EE 105: Circuits Resources and Worksheets

Prof. Yasser Khan and Prof. Rehan Kapadia

W. V. Unglaub
unglaub@usc.edu

January 28, 2025

Contents

1 Online resources

Below is a curated list of quality online resources that you may find helpful for learning, exploring, and practicing a variety of concepts in circuits and electrical engineering.

1.1 Tutorials and reference websites

- **All About Circuits**: This website features extensive text-based tutorials that cover DC/AC circuits, semiconductor devices, and more. Every topic also includes a forum at the bottom of the page where students can ask questions and interact with the community to give and receive help. For additional practice, select a topic to explore and challenge yourself in the Worksheets section.
- **Electronics-Tutorials**: This is a fantastic organized collection of tutorials covering a myriad of topics in circuits with tons of diagrams and worked-out examples featuring step-by-step derivations.
- **Khan Academy**: Features a collection of short video lessons and practice problems on the fundamentals of circuit basics, signals, and systems with a user-friendly approach suitable for beginners.

1.2 Simulation tools

- **Tinkercad Circuits**: A browser-based circuit design and simulation tool from Autodesk. This is a great tool for beginners to visualize and interact with circuits in a simulated fashion. It includes basic electronic components, measurement tools, and real-time simulation. For some introductory tutorials to get you started, check out the official playlist on YouTube.
- **Falstad Circuit Simulator**: A web-based real-time simulation interface with helpful visualizations of voltages and currents which allows students to see waveforms change dynamically. A large variety of preset examples (such as RLC circuits, op-amps, and digital logic circuits) are included for students to explore with a user-friendly drag-and-drop interface.
- **LTspice**: An excellent freeware circuit simulator developed by Analog Devices which is widely used in both academia and industry. It has a slightly steeper learning curve than the other simulators but it is capable of simulating many different types of complex circuits, making it a useful tool for learning schematic capture, waveform analysis, and advanced circuit behavior. For some introductory tutorials to get started, check out the following YouTube tutorial playlist.
- **PhET Simulations**: For absolute beginners who are particularly visual learners and are looking for hands-on learning to aid conceptual understanding, this website includes many interactive demos and simulation tools for DC and AC circuit basics. Students are able to manipulate variables in real time (e.g., resistor values, battery voltage, etc.) and see immediate effects.
- **CircuitLab**: Another browser-based circuit simulator with built-in analysis tools. Featuring a library of example circuits, it can be useful for practicing circuit design and learning how measurement tools work, such as oscilloscopes.

1.3 Discussion and forum communities

- **Electronics Stack Exchange**: A Q&A format discussion forum with a broad community of hobbyists, students, and professionals. It can be a great resource for well-structured answers to specific, technical circuit questions.
- **r/ElectricalEngineering**: An electrical engineering subreddit forum featuring discussions on coursework, textbooks, career advice, and circuit troubleshotting. It can be a good place to ask questions and see how others approach similar problems.

2 Electric circuit basics

Learning objectives:

- Understand the basic elements of circuits, including currents, voltages, and resistances.
- Understand how to use Kirchhoff's circuit laws to solve linear systems of equations for relevant quantities.
- Understand how to add resistors and capacitors in series and in parallel.

2.1 Introduction

Electronic devices are the building blocks of all electronic circuits. They consist of interconnected electronic components like resistors, capacitors, inductors, diodes, and transistors. In particular, the transistor allows for the manipulation of the flow of electrical energy to achieve specific functions, such as amplification, switching, and signal processing. The design and analysis of electronic circuits rely on several foundational principles and laws. In this section, we will introduce several concepts associated with electrical circuits, along with a set of laws essential for understanding and designing linear circuits.

Circuit analysis begins with a few fundamental concepts: voltage, current, and resistance. Along with these quantities, basic circuit elements (like resistors, voltage sources, and current sources) form the building blocks of nearly all electrical systems. By applying Ohm's law and rules for series and parallel connections, one can analyze simple circuits and build up to more complex ones.

Below is a concise overview of these essential topics.

2.2 Voltage, Current, and Resistance

Voltage (V):

- Definition: The electrical potential difference between two points in a circuit. It's a measure of how much energy (per unit charge) is required to move a charge from one point to another.

- Unit: Volt (V).

Current (I):

- Definition: The rate at which charge flows through a conductor or circuit element. If you imagine electrons (or charges) moving from a higher potential to a lower potential, the flow of this charge constitutes current.

- Unit: Ampere (A). One ampere equals one coulomb of charge passing a point per second.

Resistance (R):

- Definition: The opposition that a circuit element presents to the flow of current. Materials with high resistance resist current flow strongly, while low-resistance materials allow easier flow.

- Unit: Ohm (Ω).

2.3 Ohm's Law

Ohm's law is one of the most fundamental relationships in circuit analysis. In electronic circuits, it describes the relationship between voltage V, current I, and resistance R in a conductor. Mathematically, it is expressed as:

$$V = I \cdot R \tag{1}$$

where

- V is the voltage across the resistor (in volts),

- I is the current through the resistor (in amperes),

- R is the resistance (in ohms).

This relationship allows you to solve for any one of the three quantities (voltage, current, or resistance) if you know the other two. Ohm's Law signifies that the voltage across a resistor is directly proportional to the current passing through it, with the resistance R as the constant of proportionality. The current I is the rate at which electric charge flows through a conductor. It is measured in amperes (A), where one ampere equals one coulomb of charge passing through a point in one second. The relationship between current I and electric charge Q is:

$$I = \frac{\Delta Q}{\Delta t}, \tag{2}$$

where ΔQ represents the amount of charge passing through a point in time interval Δt. If the charge at any point in a circuit is smoothly varying with time t, the instantaneous current $I(t)$ is then defined as

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}. \tag{3}$$

2.4 Basic Circuit Elements

1. Voltage Source: A circuit element (like a battery or power supply) that provides a constant voltage difference.

2. Current Source: A circuit element designed to maintain a constant current flow.

3. Resistor: A component specifically designed to provide a precise resistance R.

4. Wires and Conductors: Usually assumed to have negligible resistance in ideal circuit analysis.

5. Switch: A mechanical or electronic device used to open or close a circuit path.

2.5 Series and Parallel Connections

In circuits, components are often combined either in series or parallel (or more complex combinations). Understanding these configurations is crucial for analyzing and simplifying circuits.

Series Connection:

- Definition: Components are in series if the same current flows through each one sequentially.

- Equivalent Resistance: If resistors $R_1, R_2, \ldots, R_n$ are in series, the total resistance R_{series} is:

$$R_{\text{series}} = R_1 + R_2 + \cdots + R_n.$$

- Voltage Division: In a series circuit, the source voltage divides across each resistor in proportion to that resistor's value.

Parallel Connection:

- Definition: Components are in parallel if they share the same voltage across each one but can carry different currents.

- Equivalent Resistance: If resistors $R_1, R_2, \ldots, R_n$ are in parallel, the total (equivalent) resistance R_{parallel} satisfies:

$$\frac{1}{R_{\text{parallel}}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}.$$

- Current Division: In parallel, the current divides among the parallel branches according to their conductances (inverses of resistances).

2.6 Simple Example

Series Circuit Example:

Suppose a 9 V battery is connected to two resistors $R_1 = 3\,\Omega$ and $R_2 = 6\,\Omega$ in series:

1. Total Resistance:

$$R_{\text{total}} = 3 + 6 = 9\,\Omega.$$

2. Circuit Current (from Ohm's law):

$$I = \frac{V_{\text{battery}}}{R_{\text{total}}} = \frac{9\,\text{V}}{9\,\Omega} = 1\,\text{A}.$$

3. Voltage Drops:

- Across R_1: $V_1 = I \times R_1 = 1 \times 3 = 3\,\text{V}$.
- Across R_2: $V_2 = I \times R_2 = 1 \times 6 = 6\,\text{V}$.

Parallel Circuit Example

If the same 9 V battery is connected across two parallel resistors $R_1 = 10\,\Omega$ and $R_2 = 15\,\Omega$:

1. Equivalent Resistance:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{10} + \frac{1}{15} = 0.1 + 0.0667 = 0.1667, \quad R_{\text{eq}} \approx 6\,\Omega.$$

2. Total Current:

$$I_{\text{total}} = \frac{9\,\text{V}}{6\,\Omega} = 1.5\,\text{A}.$$

3. Branch Currents:

- Through R_1: $I_1 = \frac{9}{10} = 0.9\,\text{A}$.
- Through R_2: $I_2 = \frac{9}{15} = 0.6\,\text{A}$.

Note that $I_1 + I_2 = 1.5\,\text{A}$, which matches the total current.

2.7 Summary

1. Ohm's Law: Central to determining relationships among voltage, current, and resistance in simple one-resistor circuits.

2. Series Circuits: Resistances add directly; current is the same through each component.

3. Parallel Circuits: Conductances (inverse of resistance) add; voltage is the same across each branch, but currents can differ.

4. Combination Circuits: Larger, more complex circuits often reduce step by step into simpler series or parallel sections—or use other methods (like Kirchhoff's laws) for a systematic approach.

By mastering these basics—voltage, current, resistance, Ohm's law, and how resistors combine—one gains a solid foundation for exploring more complex circuit topics like capacitors, inductors, operational amplifiers, and beyond.

2.8 Problem Worksheet

Click here to access problems on the basics of circuits.

3 Kirchhoff's Laws and Equivalent Circuits

Learning objectives:

- Understand Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL).
- Use KCL and KVL to solve for various quantities of interest in circuit topologies with multiple loops.
- Understand Thevenin and Norton equivalent circuits.

In electrical engineering, understanding how currents and voltages behave in a circuit is foundational. Two of the most fundamental laws that govern the behavior of currents and voltages are Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL). These laws apply to all circuits, from simple ones with just a battery and a resistor to more complex networks involving multiple components. In parallel, the concept of equivalent circuits allows us to simplify complex networks into simpler, more manageable forms without changing their external electrical behavior.

3.1 Kirchhoff's Current Law (KCL)

Statement of KCL:

Kirchhoff's Current Law states that the algebraic sum of all currents entering (or leaving) a node in a circuit is zero. In simpler terms:

$$\sum I_{\text{entering a node}} = \sum I_{\text{leaving a node}}$$

or equivalently,

$$\sum I = 0 \quad \text{(at any node).}$$

Interpretation:

A node (sometimes also called a junction) is any point where two or more circuit elements meet. According to KCL, electric charge (and thus current) does not accumulate or vanish at a node; what flows in must flow out. This reflects the principle of conservation of charge.

Example:

Suppose three wires meet at a node N. If current I_1 and I_2 flow into the node, and current I_3 flows out, then KCL implies:

$$I_1 + I_2 - I_3 = 0 \quad \Longrightarrow \quad I_3 = I_1 + I_2.$$

3.2 Kirchhoff's Voltage Law (KVL)

Statement of KVL:

Kirchhoff's Voltage Law states that the algebraic sum of all voltages around any closed loop in a circuit is zero:

$$\sum V = 0 \quad \text{(around any loop).}$$

Interpretation:

A loop is any path in a circuit that starts and ends at the same node and does not pass through any intermediate node more than once. KVL is essentially the principle of conservation of energy in a circuit: as you traverse a loop, any energy gained (from voltage sources) must be balanced by energy lost (across components like resistors).

Example:

Consider a simple series loop with a battery (voltage V_{source}) and two resistors R_1 and R_2. If I is the current flowing through them, then KVL would give:

$$V_{\text{source}} - I\,R_1 - I\,R_2 = 0.$$

3.3 Applications of Kirchhoff's Laws

1. Node-Voltage Method (KCL-based):

- Assign voltages to nodes relative to a reference (ground).
- Write KCL equations at each node using Ohm's law to express currents in terms of voltages.
- Solve the simultaneous equations for unknown node voltages.

2. Mesh-Current Method (KVL-based):

- Identify loops (meshes) in the circuit.
- Assign a current variable to each loop.
- Write KVL equations for each loop in terms of loop currents, then solve for unknowns.

These systematic methods reduce complex circuits to sets of linear equations that can be solved step by step.

3.4 Equivalent Circuits

An equivalent circuit is a simplified representation that preserves the same electrical behavior at the terminals of interest as the original, more complex network. Equivalent circuits are simplified representations of complex electrical circuits that retain the same electrical behavior as the original circuit for analysis purposes. By using an equivalent circuit, engineers can replace a complicated network of components with simpler representations that are easier to analyze on both a conceptual and mathematical level. In this regard, equivalent circuits facilitate the design of larger, more complicated systems by isolating specific sections.

Circuits can have components such as resistors and capacitors placed in series or in parallel to each other. The configuration of multiple resistors or capacitors in a circuit can be simplified using specific rules, yielding a streamlined circuit topology. A complicated linear circuit can be replaced by a single voltage source V_{th} in series with an effective resistor R_{th}. This type of equivalent circuit, called a *Thevenin* equivalent circuit, simplifies the calculation of the circuit's response to different electrical loads. A Thevenin circuit represents a circuit seen from two terminals as a single voltage source V_{th} in series with a resistor R_{th}. Another type of equivalent circuit is the *Norton* equivalent, which takes a given circuit and represents it by a current source I_{N} placed in parallel with an effective resistor R_{N}. Norton and Thevenin equivalents are related and can be converted into each other. A Norton circuit represents a circuit as a single current source I_n in parallel with a resistor R_n. These simplifications greatly aid in analyzing how a circuit interacts with external components.

Thévenin and Norton Theorems

- Thévenin's Theorem: Any linear, bilateral circuit with two output terminals can be replaced by a single voltage source V_{th} in series with a resistor R_{th}.
  - V_{th} is found by calculating the open-circuit voltage at the terminals (i.e., the voltage when no load is connected).
  - R_{th} is found by "looking back" into the circuit from those terminals with independent sources turned off (voltage sources replaced by short circuits, current sources replaced by open circuits).
- Norton's Theorem: The same circuit can also be represented by a current source I_n in parallel with a resistor R_n.
  - I_n is the short-circuit current at the terminals (i.e., the current when the terminals are directly shorted).
  - R_n is found the same way as R_{th}, and in fact $R_n = R_{th}$.

Example:

A circuit with multiple resistors and sources can often be reduced to a single source and a single resistor by calculating an open-circuit voltage (for the Thévenin voltage) and short-circuit current (for Norton or by extension for Thévenin). This makes analyzing the effect of connecting a load resistor R_L much easier.

3.5 Summary

- Kirchhoff's Current Law (KCL): The sum of currents into a node equals zero (conservation of charge).
- Kirchhoff's Voltage Law (KVL): The sum of voltage rises and drops around any closed loop is zero (conservation of energy).
- Equivalent Circuits: Simplify complex networks into simple forms (like Thévenin or Norton equivalents) that preserve external circuit behavior.

Together, Kirchhoff's Laws and the concept of equivalent circuits form the bedrock of circuit analysis. They enable engineers to break down complicated networks systematically, solve for unknown voltages and currents, and ultimately design and understand sophisticated electronic systems.

3.6 Problem Worksheet

Click here to access problems on Kirchhoff's Laws and equivalent circuits.

4 Capacitors and Inductors

Learning objectives:

- Understand the basic properties of capacitors and inductors.
- Understand how voltage and current change as functions of time in circuits featuring reactive components such as capacitors and inductors.
- Understand the basics of impedance in circuits.

Capacitors and inductors are two of the most fundamental passive components in electrical engineering. They store and release energy—capacitors store energy in an electric field, whereas inductors store energy in a magnetic field. By leveraging these storage properties, engineers design circuits to filter signals, manage power delivery, generate oscillations, and perform myriad other functions. Below is a basic overview of what capacitors and inductors are, how they behave, and how they are commonly used in electric circuits.

4.1 Capacitors

What is a Capacitor?

A capacitor is a device that stores electrical charge and in particular, stores electrical energy in the form of an electric field. This is visually depicted in Fig. 1. It typically consists of two conductive plates separated by an insulator (dielectric). When a voltage is applied across the plates, an electric field is created, and charge accumulates on each plate—positive charge on one plate, negative charge on the other.

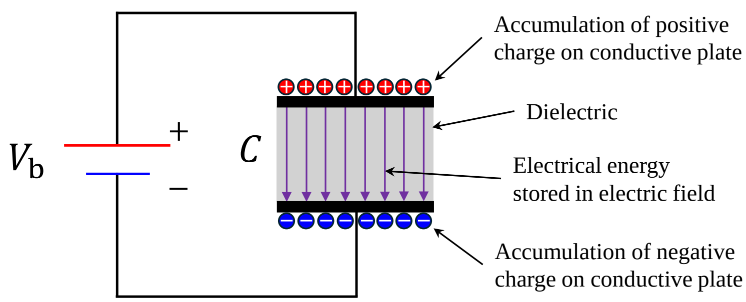

Figure 1: Schematic overview of a capacitor C, a circuit component that stores electrical energy in the electric field (purple arrows) due to the accumulation of positive (red dots) and negative (blue dots) electric charge on parallel conducting plates separated by a dielectric material (gray area).

The amount of charge Q stored on a capacitor is directly proportional to the voltage V across its plates; this relationship is given by:

$$Q = C \cdot V, \tag{4}$$

C is the capacitance of the capacitor, measured in units of farads (F).

In list form,

- Q is the charge in coulombs (C),
- V is the voltage in volts (V), and
- C is the capacitance in farads (F).

Typical practical capacitors might have values in microfarads (μF), nanofarads (nF), or picofarads (pF).

Current-Voltage Relationship:

In circuit analysis, the current $i(t)$ through a capacitor is related to the rate of change of the voltage $v_C(t)$ across it:

$$i(t) = C\,\frac{dv_C(t)}{dt}.$$

This means that when the voltage across a capacitor changes quickly, it draws (or delivers) a large current.

Energy Storage:

A charged capacitor holds energy in its electric field:

$$E_C = \tfrac{1}{2}CV^2.$$

This stored energy can be discharged later when the circuit requires it.

Common Uses:

- Decoupling/Bypass: Capacitors placed near ICs (integrated circuits) to smooth out fluctuations in the power supply line.
- Filtering: In combination with resistors or inductors, capacitors form filters (low-pass, high-pass, band-pass) to shape signal frequency content.
- Timing: RC circuits (resistor-capacitor) establish time constants in oscillators and timers (e.g., 555 timer circuits).
- Coupling/Blocking: A capacitor can allow AC signals to pass between stages of an amplifier but block DC offset.

4.2 Inductors

What is an Inductor?

An inductor (often just a coil of wire) stores energy in a magnetic field created by current flowing through it. The more turns of wire and the larger the coil (and/or the presence of a ferromagnetic core), the higher its inductance.

Inductance (L):

The inductance L of an inductor indicates how much voltage is induced for a given rate of change of current:

$$v_L(t) = L\,\frac{di_L(t)}{dt},$$

where

- $v_L(t)$ is the voltage across the inductor,
- $i_L(t)$ is the current through the inductor, and
- L is the inductance measured in henries (H).

Typical inductors might have values in microhenries (μH) or millihenries (mH).

Energy Storage:

The energy stored in an inductor's magnetic field is:

$$E_L = \tfrac{1}{2}LI^2,$$

where I is the current through the inductor.

Common Uses:

- Filtering: Inductors in combination with capacitors and/or resistors create filters that pass or block certain frequencies (e.g., in power supplies or audio crossover networks).
- Energy Storage: Inductors store energy for use in power conversion circuits (like switch-mode power supplies).

- Chokes: Inductors can choke or limit high-frequency noise in signal or power lines.
- Transformers: Two coupled inductors form a transformer for stepping voltages up or down in AC circuits.

4.3 Capacitors and Inductors in Circuits

So far, we have worked with *direct* current in which electric charge only flows in one direction. In the case of *alternating* current, electric charge periodically reverses its direction and magnitude. In AC circuits, the *impedance* is the total opposition to the flow of current and is expressed as a complex number to reflect both the resistance R and the reactance X in the circuit, respectively representing the real and imaginary components of impedance. Complex impedance allows for comprehensive analysis of circuits involving inductors and capacitors, which behave differently under alternating current compared to direct current. Mathematically, complex impedance may described in terms of the complex number $Z = R + \mathrm{i}X$, with real resistance R and imaginary reactance $\mathrm{i}X$. While resistance opposes current flow in a circuit by dissipating energy as heat, reactance opposes current flow due to *inductance*, the property of circuit components that resists changes in current through electromagnetic induction, and *capacitance*, the property of circuit components that stores electrical energy in an electric field.

Reactance and Impedance:

Both capacitors and inductors have frequency-dependent behavior:

1. Capacitive Reactance: $X_C = \frac{1}{\omega C}$, where $\omega = 2\pi f$ is the angular frequency. High frequency $\rightarrow$ low capacitive reactance.

2. Inductive Reactance: $X_L = \omega L$. High frequency $\rightarrow$ high inductive reactance.

In AC analysis, capacitors and inductors are treated as impedances:

$$Z_C = \frac{1}{j\omega C}, \quad Z_L = j\omega L,$$

where $j = \sqrt{-1}$. This allows circuit engineers to calculate voltage and current phasors just as they do with resistors (where $Z_R = R$ is constant with frequency).

Resonance (LC Circuits):

A circuit with an inductor L and capacitor C can exhibit resonance, where energy oscillates between the capacitor's electric field and the inductor's magnetic field. The resonance (natural) angular frequency is:

$$\omega_0 = \frac{1}{\sqrt{LC}},$$

and it plays a key role in filters, oscillators, and tuning circuits (like radio tuners).

4.4 Typical Applications

1. Power Supplies:
   - Filtering: Capacitors smooth out the ripple after AC is rectified to DC. Inductors may help further reduce ripple or form part of switch-mode regulators.
   - PFC (Power Factor Correction): Inductors and capacitors can correct supply power factor in industrial settings.
2. Signal Processing:
   - Filters: LC or RC filters can pass or block specific frequency ranges.
   - Oscillators: LC resonators or RC timing networks generate waveforms in audio, radio, or microcontroller circuits.
3. RF Circuits:
   - Matching Networks: Capacitors and inductors help match impedances to maximize power transfer in antenna and radio circuits.

- Tuning: By varying capacitance or inductance, engineers tune the resonance frequency of an antenna or an LC circuit.

4. Transient and Energy Storage:
   - Snubbers (RC or RLC networks) protect switching devices from voltage spikes by absorbing or dissipating energy.
   - Energy Transfer: Inductors in switch-mode power supplies store and release energy efficiently during switching cycles.

4.5 Summary

Capacitors and inductors are indispensable for any engineer working on circuits. Their frequency-dependent impedance makes them crucial in designing filters and oscillators, while their energy storage properties make them vital in power supply and transient protection circuits. By understanding how each component stores and releases energy—and how their impedance changes with frequency—engineers can build and analyze a vast array of electronic systems, from simple timers and power filters to complex communication networks.

In particular,

- Capacitors oppose changes in voltage and can deliver or absorb current based on how quickly the voltage changes across them.
- Inductors oppose changes in current and can deliver or absorb voltage based on how quickly the current changes through them.
- In DC steady state, capacitors behave like open circuits (blocking steady current after charging), and inductors behave like short circuits (allowing steady current flow).
- In AC circuits, their reactances depend on frequency: inductors' reactance goes up with frequency, while capacitors' goes down.
- Both store energy—capacitors in their electric field, inductors in their magnetic field—and release it to the circuit when needed.
- They are essential for a wide range of applications, including filtering, energy storage, power regulation, and signal generation.

4.6 Problem Worksheet

Click here to access problems on capacitors and inductors.

5 Operational Amplifiers

Learning objectives:

- Understand the basic elements of operational amplifiers (op amps).
- Understand different types of op amps.
- Understand how to use op amps to achieve different levels of signal gain in a circuit.

An *operational amplifier* (op amp) is a high-gain, DC-coupled voltage amplifier with differential inputs (inverting and non-inverting) and typically a single-ended output. In an ideal sense, it has infinite open-loop gain, infinite input impedance, zero output impedance, and zero offset voltage. While no physical op amp meets these "ideal" criteria perfectly, real op amps approach them closely enough for many practical applications. Below is a fundamental overview of op amps, including their ideal properties, typical configurations, and common uses in electronic circuits.

5.1 Basics and Ideal Model

1. Differential Inputs:
   - v_+ is the non-inverting input.
   - v_- is the inverting input.
2. Single Output: v_{out}.
3. Infinite Open-Loop Gain:
$$A_{\text{OL}} = \frac{v_{\text{out}}}{v_+ - v_-} \quad (\text{ideally, } A_{\text{OL}} \to \infty).$$
   In practice, A_{OL} might be 10^5 to 10^7 at low frequencies.
4. Infinite Input Impedance: No current flows into either input terminal in an ideal op amp.
5. Zero Output Impedance: The output can drive loads without significant voltage drop inside the op amp.

Key Consequence in Feedback Circuits: If negative feedback is applied, the difference between inputs $(v_+ - v_-)$ becomes extremely small (ideally zero), leading to the "virtual short" concept in many linear op amp configurations.

5.2 Op Amp Configurations

Inverting Amplifier:

- Setup:
  - Input signal v_{in} feeds the inverting input through an input resistor R_{in}.
  - A feedback resistor R_f connects from output to inverting input.
  - Non-inverting input is grounded (0 V).
- Gain:
$$A_v = -\frac{R_f}{R_{\text{in}}}.$$
  The negative sign indicates a 180° phase shift.

Non-Inverting Amplifier:

- Setup:
  - Input signal v_{in} feeds the non-inverting input.
  - The inverting input is connected to the output through a feedback resistor R_f, and to ground through another resistor R_g.

- Gain:

$$A_v = 1 + \frac{R_f}{R_g}.$$

Voltage Follower (Buffer):

- Setup:
  - Output is directly connected to the inverting input.
  - Signal is applied to the non-inverting input.
- Gain:

$$A_v = 1.$$

Because of the high input impedance, it acts as a buffer, isolating the source from the load.

Summing Amplifier:

- Setup:
  - Multiple input resistors $R_a, R_b, \ldots$ feed into the inverting input.
  - A single feedback resistor R_f from output to inverting input.
- Output:

$$v_{\text{out}} = -\frac{R_f}{R_a} v_a \; - \; \frac{R_f}{R_b} v_b \; - \cdots$$

(assuming the same resistor values for each input simplifies the gain calculation).

Difference Amplifier:

- Setup:
  - Two inputs v_1 and v_2 combine in a resistor network such that the output is proportional to $(v_2 - v_1)$.
- Gain (assuming symmetrical resistor values):

$$v_{\text{out}} = \left(\frac{R_2}{R_1}\right)(v_2 - v_1).$$

Integrator / Differentiator:

- Setup involves placing a capacitor in the feedback or input path to perform an integration or differentiation of the input signal:
  - **Integrator**: capacitor in the feedback path, resistor in series with input.
  - **Differentiator**: resistor in the feedback path, capacitor in series with input.
- Gain:

5.3 Practical Non-Idealities

Real op amps deviate from the ideal in several ways:

1. **Finite Open-Loop Gain**: The gain might be high (e.g., 10^5) but not infinite.
2. **Input Offset Voltage**: A small unwanted voltage that effectively appears between the inverting and non-inverting inputs.
3. **Input Bias Current**: Small currents flow into or out of the input terminals.
4. **Slew Rate**: A limit on how fast the output voltage can change (volts per microsecond, V/µs).
5. **Finite Bandwidth**: The gain typically decreases with frequency, often described by the gain-bandwidth product.

These real effects must be taken into account in high-precision or high-speed circuits.

5.4 Common Uses in Electric Circuits

1. **Signal Conditioning**: Amplifying, filtering, or buffering sensor outputs before feeding into data acquisition systems.
2. **Mathematical Operations**:
   - Adder / Subtractor: Summing amplifier or difference amplifier.
   - Integrator / Differentiator: Used in analog computation and signal processing.
3. **Comparators**: Detect if one signal is higher or lower than another (though dedicated comparator ICs often have design optimizations for faster switching).
4. **Oscillators and Filters**: With feedback networks, op amps can build active filters (low-pass, high-pass, band-pass) and produce sine, square, or other waveforms.
5. **Voltage Follower / Buffer**: Provide high-input, low-output impedance to isolate signal sources from loads.

5.5 Summary

Overall, op amps are indispensable components in analog electronics, enabling linear amplification and signal processing in an array of systems—from simple audio amplifiers to advanced instrumentation and control circuits.

- An operational amplifier is essentially a high-gain amplifier with differential inputs and a single-ended output.
- Feedback is key. By providing negative feedback, you can set precise closed-loop gains and perform a variety of linear operations on signals.
- Basic Configurations (inverting, non-inverting, summing, difference) are building blocks for countless circuits.
- Real-World Limitations (finite gain, input offset, slew rate, etc.) guide practical op amp choice and design considerations.

5.6 Problem Worksheet

Click here to access problems on operational amplifiers.

6 Problems

6.1 Electric circuit basics [return to TOC]

The following problems focus on circuit fundamentals—focusing on Ohm's law, series and parallel resistor networks, Kirchhoff's laws, power calculations, and basic RC transients. For several problems, there is accompanying Python code that may be used to calculate and in some cases plot various quantities. Each problem combines analysis (theory) with computation (Python), reinforcing crucial circuit concepts through hands-on simulation/plots. Try varying component values (resistor or capacitor) in the Python codes to deepen understanding. Always keep track of units (V, A, Ω, F, etc.) and follow safe practices when experimenting with real circuits.

6.1.1 Problem 1: Basic Ohm's Law

A resistor of $R = 10\,\Omega$ is connected to a DC voltage source of $V = 5\,\text{V}$.

1. Find the current I flowing through the resistor.
2. Calculate the power P dissipated by the resistor.

6.1.2 Problem 2: Series Resistors

You have two resistors $R_1 = 4\,\Omega$ and $R_2 = 6\,\Omega$ connected in series across a 20 V supply.

1. Find the total resistance R_{total}.
2. Calculate the current I flowing through the series combination.
3. Find the voltage drop across each resistor.

6.1.3 Problem 3: Parallel Resistors

Two resistors $R_1 = 10\,\Omega$ and $R_2 = 15\,\Omega$ are connected in parallel across a 12 V source.

1. Find the total (equivalent) resistance R_{eq}.
2. Calculate the total current drawn from the source.
3. Find the current through each resistor.

6.1.4 Problem 4: Series-Parallel Network

A 12 V battery is connected to a network of three resistors:

- $R_1 = 3\,\Omega$ in series with a parallel combination of $R_2 = 6\,\Omega$ and $R_3 = 6\,\Omega$.

1. Find the equivalent resistance R_{eq} of the circuit.
2. Calculate the total current I_{total}.
3. Determine the current through each branch of the parallel section (I_2 and I_3).

6.1.5 Problem 5: Kirchhoff's Voltage Law (KVL)

Consider a simple loop with a 9 V battery and two resistors $R_1 = 2\,\Omega$ and $R_2 = 7\,\Omega$ in series. Show that applying KVL around the loop confirms that the sum of voltage drops equals the source voltage.

1. Calculate the current in the loop.
2. Find the voltage drop across each resistor.
3. Verify that $V_battery - V_{R_1} - V_{R_2} = 0$.

6.1.6 Problem 6: Kirchhoff's Current Law (KCL)

A node N has three connected branches:

- A 10 V supply through a 5 Ω resistor into the node
- A branch from the node directly to ground through a 10 Ω resistor
- Another branch from the node to ground through a 5 Ω resistor

1. Calculate the current I_1 entering the node from the supply.
2. Calculate the currents I_2 and I_3 leaving the node through each of the 10 Ω and 5 Ω resistors.
3. Verify KCL: $I_1 = I_2 + I_3$.

Assumption: Node N has some node voltage V_N. We can write KCL + individual resistor currents to find V_N.

6.1.7 Problem 7: Power Calculation in a Resistor Network

A 12 V battery is connected to a resistor network consisting of two resistors in parallel ($R_1 = 4\,\Omega$, $R_2 = 12\,\Omega$) that is then in series with a $R_3 = 4\,\Omega$ resistor.

1. Find the total power delivered by the battery.
2. Find the power dissipated by each resistor individually.

6.1.8 Problem 8: DC Voltage Sweep in a Single Resistor

Consider a single resistor $R = 10\,\Omega$. We vary the supply voltage from 0 V to 20 V in increments of 2 V.

1. For each supply voltage, compute the current through the resistor.
2. Plot the relationship between supply voltage (x-axis) and current (y-axis).

6.1.9 Problem 9: Parametric Sweep for Parallel Resistance

We have two resistors in parallel, R_1 and R_2, across a constant 10 V supply. Let's fix $R_1 = 10\,\Omega$ and sweep R_2 from 5 Ω to 50 Ω in steps of 5 Ω. For each R_2 value, find: 1. The equivalent resistance R_{eq}.

2. The total current I_{total}.
3. Plot R_{eq} and I_{total} vs. R_2.

6.1.10 Problem 10: RC Charging Circuit (Transient)

A $10\,\Omega$ resistor and a $100\,\mu$F capacitor are connected in series to a 5 V DC supply through a switch. At $t = 0$, the switch is closed.

1. Write the expression for the capacitor voltage $v_C(t)$ during charging.
2. Plot $v_C(t)$ and the resistor current $i_R(t)$ from $t = 0$ to $t = 0.05\,\text{s}$ in Python.

6.2 Kirchhoff's Laws and Equivalent Circuits

[return to TOC]

The following problems focus on Kirchhoff's circuit laws and equivalent circuits.

- **KCL** states that the algebraic sum of currents at a node is zero.
- **KVL** states that the algebraic sum of voltage changes around any closed loop is zero.
- For more complex circuits, systematic methods (Node-Voltage Method, Mesh-Current Method) are direct applications of KCL and KVL.
- In problems involving dependent sources, track the controlling variable carefully.
- For sweeps or parametric studies, Python loops (or 'numpy' arrays) plus symbolic or direct formulas help see how voltages/currents change.
- In transient circuits (like RC), KVL becomes a differential equation. The standard solutions often appear in exponential forms.

These 10 problems cover a variety of Kirchhoff's Laws scenarios—from simple loops and nodes to multi-loop systems, dependent sources, and even a transient RC example.

6.2.1 Problem 1: Basic Single Loop (KVL)

A loop is composed of a 12 V battery and two series resistors: $R_1 = 3\,\Omega$ and $R_2 = 9\,\Omega$. Use KVL around the loop to find:

1. The current I in the loop.
2. The voltage drop across each resistor, V_{R_1} and V_{R_2}.
3. Verify that the sum of voltage rises and drops is zero.

6.2.2 Problem 2: Single Node (KCL)

A node N has three connected branches:

1. A 10 V source feeding the node through a 10 Ω resistor.
2. A resistor $R_2 = 5\,\Omega$ from the node to ground.
3. A resistor $R_3 = 10\,\Omega$ from the node to ground.

Use KCL at the node to find the node voltage V_N (measured relative to ground). Then find each branch current and verify $I_{\text{in}} = I_{\text{out}}$.

6.2.3 Problem 3: Two Voltage Sources in a Series Loop (KVL)

A loop has two voltage sources, $V_1 = 10\,\text{V}$ and $V_2 = 5\,\text{V}$, in series with two resistors $R_1 = 2\,\Omega$ and $R_2 = 3\,\Omega$. The loop orientation is such that V_1 "helps" V_2 (they add in the same polarity). Use KVL to find the loop current I, then the voltage drops across each resistor.

Circuit Orientation

- The total source voltage is $10 + 5 = 15\,\text{V}$ (since they are aiding each other).

6.2.4 Problem 4: Node Voltage Method (KCL)

We have a circuit with two nodes of interest: node A (the higher node) and the reference ground. There are three resistors:

- $R_1 = 4\,\Omega$ from a 12 V supply to node A. (The other terminal of the supply is ground.)
- $R_2 = 6\,\Omega$ from node A to ground.
- $R_3 = 12\,\Omega$ also from node A to ground.

Use the node-voltage method (which is effectively KCL at node A) to find the node voltage V_A, the current through each resistor, and verify KCL.

6.2.5 Problem 5: Two-Loop Circuit (Mesh-Current Method)

Consider a circuit with two loops sharing a common resistor R_3:

- Left Loop: 10 V source in series with resistor $R_1 = 2\,\Omega$ and resistor $R_3 = 4\,\Omega$.
- Right Loop: 5 V source in series with resistor $R_2 = 3\,\Omega$ and the same resistor R_3 (shared).

Label the left loop current as I_1 (clockwise) and the right loop current as I_2 (clockwise). Use mesh-current method (KVL in each loop, with sign convention for shared resistor).

6.2.6 Problem 6: Multi-Node Circuit (KCL, System of Equations)

We have a circuit with two nodes A and B above ground, connected as follows:

1. A 15 V supply is connected to node A via $R_1 = 5\,\Omega$.
2. Node A is connected to node B via $R_2 = 10\,\Omega$.
3. Node B is connected to ground via $R_3 = 5\,\Omega$.
4. Node A is also connected to ground via $R_4 = 15\,\Omega$.

Use KCL at both nodes A and B to find V_A and V_B (the node voltages above ground).

6.2.7 Problem 7: Circuit with Dependent Source (KCL/KVL)

A circuit has:

- A 10 V independent voltage source in series with a resistor $R_1 = 2\,\Omega$.
- A dependent voltage source of value $2I_x$ volts (voltage-controlled or current-controlled example can vary). For this problem, assume it is a current-controlled voltage source where I_x is the current through R_1.
- The dependent source is in series with another resistor $R_2 = 4\,\Omega$.
- The entire loop is closed (single loop).

Find the loop current I if $I = I_x$ is the same current going through R_1 and the dependent source. Use KVL.

6.2.8 Problem 8: Parametric Sweep (KCL) + Plotting

A single node N with respect to ground has a variable resistor R_x from node N to ground, and a fixed 12 V source feeding node N through a 10 Ω resistor. We want to:

1. Sweep R_x from 2 Ω to 20 Ω (in steps of 2 Ω).
2. For each R_x, use KCL at node N to find the node voltage V_N.
3. Plot V_N vs. R_x.

6.2.9 Problem 9: Single Loop with Variable Resistor (KVL) + Plotting

A simple loop has a 9 V battery in series with two resistors: $R_1 = 3\,\Omega$ and a variable resistor R_x. We will:

1. Sweep R_x from 1 Ω to 10 Ω.
2. Use KVL to find the loop current for each R_x.
3. Plot the current I and the voltage drop across R_x vs. R_x.

6.2.10 Problem 10: RC Charging Circuit (KVL in Transient Analysis)

A resistor $R = 10\,\Omega$ and capacitor $C = 100\,\mu\text{F}$ are in series with a 5 V source. At $t = 0$, the switch closes, and the capacitor is uncharged initially. Use KVL to write the capacitor voltage $v_C(t)$ and resistor current $i_R(t)$ as functions of time. Plot both over the interval $0 \leq t \leq 0.03$ s.

6.3 Capacitors and Inductors

[return to TOC]

The following 10 problems span fundamental to intermediate concepts involving capacitors and inductors, with both transient and frequency-domain perspectives.

- Capacitors store energy as an electric field, governed by $Q = CV$ and $i = C\frac{dv_C}{dt}$.
- Inductors store energy as a magnetic field, governed by $v_L = L\frac{di_L}{dt}$ and i continuity.
- Transient solutions for RC and RL (and RLC) circuits typically involve exponentials.
- Resonance in LC or RLC circuits is characterized by natural frequency $\omega_0 = 1/\sqrt{LC}$.
- For AC analysis, reactances $X_C = 1/(\omega C)$ and $X_L = \omega L$ help determine currents and voltages in the frequency domain.
- The quality factor (Q) relates energy storage to energy loss per cycle in resonant circuits.

6.3.1 Problem 1: Basic Capacitor Charge and Voltage

A capacitor $C = 10\,\mu\text{F}$ is charged to a voltage of $V = 5\,\text{V}$.

1. Find the charge Q stored on the capacitor.

2. If the capacitor is then disconnected from the voltage source and allowed to discharge through a $1\,\text{M}\Omega$ resistor, find the time constant τ of the discharge.

6.3.2 Problem 2: Capacitors in Series and Parallel

You have three capacitors:

- $C_1 = 5\,\mu\text{F}$
- $C_2 = 10\,\mu\text{F}$
- $C_3 = 15\,\mu\text{F}$

1. First, connect C_1 and C_2 in series, and then place that series combination in parallel with C_3. Calculate the equivalent capacitance C_{eq}.

2. If a 12 V DC source is applied to this network, find the total stored energy in the resulting equivalent capacitor.

6.3.3 Problem 3: Inductor Voltage and Current Relationship

An inductor $L = 5\,\text{mH}$ has a time-varying current described by:

$$i(t) = 2\,\sin(1000\,t)\,\text{A}.$$

Find the voltage across the inductor $v_L(t)$. Recall that $v_L(t) = L\frac{di(t)}{dt}$.

6.3.4 Problem 4: RC Charging Transient (with Plot)

A resistor $R = 10\,\Omega$ is in series with a capacitor $C = 100\,\mu\text{F}$ and a 5 V DC supply. At $t = 0$, the capacitor is uncharged and the switch is closed.

1. Write the expression for the capacitor voltage $v_C(t)$ over time.

2. Plot $v_C(t)$ and the charging current $i(t)$ from $t = 0$ to $t = 0.02\,\text{s}$.

6.3.5 Problem 5: RL Discharge Transient

A 10 V source is connected to an inductor $L = 0.1\,\text{H}$ and resistor $R = 10\,\Omega$ in series. The circuit is allowed to reach steady state. Then at $t = 0$, the source is suddenly replaced by a short circuit (effectively 0 V). Derive the expression for the current $i(t)$ through the inductor for $t \geq 0$.

6.3.6 Problem 6: Energy Stored in Capacitor and Inductor

1. A capacitor of $C = 50\,\mu\text{F}$ is charged to 12 V. Find the energy stored.

2. An inductor of $L = 2\,\text{H}$ carries a current of 3 A. Find the energy stored.

6.3.7 Problem 7: LC Resonant Frequency

An LC circuit has an inductor $L = 0.5\,\text{H}$ and a capacitor $C = 200\,\mu\text{F}$. Calculate the resonant (natural) angular frequency ω_0 and the resonant frequency f_0 in Hz. Use:

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad f_0 = \frac{\omega_0}{2\pi}.$$

6.3.8 Problem 8: Series RLC Circuit Damping

A series RLC circuit has $R = 50\,\Omega, L = 0.2\,\mathrm{H}, C = 100\,\mu\mathrm{F}$. Determine whether the circuit is underdamped, critically damped, or overdamped by calculating the damping factor ζ (often using the standard form or comparing α and ω_0).

- The standard form can be: $\alpha = \frac{R}{2L}$ and $\omega_0 = \frac{1}{\sqrt{LC}}$.
- Compare α and ω_0:
  – If $\alpha > \omega_0$, overdamped.
  – If $\alpha = \omega_0$, critically damped.
  – If $\alpha < \omega_0$, underdamped.

6.3.9 Problem 9: AC Circuit with Capacitor (Phasor Analysis + Plotting)

Consider a simple AC circuit: a 10 V RMS source at variable frequency f driving a capacitor $C = 1\,\mu\mathrm{F}$.

1. Express the capacitive reactance $X_C = \frac{1}{\omega C}$.
2. Compute the current magnitude $I_{\mathrm{rms}} = \frac{V_{\mathrm{rms}}}{X_C}$ as a function of frequency from $10\,\mathrm{Hz}$ to $10\,\mathrm{kHz}$.
3. Plot I_{rms} vs. f on a log-log scale.

6.3.10 Problem 10: RLC Parallel Resonance

An RLC parallel circuit has the following elements in parallel:

- $L = 1\,\mathrm{H}$
- $C = 1\,\mu\mathrm{F}$
- $R = 2\,\mathrm{k}\Omega$

1. Calculate the resonance frequency $\omega_0 = 1/\sqrt{LC}$.
2. Calculate the bandwidth $\Delta\omega$ given by $\frac{1}{RC}$ for a high-Q parallel RLC approximation (assuming R is large).
3. Estimate the quality factor $Q = \frac{\omega_0}{\Delta\omega}$.

6.4 Operational Amplifiers [return to TOC]

The following problems provide a solid overview of introductory op amp circuits—including standard linear amplifiers, active filters (integrators/differentiators), comparators, and real-life limitations. A few problems involve plotting to visualize op amp behavior in the time or frequency domain. Topics covered include:

- Ideal Op Amp Assumptions: Infinite open-loop gain, infinite input impedance, zero output impedance, zero input bias current, and zero offset voltage (unless specified) make the standard formulas straightforward.
- Inverting vs. Non-inverting: Know the standard gain formulas:
  * Inverting: $v_{\text{out}} = -\frac{R_f}{R_{\text{in}}} v_{\text{in}}$.
  * Non-inverting: $v_{\text{out}} = \left(1 + \frac{R_f}{R_{\text{gnd}}}\right) v_{\text{in}}$.
- Summing and Difference Amplifiers: Are extensions of the inverting and differential concepts.
- Integrator & Differentiator: Involve capacitors in feedback or series, producing time-domain operations on signals.
- Comparator: Op amp without feedback (or with very high gain) used to compare an input to a reference threshold.
- Instrumentation Amplifier: Combines three op amps with matched resistors, giving high common-mode rejection and precise gain.
- Slew Rate: Non-ideal effect limiting how fast the output can change.
- Input Offset: Another real-world op amp effect causing a small output voltage even with zero input.

6.4.1 Problem 1: Inverting Amplifier

You have an inverting amplifier with feedback resistor $R_f = 100\,\text{k}\Omega$ and input resistor $R_{\text{in}} = 10\,\text{k}\Omega$. The input signal is $v_{\text{in}} = +0.1\,\text{V}$. Assume an ideal op amp (infinite open-loop gain, zero input current, etc.).

1. Find the closed-loop voltage gain A_v.
2. Determine the output voltage v_{out}.
3. Verify the direction (sign) of the output signal.

6.4.2 Problem 2: Non-Inverting Amplifier

A non-inverting amplifier has $R_1 = 4.7\,\text{k}\Omega$ from the op amp output to the inverting input, and $R_2 = 1\,\text{k}\Omega$ from the inverting input to ground. The signal is applied to the non-inverting input.

1. Find the closed-loop gain A_v.
2. If $v_{\text{in}} = +0.2\,\text{V}$, compute v_{out}.

6.4.3 Problem 3: Summing Amplifier

A summing amplifier (inverting summer) has three input resistors (R_a, R_b, R_c) all equal to $10\,\text{k}\Omega$ and a single feedback resistor $R_f = 10\,\text{k}\Omega$. The three inputs are:

$$v_a = 1\,\text{V}, \quad v_b = 0.5\,\text{V}, \quad v_c = -2\,\text{V}.$$

1. Find the expression for the output voltage v_{out}.
2. Calculate v_{out} numerically.

6.4.4 Problem 4: Difference Amplifier

A difference amplifier has four resistors forming the classic configuration:
- $R_1 = 10\,\text{k}\Omega$ from the inverting input to the output node,
- $R_2 = 10\,\text{k}\Omega$ from the inverting input to v_1,
- $R_3 = 10\,\text{k}\Omega$ from the non-inverting input to ground,
- $R_4 = 10\,\text{k}\Omega$ from v_2 to the non-inverting input.

Inputs are $v_1 = 2\,\text{V}$ and $v_2 = 5\,\text{V}$. Assume an ideal op amp.

1. Write the standard difference amplifier formula.
2. Calculate v_{out} given these resistor values and inputs.

6.4.5 Problem 5: Op Amp Integrator (Step Response) – With Plot

Consider an ideal op amp integrator with input resistor $R = 10\,\text{k}\Omega$ and capacitor $C = 0.1\,\mu\text{F}$ in the feedback path. A step input of +1 V is applied at $t = 0$. Derive the output $v_{\text{out}}(t)$ and generate a plot for $0 \le t \le 5\,\text{ms}$.

6.4.6 Problem 6: Op Amp Differentiator (Sinusoidal Input) – With Plot

An ideal inverting differentiator has capacitor $C = 0.01\,\mu\text{F}$ in series with the input and resistor $R = 10\,\text{k}\Omega$ in the feedback path. The input is a sine wave $v_{\text{in}}(t) = 2\sin(2000\pi t)\,\text{V}$. Plot the output $v_{\text{out}}(t)$ for one period (0 to 1 ms).

6.4.7 Problem 7: Basic Comparator

An op amp is used as a comparator with the non-inverting input tied to 0 V (ground) and the inverting input receiving a variable input v_{in}. Assume the output saturates at $\pm 12\,\text{V}$ for an ideal comparator-like op amp. For each of these input values $[-1\,\text{V}, 0\,\text{V}, +3\,\text{V}]$, determine the comparator output.

6.4.8 Problem 8: Instrumentation Amplifier (Ideal)

A classic 3-op-amp instrumentation amplifier has an internal gain resistor R_G. The gain formula (for the differential input $(v_2 - v_1)$) is:

$$A = 1 + \frac{2R}{R_G},$$

where each of the other matched resistors is R. Suppose each matched resistor is $R = 10\,\text{k}\Omega$, and $R_G = 2\,\text{k}\Omega$. The differential input is $(v_2 - v_1) = 0.1\,\text{V}$. Find the output.

6.4.9 Problem 9: Slew Rate Limitation

An op amp has a slew rate of $0.5\,\text{V}/\mu\text{s}$. You want the output to step from 0 V to 10 V.

1. How long does it take (ideally) for the op amp output to reach 10 V under slew rate limitation?
2. If you need it in 5 μs or less, what minimum slew rate is required?

6.4.10 Problem 10: Input Offset Voltage Effect

An op amp inverting amplifier has $R_{\text{in}} = 10\,\text{k}\Omega$ and $R_f = 100\,\text{k}\Omega$. Suppose the amplifier is ideal except for a small input offset voltage $v_{\text{os}} = 2\,\text{mV}$ effectively appearing at the op amp input. If $v_{\text{in}} = 0\,\text{V}$, estimate the output offset $v_{\text{out, offset}}$.

> (Hint: in an inverting amplifier, an input offset voltage at the input terminals appears effectively as though it were at the non-inverting input with the inverting input at virtual ground.)

7 Solutions to problems

7.1 Electric circuit basics

7.1.1 Solution 1: Basic Ohm's Law

1. Current: By Ohm's law, $I = \frac{V}{R} = \frac{5}{10} = 0.5\,\mathrm{A}$.
2. Power: $P = V \times I = 5 \times 0.5 = 2.5\,\mathrm{W}$.

7.1.2 Solution 2: Series Resistors

1. Total Resistance: For series connection,

$$R_{\text{total}} = R_1 + R_2 = 4 + 6 = 10\,\Omega$$

2. Current: Using Ohm's law on the entire series circuit,

$$I = \frac{V}{R_{\text{total}}} = \frac{20}{10} = 2\,\mathrm{A}$$

3. Voltage Drops:
- $V_1 = I \times R_1 = 2 \times 4 = 8\,\mathrm{V}$
- $V_2 = I \times R_2 = 2 \times 6 = 12\,\mathrm{V}$

7.1.3 Solution 3: Parallel Resistors

1. Equivalent Resistance:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{10} + \frac{1}{15} = 0.1 + 0.0667 = 0.1667 \quad \Longrightarrow \quad R_{\text{eq}} \approx 6\,\Omega$$

2. Total Current:

$$I_{\text{total}} = \frac{V}{R_{\text{eq}}} \approx \frac{12}{6} = 2\,\mathrm{A}$$

3. Currents Through Each Resistor:
- $I_1 = \frac{V}{R_1} = \frac{12}{10} = 1.2\,\mathrm{A}$
- $I_2 = \frac{V}{R_2} = \frac{12}{15} = 0.8\,\mathrm{A}$

7.1.4 Solution 4: Series-Parallel Network

1. Equivalent Resistance of R_2 and R_3 in parallel:

$$R_{23} = \frac{R_2 R_3}{R_2 + R_3} = \frac{6 \times 6}{6 + 6} = \frac{36}{12} = 3\,\Omega$$

2. Total Circuit Resistance:

$$R_{\text{eq}} = R_1 + R_{23} = 3 + 3 = 6\,\Omega$$

3. Total Current:

$$I_{\text{total}} = \frac{12}{6} = 2\,\mathrm{A}$$

4. Current Through R_2 and R_3: Both have 12 - (voltage drop across R_1), but we can simplify as they share the same voltage in parallel.
- Voltage drop across $R_1 = I_{\text{total}} \times R_1 = 2 \times 3 = 6\,\mathrm{V}$.
- Voltage across parallel branch $= 12 - 6 = 6\,\mathrm{V}$.
- $I_2 = \frac{6}{6} = 1\,\mathrm{A}$, $I_3 = \frac{6}{6} = 1\,\mathrm{A}$.

7.1.5 Solution 5: Kirchhoff's Voltage Law (KVL)

1. Loop Current:

$$I = \frac{9\,\mathrm{V}}{2 + 7} = 1\,\mathrm{A}$$

2. Voltage Drops: - $V_{R_1} = I \times R_1 = 1 \times 2 = 2\,\mathrm{V}$ - $V_{R_2} = I \times R_2 = 1 \times 7 = 7\,\mathrm{V}$
3. KVL Check:

$$9\,\mathrm{V} - 2\,\mathrm{V} - 7\,\mathrm{V} = 0$$

7.1.6 Solution 6: Kirchhoff's Current Law (KCL)

1. Let V_N be the node voltage at N. The current from the 10 V supply through the 5 Ω resistor is:

$$I_1 = \frac{10 - V_N}{5}$$

2. Current through the 10 Ω resistor to ground:

$$I_2 = \frac{V_N - 0}{10} = \frac{V_N}{10}$$

3. Current through the 5 Ω resistor to ground:

$$I_3 = \frac{V_N - 0}{5} = \frac{V_N}{5}$$

4. By KCL at node N: $I_1 = I_2 + I_3$. Hence:

$$\frac{10 - V_N}{5} = \frac{V_N}{10} + \frac{V_N}{5}$$

Multiply by 10 to clear denominators:

$$2(10 - V_N) = V_N + 2V_N \quad \Longrightarrow \quad 20 - 2V_N = 3V_N \quad \Longrightarrow \quad 5V_N = 20 \quad \Longrightarrow \quad V_N = 4\,\mathrm{V}$$

5. Substitute $V_N = 4\,\mathrm{V}$ back into the current expressions: - $I_1 = \frac{10-4}{5} = \frac{6}{5} = 1.2\,\mathrm{A}$ - $I_2 = \frac{4}{10} = 0.4\,\mathrm{A}$ - $I_3 = \frac{4}{5} = 0.8\,\mathrm{A}$ - Check: $I_2 + I_3 = 0.4 + 0.8 = 1.2\,\mathrm{A} = I_1$.

7.1.7 Solution 7: Power Calculation in a Resistor Network

1. Equivalent Resistance: - Parallel of R_1 and R_2:

$$R_{12} = \frac{4 \times 12}{4 + 12} = \frac{48}{16} = 3\,\Omega$$

- Total: $R_{\mathrm{eq}} = R_{12} + R_3 = 3 + 4 = 7\,\Omega$
2. Total Current:

$$I_{\mathrm{total}} = \frac{12}{7} \approx 1.714\,\mathrm{A}$$

3. Power Delivered by Battery:

$$P_{\mathrm{total}} = V \times I_{\mathrm{total}} = 12 \times \frac{12}{7} = \frac{144}{7} \approx 20.57\,\mathrm{W}$$

4. Voltage Drop on R_3:

$$V_3 = I_{\mathrm{total}} \times R_3 = \left(\frac{12}{7}\right) \times 4 = \frac{48}{7}\,\mathrm{V}$$

5. Power in R_3:

$$P_3 = I_{\mathrm{total}}^2 \times R_3 = \left(\frac{12}{7}\right)^2 \times 4 = \frac{144}{49} \times 4 = \frac{576}{49}\,\mathrm{W} \approx 11.76\,\mathrm{W}$$

6. Voltage across Parallel Branch ($R_1 \| R_2$):

$$V_{12} = 12 - \frac{48}{7} = \frac{84 - 48}{7} = \frac{36}{7}\,\mathrm{V}$$

7. Current in R_1 and R_2: - $I_1 = \frac{V_{12}}{R_1} = \frac{36/7}{4} = \frac{36}{28} = \frac{9}{7} \approx 1.2857\,\mathrm{A}$ - $I_2 = \frac{V_{12}}{R_2} = \frac{36/7}{12} = \frac{36}{84} = \frac{3}{7} \approx 0.4286\,\mathrm{A}$
8. Power in R_1:

$$P_1 = I_1^2 \times R_1 = \left(\frac{9}{7}\right)^2 \times 4 \approx 6.61\,\mathrm{W}$$

9. Power in R_2:

$$P_2 = I_2^2 \times R_2 = \left(\frac{3}{7}\right)^2 \times 12 \approx 2.20\,\mathrm{W}$$

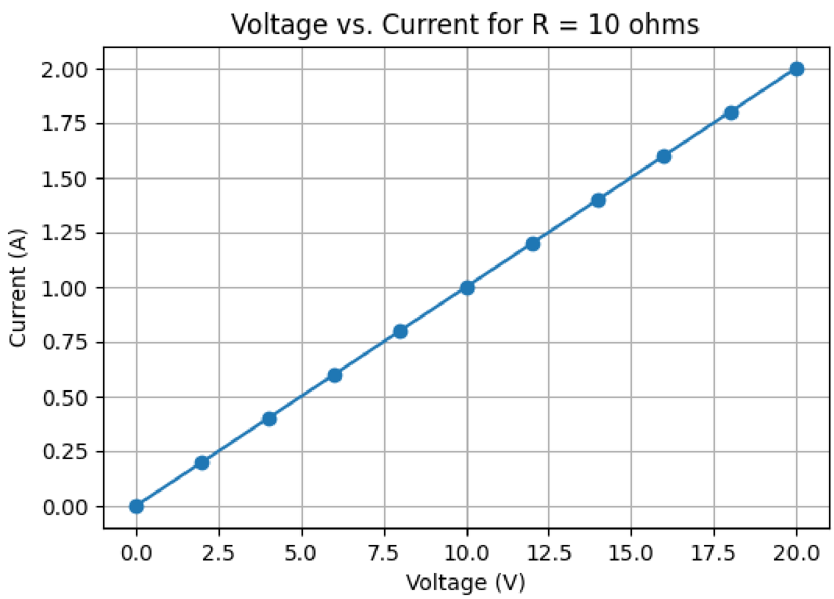

Figure 2: Solution for problem 8.

7.1.8 Solution 8: DC Voltage Sweep in a Single Resistor

- Ohm's law: $I = \frac{V}{R}$.
- As V goes from 0 to 20 V (0, 2, 4, ..., 20), the current changes proportionally from 0 to 2 A.
- Solution plot is shown in Fig. 4

7.1.9 Solution 9: Parametric Sweep for Parallel Resistance

1. Use parallel resistance formula:

$$R_{\text{eq}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

2. $I_{\text{total}} = \frac{10\,\text{V}}{R_{\text{eq}}}$.
3. Plot against R_2.

- Solution plot is shown in Fig. ??

7.1.10 Solution 10: RC Charging Circuit (Transient)

1. Capacitor Voltage Over Time:

$$v_C(t) = V_{\text{final}}\big(1 - e^{-\frac{t}{RC}}\big) = 5\big(1 - e^{-\frac{t}{(10)(100\times10^{-6})}}\big)$$

where $RC = 10 \times 100 \times 10^{-6} = 10^{-3}\,\text{s} = 0.001\,\text{s}$.

2. Resistor Current Over Time:

$$i_R(t) = \frac{v_R(t)}{R} = \frac{V_{\text{source}} - v_C(t)}{R} = \frac{5 - 5\big(1 - e^{-t/0.001}\big)}{10} = \frac{5e^{-t/0.001}}{10} = 0.5\,e^{-1000\,t}$$

- Solution plot is shown in Fig. ??

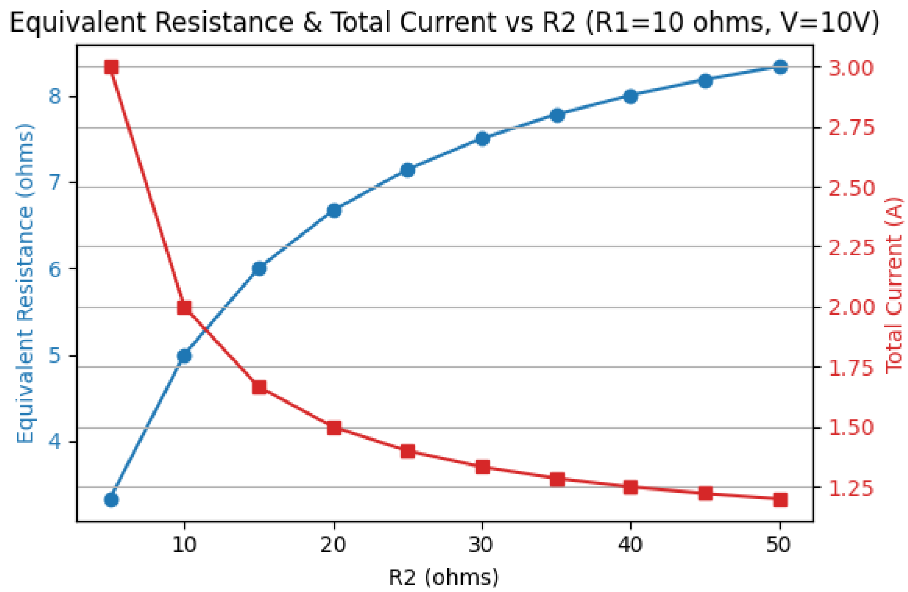

Figure 3: Solution for problem 9.

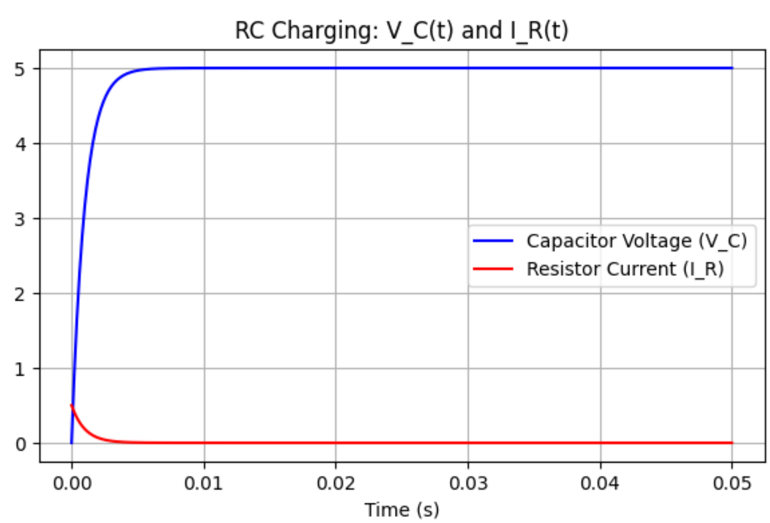

Figure 4: Solution for problem 10.

7.2 Kirchhoff's Laws and Equivalent Circuits

7.2.1 Solution 1: Basic Single Loop (KVL)

1. Apply KVL:
$$+12\ (\text{battery}) - IR_1 - IR_2 = 0$$
2. Solve for I:
$$12 - I(3+9) = 0 \quad \Longrightarrow \quad I = \frac{12}{3+9} = \frac{12}{12} = 1\,\text{A}$$
3. Voltage drops: - $V_{R_1} = I \times R_1 = 1 \times 3 = 3\,\text{V}$ - $V_{R_2} = I \times R_2 = 1 \times 9 = 9\,\text{V}$
4. Check KVL:
$$12 - 3 - 9 = 0 \quad \checkmark$$

7.2.2 Solution 2: Single Node (KCL)

1. Let the node voltage be V_N. The current from the 10 V source to the node is
$$I_1 = \frac{10 - V_N}{10}.$$
2. The node has two currents to ground:
$$I_2 = \frac{V_N}{5}, \quad I_3 = \frac{V_N}{10}.$$
3. KCL at the node:
$$I_1 = I_2 + I_3 \quad \Longrightarrow \quad \frac{10 - V_N}{10} = \frac{V_N}{5} + \frac{V_N}{10}.$$
4. Multiply through by 10:
$$10 - V_N = 2V_N + V_N \quad = 3V_N \quad \Longrightarrow \quad 10 = 4V_N \quad \Longrightarrow \quad V_N = 2.5\,\text{V}.$$
5. Branch Currents: - $I_1 = \frac{10-2.5}{10} = 0.75\,\text{A}$ - $I_2 = \frac{2.5}{5} = 0.5\,\text{A}$ - $I_3 = \frac{2.5}{10} = 0.25\,\text{A}$
6. Verify: $I_1 = I_2 + I_3 = 0.5 + 0.25 = 0.75\,\text{A}$.

7.2.3 Solution 3: Two Voltage Sources in a Series Loop (KVL)

1. KVL around the loop:
$$(V_1 + V_2) - IR_1 - IR_2 = 0 \quad \Longrightarrow \quad 15 - I(2+3) = 0.$$
2. Solve for I:
$$I = \frac{15}{5} = 3\,\text{A}.$$
3. Voltage Drops: - $V_{R_1} = I \times R_1 = 3 \times 2 = 6\,\text{V}$ - $V_{R_2} = I \times R_2 = 3 \times 3 = 9\,\text{V}$
4. Check: $6 + 9 = 15\,\text{V}$, which matches $V_1 + V_2$.

7.2.4 Solution 4: Node Voltage Method (KCL)

1. Let V_A be the voltage of node A.
2. The current from the 12 V source to A is
$$I_1 = \frac{12 - V_A}{R_1}.$$
3. The currents leaving A to ground:
$$I_2 = \frac{V_A}{R_2}, \quad I_3 = \frac{V_A}{R_3}.$$
4. KCL at node A:
$$I_1 = I_2 + I_3 \quad \Longrightarrow \quad \frac{12 - V_A}{4} = \frac{V_A}{6} + \frac{V_A}{12}.$$
5. Multiply through by 12 to clear denominators:
$$3(12 - V_A) = 2V_A + V_A \quad \Longrightarrow \quad 36 - 3V_A = 3V_A \quad \Longrightarrow \quad 36 = 6V_A \quad \Longrightarrow \quad V_A = 6\,\text{V}.$$
6. Branch currents: - $I_1 = \frac{12-6}{4} = \frac{6}{4} = 1.5\,\text{A}$. - $I_2 = \frac{6}{6} = 1\,\text{A}$. - $I_3 = \frac{6}{12} = 0.5\,\text{A}$.
7. Check: $I_1 = 1.5\,\text{A}$ and $I_2 + I_3 = 1 + 0.5 = 1.5\,\text{A}$. KCL holds.

7.2.5 Solution 5: Two-Loop Circuit (Mesh-Current Method)

1. KVL in the left loop:

$$10 - I_1 R_1 - (I_1 - I_2) R_3 = 0$$

Because the current through R_3 is $I_1 - I_2$ if both are flowing in the clockwise direction in their respective loops.
2. KVL in the right loop:

$$5 - I_2 R_2 - (I_2 - I_1) R_3 = 0$$

Note that $(I_2 - I_1) R_3$ is the drop in R_3 for the right loop orientation.
3. Plug in $R_1 = 2, R_2 = 3, R_3 = 4$. The system becomes:
Left loop:

$$10 - 2I_1 - 4(I_1 - I_2) = 0 \quad \Longrightarrow \quad 10 - 2I_1 - 4I_1 + 4I_2 = 0 \quad \Longrightarrow \quad -6I_1 + 4I_2 = -10.$$

Right loop:

$$5 - 3I_2 - 4(I_2 - I_1) = 0 \quad \Longrightarrow \quad 5 - 3I_2 - 4I_2 + 4I_1 = 0 \quad \Longrightarrow \quad 4I_1 - 7I_2 = -5.$$

4. We have two linear equations:

$$\begin{cases} -6I_1 + 4I_2 = -10, \\ 4I_1 - 7I_2 = -5. \end{cases}$$

5. Solve for I_1 and I_2.
Once I_1 and I_2 are known, you can find any voltage drops or branch currents.

7.2.6 Solution 6: Multi-Node Circuit (KCL, System of Equations)

1. Let node A have voltage V_A, node B have voltage V_B.
2. KCL at node A:
Inflow from supply: $I_1 = \dfrac{15 - V_A}{5}$. Outflows:

$$I_{A \to B} = \frac{V_A - V_B}{10}, \quad I_{A \to ground} = \frac{V_A}{15}.$$

So,

$$\frac{15 - V_A}{5} = \frac{V_A - V_B}{10} + \frac{V_A}{15}.$$

3. KCL at node B:
Inflow from node A: $I_{B \leftarrow A} = \dfrac{V_A - V_B}{10}$. Outflow to ground:

$$I_{B \to ground} = \frac{V_B}{5}.$$

So,

$$\frac{V_A - V_B}{10} = \frac{V_B}{5}.$$

4. Solve the system for V_A, V_B.

7.2.7 Solution 7: Circuit with Dependent Source (KCL/KVL)

1. Let loop current be I. Then $I_x = I$.
2. KVL around the loop:

$$+10 - (I \cdot R_1) - [2I_x] - (I \cdot R_2) = 0.$$

But $I_x = I$. So the dependent voltage source is $2I$ V.
3. Simplify:

$$10 - 2I - 2I - 4I = 0 \quad \Longrightarrow \quad 10 - (2I + 2I + 4I) = 0,$$

$$10 - 8I = 0 \quad \Longrightarrow \quad I = \frac{10}{8} = 1.25\,\text{A}.$$

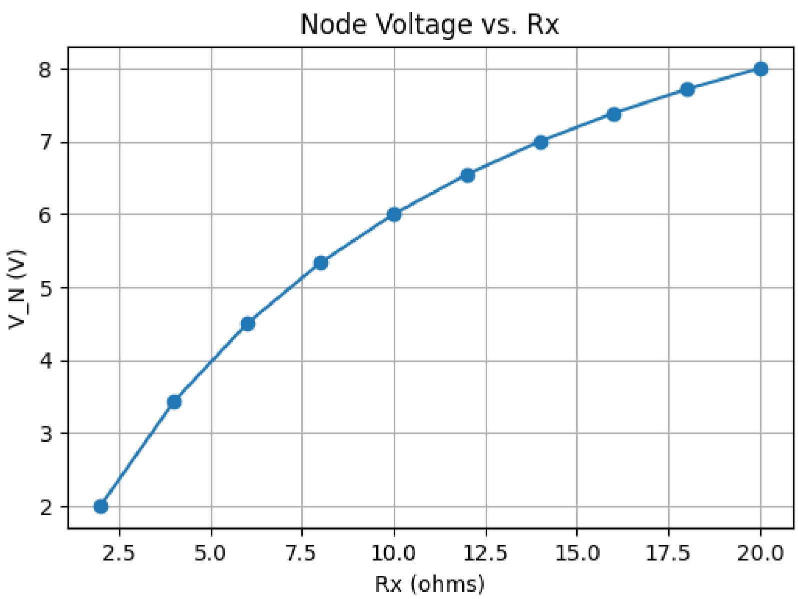

Figure 5: Solution for problem 8.

7.2.8 Solution 8: Parametric Sweep (KCL) + Plotting

1. Let V_N be the node voltage. Then the current from the 12 V source through 10 Ω is $\frac{12-V_N}{10}$.
2. The current to ground through R_x is $\frac{V_N}{R_x}$.
3. KCL: $\frac{12-V_N}{10} = \frac{V_N}{R_x}$. Solve for V_N for each R_x.

- Solution plot is shown in Fig. 5

7.2.9 Solution 9: Single Loop with Variable Resistor (KVL) + Plotting

1. KVL: $9 - I \cdot 3 - I \cdot R_x = 0$.
2. Solve for I: $I = \frac{9}{3+R_x}$.
3. Voltage drop across R_x: $V_{R_x} = I \cdot R_x$.

- Solution plot is shown in Fig. 6

7.2.10 Solution 10: RC Charging Circuit (KVL in Transient Analysis)

Key Equations:
- KVL around the loop: $V_{source} - v_R(t) - v_C(t) = 0$.
- $v_R(t) = R\,i(t)$, and for a charging capacitor, $i(t) = C\frac{dv_C(t)}{dt}$.
- The standard solution:

$$v_C(t) = 5\left(1 - e^{-\frac{t}{RC}}\right), \quad i_R(t) = \frac{v_R(t)}{R} = \frac{5 - v_C(t)}{R}.$$

- Solution plot is shown in Fig. 7

Analysis:
- KVL yields $5 - i_R \cdot 10 - v_C = 0$.
- The standard exponential charging solution for the capacitor and corresponding resistor current is shown by the plotted functions.

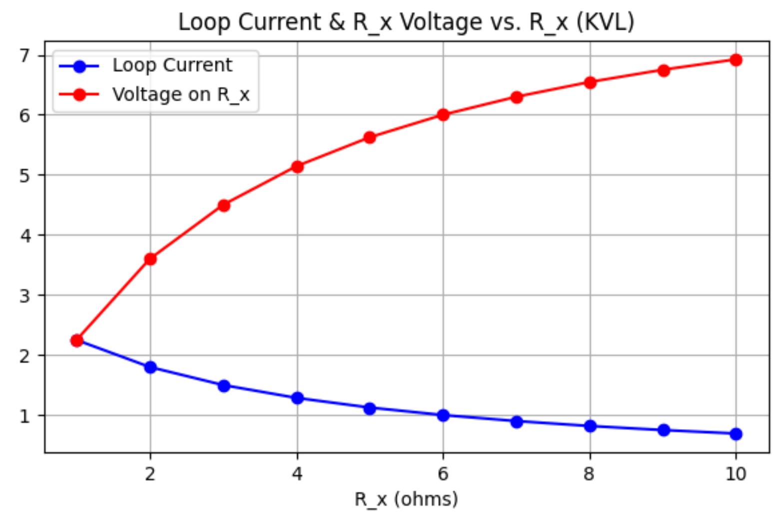

Figure 6: Solution for problem 9.

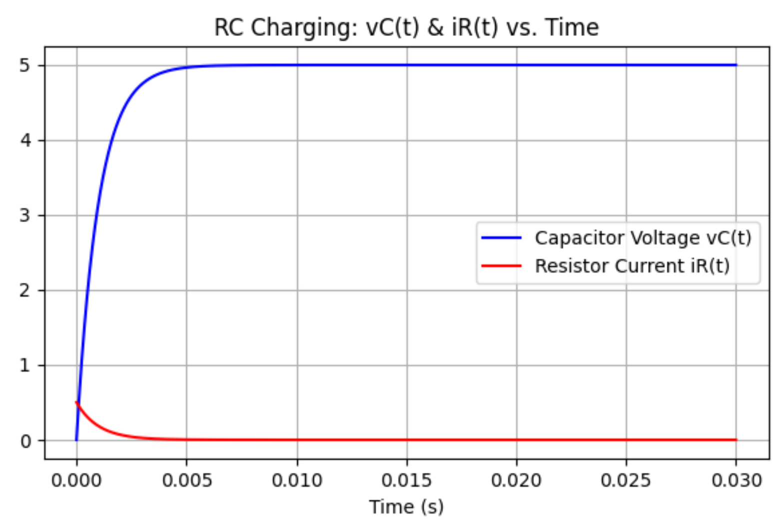

Figure 7: Solution for problem 10.

7.3 Capacitors and Inductors

7.3.1 Solution 1: Basic Capacitor Charge and Voltage

1. Charge:
$$Q = C \times V = 10 \times 10^{-6}\,\mathrm{F} \times 5\,\mathrm{V} = 50 \times 10^{-6}\,\mathrm{C} = 50\,\mu\mathrm{C}.$$

2. Time Constant:
$$\tau = R \times C = (1 \times 10^{6}\,\Omega) \times (10 \times 10^{-6}\,\mathrm{F}) = 10\,\mathrm{s}.$$

7.3.2 Solution 2: Capacitors in Series and Parallel

1. Series combination of C_1 and C_2:
$$\frac{1}{C_{12}} = \frac{1}{C_1} + \frac{1}{C_2} \implies C_{12} = \left(\frac{1}{5} + \frac{1}{10}\right)^{-1} \mu\mathrm{F} = (0.2 + 0.1)^{-1}\,\mu\mathrm{F} = (0.3)^{-1}\,\mu\mathrm{F} = \frac{1}{0.3}\,\mu\mathrm{F} \approx 3.33\,\mu\mathrm{F}.$$

2. Parallel with C_3:
$$C_{\mathrm{eq}} = C_{12} + C_3 = 3.33\,\mu\mathrm{F} + 15\,\mu\mathrm{F} = 18.33\,\mu\mathrm{F}.$$

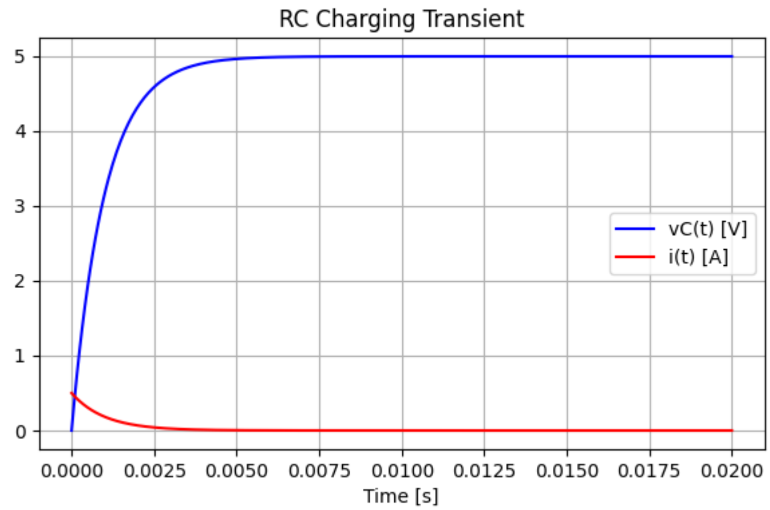

Figure 8: Solution for problem 4.

3. Energy stored in a capacitor:

$$E = \frac{1}{2} C_{\text{eq}} V^2 = \frac{1}{2} \times 18.33 \times 10^{-6}\,\text{F} \times (12)^2 \approx 1.32 \times 10^{-3}\,\text{J}.$$

7.3.3 Solution 3: Inductor Voltage and Current Relationship

1. Current:

$$i(t) = 2\sin(1000t).$$

2. Time derivative:

$$\frac{d}{dt}\big[2\sin(1000t)\big] = 2 \times 1000\cos(1000t) = 2000\cos(1000t).$$

3. Voltage:

$$v_L(t) = L\frac{di(t)}{dt} = 5 \times 10^{-3}\,\text{H} \times 2000\cos(1000t) = 10\,\cos(1000t)\,\text{V}.$$

7.3.4 Solution 4: RC Charging Transient (with Plot)

1. Time constant: $\tau = RC = 10 \times 100 \times 10^{-6} = 10^{-3}\,\text{s}$.
2. Capacitor Voltage (standard charging formula):

$$v_C(t) = 5\big(1 - e^{-t/\tau}\big).$$

3. Current:

$$i(t) = \frac{5 - v_C(t)}{R} = \frac{5 - 5(1 - e^{-t/\tau})}{10} = \frac{5e^{-t/\tau}}{10} = 0.5\,e^{-1000t}\,\text{A}.$$

- Solution plot is shown in Fig. 8

7.3.5 Solution 5: RL Discharge Transient

1. Steady-state current before $t = 0$: The inductor eventually behaves like a short circuit in DC steady-state. So

$$I_{0^-} = \frac{10\,\text{V}}{10\,\Omega} = 1\,\text{A}.$$

2. For $t \geq 0$ with the source replaced by a short (0 V across the series), we have:

$$L\frac{di}{dt} + Ri = 0.$$

The standard solution is:

$$i(t) = I_{0^-} e^{-\frac{R}{L}t} = 1 \times e^{-\frac{10}{0.1}t} = e^{-100t}.$$

7.3.6 Solution 6: Energy Stored in Capacitor and Inductor

1. Capacitor Energy:

$$W_C = \frac{1}{2}CV^2 = \frac{1}{2} \times 50 \times 10^{-6} \times (12)^2 = 25 \times 10^{-6} \times 144 = 3.6 \times 10^{-3}\,\text{J}.$$

2. Inductor Energy:

$$W_L = \frac{1}{2}LI^2 = \frac{1}{2} \times 2 \times (3)^2 = 1 \times 9 = 9\,\text{J}.$$

7.3.7 Solution 7: LC Resonant Frequency

1. Angular frequency:

$$\omega_0 = \frac{1}{\sqrt{0.5 \times 200 \times 10^{-6}}} = \frac{1}{\sqrt{0.5 \times 200 \times 10^{-6}}}.$$

Numerically,

$$0.5 \times 200 \times 10^{-6} = 100 \times 10^{-6} = 1 \times 10^{-4}.$$

$$\sqrt{1 \times 10^{-4}} = 10^{-2} = 0.01, \quad \omega_0 = \frac{1}{0.01} = 100\,\text{rad/s}.$$

2. Frequency:

$$f_0 = \frac{\omega_0}{2\pi} = \frac{100}{2\pi} \approx 15.92\,\text{Hz}.$$

7.3.8 Solution 8: Series RLC Circuit Damping

1. $\alpha = \dfrac{R}{2L} = \dfrac{50}{2 \times 0.2} = \dfrac{50}{0.4} = 125\,\text{rad/s}.$
2. $\omega_0 = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{0.2 \times 100 \times 10^{-6}}} = \dfrac{1}{\sqrt{20 \times 10^{-6}}}.$

$$20 \times 10^{-6} = 2 \times 10^{-5}, \quad \sqrt{2 \times 10^{-5}} \approx 4.47 \times 10^{-3}. \quad \omega_0 \approx \frac{1}{4.47 \times 10^{-3}} \approx 224\,\text{rad/s}.$$

3. Compare $\alpha = 125$ and $\omega_0 \approx 224$:

$$\alpha < \omega_0 \implies \text{Underdamped}.$$

7.3.9 Solution 9: AC Circuit with Capacitor (Phasor Analysis + Plotting)

1. $\omega = 2\pi f$.
2. $X_C(f) = \frac{1}{2\pi f C}$.
3. $I_{\text{rms}}(f) = \frac{V_{\text{rms}}}{X_C(f)} = V_{\text{rms}} \cdot 2\pi f C$.

- Solution plot is shown in Fig. 9

7.3.10 Solution 10: RLC Parallel Resonance

1. Resonance frequency:

$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 1 \times 10^{-6}}} = \frac{1}{10^{-3}} = 1000\,\text{rad/s}.$$

2. Bandwidth (approx.):

$$\Delta\omega \approx \frac{1}{RC} = \frac{1}{2000 \times 1 \times 10^{-6}} = \frac{1}{2 \times 10^{-3}} = 500\,\text{rad/s}.$$

3. Quality factor:

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{1000}{500} = 2.$$

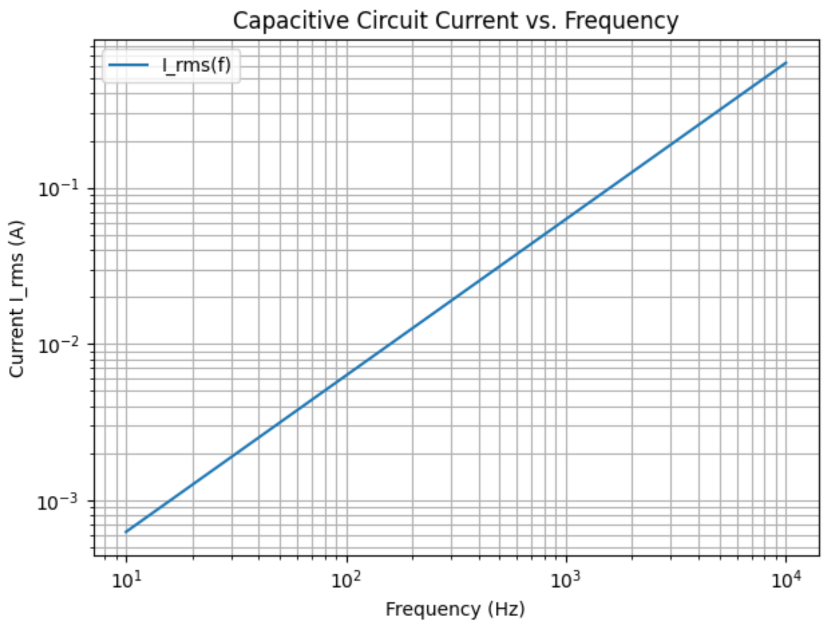

Figure 9: Solution for problem 9.

7.4 Operational Amplifiers

7.4.1 Solution 1: Inverting Amplifier

1. Voltage Gain of an inverting amplifier:

$$A_v = -\frac{R_f}{R_{\text{in}}} = -\frac{100\,000}{10\,000} = -10.$$

2. Output Voltage:

$$v_{\text{out}} = A_v \cdot v_{\text{in}} = -10 \times 0.1\,\text{V} = -1.0\,\text{V}.$$

3. The negative sign indicates a 180° phase inversion: a positive input leads to a negative output.

7.4.2 Solution 2: Non-Inverting Amplifier

1. Gain of a non-inverting amplifier:

$$A_v = 1 + \frac{R_1}{R_2} = 1 + \frac{4.7\,\text{k}\Omega}{1\,\text{k}\Omega} = 1 + 4.7 = 5.7.$$

2. Output:

$$v_{\text{out}} = 5.7 \times 0.2\,\text{V} = 1.14\,\text{V}.$$

7.4.3 Solution 3: Summing Amplifier

1. Summing amplifier formula (inverting type):

$$v_{\text{out}} = -\frac{R_f}{R_a}(v_a + v_b + v_c),$$

assuming $R_a = R_b = R_c$ and they all equal R_f.
2. Here, $R_f = R_a = 10\,\text{k}\Omega$. So,

$$v_{\text{out}} = -(v_a + v_b + v_c).$$

3. Numerically:

$$(v_a + v_b + v_c) = 1 + 0.5 + (-2) = -0.5.$$
$$v_{\text{out}} = -(-0.5) = +0.5\,\text{V}.$$

(Note that it's "inverting" in principle, but the sum of inputs is negative, flipping the sign again.)

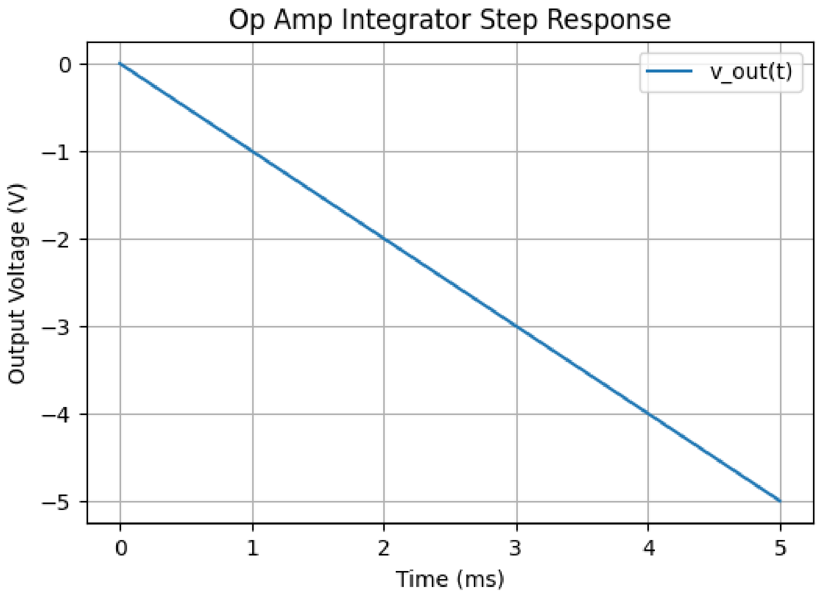

Figure 10: Solution for problem 5.

7.4.4 Solution 4: Difference Amplifier

1. Difference Amplifier Formula (assuming $R_1 = R_2$ and $R_3 = R_4$):

$$v_{\text{out}} = \left(\frac{R_1}{R_2}\right)(v_2 - v_1).$$

If $R_1 = R_2$ and $R_3 = R_4$, it simplifies to $v_{\text{out}} = v_2 - v_1$.
2. Here, $v_{\text{out}} = 5\,\text{V} - 2\,\text{V} = 3\,\text{V}$.

7.4.5 Solution 5: Op Amp Integrator (Step Response) – With Plot

1. Integrator Equation (ideal inverting integrator):

$$v_{\text{out}}(t) = -\frac{1}{RC}\int_0^t v_{\text{in}}(\tau)\,d\tau.$$

2. For a step $v_{\text{in}} = +1\,\text{V}$ (for $t \geq 0$):

$$v_{\text{out}}(t) = -\frac{1}{10{,}000 \times 0.1 \times 10^{-6}}\int_0^t 1\,d\tau = -\frac{1}{10{,}000 \times 0.1 \times 10^{-6}}\cdot t = -\frac{1}{10{,}000 \times 10^{-7}}\cdot t = -\frac{1}{10^{-3}}\cdot t = -1000\,t.$$

So $v_{\text{out}}(t) = -1000\,t$ volts, for t in seconds.
At $t = 5\,\text{ms}$, $v_{\text{out}}(5 \times 10^{-3}) = -1000 \times 0.005 = -5\,\text{V}$.
- Solution plot is shown in Fig. 10.

7.4.6 Solution 6: Op Amp Differentiator (Sinusoidal Input) – With Plot

1. Differentiator Equation (ideal inverting differentiator):

$$v_{\text{out}}(t) = -R\,C\,\frac{d}{dt}v_{\text{in}}(t).$$

2. The input is $2\sin(2000\pi t)$.
- The derivative $\frac{d}{dt}[\sin(\omega t)] = \omega\cos(\omega t)$.
- So $\frac{d}{dt}[2\sin(2000\pi t)] = 2 \times 2000\pi\cos(2000\pi t)$.
3. Hence:

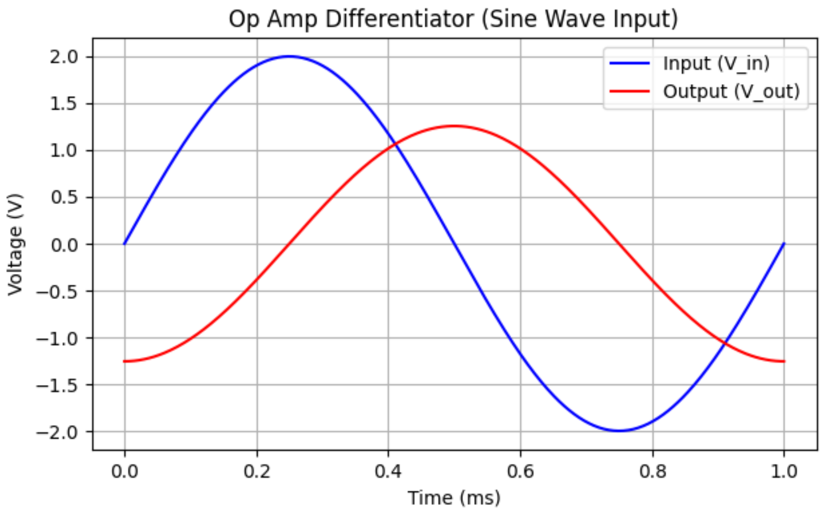

Figure 11: Solution for problem 6.

$$v_{\text{out}}(t) = -RC\Big[2 \cdot 2000\pi \cos(2000\pi t)\Big].$$

With $R = 10{,}000\,\Omega$ and $C = 0.01 \times 10^{-6}$ F:

$$RC = 10{,}000 \times 0.01 \times 10^{-6} = 10{,}000 \times 10^{-8} = 10^{-4}.$$

$$v_{\text{out}}(t) = -10^{-4} \times 2 \times 2000\pi \cos(2000\pi t) = -0.4\pi \cos(2000\pi t)\,\text{V}.$$

Numerically, $0.4\pi \approx 1.2566$, so $v_{\text{out}}(t) \approx -1.2566 \cos(2000\pi t)$.
- Solution plot is shown in Fig. 11.

7.4.7 Solution 7: Basic Comparator

1. Comparator Rule:

$$v_{\text{out}} = \begin{cases} +V_{\text{sat}} & \text{if } v_- < v_+, \\ -V_{\text{sat}} & \text{if } v_- > v_+. \end{cases}$$

But here, $v_+ = 0\,\text{V}$.
- If $v_{\text{in}} < 0$, the inverting input is negative relative to + input, so output saturates negative.
- If $v_{\text{in}} > 0$, output saturates positive.
2. For the three test voltages:
- $v_{\text{in}} = -1\,\text{V} < 0 \implies v_{\text{out}} = -12\,\text{V}$.
- $v_{\text{in}} = 0\,\text{V} \implies$ ideally, at threshold, it might latch either way (practically ≈ 0 or hysteresis needed). But in an ideal sense, $v_- = v_+$, so output is uncertain or 0 V if we allow a linear region. Usually for a real comparator, any offset might tip it to + or -. Let's assume 0 V is the boundary.
- $v_{\text{in}} = +3\,\text{V} > 0 \implies v_{\text{out}} = +12\,\text{V}$.

7.4.8 Solution 8: Instrumentation Amplifier (Ideal)

1. Instrumentation Amplifier Gain:

$$A = 1 + \frac{2R}{R_G} = 1 + \frac{2 \times 10{,}000}{2{,}000} = 1 + \frac{20{,}000}{2{,}000} = 1 + 10 = 11.$$

2. The output is:

$$v_{\text{out}} = A\,(v_2 - v_1) = 11 \times 0.1\,\text{V} = 1.1\,\text{V}.$$

7.4.9 Solution 9: Slew Rate Limitation

1. Time to swing $\Delta V = 10\,\mathrm{V}$ with slew rate $\mathrm{SR} = 0.5\,\mathrm{V}/\mu\mathrm{s}$:

$$t = \frac{\Delta V}{\mathrm{SR}} = \frac{10\,\mathrm{V}}{0.5\,\mathrm{V}/\mu\mathrm{s}} = 20\,\mu\mathrm{s}.$$

2. If we want it in $\leq 5\,\mu\mathrm{s}$:

$$\mathrm{SR}_{\mathrm{min}} = \frac{10\,\mathrm{V}}{5\,\mu\mathrm{s}} = 2\,\mathrm{V}/\mu\mathrm{s}.$$

7.4.10 Solution 10: Input Offset Voltage Effect

1. For an ideal inverting amplifier, the inverting input is at virtual ground (0 V). A small offset v_{os} effectively means the difference between the op amp inputs is v_{os}.
2. In a simple approximation, you can think of the offset as an effective input that gets amplified by $-R_f/R_{\mathrm{in}}$.
3. So:

$$v_{\mathrm{out,\,offset}} \approx -\left(\frac{R_f}{R_{\mathrm{in}}}\right) v_{\mathrm{os}} = -\left(\frac{100\,000}{10\,000}\right) \times 2\,\mathrm{mV} = -10 \times 2\,\mathrm{mV} = -20\,\mathrm{mV}.$$