

Stack & Queue

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Stack \rightarrow A stack is a list of elements in which an element may be inserted or deleted only at one end, called the top of the stack.

This means, in particular, that elements are removed from a stack in the reverse order of that in which they were inserted into the stack. Accordingly, stacks are also called Last-in-First-out (LIFO) lists.

Special terminology is used for 2 basic operations associated with stacks.

a) "Push" is the term used to insert an element into a stack.

b) "Pop" is the term used to delete an element from a stack.

Algorithm \rightarrow

PUSH(STACK, TOP, MAXSTK, ITEM)

S1: \rightarrow IF $TOP \geq MAXSTK - 1$, then

Print "Overflow" Δ return

S2: \rightarrow Set $TOP = TOP + 1$

S3: \rightarrow Set $STACK[TOP] = ITEM$

S4: \rightarrow Return

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POP(STACK, TOP, ITEM)

S1: \rightarrow IF TOP = -1, then

Print "Underflow" & Return

S2: \rightarrow Set ITEM = STACK [TOP]

S3: \rightarrow Set TOP = TOP - 1

S4: \rightarrow Return

Polish Notation: \rightarrow The process of writing the operators of an expression either before their operands or after them is called the Polish Notation. The Polish notations are classified into 3 categories.
i) Infix, ii) Prefix iii) Postfix.

i) Infix: \rightarrow When the operators exists b/w 2 operands then the expression is called infix expression.

Ex- $(a+b)*c$, $a+(b*c)$.

ii) Prefix: \rightarrow When the operators are written before their operands then the resulting expression is called the prefix expression.

Ex: $(A+B)*C = [+AB]*C = *+ABC$

$A+(B*C) = A+[*BC] = +A*BC$

$(A+B)/(C-D) = (+AB)/(-CD) = /+AB-CD$

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i) Postfix: \rightarrow When the operators come after their operands the resulting expression is called the reverse Polish notation or Postfix expression.

Ex- $A + (B * C) = A + (BC *) = ABC * +$

Evaluation of a Postfix Expression:

Suppose P is an arithmetic expression written in postfix notation.

Algorithm:

S1: \rightarrow Add ')' at the end of P.

S2: \rightarrow Scan P from left to right & repeat S3 & S4 for each element of P until ')' is encountered.

S3: \rightarrow If an operand is encountered, push it on stack.

S4: \rightarrow If an operator \otimes is encountered, then

(a) Remove the 2 top elements of stack, where A is top & B is next to top element

(b) Evaluate $B \otimes A$

(c) Push the result of (b) back on stack

S5 $\begin{matrix} \text{end of if} \\ \text{end of loop} \\ \text{Exit.} \end{matrix}$

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Ex → Consider the following arithmetic exp P, written in postfix notation:

$$P: 5, 6, 3 +, *, 12, 4, / -$$

Symbol	STACK
1) 5	5
2) 6	5, 6
3) 2	5, 6, 2
4) +	5, 8
5) *	40
6) 12	40, 12
7) 4	40, 12, 4
8) /	40, 3
9) -	37
10))	

Ex → Consider the following arithmetic exp P, written in postfix notation:

$$P: 12, 3, 3 -, /, 2, 1, 5, +, *, +$$

Evaluate P using algo.

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P: 12, 7, 3, -, /, 2, 1, 5, +, *, +

Symbol	STACK
1) 12	12
2) 7	12, 7
3) 3	12, 7, 3
4) -	12, 4
5) /	3
6) 2	3, 2
7) 1	3, 2, 1
8) 5	3, 2, 1, 5
9) +	3, 2, 6
10) *	3, 12
11) +	15
12))	

Transforming Infix Expression into Postfix Expression:

Suppose Q is an arithmetic expression written in infix notation. This algorithm finds the equivalent Postfix expression P.

S1: Push "(" onto STACK & add ")" to the end of Q.

S2: Scan Q from left to right & repeat steps 3 to step 6 for each element of Q until STACK is empty.

S3: If an operand is encountered, add it to P.

⑥ S4: \rightarrow If a left parenthesis is encountered, push it onto STACK.

S5: \rightarrow If an operator (\otimes) is encountered, then
① Repeatedly Pop from STACK & add to P each operator which has the same precedence as or higher precedence than \otimes .
② Add \otimes to STACK.
end of if.

S6: \rightarrow If a ")" is encountered, then
① Repeatedly Pop from STACK & add to P each operator until a left parenthesis is encountered.
② Remove the "C". [Don't add 'C' to P].
end of if.
end of loop.

S7: \rightarrow Exit.

Ex: Consider the following arithmetic expression Q:
$$Q: A + (B * C - (D / E) \uparrow F) * G * H$$

Using the algorithm transform Q into its equivalent postfix expression P.

<u>Soln:</u>	<u>Symbol</u>	<u>STACK</u>	<u>Expression P</u>
1>	A	C	A
2>	+	C+	A
3>	C	C+C	A
4>	B	C+C	AB
5>	*	C+C*	AB
6>	C	C+C*	ABC
7>	-	C+C-	ABC*
8>	C	C+C-C	ABC*
9>	D	C+C-C	ABC*D
10>	/	C+C-C/	ABC*D
11>	E	C+C-C/	ABC*D E
12>	↑	C+C-C/↑	ABC*D E ↑
13>	F	C+C-C/↑	ABC*D E F ↑
14>)	C+C-	ABC*D E F T /
15>	*	C+C-*	ABC*D E F T /
16>	G	C+C-*	ABC*D E F T / G
17>)	C+	ABC*D E F T / G*-
18>	*	C+*	ABC*D E F T / G*-
19>	H	C+*	ABC*D E F T / G*-H
20>)	-	ABC*D E F T / G*-H*+

- ⑧ Ex Consider the following infix expression

$$Q: ((A+B)*D)\uparrow(E-F)$$

Soln:

Symbol	STACK	Expression P
1> C	CC	
2> C	CCC	
3> A	CCC	A
4> +	CCC+	A
5> B	CCC+	AB
6>)	CC	AB+
7> *	CC*	AB+
8> D	CC*	AB+D
9>)	C	AB+D*
10> ↑	C↑	AB+D*
11> C	C↑C	AB+D*
12> E	C↑C	AB+D*E
13> -	C↑C-	AB+D*E
14> F	C↑C-	AB+D*EF
15>)	C↑	AB+D*EF-
16>)	-	AB+D*EF-↑

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Queue:> A queue is a linear list of elements in which deletions can take place only at one end called the front & insertion can take place only at other end called rear.

Queues are also called first-in-first-out (FIFO) list, since the first element in a queue will be the first element out of the queue.

Linear Queue Insertion

LQInsert(Queue, Element, Size)

S1:> If front = 0 & rear = size - 1 then
print "Overflow" & exit.

S2:> If front = -1 then
front = 0, rear = 0

S3:> Else
rear = rear + 1

S4:> Queue[rear] = element

S5:> Return.

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Deletion:

CQDelete(Queue Element)

S1: \rightarrow If front = -1 then

 print "Underflow" & exit

S2: \rightarrow Element = Queue[front]

S3: \rightarrow If front = rear then

 front = -1 & rear = -1

S4: \rightarrow else

 front = front + 1

S5: \rightarrow Return (Element).

Circular Queue:

Let we have an array Q that contains n elements in which Q[0] comes after Q[n-1] in the array. When this technique is used to construct a queue then the queue is called circular queue.

Insertion:

CQInsert(Queue, Size, Item)

S1: \rightarrow If front = 0 & rear = size - 1 or front = rear + 1

 then Print "Overflow" & exit

S2: \rightarrow If front = -1 then

 front = 0 & rear = 0

S3: else if rear = size - 1 then
 rear = 0

S4: else

 rear = rear + 1

S5: Queue[rear] = element

S6: Return.

Deletion:

CQ Delete (Queue, size, Element)

S1:→ If front = -1 then

 print "Underflow" & exit

S2:→ Element = Queue [front]

S3:→ If front = rear then
 front = -1 & rear = -1

S4:→ else if front = size - 1 then

 Front = 0

S5:→ else

 Front = Front + 1

S6:→ Return (Element)

Dequeue: A deamee is a linear list in which elements can be added or removed at either end but not in the middle. The term deamee is a contraction of the name double-ended queue.

There are 2 types of deamee.

i) Input-restricted deamee &

ii) Output-restricted deamee.

i) Input-restricted deamee: An input restricted deamee is a deamee which allows insertions at only one end of the list but also allows deletions at both ends of the list.

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i) Output-restricted deque → An output-restricted deque is a deque which allows deletion at only one end of the list but allows insertion at both ends of the list.

Algorithm to insert any element in a deque →

Insert-front() algo:

Ins-f(Deque, item, size)

S1: → If $\text{front} = 0 \wedge \text{rear} = \text{size} - 1$ or $\text{front} = \text{rear} + 1$ then
print "Overflow" & exit.

S2: → If $\text{front} = -1 \wedge \text{rear} = -1$ then
 $\text{front} = 0 \wedge \text{rear} = 0$

S3: → Else if $\text{front} = 0$ then
 $\text{front} = \text{size} - 1$

S4: → Else $\text{front} = \text{front} + 1$

S5: → Deque[front] = item

S6: → Return.

Insert-rear() algo:

Ins-r(Deque, item, size)

S1: → If $\text{front} = 0 \wedge \text{rear} = \text{size} - 1$ or $\text{front} = \text{rear} + 1$ then
print "Overflow" & exit

S2: → If $\text{front} = -1 \wedge \text{rear} = -1$ then
 $\text{front}, \text{rear} = 0$

S3: → Else if $\text{rear} = \text{size} - 1$ then
 $\text{rear} = 0$

S4: → Else $\text{rear} = \text{rear} + 1$

S5: → Deque[rear] = item

S6: → Return.

Algorithm to delete an element from the deque:

delet-front() algo:

del-F(Deque, item, size)

S1: → If $\text{front} = -1 \& \text{rear} = -1$ then
print "Underflow" & exit

S2: → item = Deque[front]

S3: → if $\text{front} = \text{rear}$ then
 $\text{front} = -1 \& \text{rear} = -1$

S4: → else if $\text{front} = \text{size} - 1$ then
 $\text{front} = 0$

S5: → else $\text{front} = \text{front} + 1$

S6: → Return(item)

delet-rear() algo:

del-R(Deque, item, size)

S1: → If $\text{front} = -1 \& \text{rear} = -1$ then
print "Underflow" & Exit.

S2: → item = Deque[rear]

S3: → if $\text{front} = \text{rear}$ then
 $\text{front} = -1 \& \text{rear} = -1$

S4: → else if $\text{rear} = 0$ then
 $\text{rear} = \text{size} - 1$

S5: → else
 $\text{rear} = \text{rear} - 1$

S6: → Return(item).

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Priority Queue: \rightarrow A priority queue is collection of elements such that each element has been assigned a priority & such that the order in which elements are deleted & processed comes from the following rules:

- ii) An elements with the same priority are processed according to order in which they were added to the queue.
- i) An element of higher priority is processed before any element of lower priority.

There are 2 types of priority queue.

i) Ascending Priority Queue: \rightarrow An ascending priority queue is a collection of items into which items can be inserted arbitrarily & from which only the smallest item can be removed.

A queue may be viewed as ascending priority queue whose elements are ordered by time of insertion. The element that was inserted 1st has the lowest insertion time value & is the only item that can be retrieved.

i) Descending Priority Queue (15)
An descending priority queue is a collection of items into which item can be inserted arbitrarily & from which only the largest item can be removed.

A stack may be viewed as descending priority queue whose elements are ordered by time of insertion. The element that was inserted last has the greatest insertion time value & is the only item that can be retrieved.