BME 504/BKN 504: Lecture 2: Muscle as viscoelastic actuator. [1]

Hill-Type model $\rightarrow$ lumped parameter model

Figure - contractile element

<u>Isometric Muscle Contraction</u>

1. $\boxed{x_1 + x_2}$ = constant
2. spring force: $\boxed{F = kx_2}$
3. damper force: $\boxed{F = B\dot{x_1}}$
   Contractile element force = f_o

   Figure - f_o vs t
4. Damper + C.E. force: $\boxed{F = B\dot{x_1} + f_o}$
   because of isometric contraction
5. $x_1 + x_2 = constant \rightarrow \boxed{\dot{x_1} = \dot{x_2}}$
6. $\dot{F} = k\dot{x_2} \rightarrow \boxed{\dot{x_2} = \dot{F}/k}$
   $\dot{x_1} = -\dot{F}/k \rightarrow F = -(B/k)\dot{F} + f_o$
   $\Longrightarrow \boxed{\dot{F} + (k/b)F = (k/b)f_o} \leftarrow$ forcing function

Use Laplace transforms:

$L[x(t)] = X(s)$

$L[\dot{x}(t)] = sX(s) - x(0) \leftarrow$ initial condition

$L[step] = 1/s$

$L[\dot{F} + (k/B)F = (k/b)f_o]$

$= sF(s) - f(0) + (k/b)F(s) = (k/b)f_o(1/s)$

initial condition: $f(0) = 0$, muscle at rest

$F(s)[s + (k/B)] = (k/B)(f_o/s)$

$F(s) = \frac{k}{B} f_o \frac{1}{s*s+(k/B)}$

$L^{-1}[\frac{1}{s(s+a)}] = 1/a(1 - e^{-at})$

$F(t) = \frac{k}{B} f_o \frac{B}{k} [1 - e^{(k/B)t}]$

$F(t) = f_o[1 - e^{(k/B)t}]$

Figure - f_o vs t

Behavior when neural drive turns off: @ $t = 0, f_o = 0$, need to look at the homogenous solution

$c < t < c + a$

$\dot{F} + (k/B)F = 0$

$L[\dot{F} + (k/B)F = 0] \longrightarrow sF(s) - f(0) + (k/B)F(s) = 0$

$F(s)[s + \frac{k}{B}] = f(0) = F(c)$

$F(s) = \frac{F(c)}{s+k/B}$

$L^{-1}[\frac{1}{s+a}] = e^{-at}$

$\boxed{F(t) = F(c)e^{-(k/B)t}}$

Figure - muscle twitch, Fundamental unit of force control