pareto Optimal payoff should be similar for both. Now, we assume that when $w1 < 0.5$ and $w2 > 0.5$, which means that a startup is valued less than the other startup, then the undervalued startup should invest the least amount as to be able to participate in the contest. However, for the startup with the highest value, this means that the higher the investment in the R&D, then the more of a certainty that this startup will win the contest.

Discussion

In such a scenario where two firms are competing for the seed money to be split between them, Nash equilibrium as well as the Pareto Optimal solution showcased a variety of interesting solution. Looking into the Nash equilibria first, it has been seen that the Nash equilibrium is stable and tends to increase as the payout increases. The investigation was done for an increasing payout, which will be known and the equilibrium point generally increases as the payout increases. Even with the maximum payout, the amount only goes up to a little above the half and that is for the aim to win a little more. After that, the companies do not find it worthy or safe enough to invest. Further investigation into the domains of the trend in Nash equilibrium showed that the equilibrium point increases linearly with the known payout. However, as has been told before that the money was supposed to be split so we can definitely expect the Pareto Optimal solution to provide us with the best option of collaboration and split of money and investment between the two startups. The pareto optimal solution, however, tends to seem to go towards saturation as the payoff becomes higher. That means that the higher the payoff, the Nash equilibrium and the Pareto optimal solution tend to start converging as can be seen from the 200k payout and they intersect. This can be explained with the Nash equilibrium as for that payout, both the firms aimed for a little more than 50:50 split of the money. As to investing to a total of 55 percent in the aim to justify spending a little more for a better shot at winning. Above that, they don't think it is worth it enough and that is where the Pareto Optimal and Nash equilibria intersect .

As it has been shown above, the data or the model is highly sensitive to the amount of payout. It would be an interesting game of competition where users are given a range as big as 150 k and then asked to compete. Since the model seems to be so receptive to the amount of payout (linear relation shown above), it would be nice to investigate the incentive and possibility of collaboration, if there may exist between the firms. Since they won't know for sure what the payout could be, would they be better of collaborating their expertise in that case and splitting the money accordingly, at least they would be somewhat sure shot of getting a substantial compensation for the work done rather than risking losing to a better technology and getting much less.

The equilibria and the modelling equation of the crowdsourcing competition presented do have a lot of assumptions that could generally shift the situation of the Nash equilibrium in each of the payout. Right now, the competition modelled was only to check for compactness that the device was offering. They were only being evaluated on how well they could "compact down" the solution to make is sizeable and cheap enough. However, it has to be seen that different companies have a different advantage. Some might be better in size of the solution while the other firm might be better at producing newer Haptic textures and experiences. For a company with no prior experience to actually work on the device itself, their cost of incurring or producing a similar or closer outcome to one with experience will be more. They have to invest more in R&D of something that they are not efficient in so Nash equilibrium might actually be a much higher number than the one obtained for them to be competitive with a firm who works. Moreover, evaluation can also include the software accessibility, ease of use, cost while right now it is only based on compactness. Addition of other elements in the evaluation could affect the stability of the equilibrium and the linearity of the effect of payout with the equilibrium point. To put it in easier words, right now there is one parameter but in real life, the firms might have a higher dimensional strategy in mind they would want to get an overall advantage with as opposed to a single parameter the stakeholder wants to judge them on. So, this is a simplification from stakeholder by narrowing the scope which could put an overall company's strategy at risk.

Another general effect on the stability of this equilibrium is the fact that there is not a lot of research data that exists in such systems as of yet. The data has been applied generally from the assistive technology industry with some use of data from sensory systems being developed for sensory perceptions for blind people. A lot of assistive technology is for impaired people while the goal here is to use these to assist normal people to learn better so the data might be skewed and might change the results of the equilibrium.

CONCLUSION

The game theoretical modelling used to model the crowdsourcing competition has shown with reasonable success, how the stake holders and the firms interact. The firms can use such a strategy to see how much that can or should possibly invest in such a situation where the model for them is so highly sensitive to the amount of seed money that they will get. It seems from the situation that Pareto Optima solutions tends to give a higher utility to the firms in terms of collaboration together and gain more advantage from it, as compared to risking investments and having sunk cost on their hand.

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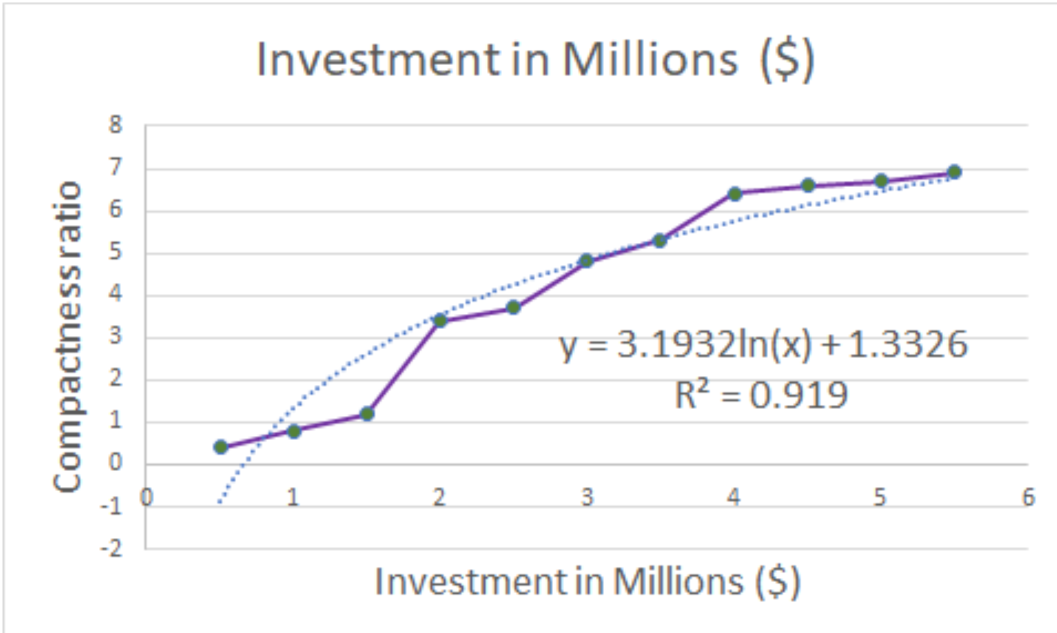
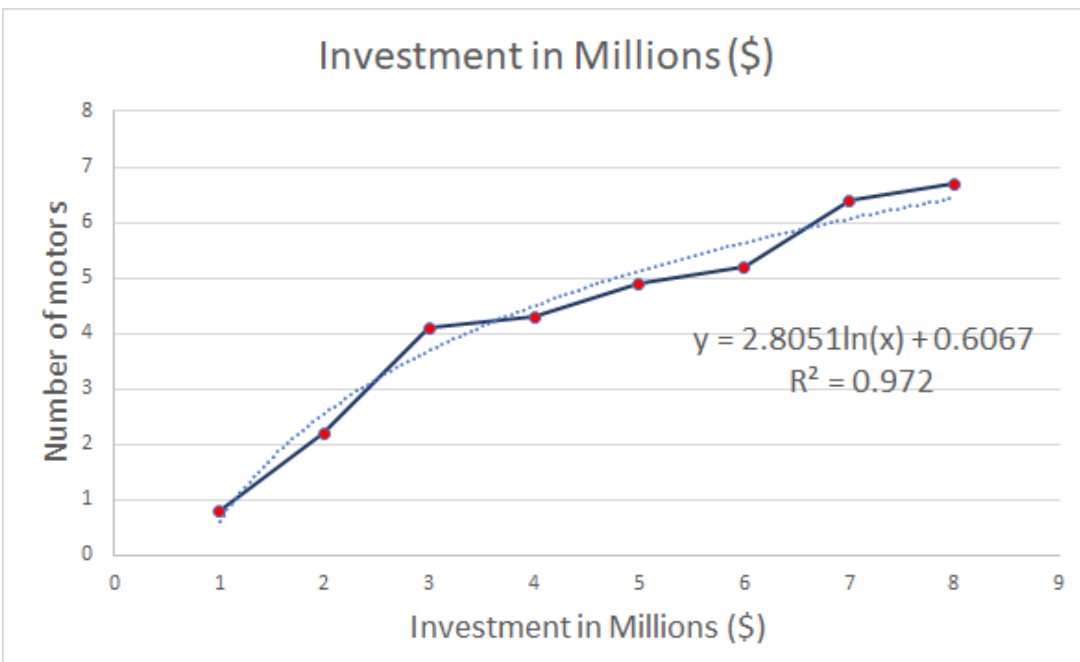
Appendix A

Compact ratio	Investment in Millions ($)
0.5	0.4
1	0.8
1.5	1.2
2	3.4
2.5	3.7
3	4.8
3.5	5.3
4	6.4
4.5	6.6
5	6.7
5.5	6.9





APPENDIX B

```
clc
clear all
close
```

```
c1=5:5:2000;
c2=5:5:2000;
payout=50000:25000:250000;
a=3.1932;
b=1.3326
list1=a.*log(c2)+b;
list2=a.*log(c1.*c2)+2*b;
list3=a.*log(c1)+b;
BR1(1,:)=(payout(1)*(a.*(list1))./(c1.*(list2).^2))-1;
BR1(2,:)=(payout(2)*(a.*(list1))./(c1.*(list2).^2))-1;
BR1(3,:)=(payout(3)*(a.*(list1))./(c1.*(list2).^2))-1;
BR1(4,:)=(payout(4)*(a.*(list1))./(c1.*(list2).^2))-1;
BR1(5,:)=(payout(5)*(a.*(list1))./(c1.*(list2).^2))-1;
BR1(6,:)=(payout(6)*(a.*(list1))./(c1.*(list2).^2))-1;
BR1(7,:)=(payout(7)*(a.*(list1))./(c1.*(list2).^2))-1;
BR1(8,:)=(payout(8)*(a.*(list1))./(c1.*(list2).^2))-1;
BR1(9,:)=(payout(9)*(a.*(list1))./(c1.*(list2).^2))-1;
BR2(1,:)=(payout(1)*(a.*(list3))./(c2.*(list2).^2))-1;
BR2(2,:)=(payout(2)*(a.*(list3))./(c2.*(list2).^2))-1;
BR2(3,:)=(payout(3)*(a.*(list3))./(c2.*(list2).^2))-1;
BR2(4,:)=(payout(4)*(a.*(list3))./(c2.*(list2).^2))-1;
BR2(5,:)=(payout(5)*(a.*(list3))./(c2.*(list2).^2))-1;
BR2(6,:)=(payout(6)*(a.*(list3))./(c2.*(list2).^2))-1;
BR2(7,:)=(payout(7)*(a.*(list3))./(c2.*(list2).^2))-1;
BR2(8,:)=(payout(8)*(a.*(list3))./(c2.*(list2).^2))-1;
BR2(9,:)=(payout(9)*(a.*(list3))./(c2.*(list2).^2))-1;

%gradient
[dx1,dy1]=gradient(BR1);
[ddx1,ddy1]=gradient(dx1);
[dx2,dy2]=gradient(BR2);
[ddx2,ddy2]=gradient(dx2);

%
%  plot(c1/1000,BR1)
%  ylim([-1 0])
%  xlabel("Cost Strategy of Player C1(in k)")
```

```
%  ylabel("Best Response BR1(C2)")
% figure
% plot(c2/1000,BR2)
% ylim([-1 0])
% xlabel("Cost Strategy of Player C2(in k)")
% ylabel("Best Response BR2(C1)")
% % figure
%contourf(BR1)
%ylim([0 9])

plot(c1,BR1(9,:),BR2(9,:),c2)
xlim([0 200])
title("Nash Equilibrium")
xlabel("Prototype and R&D Development Cost by Firm 2")
legend("Best Response 1","Best Response 2")
ylabel("Prototype and R&D Development Cost by Firm 1")

ylim([0 200])
```