• Midterm solution:

1. a) $15 \parallel 5 = \frac{15 \times 5}{20} = \frac{15}{4} + 10 = \frac{55}{4} \parallel 5 = \frac{\frac{55}{4} \times 5}{\frac{55}{4} + 5} =$

$$\frac{\frac{55}{4} \times 5}{\frac{75}{4}} = \frac{55 \times 5}{75} = \frac{55}{15} = \frac{11}{3} + 10 = \frac{41}{3} \parallel 5$$

$$\frac{\frac{41}{3} \times 5}{\frac{41}{3} + 5} = \frac{41 \times 5}{56} = 3.6\ K\Omega$$

b)
$$\left.\begin{array}{l}
N_1 : I_{total} = I_6 + I_1 \\
N_2 : I_1 = I_5 + I_2 \\
N_3 : I_2 = I_4 + I_3 \\
N_4 : I_4 + I_3 + I_7 = 0 \\
N_5 : I_5 + I_8 = I_7 \\
N_6 : I_6 = I_{total} + I_8
\end{array}\right\} \text{KCL}$$

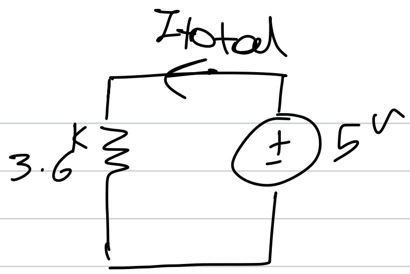
L_1: $5 = 5I_6 \Rightarrow I_6 = 1mA$

L_2: $5I_1 + 5I_5 - 5I_8 - 5I_6 = 0 \Rightarrow I_1 + I_5 - I_8 = 1$

L_3: $5I_2 + 5I_4 - 5I_7 - 5I_5 = 0 \Rightarrow I_2 + I_4 = I_7 + I_5$

L_4: $15I_3 = 5I_4 \Rightarrow 5I_3 = I_4$

c) $R_{total} = 3.6\ K\Omega$



$I_{total} = 1.39\ mA$

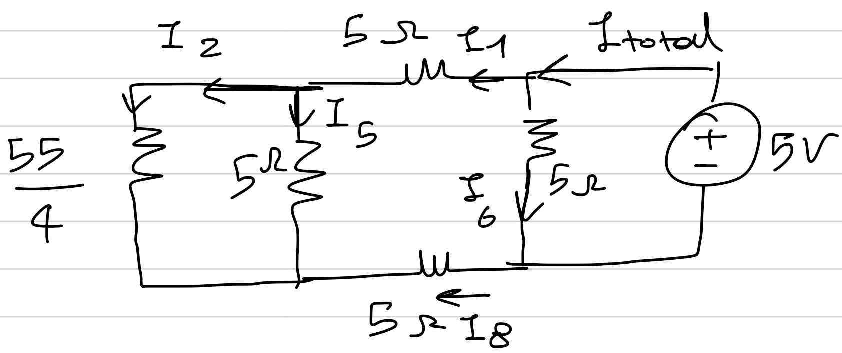
$\boxed{V_{N_6} = 0}$, $\boxed{V_{N_1} = 5V}$

$\boxed{I_6 = 1mA} \Rightarrow \boxed{V_{N_1} = 5^{K\Omega} \times 1^{mA} = 5V}$

$5 = (I_1 + I_5 - I_8)5 \Rightarrow I_1 + I_5 - I_8 = 1$

$I_{total} = I_6 + I_1 \Rightarrow I_1 = 1.39 - 1 = \boxed{0.39\ mA = I_1}$

$V_{N_1} - V_{N_2} = 5 \times 0.39 = 1.95 \Rightarrow V_{N_2} = 5 - 1.95 = \boxed{3.05V = V_{N_2}}$



$I_2 = I_1 \times \frac{5\Omega}{5 + \frac{55}{4}} = \boxed{0.104 = I_2}$

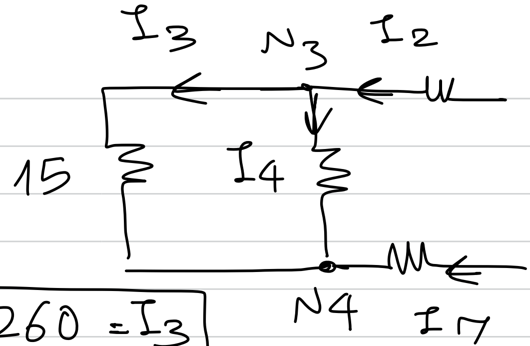
$I_1 = I_2 + I_5 \Rightarrow I_5 = 0.39 - 0.104 = \boxed{0.286 = I_5}$

$I_2' + I_5 + I_8 = 0 \Rightarrow \boxed{I_8 = -0.39}$

$5 \times I_5 = V_{N_2} - V_{N_5} \Rightarrow 1.43 = 3.05 - V_{N_5} \Rightarrow \boxed{V_{N_5} = 1.62V}$

$I_5 + I_8 = I_7 \Rightarrow$

$0.286 - 0.39 \Rightarrow \boxed{I_7 = -0.104}$



$I_3 = I_2 \times \dfrac{5}{5+15} = 0.104 \times \dfrac{1}{4} = \boxed{0.0260 = I_3}$

$I_2 = I_3 + I_4 \Rightarrow I_4 = 0.104 - 0.026 = \boxed{0.0780 = I_4}$

$V_{N2} - V_{N3} = 5 I_2 \Rightarrow 3.05 - 5 \times 0.104 = \boxed{2.53 = V_{N3}}$

$V_{N3} - V_{N4} = 5 \times I_4 \Rightarrow 2.53 - V_{N4} = \boxed{2.14 = V_{N4}}$

$V_{N1} = 5V$

$V_{N2} = 3.05$

$V_{N3} = 2.53$

$V_{N4} = 2.14$

$V_{N5} = 1.62$

$V_{N6} = 0$

$I_{total} = 1.39mA$

$I_1 = 0.39\ mA$

$I_2 = 0.104$

$I_3 = 0.0260$

$I_4 = 0.078$

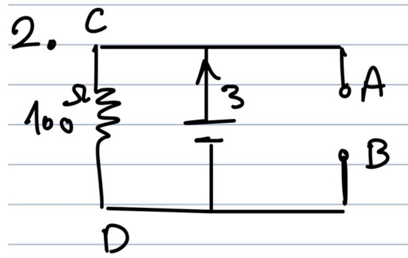
$I_5 = 0.286$

$I_6 = 1mA$

$I_7 = -0.104$

$I_8 = -0.39$

Q_2:



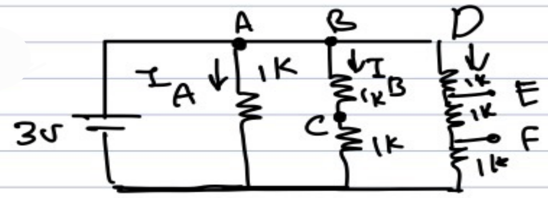
$V_{AB} = 3V$

$I_{AB} = 0$

$I_{CD} = ? \Rightarrow V_{CD} = R_{CD} I_{CD} \Rightarrow$

$I_{CD} = \dfrac{V_{CD}}{R_{CD}} = \dfrac{3V}{100} = 0.03A$

Q_3:



simplify ↓

=

$\frac{6}{5} \parallel 1 = \dfrac{\frac{6}{5} \times 1}{\frac{6}{5} + 1}$

$= \dfrac{6}{11}$

$\dfrac{3 \times 2}{5} = \dfrac{6}{5}$

⇓

$i_{total} = \dfrac{3}{\frac{6}{11}} = \dfrac{11}{2} = 5.5A$

$\Rightarrow \underline{I_A} = 5.5^A \times \dfrac{\frac{6}{5}}{\frac{6}{5} + 1} = 3\ mA$

$I' = 5.5^A \times \dfrac{1}{1 + \frac{6}{5}} = 2.5mA$

0.5A ↓

$\Rightarrow \underline{I_B} = \underline{I_C} = 2.5 \times \dfrac{3}{5} = 1.5mA$

$\underline{I_D} = \underline{I_E} = \underline{I_F} = 2.5 \times \dfrac{2}{5} = 1\ mA$

$V_A = V_B = V_D = 3V$

$V_C = 1.5V$, $V_E = 2V$, $V_F = 1V$

Q4:

a) no current flows since $V_{source} < V_{th}$

$I_D = 0$

b) from the LED characteristic:

$I_{Diode} = I_R = 5\,mA \longrightarrow V_F = 2V$

$V_{source} = V_F + V_{Resistor} = 2 + RI = 2 + 100\,\Omega \times 5\times10^{-3} \Rightarrow$

$V_{source} = 2.5V$

c) optical power emitted = Power emitted per electron x number of electrons

$\frac{E}{t}$

Energy of a single photon $= \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times 3\times10^{8}}{630\times10^{-9}} = 3.15\times10^{-19}\,J$

$E_{photon} = E_{electron} \Rightarrow I = qn \Rightarrow \frac{5\times10^{-3}}{1.6\times10^{-19}} = 3.125\times10^{16}\ \frac{electron}{s}$

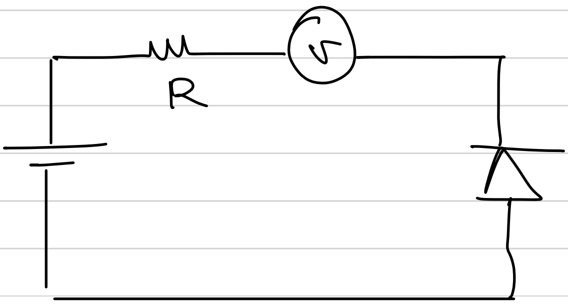
$n_{photon/s} = \eta \times n_{electron/s} = \frac{2}{10} \times 3.125\times10^{16} = 6.242\times10^{15}$

$P_{optical} = E \times n_{photon} = 3.15\times10^{-19} \times 6.24\times10^{15} = 1.96\times10^{-3} = 1.9\,mW$

Q5:

a) the left configuration is more effective in capturing light since it is in reversed bias. In reversed bias the depletion region is widened, which allows it to capture light effectively.

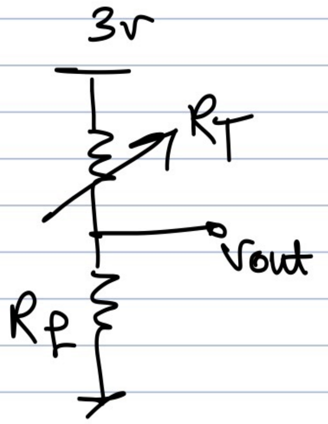
b)



$V = RI \Rightarrow I_{ph} = \frac{V_R}{R}$

we measure volt through volt meter & we know the value of R

Q6 :



$R_T = f(V_{out}) \Rightarrow$

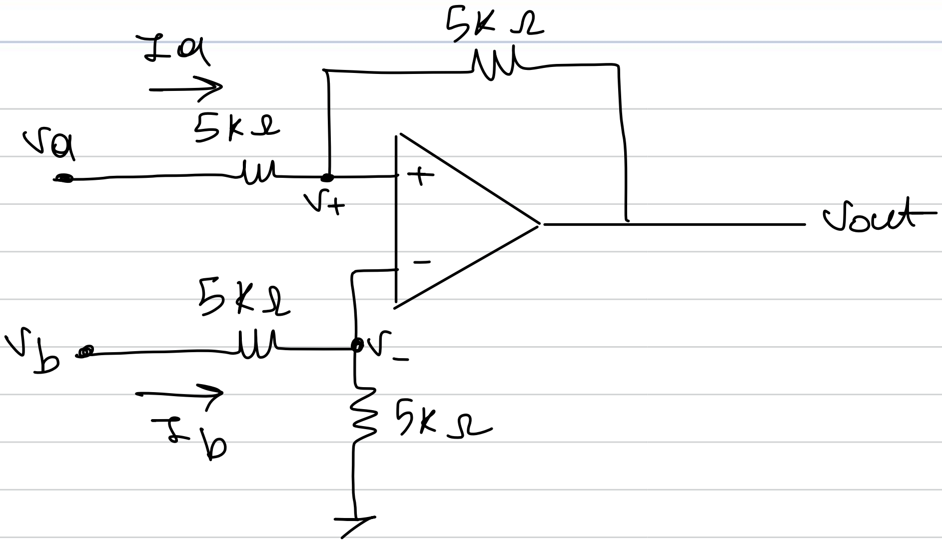
$V_{out} = R_f \cdot I$

$3 = (R_T + R_f) I \Rightarrow$

$R_T = \frac{3}{I} - R_f \Rightarrow R_T = \frac{3R_F}{V_{out}} - R_F \Rightarrow$

$R_T = R_F \left(\frac{3}{V_{out}} - 1 \right)$

Q7 :



a)

$V_+ = V_-$, $V_- = 5 I_b = \frac{V_b}{2}$

$I_b = \frac{V_b - V_-}{5}$

$V_b = 10 I_b \Rightarrow I_b = \frac{V_b}{10}$

b) $v_a - v_+ = 5 I_a \Rightarrow I_a = \dfrac{v_a - v_+}{5}$

$v_+ - v_{out} = 5 I_f \Rightarrow I_f = \dfrac{v_+ - v_{out}}{5}$

we know $v_+ = v_- \Rightarrow I_a = \dfrac{v_a - \frac{v_b}{2}}{5} = \dfrac{2v_a - v_b}{10^k}$

$I_f = \dfrac{\frac{v_b}{2} - v_{out}}{5} = \dfrac{v_b - 2v_{out}}{10^k}$

c) $\left.\begin{array}{l} v_{out} = \dfrac{v_b}{2} - 5 I_f \\ I_a = I_f \Rightarrow \dfrac{v_a - \frac{v_b}{2}}{5} \end{array}\right\} \Rightarrow V_{out} = \dfrac{V_b}{2} - 5\left(\dfrac{V_a - \frac{V_b}{2}}{5}\right) \Rightarrow$

$V_{out} = \dfrac{V_b}{2} - V_a + \dfrac{V_b}{2} = V_b - V_a$

d) $V_{out} = V_b - V_a$

$\downarrow$

the circuit perform a difference operation between the input voltages, It's a form of differential amplifier with $R_f = R_a$

Q8: $I = I_0\left(e^{\frac{qV}{nKT}} - 1\right) \Rightarrow$

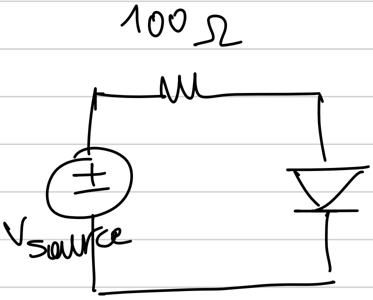
$I_{min} = 0 \Rightarrow e^{\frac{qV}{nKT}} = 1 \Rightarrow \frac{qV}{nKT} = \ln(1) = 0 \Rightarrow V_{min} = 0$

$I_{max} = 10\times10^{-3} = 1\times10^{-21} \times e^{\frac{qV}{nKT}} \Rightarrow$

$e^{\frac{qV}{nKT}} = 10^{19} \Rightarrow \frac{qV}{nKT} = 19\ln(10) = 43.74$

$\Rightarrow V = 2\times1.38\times10^{-23}\times300\times43.74\times\frac{1}{1.6\times10^{-19}} \Rightarrow$

$V_{LED} = 2.26\times10^{4}\times10^{-23}\times10^{19} = 2.26\,V$



$V_{source} = V_R + V_{LED} \Rightarrow$

$V_R = R\times I = 100\times10\times10^{-3} = 1\,V$

$V_{source_{max}} = 1 + 2.26 = 3.26$

$V_{source_{min}} = 0$

b) Photon Energy: $E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{532\times10^{-9}} = 3.73\times10^{-19}\,J$

electrons per second: $I = nq \Rightarrow n = \frac{10\times10^{-3}}{1.6\times10^{-19}} = 6.25\times10^{16}\,\frac{elec}{s}$

photons per second: $n_{photon} = \eta\times n_{electron} = 0.43\times6.25\times10^{16}$

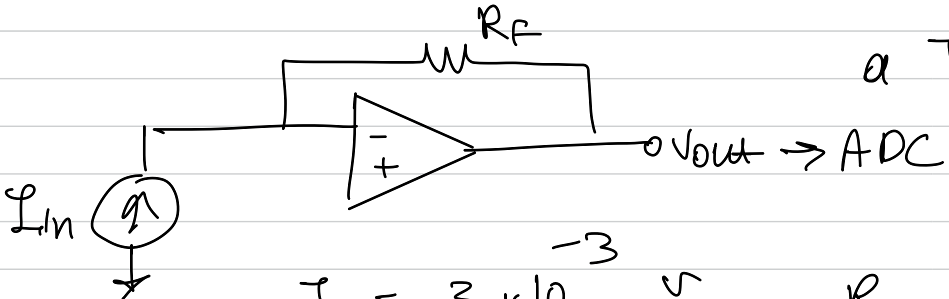
$= 2.68\times10^{16}\,\frac{photons}{s}$

optical power => $P = n_{photon} \times E = 2.68 \times 10^{16} \times 3.73 \times 10^{-19} = 10.04 \, mW$

$I_{photodiode} = R_\lambda \times P_{optical}$ → based on the curve $R_\lambda \cong 0.3 \, \frac{A}{W}$

$I_{ph} = 0.3 \times 10.04 \times 10^{-3} = 0.003 = 3mA$

Now we need to convert this current to 10 v using a Transimpedance Amplifier.



$$I = 3 \times 10^{-3} = \frac{v}{R} \Rightarrow R = \frac{10}{3 \times 10^{-3}} = 3.3K\Omega$$

we need $R_F = 3.3K\Omega$ but we want to design our circuit using 100Ω resistors so we nee 33, 100Ω resistors in series.