Is Optimal Control a Panacea?

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University of Southern California

Now in University of Washington

Is optimal control a panacea?

Outline

Part I Model Based Nonlinear Optimal Control.

Constraint Iterative optimal control methods

Application on the neuromuscular model of the index finger

Conclusions.

Part II Path Integral Control Beyond Linearization and Internals Models.

Path Integral Stochastic Optimal Control

Application to a variety of dynamical systems

Conclusions.

Part III General comments about control under uncertainty.

Part I:
Model Based Nonlinear Optimal Control

Index Finger Biomechanics

Flexor Digitorum Superficialis(FDS)

Flexor Digitorum Profundus (FDP)

Extensor Communis(EC)

Extensor Indicis(EI)

Radial Interosseous(RI)

Ulnar Interosseous(UI)

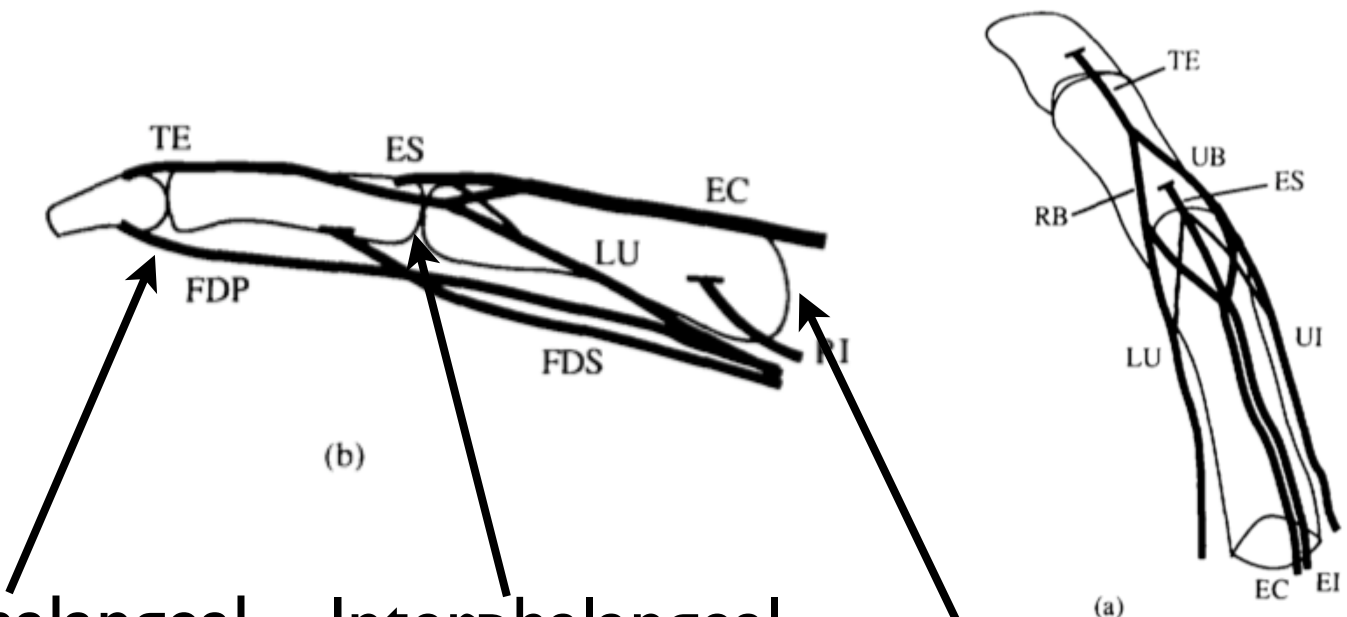
Lubrical(LU)

Terminal Extensor(TE)

Radial Band(RB)

Ulnar Band(UB)

Extensor Slip(ES)



Interphalangeal Distal

Interphalangeal Proximal

Metacarpophalangeal Proximal

N. Brook, J. Mizrahi, J. Dayan, 1995

Index Finger Model

$$\mathbf{T} = \mathbf{M}(\theta)\ \mathbf{F}$$

#torques x #tendons

$$\ddot{\boldsymbol{\theta}} = -\mathbf{I}\left(\boldsymbol{\theta}\right)^{-1}\left(\mathbf{C}\left(\boldsymbol{\theta},\dot{\boldsymbol{\theta}}\right) + \mathbf{B}\left(\boldsymbol{\theta}\right)\dot{\boldsymbol{\theta}} + \mathbf{T}\right)$$

$$\dot{\mathbf{F}} = -\frac{1}{\tau}\left(\mathbf{F} - \mathbf{u}\right)$$

$$J(\mathbf{x},\mathbf{u}) = \phi(\mathbf{x}(t_N)) + \int_0^{t_N} \mathbf{u}^T \mathbf{R} \mathbf{u} dt$$

E.A. Theodorou and FJ.Valero-Cuevas. EMBS2010

E.A. Theodorou, E. Todorov, and Francisco J.Valero-Cuevas. ASME 2010

E.A. Theodorou, Y. Tassa and E. Todorov. (ACC-2010).

E.A. Theodorou, E. Todorov, FJ Valero Cuevas. (ACC-2011)

Valero-Cuevas, F. J., Hoffmann, H., Kurse, M. U., Kutch, J. J., and Theodorou, E. A., 2009.. IEEE Reviews in Biomedical Engineering,

Nonlinear Stochastic Optimal Control

Given an initial control u(t), generate a trajectory x(t).

Start

Goal

Step 1

E. Tododorov, W. Li ACC, ILQG, 2005.

D.H. Jacobson, D.Q. Mayne, Differential Dynamic Programming, 1970.

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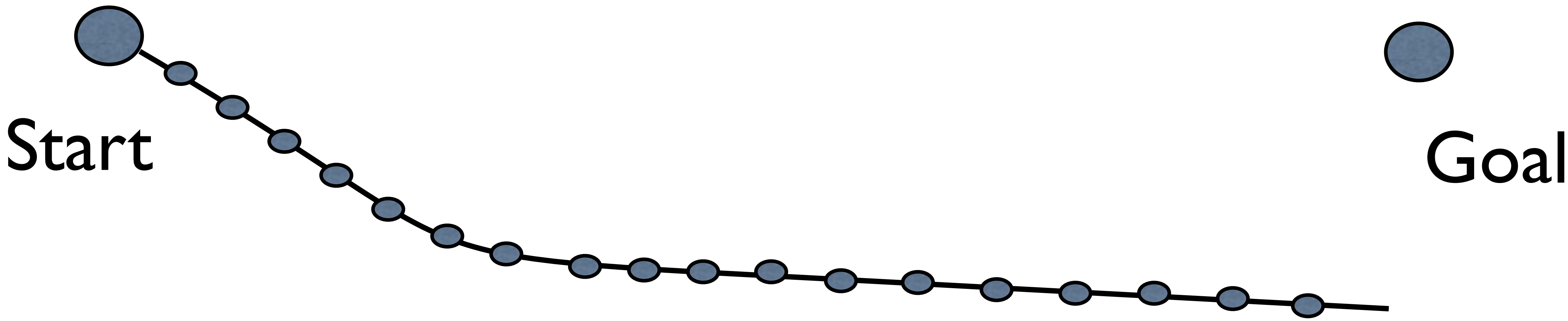

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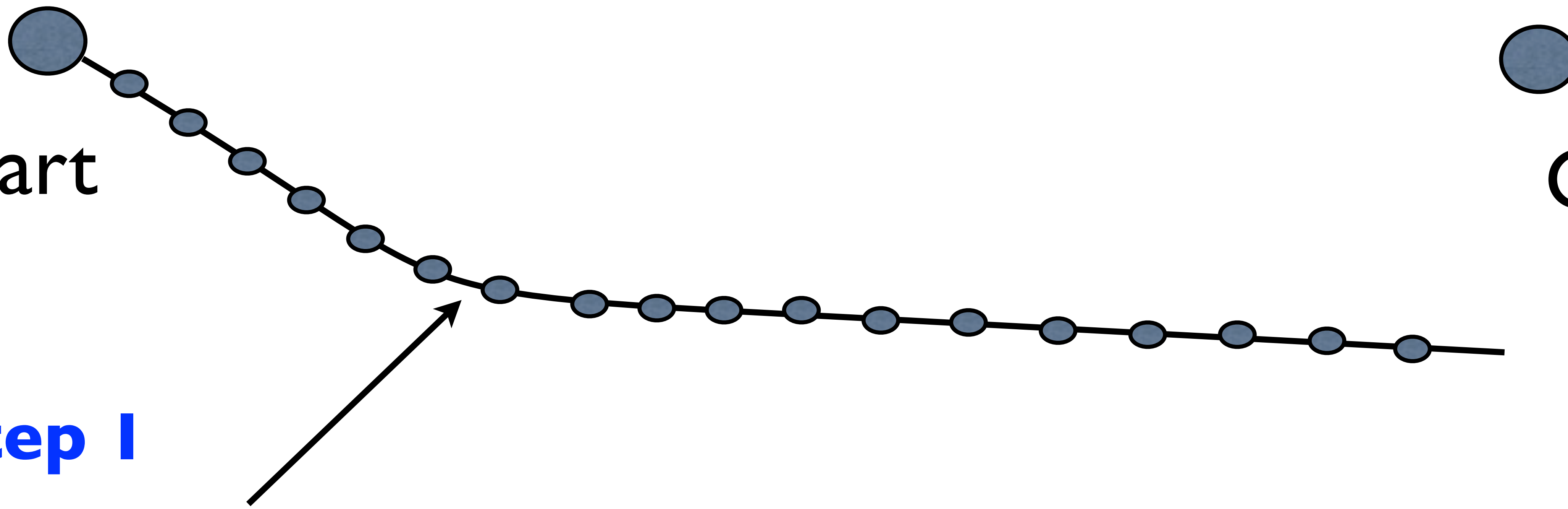

Goal

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Linear
Approximation
dynamics

E. Tododorov, W. Li ACC, ILQG, 2005.

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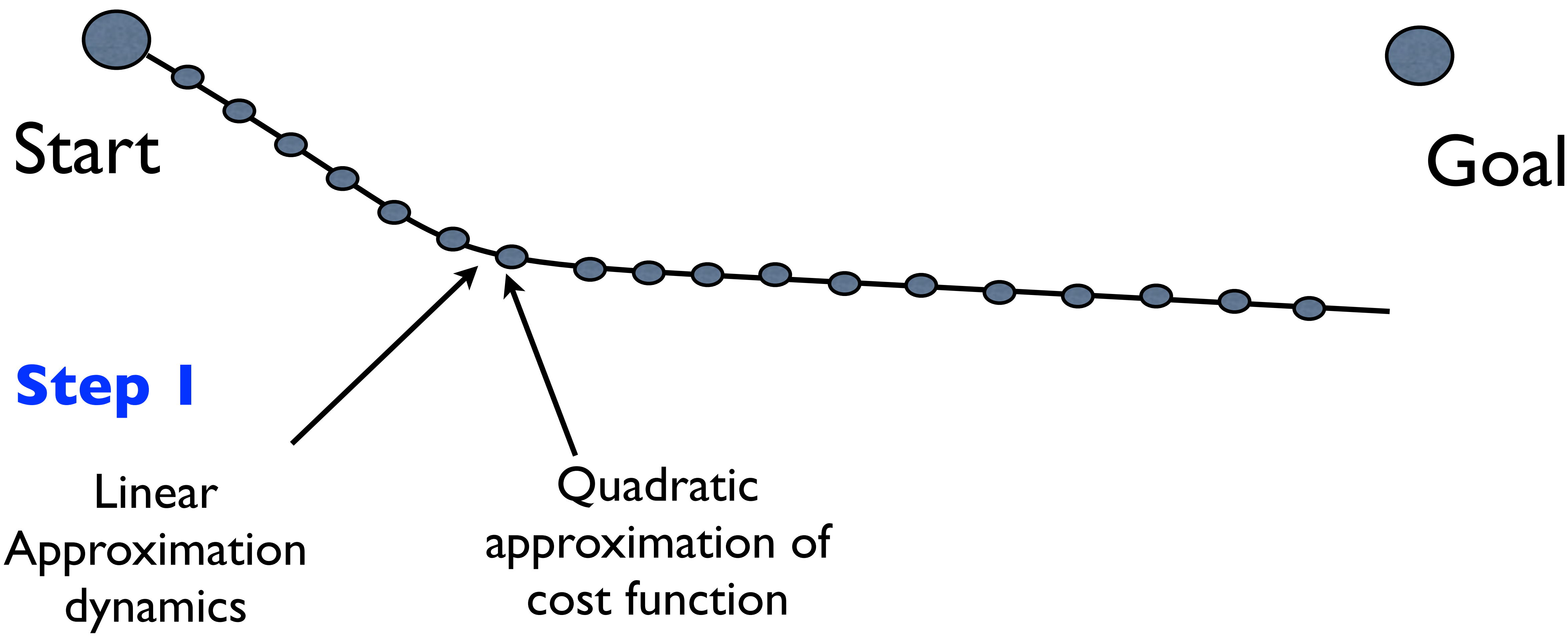

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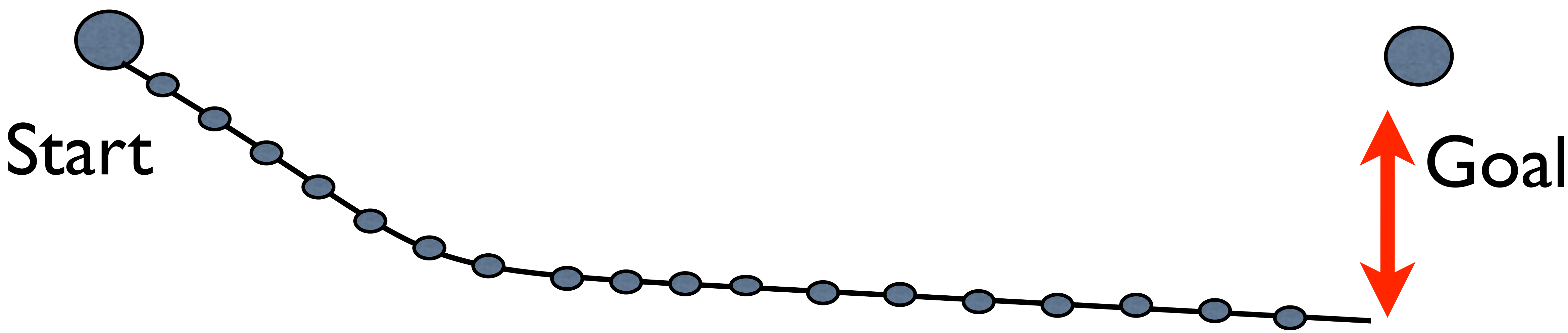
Nonlinear Stochastic Optimal Control



Step 2. Back-propagate the quadratic approximation of the Cost-to-go

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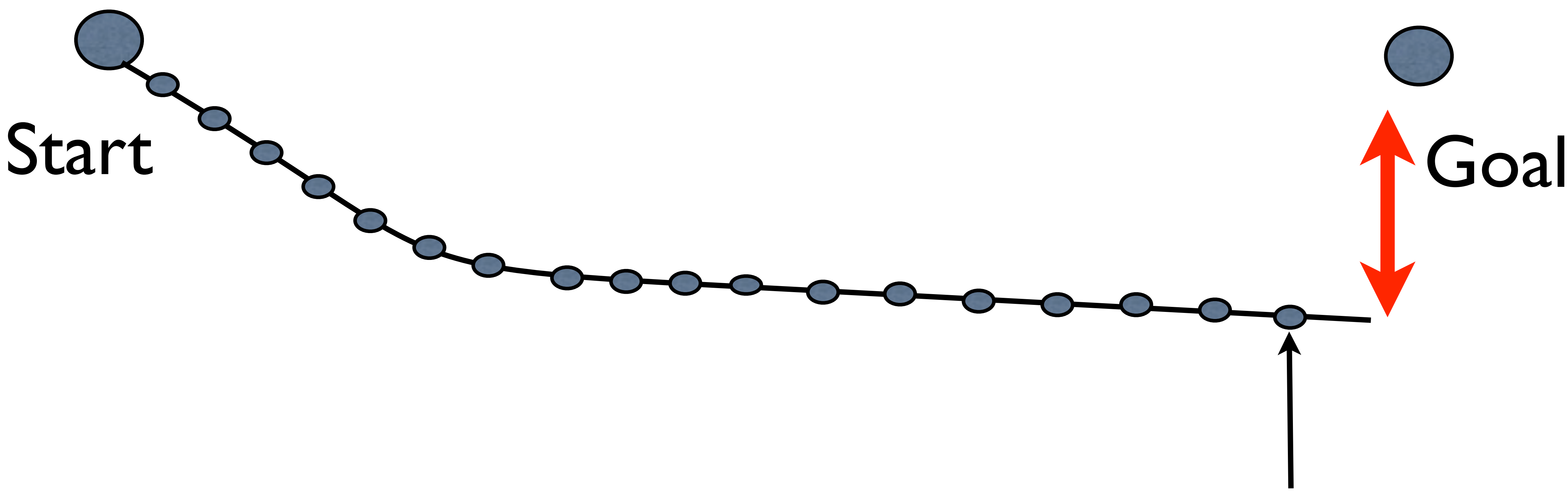
Nonlinear Stochastic Optimal Control



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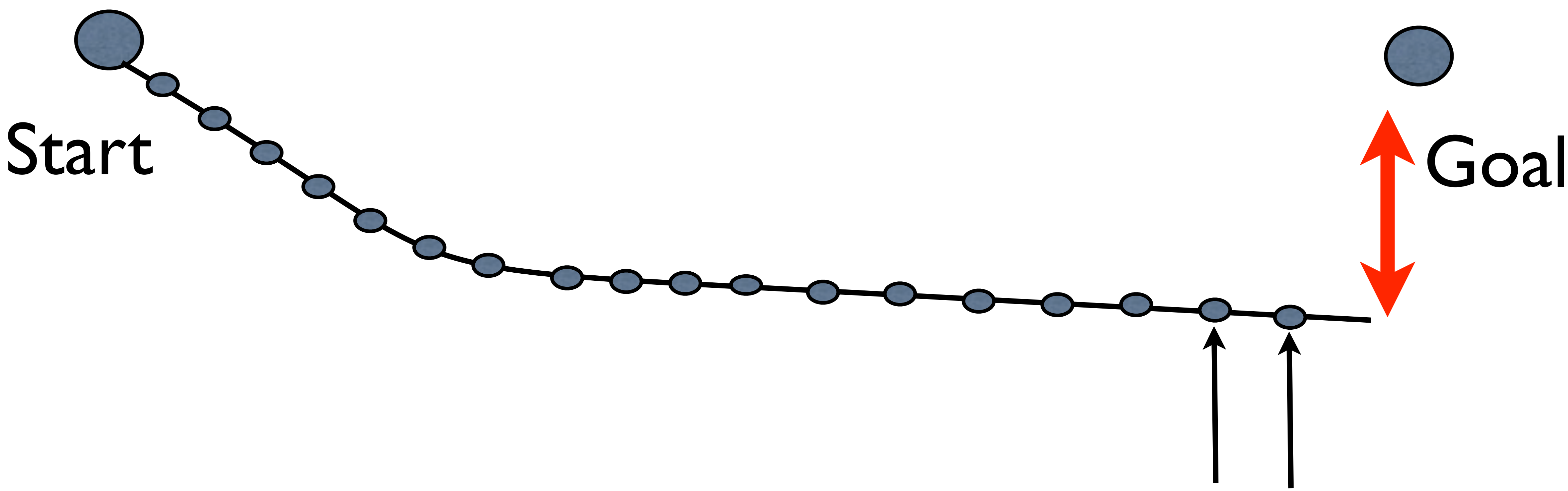

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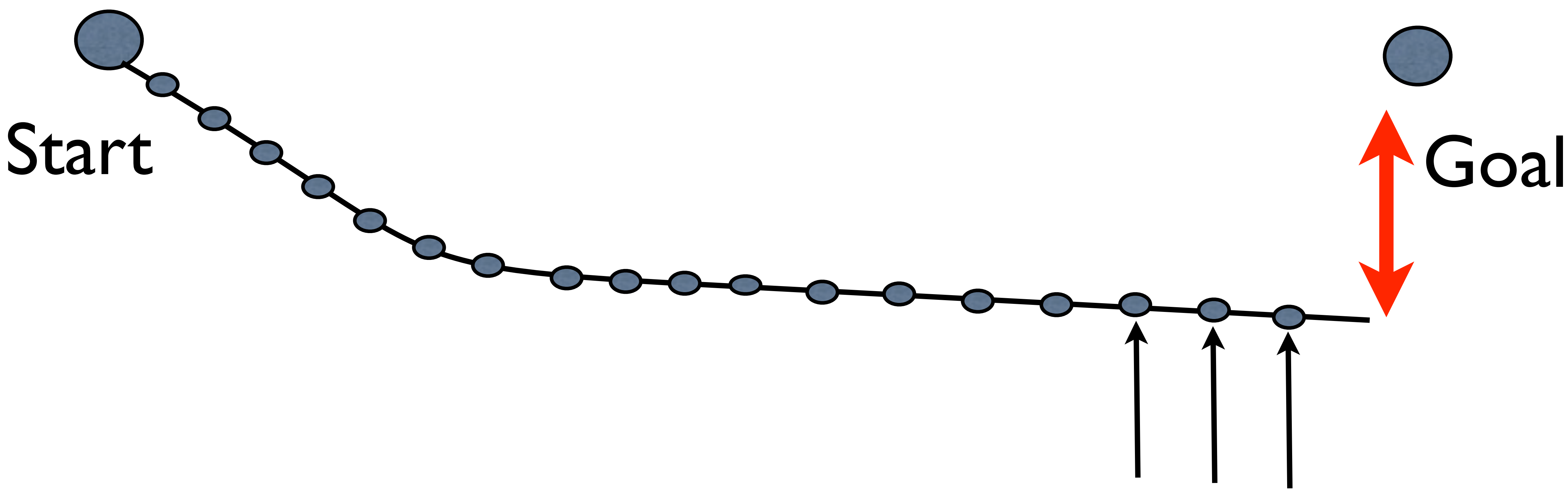

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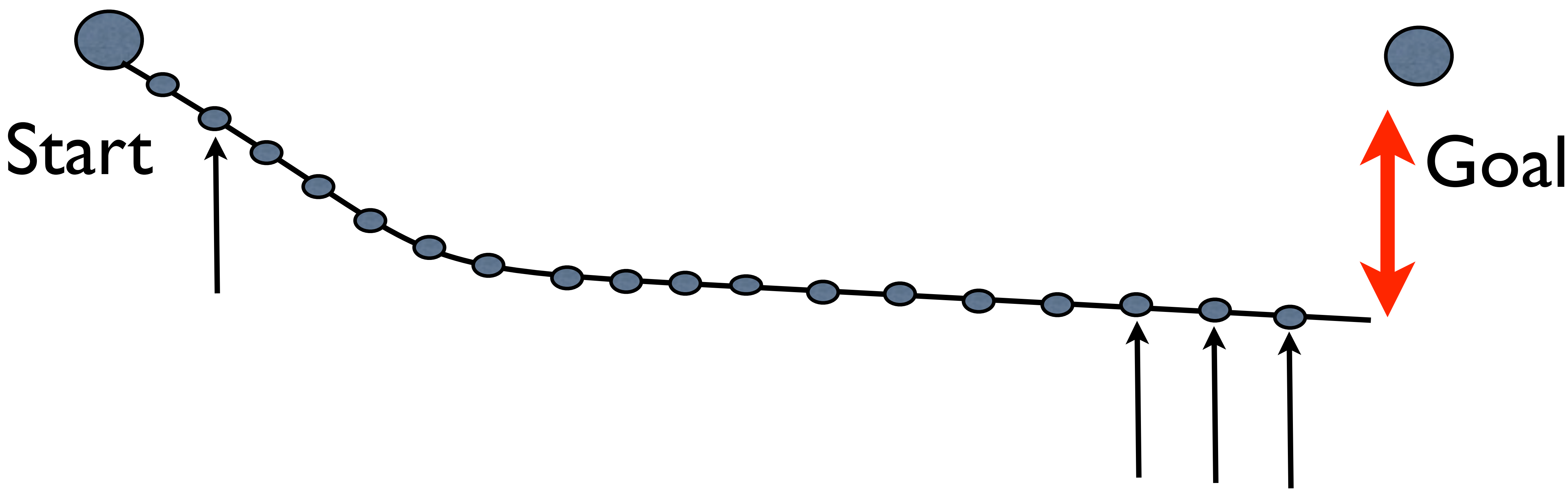

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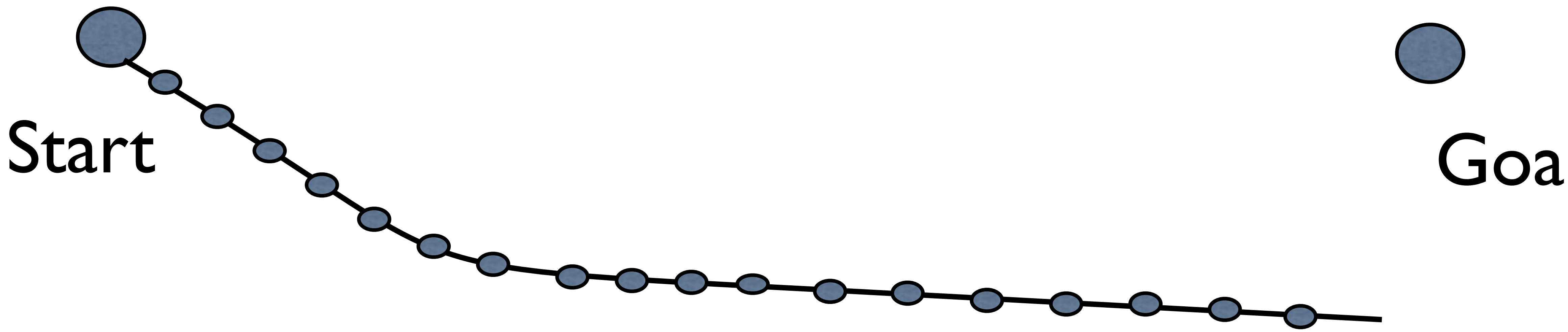

Step 3: Update the controls and the trajectory:

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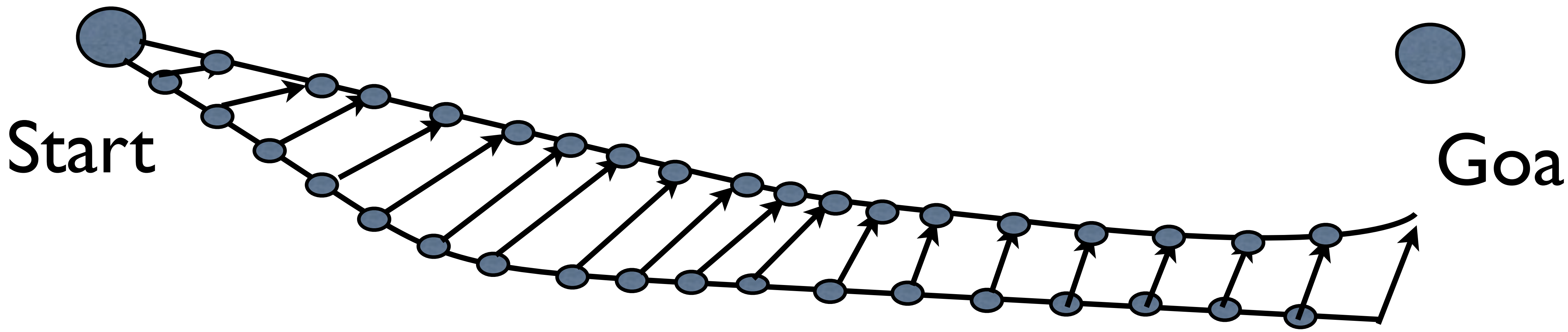

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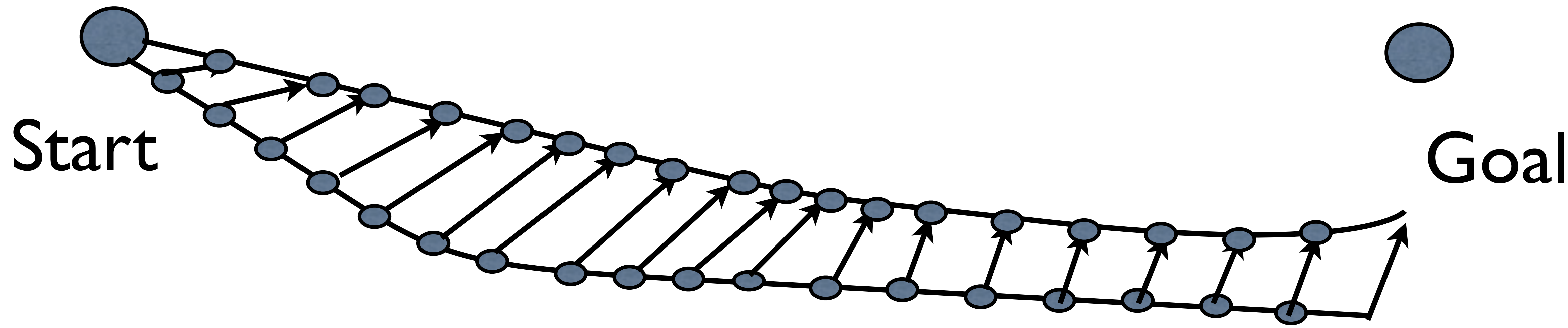

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What you get at the end

Control gains L(t)

Optimal Trajectory x(t)

Optimal feedforward controls u(t)

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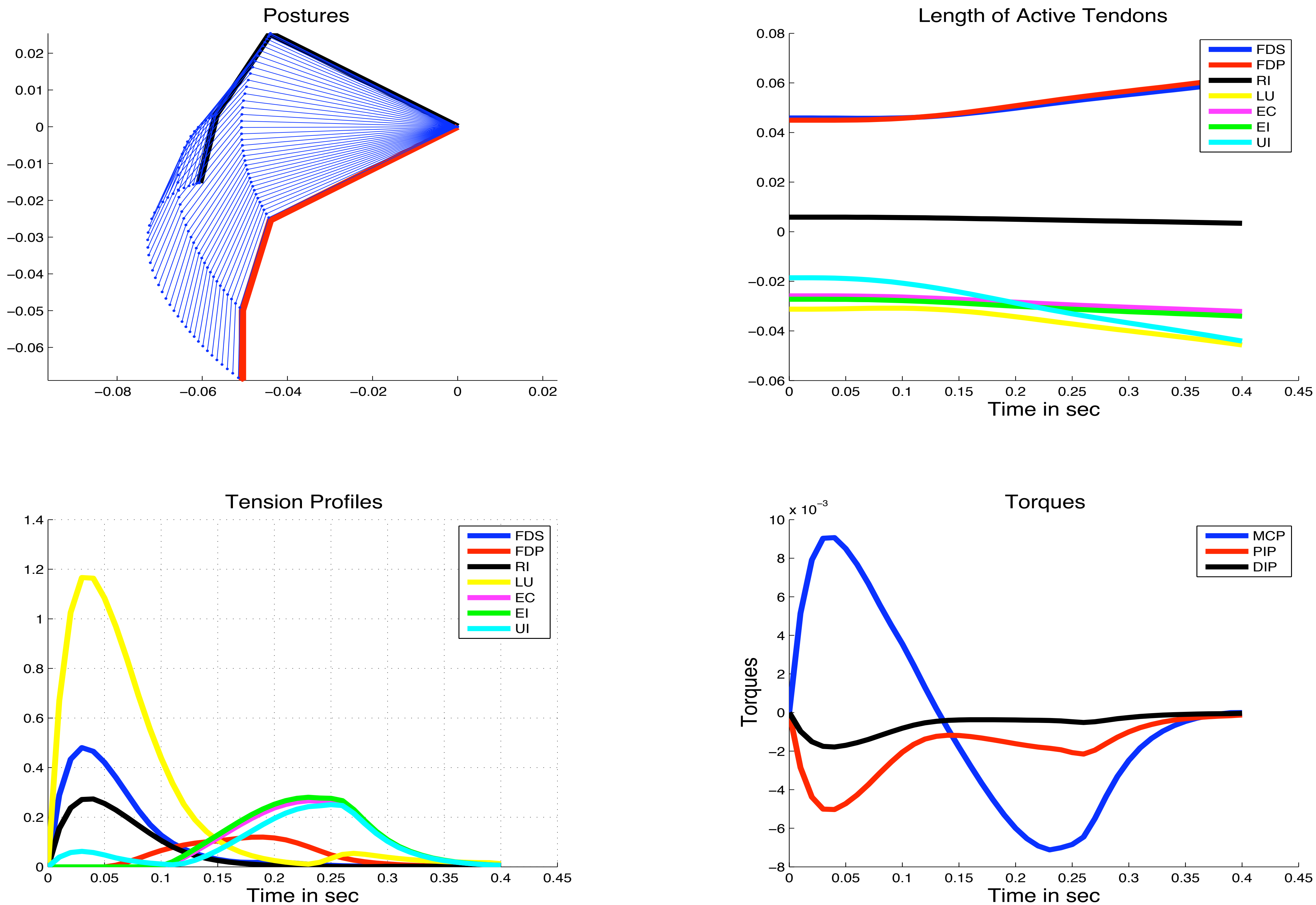
Application on the index finger



Conclusions

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Sensitivity increases due to linearization and hard constrains.

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What you get at the end

Control gains L(t)

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Part 2:
Beyond linearization and internals model:
Path integral control approach

Path Integral Optimal Control

Dynamics: $$\mathbf{dx} = (\mathbf{f}(\mathbf{x}_t) + \mathbf{G}(\mathbf{x})\mathbf{u})\,dt + \mathbf{B}(\mathbf{x})\mathbf{L}d\omega$$

Cost: $$J(\mathbf{x},\mathbf{u}) = \phi(\mathbf{x}(t_N)) + \int_{t_0}^{t_N} q(\mathbf{x},t) + \mathbf{u}^T\mathbf{R}\mathbf{u}\;dt$$

Value Function: $$V(\mathbf{x},t) = \min_{\mathbf{u}} E\Big(J(\mathbf{x},\mathbf{u})\Big)$$

Optimal Control: $$\mathbf{u}(\mathbf{x},t) = -\mathbf{R}^{-1}\mathbf{G}^T grad\Big(V(\mathbf{x},t)\Big)$$

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$$-\partial_t V_t = q_t + (\nabla_{\mathbf{x}} V_t)^T \mathbf{f}_t - \frac{1}{2}(\nabla_{\mathbf{x}} V_t)^T \mathbf{G}_t \mathbf{R}^{-1} \mathbf{G}_t^T (\nabla_{\mathbf{x}} V_t) + \frac{1}{2} trace\left((\nabla_{\mathbf{xx}} V_t)\mathbf{B}_t \boldsymbol{\Sigma}_{\boldsymbol{\epsilon}} \mathbf{B}_t^T\right)$$

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We derive the path integral optimal controller.

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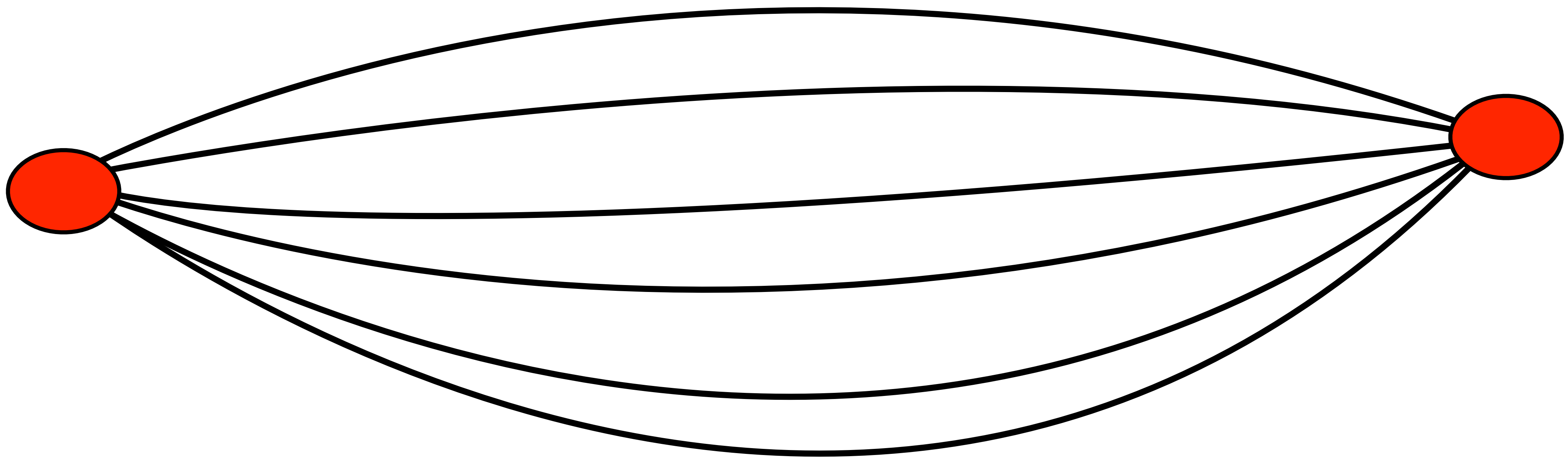
Path integral optimal control can be model based, semi-model based, model free.

Path Integral Optimal Control



E. Theodorou, J. Buchli, S.Schaal ICRA 2010

E. Theodorou, J. Buchli, S.Schaal AISTATS 2010

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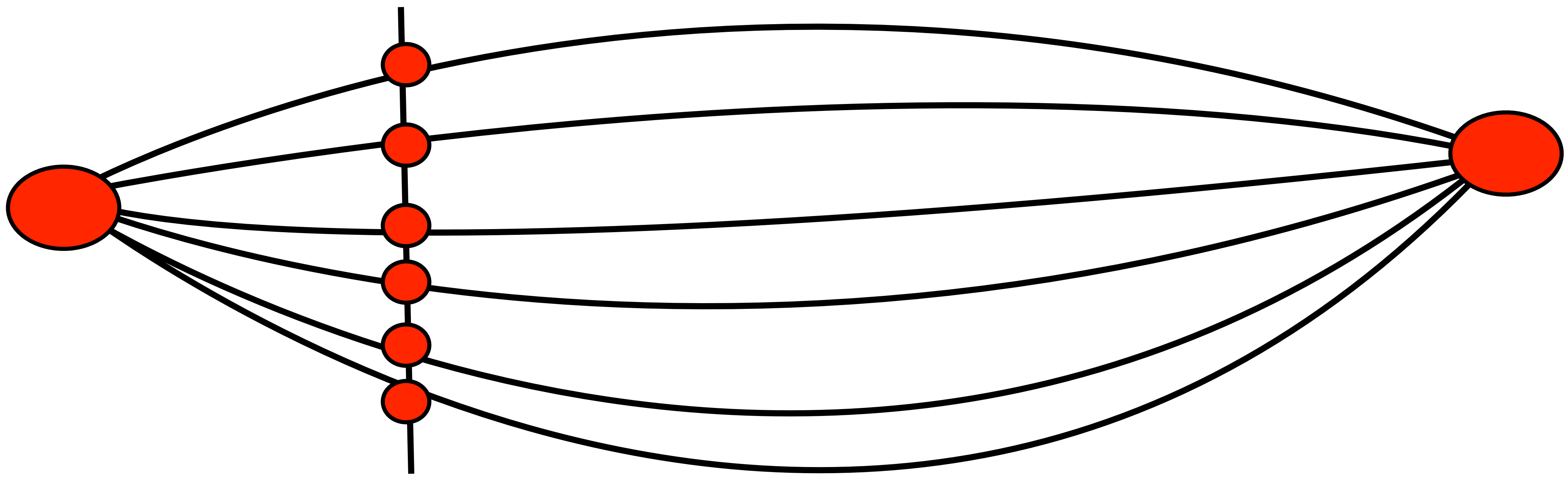

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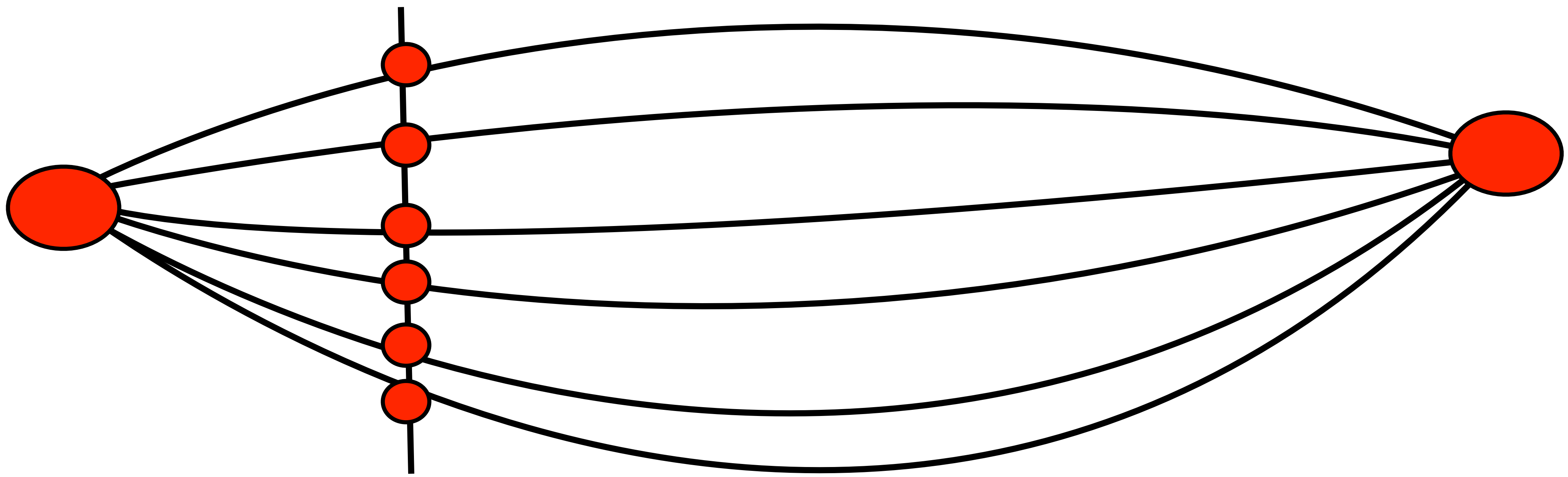

Cost $J(\mathbf{x}_j(t:N))$

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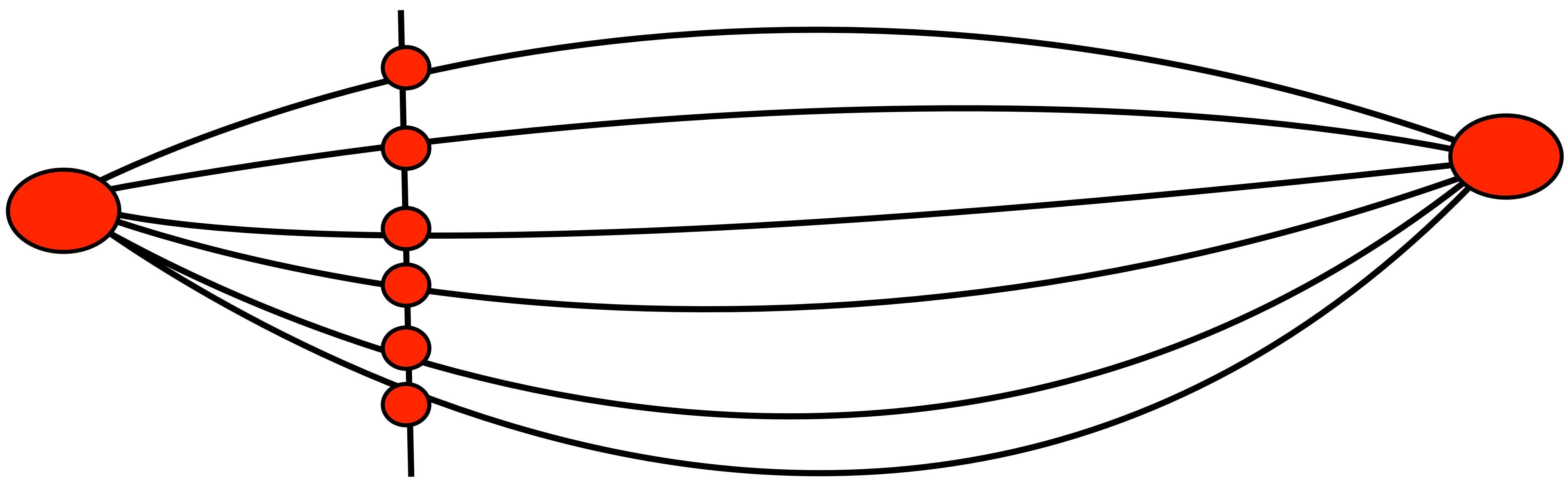

Cost $J(\mathbf{x}_j(t:N))$

Probability $P(\mathbf{x}_j(t:N)) = \frac{e^{-\frac{1}{\lambda}P(\mathbf{x}_j(t:N))}}{\int e^{-\frac{1}{\lambda}P(\mathbf{x}_j(t:N))}d\mathbf{x}}$

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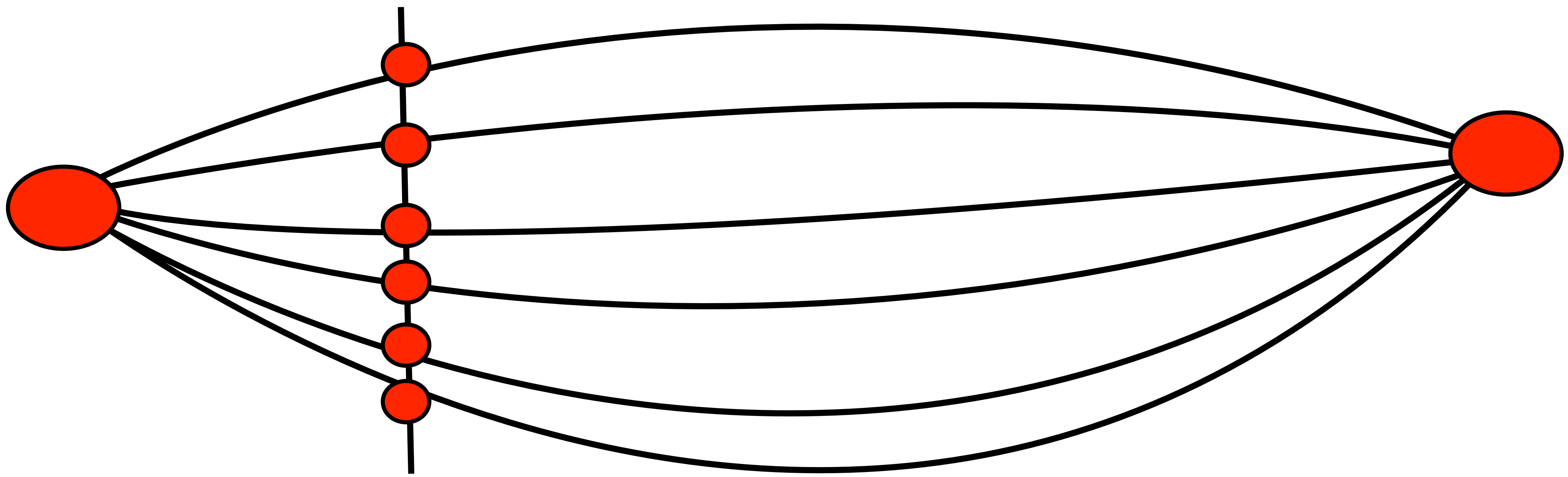

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Local controls $\delta\mathbf{u}(t) = \sum_j P(\mathbf{x}_j(t:N))\ \delta\mathbf{u}_j(t)$

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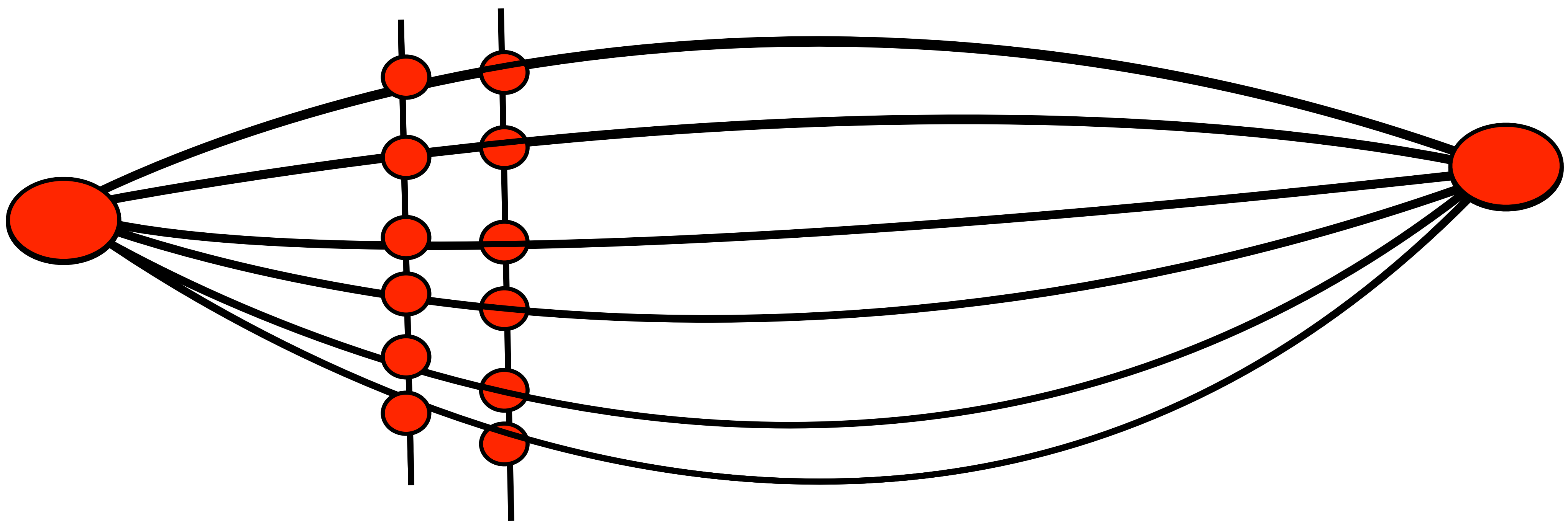
Path Integral Optimal Control



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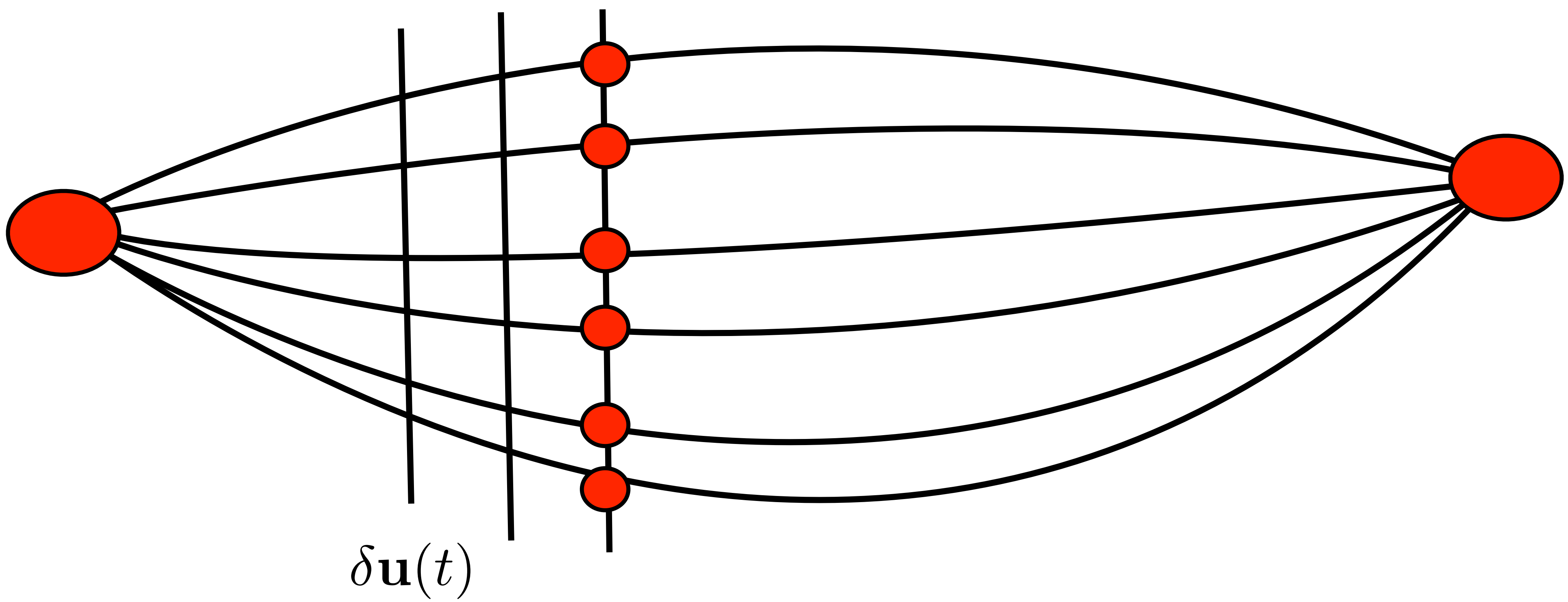

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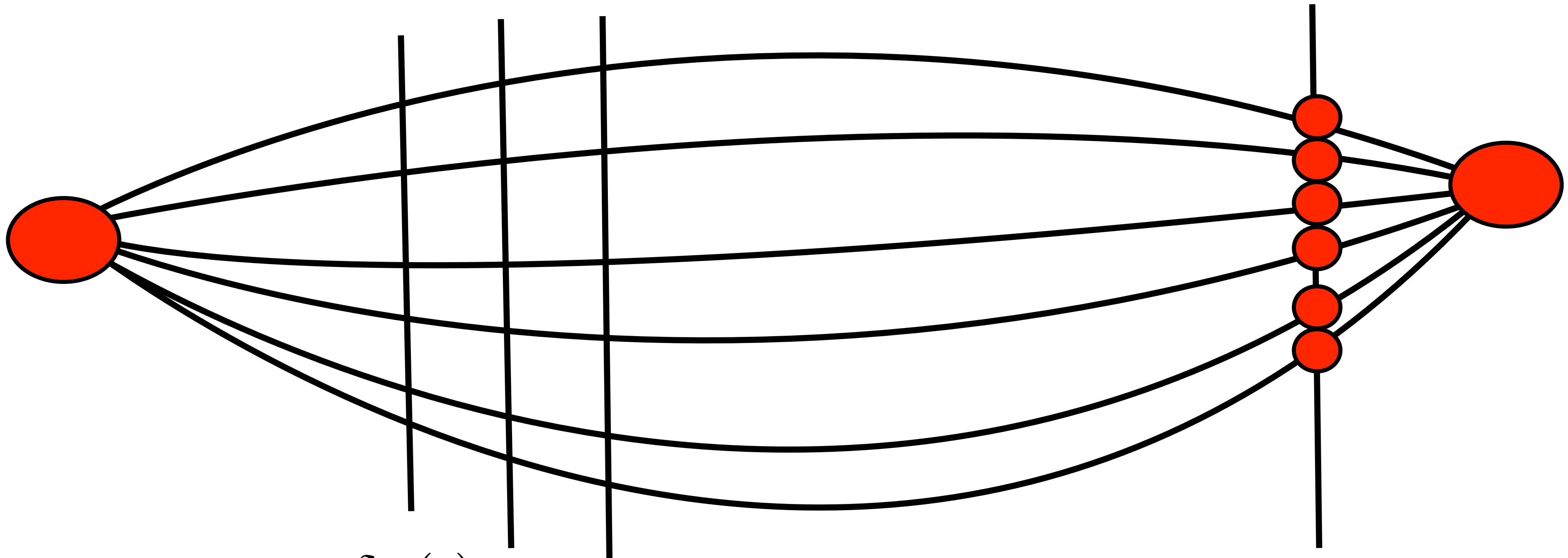

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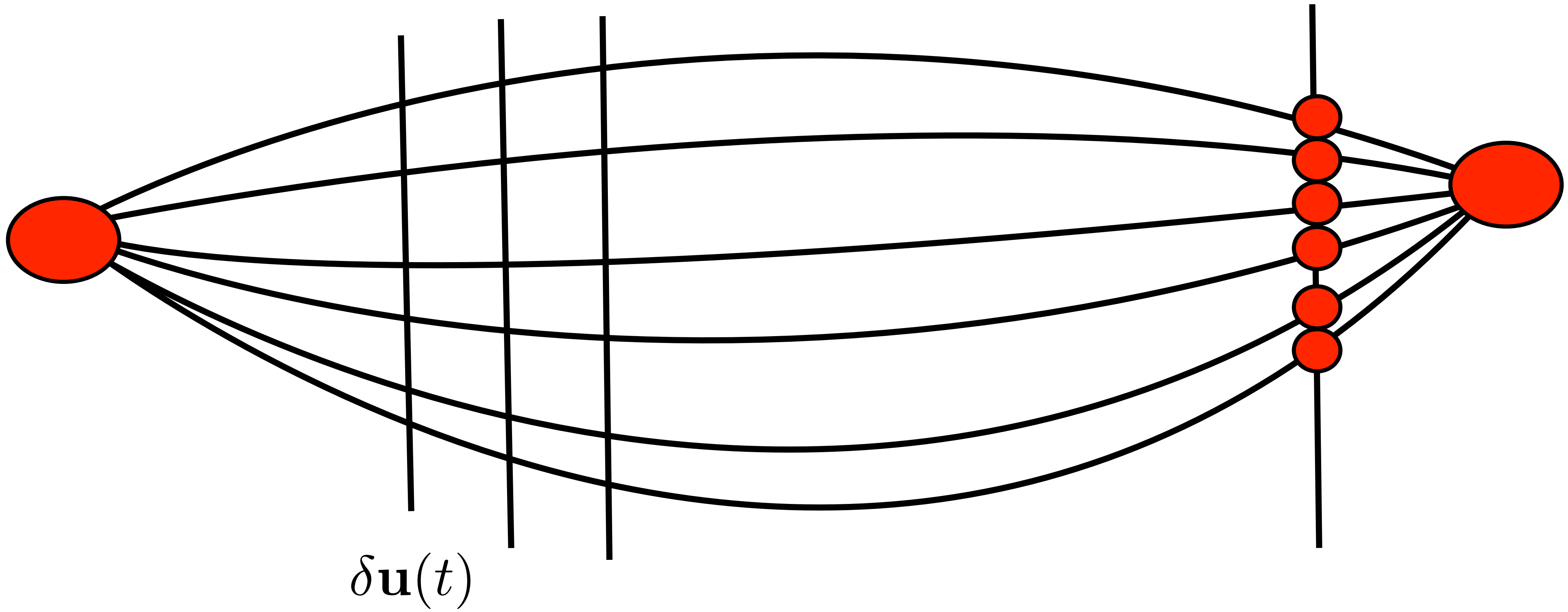


Cost $\quad J(\mathbf{x}_j(t:N))$

Probability $\quad P(\mathbf{x}_j(t:N)) = \dfrac{e^{-\frac{1}{\lambda}P(\mathbf{x}_j(t:N))}}{\int e^{-\frac{1}{\lambda}P(\mathbf{x}_j(t:N))}d\mathbf{x}}$

Local controls $\quad \delta\mathbf{u}(t) = \sum_j P(\mathbf{x}_j(t:N))\ \delta\mathbf{u}_j(t)$

$$\mathbf{u}_{new} = \mathbf{u}_{old} + \delta\mathbf{u}$$

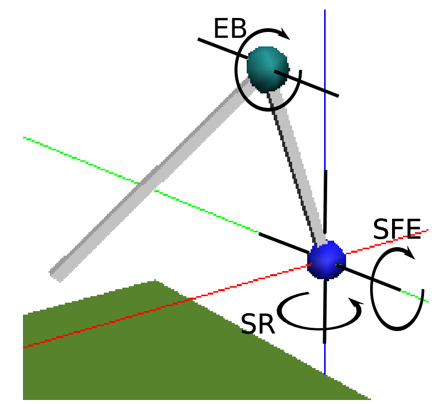
E. Theodorou, J. Buchli, S.Schaal ICRA 2010

E. Theodorou, J. Buchli, S.Schaal AISTATS 2010

E. Theodorou, J. Buchli, S.Schaal JMLR 2010

Path Integral Optimal Control

$$J(\mathbf{x},\mathbf{u},t) = \phi(\mathbf{x}(t_N)) + \int \left(q(\mathbf{x}) + \left|\left|\frac{\partial \mathbf{u}}{\partial \mathbf{x}}\right|\right|_p + \left|\left|\frac{\partial \mathbf{u}}{\partial t}\right|\right|_p \right) dt$$



Variable Stiffness Control (Gain Scheduling)

$$J(\mathbf{x},\mathbf{u},t) = \int \left(w_{accel}||\ddot{\mathbf{x}}(t)|| + w_{target}C(t) + w_{gain}\sum_{i}^{\#DOFs} K^{i}_{P,t} \right) dt$$

Acceleration Joint Angles Control Gains

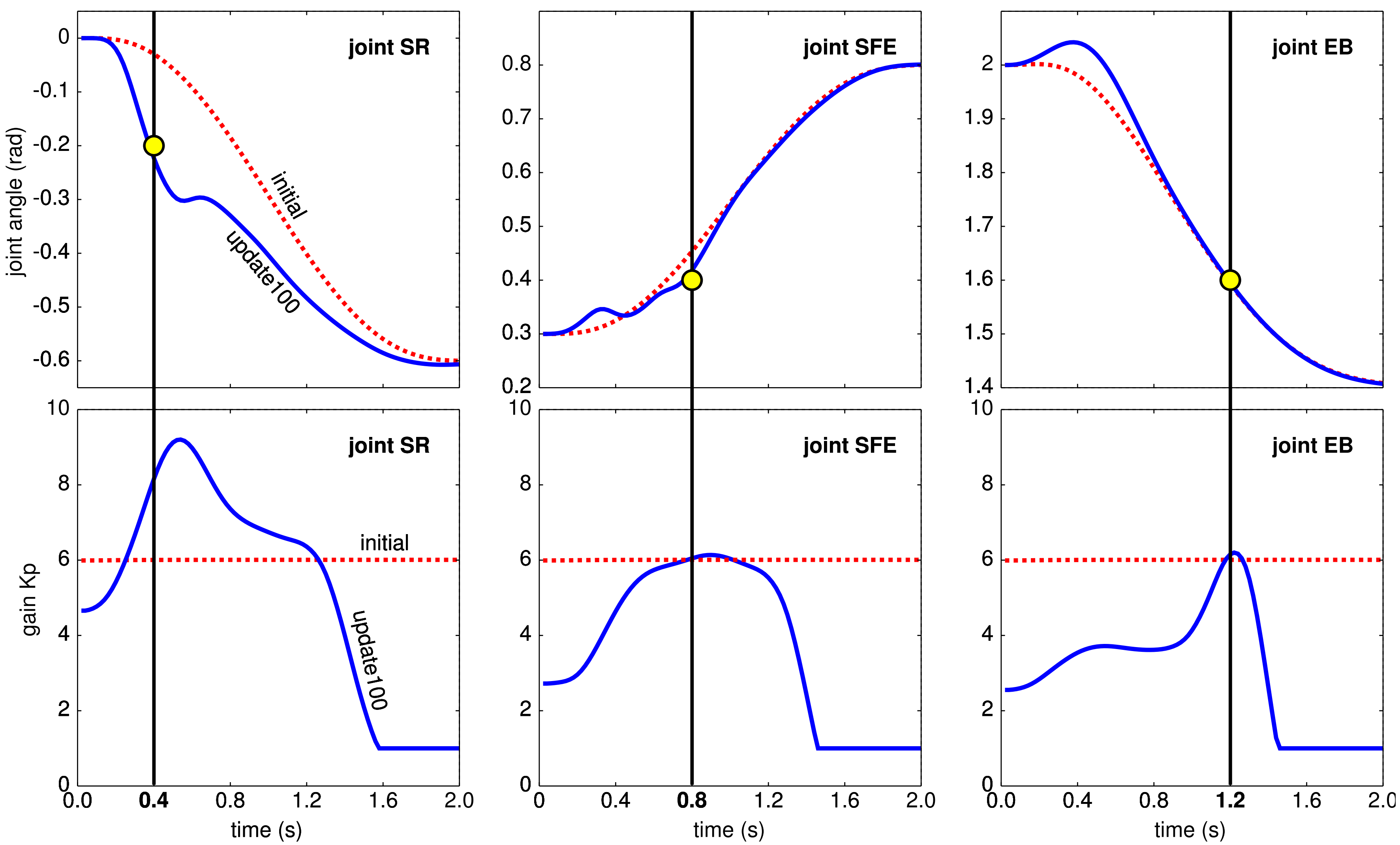
J. Buchli, E.Theodorou, F. Stulp,S. Schaal RSS2010

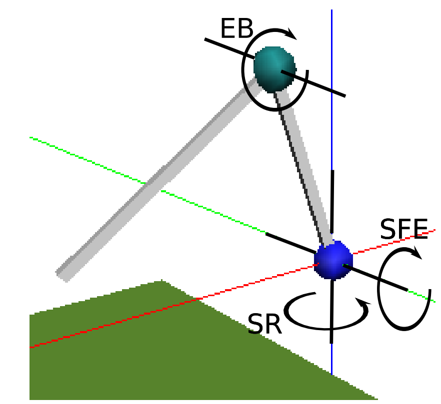
J. Buchli, F. Stulp,E.Theodorou,S. Schaal IJRR 2010

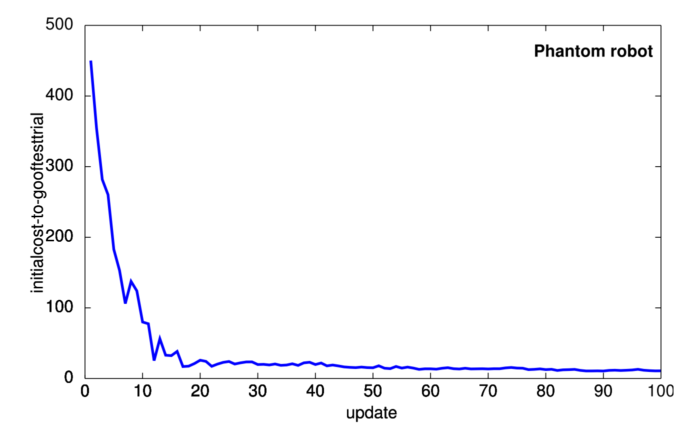
F. Stulp, E.Theodorou, J. Buchli S. Schaal Humanoids 2010

Path Integral Optimal Control

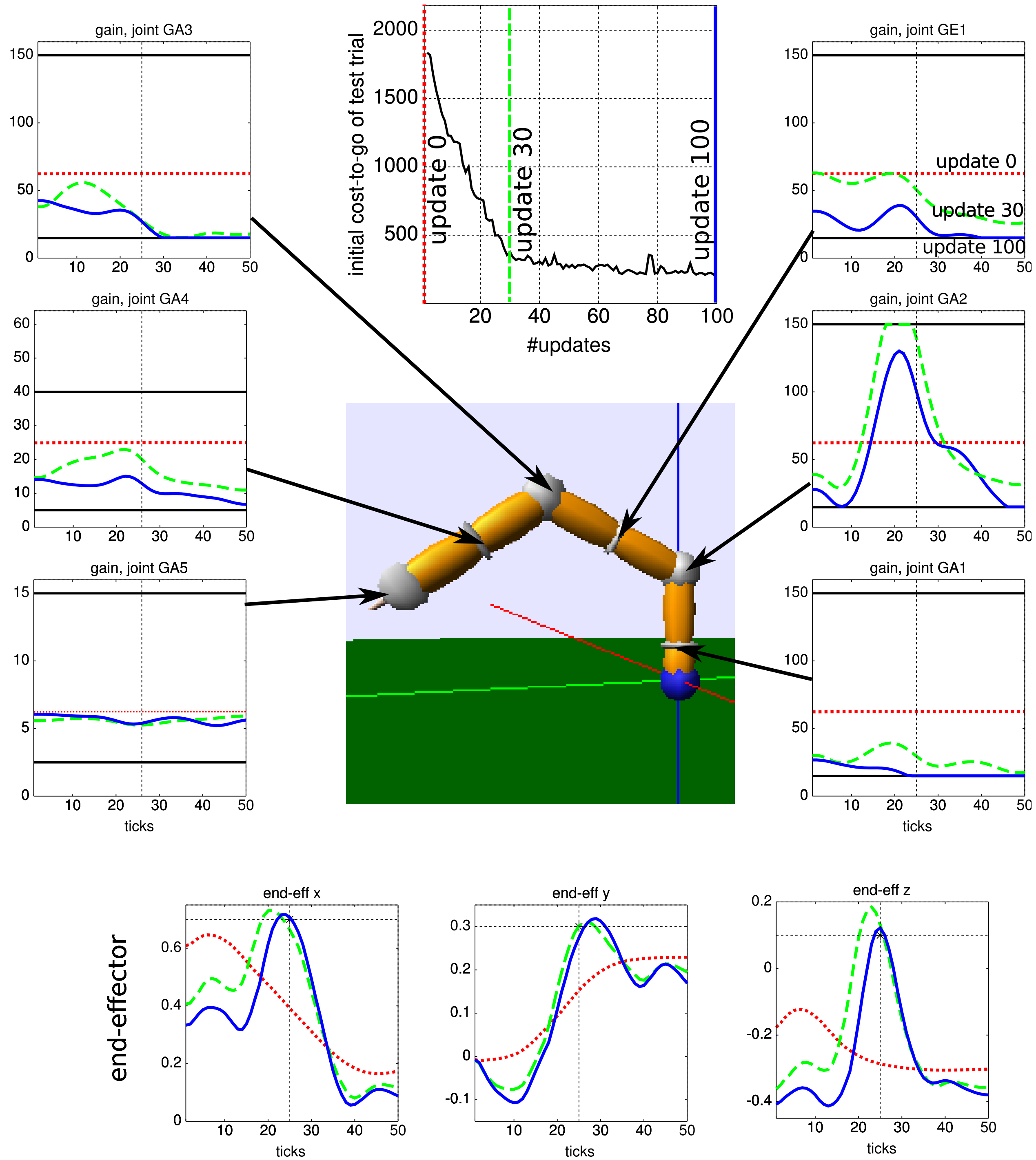
Variable Stiffness Control (Gain Scheduling)



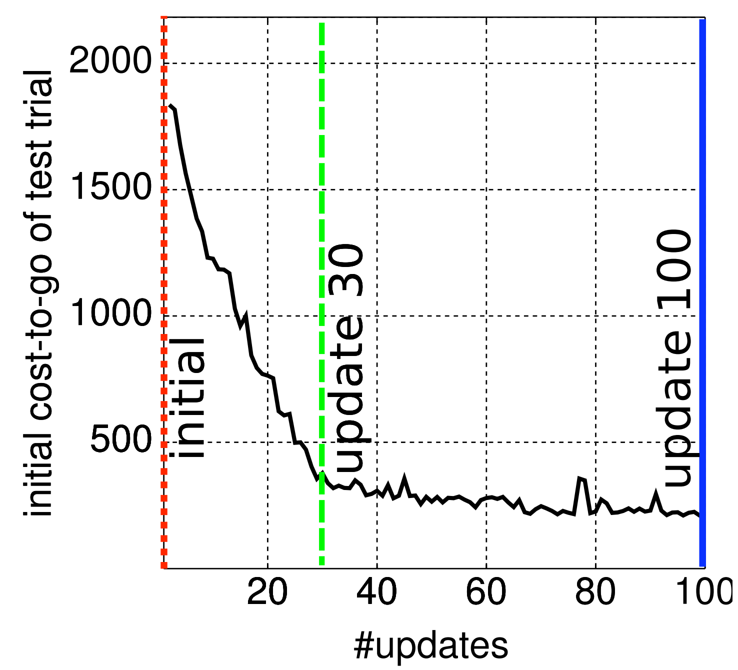




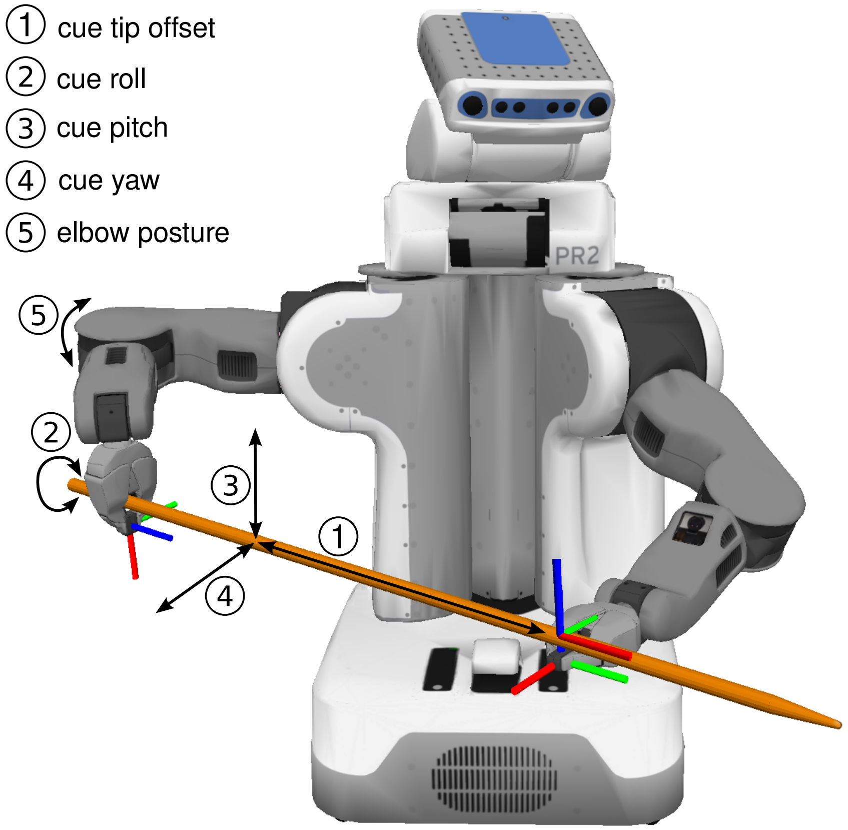
Path Integral Optimal Control



$$C(t) = \delta(t - 0.5)(\mathbf{x} - [\ 0.7\ 0.3\ 0.1]^T)$$



Path Integral Optimal Control

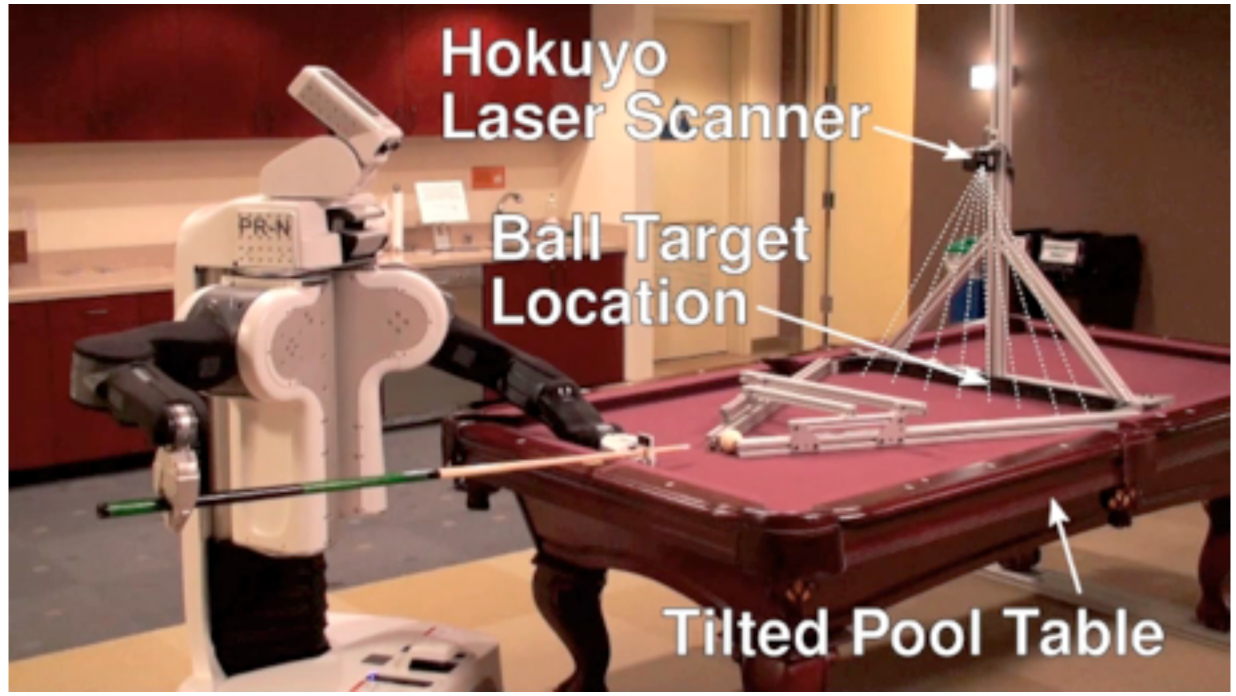


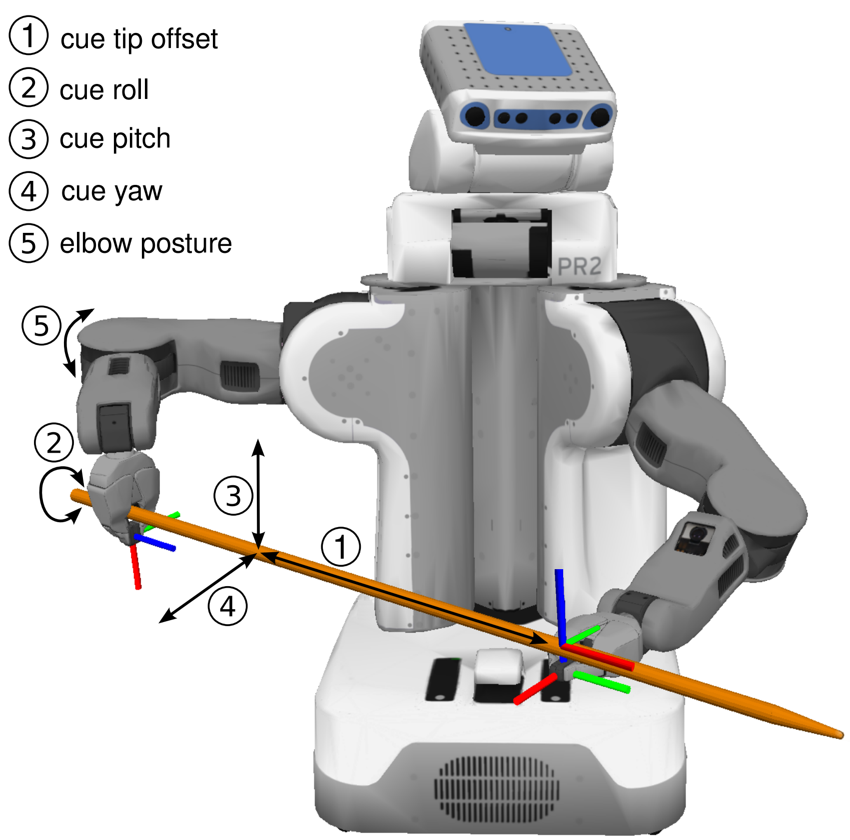
Learning Pool Stroke

$$q(x_t) = w_1 \times Time + w_2 \times Error + w_3 \times Displacement$$

P. Pastor, M. Kalakrishnan, S. Chitta, E.A. Theodorou, S. Schaal Skil, ICRA 2011

Path Integral Optimal Control





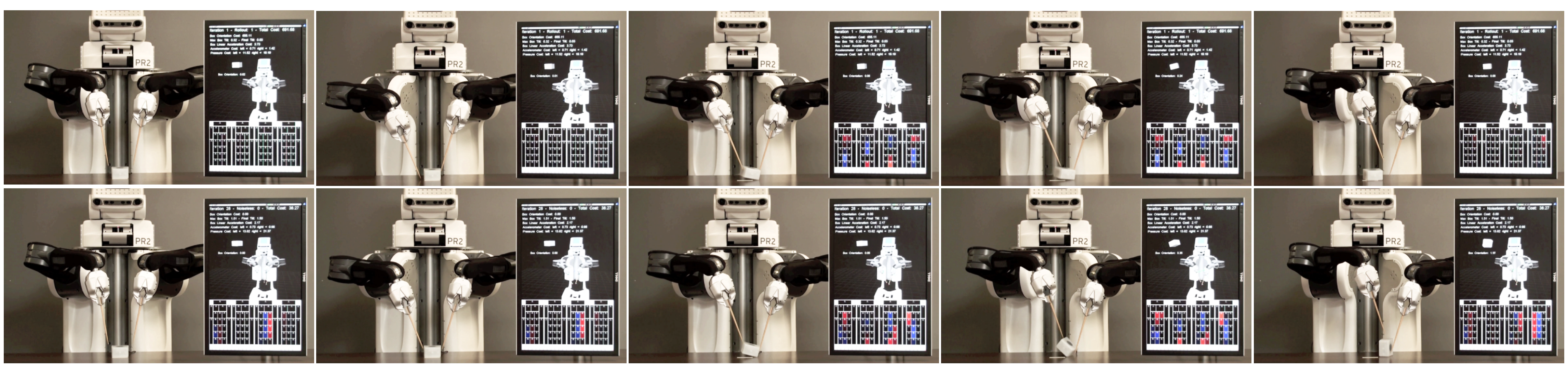
Learning Pool Stroke

$$
\begin{aligned}
q(x_t) &= w_1 \times Time + w_2 \times Error \\
&+ w_3 \times Displacement
\end{aligned}
$$

P. Pastor, M. Kalakrishnan, S. Chitta, E.A. Theodorou, S. Schaal Skil, ICRA 2011

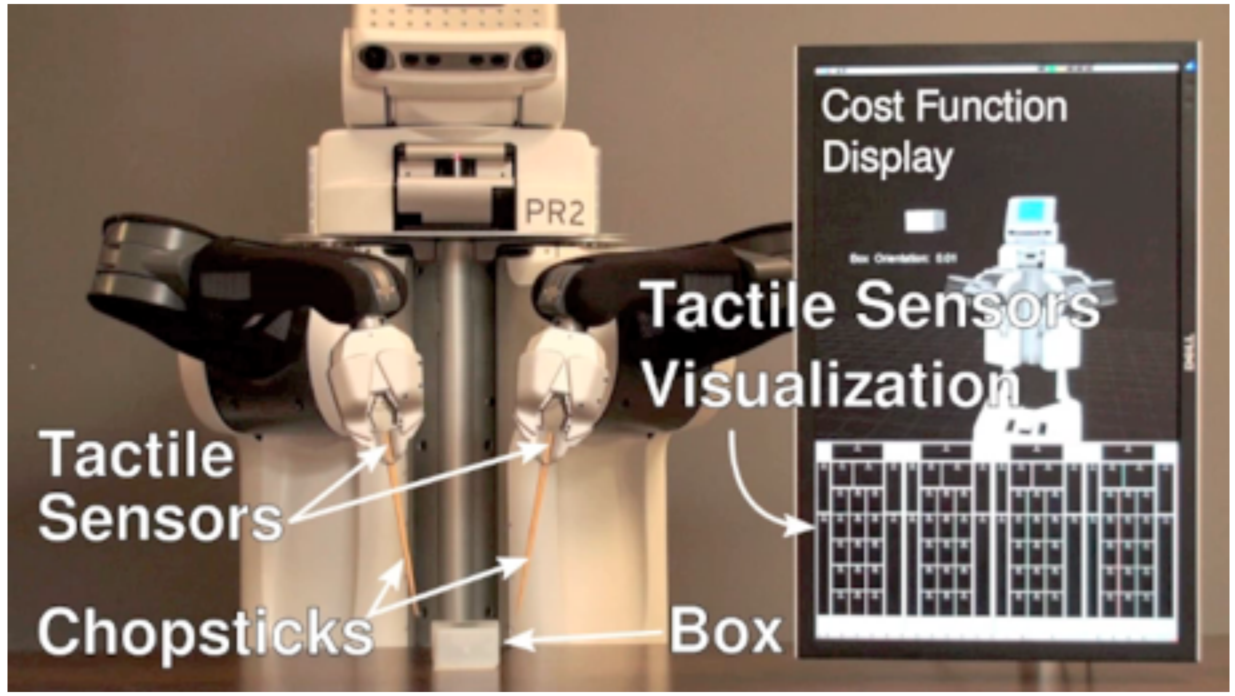
Path Integral Optimal Control

$$C(t) = w_1 \times (\text{Box Acceleration}) + w_2 \times (\text{Force}) + w_3 \times (\text{Gripper acceleration})$$
$$\phi(t) = w_4 \times (\text{Terminal state error})$$



P. Pastor, M. Kalakrishnan, S. Chitta, E.A. Theodorou, S. Schaal Skil, ICRA 2011

Path Integral Optimal Control



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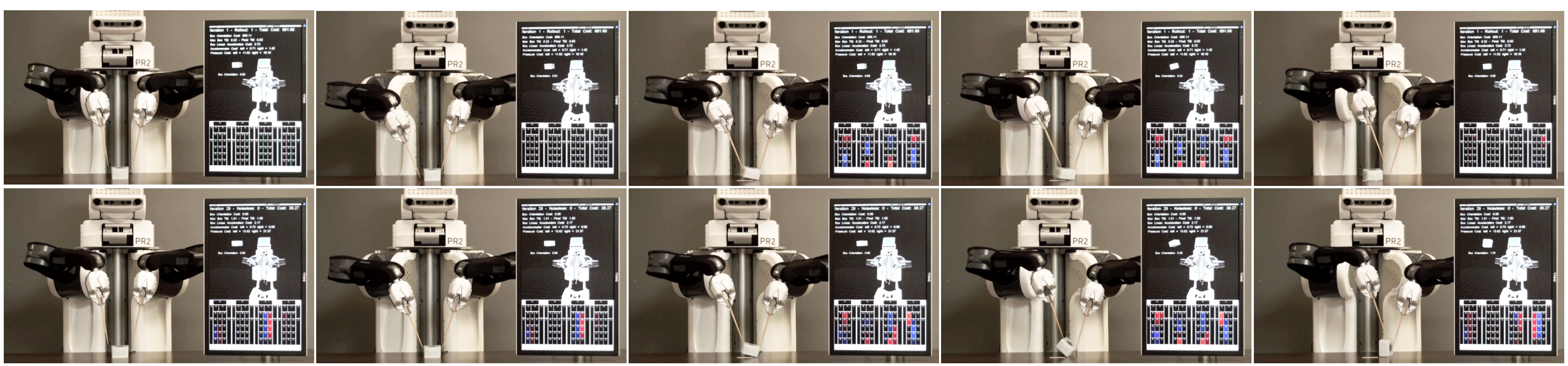

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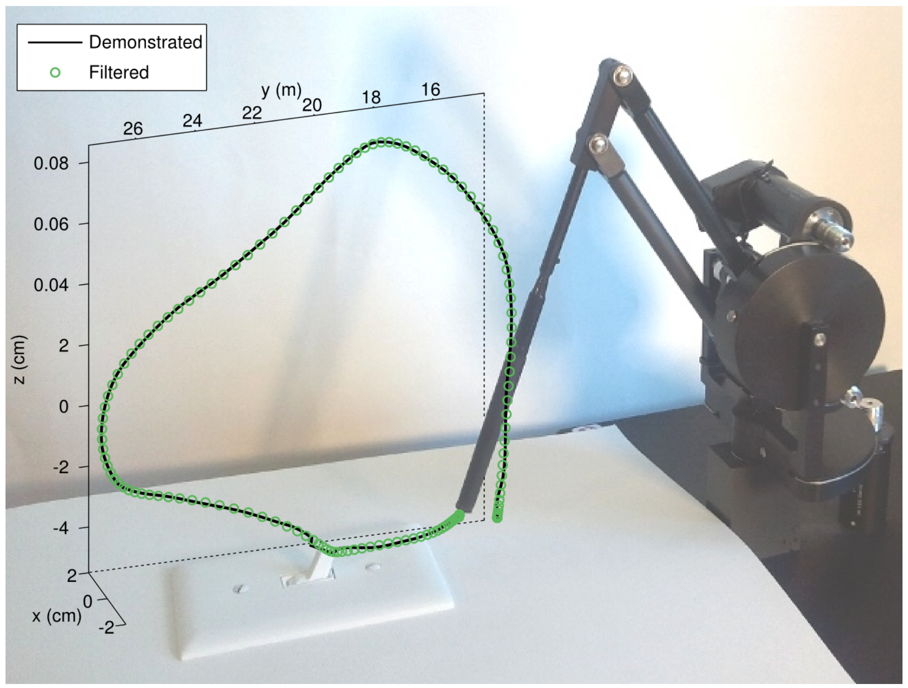
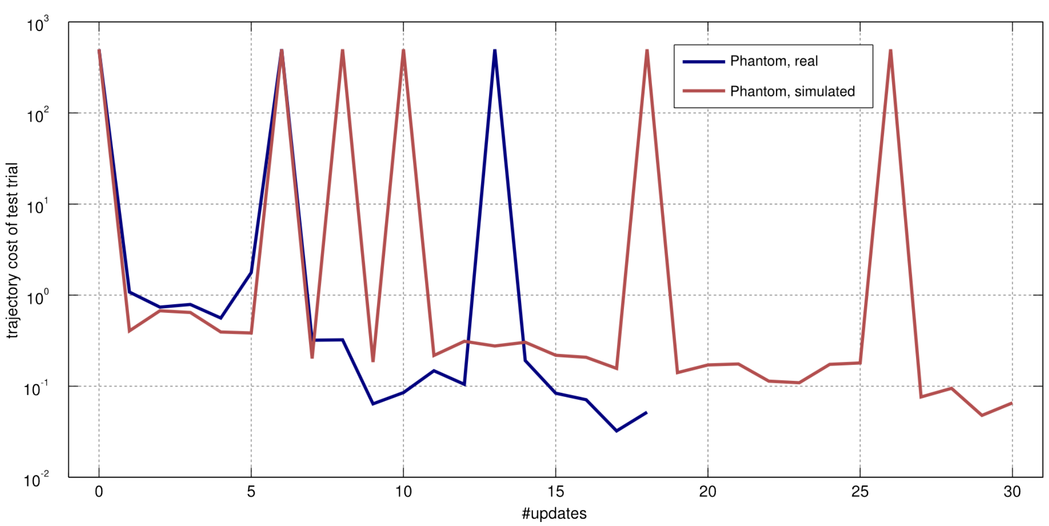
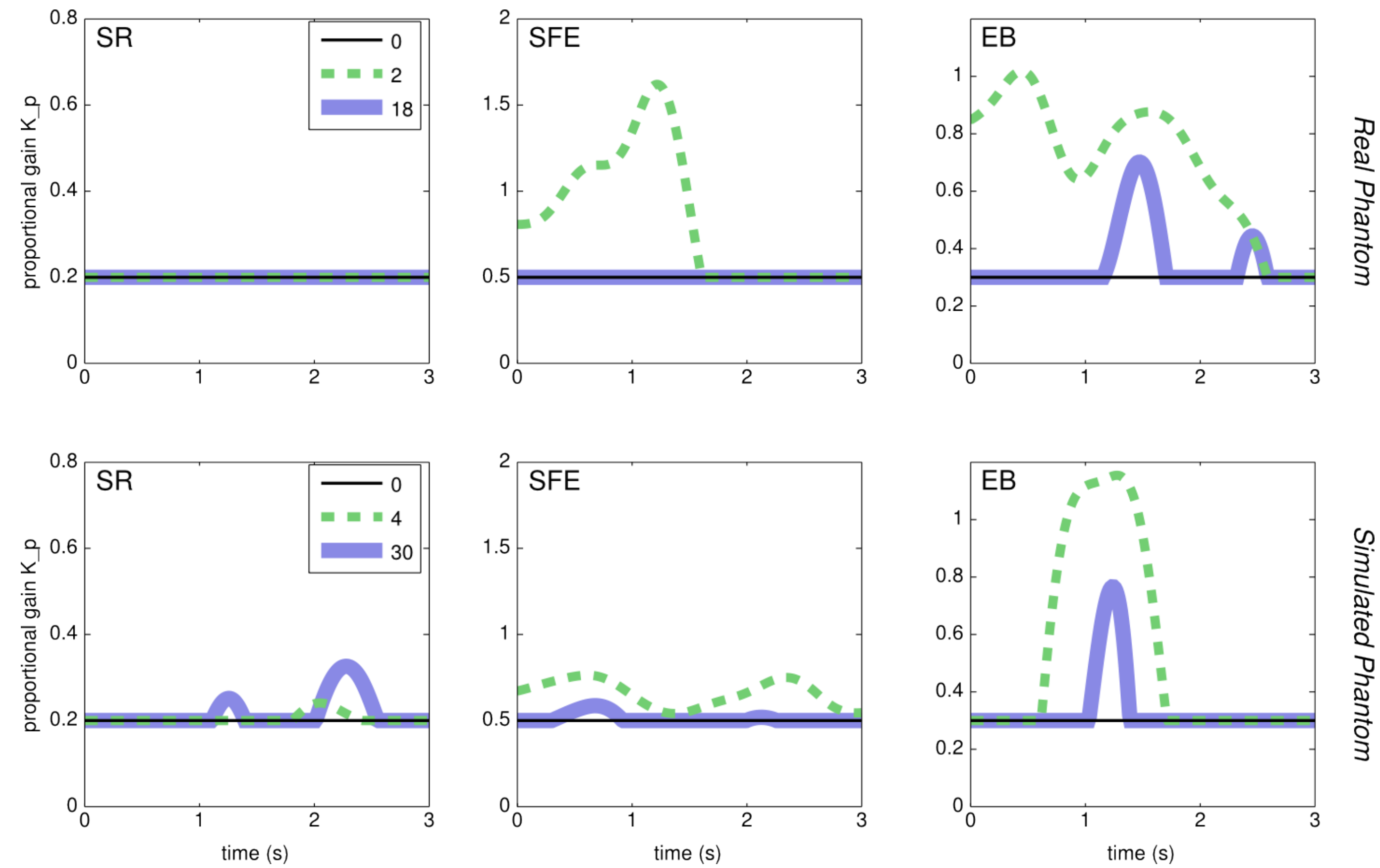
Path Integral Optimal Control

Task Goal: Flipping the switch







J. Buchli, F. Stulp,E.Theodorou,S. Schaal IJRR (Accepted)

Path Integral Optimal Control

Jump over the barrier - Little Dog

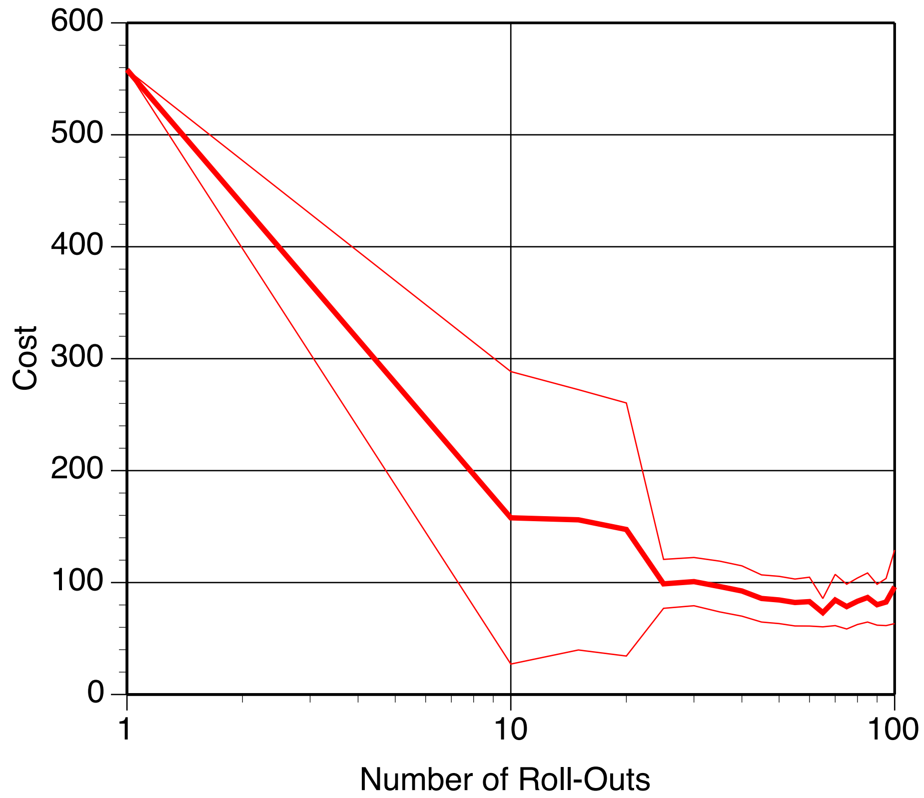
$$r_t = r_{roll} + r_{yaw} + \sum_{i=1}^{d} \left(a_1 f_{i,t}^2 + 0.5 a_2 \; \boldsymbol{\theta}_i^T \boldsymbol{\theta} \right) \; (a_1 = 1.e - 6, a_2 = 1.e - 8)$$

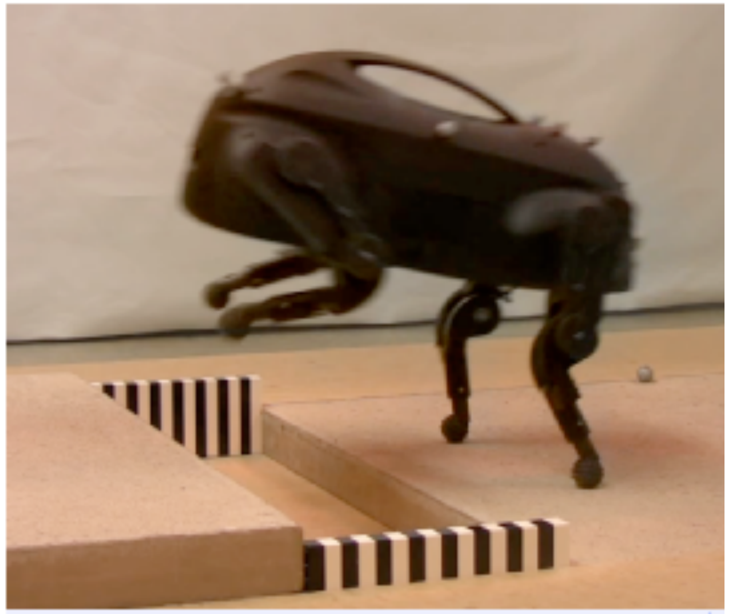
$$r_{roll} = \begin{cases} 100 * (|roll_t| - 0.3)^2, & \text{if } (|roll_t| > 0.3) \\ 0, & otherwise \end{cases}$$

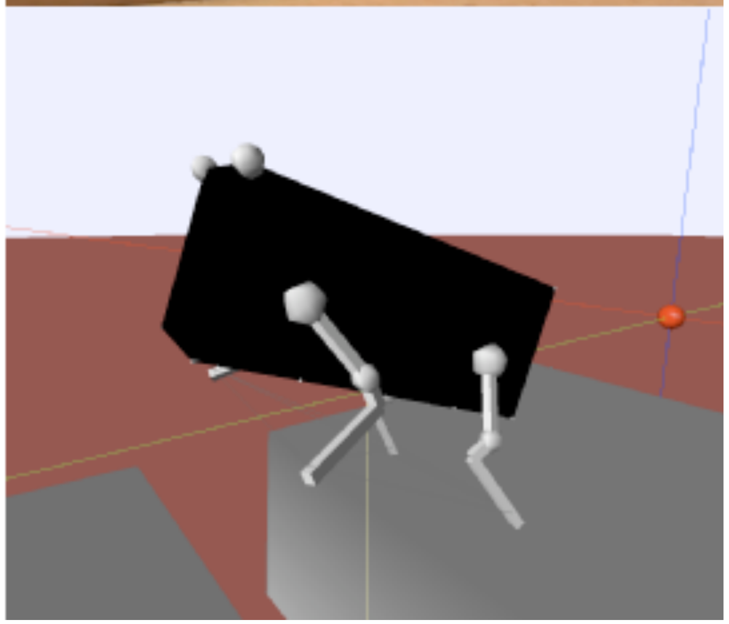
$$r_{yaw} = \begin{cases} 100 * (|yaw_t| - 0.1)^2, & \text{if } (|yaw_t| > 0.1) \\ 0, & otherwise \end{cases}$$

$$\phi_{t_N} = 50000(goal - x_{nose})^2$$

Path Integral Optimal Control







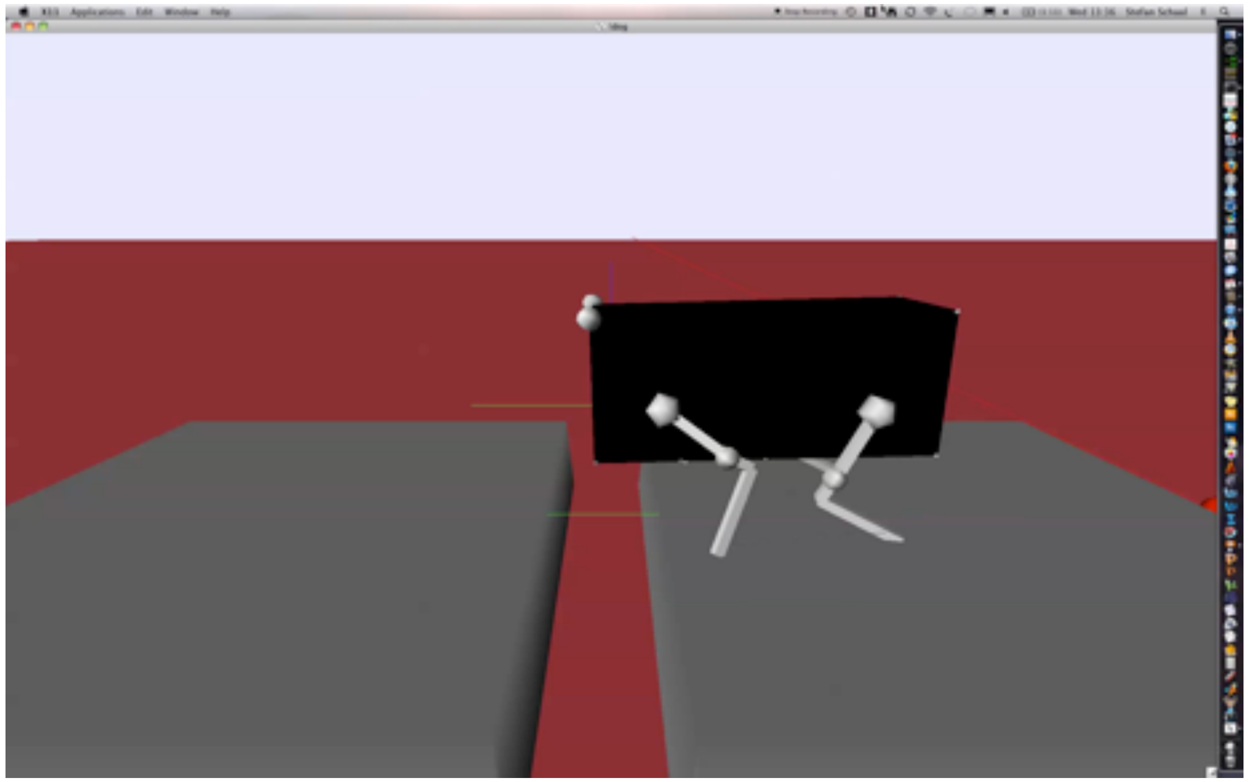
Path Integral Optimal Control

Before Learning

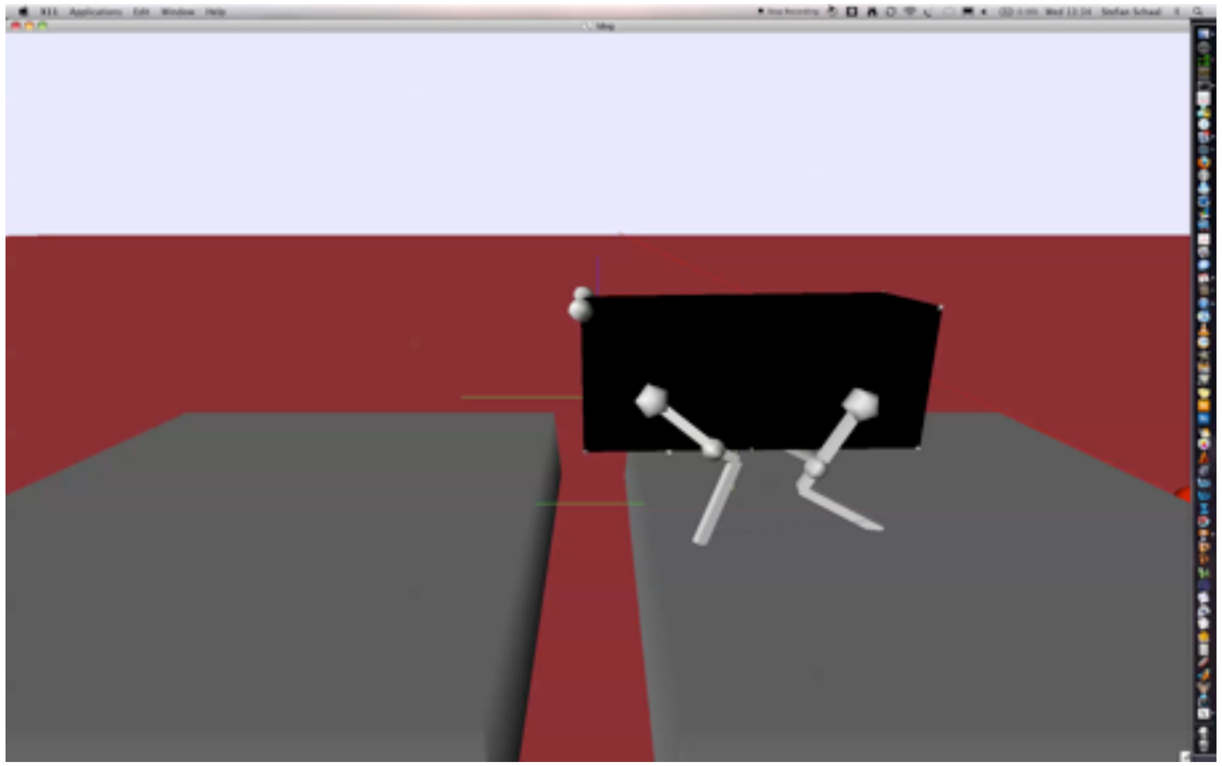
After Learning

Path Integral Optimal Control

Before Learning



After Learning



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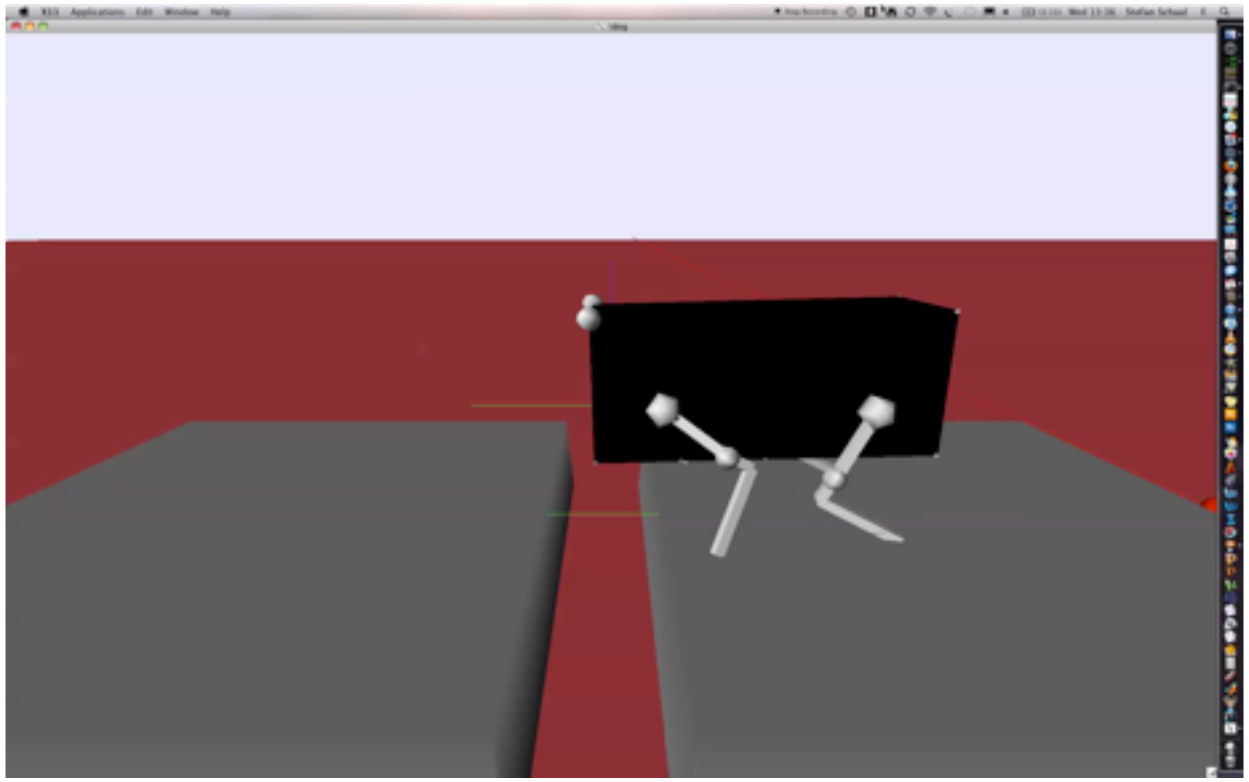
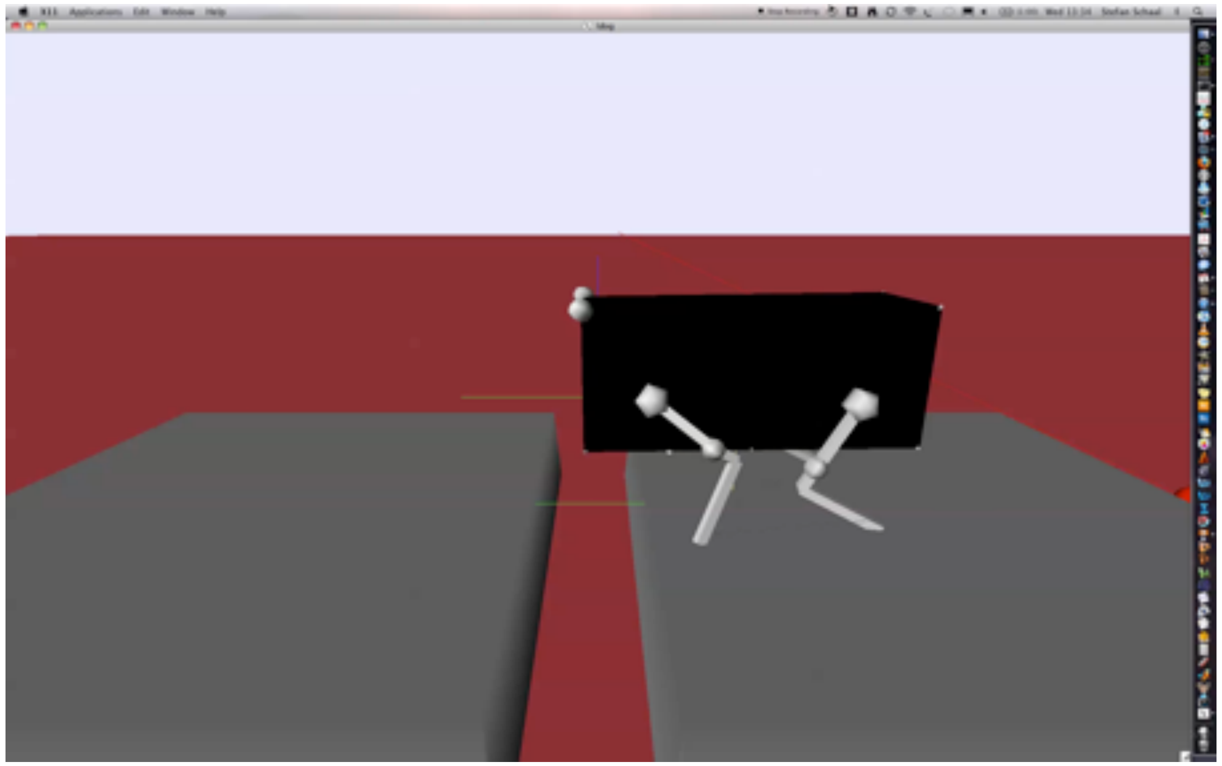


After Learning



Part 2
Beyond Linearization and internal models: Path Integral Optimal Control

Advantages!

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1) Model Free & Model Based

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General comments about control under uncertainty.

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PIs

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