# Online material for Fundamentals of Neuromechanics v1.0 August 2015 

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## Exercises: Ch. 2 Limb Kinematics

1. Re-write Eqs. 2.6 to 2.9 using 5 homogeneous transformation matrices.
2. Find the homogeneous transformation matrices for a limb based on the one shown in Fig. 2.3, but whose first DOF can also rotate in and out of the plane (i.e., rotate about the $\vec{j}_{0}$ axis.).
3. For the planar 2-link, 2-DOF limb shown in Fig. 2.3, find the Jacobian using the forward kinematic model in Eq. 2.30.
4. For Ex. 3, find the instantaneous endpoint velocities, $\dot{\vec{x}}$, for the configurations $q_{1}=q_{2}=0$ and $q_{1}=0, q_{2}=90^{\circ}$ when $\dot{q}_{1}=\dot{q}_{2}=1 \frac{\mathrm{deg}}{\mathrm{s}}$. Compare and contrast the results. Do they make sense?
5. For the planar 2-link, 2-DOF limb shown in Fig. 2.3, compare and contrast the forward kinematic models in Eqs. 2.30 and 3.13. What assumptions are being made, and what kinds of research questions can/cannot be asked with each of them?
6. For the planar 2-link, 2-DOF limb shown in Fig. 2.3, write all possible forward kinematic models.
7. For the planar 2-link, 2-DOF limb shown in Fig. 2.3, find the Jacobians for all possible forward kinematic models found in Ex. 6.

## Exercises: Ch. 3 Limb Mechanics

1. For the planar 3-link, 3-DOF limb shown in Fig. 3.3, find the Jacobian for the forward kinematic model in Eq.3.22.
2. The planar 3-link, 3-DOF limb shown in Fig. 3.3 has the associated Eqs. 3.20 and 3.21 . Describe what are the vectors $\vec{\tau}$ and $\vec{w}$.
3. For the planar 3-link, 3-DOF limb shown in Fig. 3.3, find the symbolic expressions for $J(\vec{q})$ and $J(\vec{q})^{T}$.
4. For Ex. 3, use a a mathematical package to find the symbolic expression for $J(\vec{q})^{-T}$.
5. Given the numerical examples in Sec. 3.4, draw the limb and the different vectors to explain why and how those results make sense.
6. Modify the MATLAB code in file J2D2DOF.m to explore the effect of changing posture on the input-output relationships of the system.
7. Modify the MATLAB code in file J2D2DOF.m to work for the planar 3-link, 3-DOF limb shown in Fig. 3.3.
8. Use Ex. 7 to explore the effect of changing posture on the input-output relationships of the system.

## Exercises: Ch. 4 Tendon-Driven Limbs

1. Based on Table 6.1, create the moment arm matrix for the 5 -DOF arm model, as in equation 4.21.
2. Based on Ex. 1, create the moment arm matrix for the 5 DOF arm, as in equation 4.26.
3. Based on Exs. 4 and 2, calculate the tendon excursions for that straight line motion of the hand, relative to the starting point.

## Exercises: Ch. 5 Underdetermined Control

1. Based on the information provided, solve the Diet Problem to obtain the solution shown in Eq. 5.17
2. Use the information provided in Sec. 5.3 to find a solution to maximize upward vertical force for a posture and limb parameters of your choice.

## Exercises: Ch. 6 Overdetermined Control

1. Based on Eqs. 6.1 to 6.7 , find the homogeneous transformation matrices for the 5-DOF arm model shown in Fig. 6.1.
2. Based on Ex. 1, find the forward kinematic model of the 5 -DOF arm model shown in Fig. 6.1.
3. Based on Ex. 2, find the Jacobian of the 5-DOF arm model shown in Fig. 6.1.
4. Use the numerical method described in the MATLAB code simple_inverse_kinematics.m to solve the inverse kinematic problem for a straight line motion of the hand in 3D space for the the 5-DOF arm model shown in Fig. 6.1.

## Numerical code

## Inverse kinematics

The simple_inverse_kinematics.m script solves a simple inverse kinematic problem using a closed form analytical approach.

## 5-DOF 17-muscle arm model

The FiveDOF_model_frisbee_throw.m script calculates muscle fiber lengths and velocities for a 5DOF, 17-muscle arm model as per Figs 6.3 and 6.6.

## Jacobian

The J2D2DOF.m script calculates the symbolic and numerical versions of the Jacobian for a planar 2-link 2-DOF limb.

## N -cube

The ncube.m script returns the vertices of an N -cube for dimensions 3 or higher.

## Zonotope

The zonotope_multi_N_2D.m script shows how to map N-dimensional ncubes into 2 D and 3 D via a random H matrix of dimensions 2 x N and $3 \times \mathrm{N}$, respectively. It then plots the convex hulls of the zonotopes when considering the input to be between 3 and 8 dimensions. That is, a system having 3 to 8 muscles. This function was used to create Fig. 7.9.

