EE 105: **Inverting and Non-inverting Amplifiers**
Resources and Worksheets

Prof. Yasser Khan and Prof. Rehan Kapadia

W. V. Unglaub
unglaub@usc.edu

March 4, 2025

Contents

1 Online resources (expanded)

Below is a curated list of quality online resources that you may find helpful for learning, exploring, and practicing a variety of concepts related to semiconductor devices.

1.1 Tutorials and reference websites

- **All About Circuits**: This website features extensive text-based tutorials that cover DC/AC circuits, semiconductor devices, and more. Every topic also includes a forum at the bottom of the page where students can ask questions and interact with the community to give and receive help. A comprehensive, hyperlinked table of contents on semiconductor devices is available here, and an online textbook on analog devices may be found here, with a corresponding table of contents toward the bottom of the page. For additional practice, select a topic to explore and challenge yourself in the Worksheets section.
- **Electronics-Tutorials** (*corrected hyperlink*): This is a fantastic organized collection of tutorials covering a myriad of topics in circuits with tons of diagrams and worked-out examples featuring step-by-step derivations. For semiconductor devices, relevant sections include semiconductor basics, diodes, and transistors.
- **Khan Academy**: Features a collection of short video lessons and practice problems on the fundamentals of circuit basics, signals, and systems with a user-friendly approach suitable for beginners. In regard to semiconductor devices, a set of video lectures on diodes may be found here.
- **LibreTexts**: LibreTexts is a fantastic online textbook resource for a variety of topics, including theory and worked out examples. A section on the basic physics concepts behind semiconductor devices may be found here, while a set of general textbook modules on semiconductors, including band theory, extrinsic vs. intrinsic semiconductors, metal-oxide-semiconductors (MOS), and diodes, may be found here.
- **MIT OpenCourseWare (OCW)**: Free, self-paced course materials available from MIT, including detailed lecture notes, problem sets, exams with solutions, and video lectures. For semiconductor devices, courses **6.002** (*Circuits and Electronics*, link here) and **6.012** (*Microelectronic Devices and Circuits*, links here, here, and here).
- **Selected problems by BarcelonaTech**: A collection of solved examples on semiconductor and electronic device physics, suitable for practice and deeper understanding.
- **Wikipedia**: A great resource for introductory and surface-level overviews of topics related to electronics and semiconductor devices. Examples of entries relevant to semiconductor devices include MOSFET, CMOS, and subthreshold conduction.

1.2 Simulation tools

- **Tinkercad Circuits**: A browser-based circuit design and simulation tool from Autodesk. This is a great tool for beginners to visualize and interact with circuits in a simulated fashion. It includes basic electronic components, measurement tools, and real-time simulation. For some introductory tutorials to get you started, check out the official playlist on YouTube.
- **Falstad Circuit Simulator**: A web-based real-time simulation interface with helpful visualizations of voltages and currents which allows students to see waveforms change dynamically. A large variety of preset examples (such as RLC circuits, op-amps, and digital logic circuits) are included for students to explore with a user-friendly drag-and-drop interface.
- **LTspice**: An excellent freeware circuit simulator developed by Analog Devices which is widely used in both academia and industry. It has a slightly steeper learning curve than the other simulators but it is capable of simulating many different types of complex circuits, making it a useful tool for learning schematic capture, waveform analysis, and advanced circuit behavior. For some introductory tutorials to get started, check out the following YouTube tutorial playlist.
- **PhET Simulations**: For absolute beginners who are particularly visual learners and are looking for hands-on learning to aid conceptual understanding, this website includes many interactive demos and simulation tools for DC and AC circuit basics. Students are able to manipulate variables in real time (e.g., resistor values, battery voltage, etc.) and see immediate effects.

- **CircuitLab**: Another browser-based circuit simulator with built-in analysis tools. Featuring a library of example circuits, it can be useful for practicing circuit design and learning how measurement tools work, such as oscilloscopes.

1.3 Discussion and forum communities

- **Electronics Stack Exchange**: A Q&A format discussion forum with a broad community of hobbyists, students, and professionals. It can be a great resource for well-structured answers to specific, technical circuit questions.
- **r/ElectricalEngineering**: An electrical engineering subreddit forum featuring discussions on coursework, textbooks, career advice, and circuit troubleshotting. It can be a good place to ask questions and see how others approach similar problems.

2 Brief overview of amplifiers

Operational amplifiers (op amps) are fundamental and versatile building blocks in analog electronics that can be configured to perform a wide range of tasks, such as amplification, filtering, integration, and differentiation. At their core, op amps are high-gain voltage amplifiers with differential inputs (inverting and non-inverting) and a single-ended output. They are typically used in closed-loop configurations with feedback networks that precisely set the overall circuit performance. When used with external feedback networks, they can be configured to perform a wide range of linear operations such as voltage amplification, filtering, summation, integration, and differentiation.

Key Characteristics of Op Amps:

1. **High Open-Loop Gain**: In an ideal op amp, this gain is considered infinite. In practice, it is large but finite (e.g., 10^5 to 10^7 in low-frequency ranges).
2. **Differential Inputs**: The op amp has two inputs: the inverting input (marked with a “”) and the non-inverting input (marked with a “+”).
3. **High Input Impedance / Low Output Impedance**: Ideally, no current flows into the inputs, and the output can deliver or absorb current with minimal voltage drop.
4. **Linear Operation via Negative Feedback**: By feeding back part of the output signal to the inverting input, designers can create circuits with stable and predictable gains determined by external components (usually resistors).

2.1 Inverting amplifiers

In an inverting amplifier configuration, the input signal is applied to the inverting input through an input resistor R_{in}, while the non-inverting input is connected to ground (or held at a reference voltage), and a feedback resistor R_f is connected from the output back to the inverting input. Because of the high open-loop gain and the action of negative feedback, the inverting input is held at a potential very close to ground—a condition known as a *virtual ground*. The gain of the inverting amplifier is determined solely by the ratio of the feedback resistor R_f to the input resistor R_{in}, given by

$$\boxed{\text{Gain} = -\frac{R_f}{R_{\text{in}}}.} \tag{1}$$

This configuration produces an output that is 180° out of phase with the input. Although the inverting amplifier offers predictable gain and simplicity, its input impedance is approximately equal to R_{in}, which may be lower than desired in some applications.

Key Points:

- **Virtual Ground** Because of the large open-loop gain and the negative feedback, the inverting input is virtually at ground potential, even though it is not directly connected to ground. This concept is called a “virtual ground.”
- **Gain**: The closed-loop gain is set by the ratio of R_f to R_{in}:
$$\text{Gain} = -\frac{R_f}{R_{\text{in}}}.$$
The negative sign indicates the output signal is inverted (180° phase shift).
- **Input Impedance**: In this configuration, the input impedance is approximately R_{in}.
- **Use Cases**: The inverting amplifier is often used when precise, stable gain is required, and when summing multiple input signals (using multiple input resistors into the same inverting node).

2.2 Non-inverting amplifiers

In contrast, the non-inverting amplifier configuration applies the input signal directly to the non-inverting input, while the inverting input is connected to a feedback network consisting of a voltage divider (formed by a resistor R_f from output to inverting input, and a resistor R_1 from inverting input to ground) that feeds back a portion of the output signal. The gain of a non-inverting amplifier is given by

$$\boxed{\text{Gain} = 1 + \frac{R_f}{R_1},} \tag{2}$$

where R_1 is the resistor connected from the inverting input to ground. One of the key advantages of the non-inverting amplifier is its high input impedance, which makes it ideal for interfacing with high-impedance signal sources. Additionally, since the output is in phase with the input, it avoids the phase inversion seen in the inverting configuration.

Key Points:

- **Gain**: The closed-loop gain is given by

$$\text{Gain} = 1 + \frac{R_f}{R_1}.$$

  There is no phase inversion: the output is in phase with the input.
- **High Input Impedance**: Because the input is tied to the non-inverting terminal of the op amp (which ideally draws negligible current), the circuit's input impedance is very high.
- **Use Cases**: The non-inverting amplifier is commonly employed when a high input impedance is necessary, such as buffering a high-impedance sensor output. It is also used when preserving signal phase is important.

2.3 Overview and applications

Both configurations are essential in analog circuit design:

Inverting amplifiers are widely used for precise gain control, summing functions, and when phase inversion is either desired or inconsequential. They are particularly useful when the source impedance is low.

Non-inverting amplifiers excel when a high input impedance is required or when maintaining the phase of the input signal is critical. They are commonly found in sensor interfaces and signal buffering applications.

In practice, designers must also account for non-ideal characteristics of real op amps—such as finite bandwidth, input bias currents, offset voltages, and slew rate limitations—which can influence the performance of the amplifier. Despite these limitations, with proper design and compensation techniques, op amp circuits remain a cornerstone of analog electronics.

2.3.1 Practical considerations

Real op amps deviate from the ideal model in several ways:

- **Finite Open-Loop Gain and Bandwidth**: Op amp gain decreases with increasing frequency.
- **Input Bias and Offset Currents**: Tiny currents flow into the input terminals, which can create offset voltages across external resistors.
- **Slew Rate**: The maximum rate at which the output can change limits high-speed or large-amplitude signals.
- **Power Supply Limitations**: The output voltage swing is typically constrained by the supply voltages (e.g., ±15V, 0˘5V, etc.).

Despite these non-idealities, operational amplifiers remain indispensable in modern electronics. By choosing appropriate op amps and carefully designing feedback networks (including resistors, capacitors, or more complex networks), a vast array of analog processing functions can be reliably implemented.

2.4 Problem Worksheets

Click here to access a first set of computational and conceptual problems on the basics of inverting and non-inverting op-amp amplifiers. Click here to access a second set of computational and conceptual problems on the basics of inverting and non-inverting op-amp amplifiers.

3 Problems

3.1 Amplifiers: Problem Set 1

[return to TOC]

The following first set of problems focus on the fundamentals of operational amplifiers and include a mix of computational and conceptual exercises covering inverting and non-inverting amplifiers. Specifically, below is a set of 20 problems on op-amp amplifiers (inverting, non-inverting, summing, differential, etc.) along with fully worked solutions. Problems 1–10 are quantitative/computational and include Python code to illustrate calculations and plots. Problems 11–20 are conceptual, with detailed explanations. This problem set provides a mix of computational and conceptual exercises to reinforce understanding of op amp behavior, design trade-offs, and practical design considerations.

3.1.1 Problem 1: Inverting Amplifier Design

Design an inverting amplifier that has a closed-loop gain of –10. If the input resistor is $R_{\text{in}} = 10\,\text{k}\Omega$, determine the feedback resistor R_f. Then, for an ideal op amp, plot the output voltage versus input voltage when the input varies from –1V to 1V.

3.1.2 Problem 2: Non-inverting Amplifier Design

Design a non-inverting amplifier with a desired gain of 5. Given that the resistor connected to ground is $R_1 = 10\,\text{k}\Omega$, find the feedback resistor R_f. Then plot the output voltage versus input voltage for input values between –1V and 1V.

3.1.3 Problem 3: Frequency Response of an Inverting Amplifier with Finite Bandwidth

An inverting amplifier is designed for a nominal closed-loop gain of –20 using $R_{\text{in}} = 1\,\text{k}\Omega$ and $R_f = 20\,\text{k}\Omega$. The op amp has a gain-bandwidth product (GBW) of 1MHz. Assuming that the open-loop gain falls off as

$$A_{\text{ol}}(f) = \frac{\text{GBW}}{f},$$

derive an expression for the effective closed-loop gain and determine its –3dB cutoff frequency. Then, plot the gain magnitude (in dB) versus frequency (10Hz to 1MHz).

3.1.4 Problem 4: Non-inverting Amplifier with a Low-Pass Filter

A non-inverting amplifier is designed with a gain of 10. With $R_1 = 10\,\text{k}\Omega$, choose R_f accordingly. Then, to implement a low-pass filter, a capacitor $C = 100\,\text{pF}$ is connected in parallel with R_f.

1. Determine R_f.
2. Find the –3dB cutoff frequency where the capacitor significantly lowers the effective feedback impedance.
3. Plot the frequency response (magnitude of closed-loop gain) versus frequency.

3.1.5 Problem 5: Summing Amplifier

Design a summing amplifier (using an inverting configuration) that sums two input voltages. Use two input resistors $R_1 = R_2 = 10\,\text{k}\Omega$ and a feedback resistor $R_f = 20\,\text{k}\Omega$.

1. Calculate the output voltage when $V_1 = 1\,\text{V}$ and $V_2 = 2\,\text{V}$.
2. Write Python code that computes the output voltage for various combinations of V_1 and V_2.

3.1.6 Problem 6: Differential Amplifier

Design a differential amplifier whose output is

$$V_{\text{out}} = \frac{R_2}{R_1}(V_2 - V_1)$$

with a differential gain of 5. Assume $R_1 = R_3 = 10\,\mathrm{k\Omega}$. Determine R_2 and R_4 (with $R_2 = R_4$) and then compute the output when $V_2 = 3\,\mathrm{V}$ and $V_1 = 1\,\mathrm{V}$. Also, write Python code to compute V_{out} for a range of differential inputs.

3.1.7 Problem 7: Resistor Tolerance Effects on Gain

For an inverting amplifier with ideal gain

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_f}{R_{\text{in}}}$$

using $R_{\text{in}} = 10\,\mathrm{k\Omega}$ and $R_f = 100\,\mathrm{k\Omega}$ (ideal gain –10), determine how a $\pm 5\%$ tolerance in resistor values affects the gain. Write Python code to simulate many samples (using random variations) and plot a histogram of the resulting closed-loop gains.

3.1.8 Problem 8: Effect of Input Bias Currents

An inverting amplifier uses $R_{\text{in}} = 10\,\mathrm{k\Omega}$ and $R_f = 100\,\mathrm{k\Omega}$. The op amp has an input bias current $I_b = 50\,\mathrm{nA}$.

1. Estimate the output offset voltage due to the bias current (assuming no compensation resistor).
2. Then, write Python code to compute and plot the output offset voltage for bias currents varying from 10nA to 100nA.

3.1.9 Problem 9: Output Voltage Swing and Clipping

An inverting amplifier has $R_{\text{in}} = 10\,\mathrm{k\Omega}$ and $R_f = 100\,\mathrm{k\Omega}$ (ideal gain of –10). With supply voltages $\pm 15\,\mathrm{V}$, determine the maximum undistorted input range (i.e. the maximum V_{in} before clipping occurs). Then, plot the ideal output voltage versus input voltage and indicate the clipping limits.

3.1.10 Problem 10: Noise Gain Analysis

For an inverting amplifier, the noise gain is defined as

$$\text{Noise Gain} = 1 + \frac{R_f}{R_{\text{in}}}.$$

Consider an inverting amplifier with $R_{\text{in}} = 2\,\mathrm{k\Omega}$ and $R_f = 18\,\mathrm{k\Omega}$ (ideal noise gain = 10). Assuming a single-pole open-loop response with GBW = 2MHz, plot the effective closed-loop gain (as an approximation of the noise gain) versus frequency.

3.1.11 Problem 11: Inverting vs. Non-inverting Configurations

Explain the primary differences between inverting and non-inverting op amp configurations, particularly in terms of their gain formulas, input impedance, and phase relationships.

3.1.12 Problem 12: Virtual Ground in the Inverting Amplifier

Discuss the concept of a virtual ground in an inverting amplifier and explain how it simplifies circuit analysis.

3.1.13 Problem 13: Finite Open-Loop Gain Effects

Describe how the finite open-loop gain of a real op amp affects the closed-loop gain of both inverting and non-inverting amplifiers.

3.1.14 Problem 14: Negative Feedback Stabilization

Explain how negative feedback in op amp circuits stabilizes the gain and improves linearity.

3.1.15 Problem 15: Common-Mode Rejection Ratio (CMRR)

Discuss the importance of the common-mode rejection ratio (CMRR) in differential op amp circuits.

3.1.16 Problem 16: Frequency Compensation Techniques

Describe common frequency compensation techniques used in op amp circuits to ensure stability and control bandwidth.

3.1.17 Problem 17: Slew Rate

Explain the concept of slew rate in op amps and its impact on high-speed amplifier performance.

3.1.18 Problem 18: Input Bias Current and Offset Voltage

Discuss how input bias current and input offset voltage affect op amp circuits and describe methods to mitigate these effects.

3.1.19 Problem 19: Inverting vs. Non-inverting: Noise and Linearity

Compare the advantages and disadvantages of inverting and non-inverting op amp circuits in terms of noise performance and linearity.

3.1.20 Problem 20: Limitations of the Ideal Op Amp Model

Discuss the limitations of the ideal op amp assumptions in practical circuit design and explain how non-idealities are modeled.

3.2 Amplifiers: Problem Set 2

[return to TOC]

The following second set of problems also focus on the fundamentals of operational amplifiers and include a mix of computational and conceptual exercises covering inverting and non-inverting amplifiers. Specifically, below is a collection of 20 practice problems on operational amplifier (op amp) circuits, primarily focused on inverting and non-inverting amplifier configurations. Problems 1–10 are quantitative/computational and include Python code; problems 11–20 are more conceptual, discussing theory and design considerations in op amp circuits. These problems and their worked solutions can be used as extra practice. The Python scripts demonstrate numerical calculations, simulations of tolerance and bias effects, and frequency response approximations. By working through both the quantitative and conceptual problems, students can gain a deeper understanding of inverting and non-inverting op amp circuits, practical design issues, and real-world non-idealities.

3.2.1 Problem 1: Basic Inverting Amplifier – Gain and Output Plot

1. You have an inverting amplifier with an input resistor $R_{\text{in}} = 5\,\text{k}\Omega$ and a feedback resistor $R_f = 50\,\text{k}\Omega$. Determine the closed-loop gain of the amplifier.
2. Plot the output voltage versus input voltage over the range $-1\,\text{V} \leq V_{\text{in}} \leq 1\,\text{V}$ for the ideal case.

3.2.2 Problem 2: Non-Inverting Amplifier – Gain Design

You want to design a non-inverting amplifier with a voltage gain of 8. You choose $R_1 = 10\,\text{k}\Omega$ for the resistor from the inverting input to ground. Find the feedback resistor R_f so that the closed-loop gain is 8. Then, plot the output voltage from $-2\,\text{V}$ to $2\,\text{V}$ input.

3.2.3 Problem 3: Three-Input Summing Amplifier

Design a summing amplifier that sums three input voltages V_1, V_2, V_3 using the same resistor $R_{\text{in}} = 10\,\text{k}\Omega$ for each input and $R_f = 30\,\text{k}\Omega$.

1. Write the expression for the output voltage V_{out}.
2. Calculate V_{out} if $V_1 = 1.0\,\text{V}, V_2 = 0.5\,\text{V}, V_3 = -0.5\,\text{V}$.
3. Write Python code that scans various combinations of (V_1, V_2, V_3) to produce a 3D plot (or contour plot) of the output.

3.2.4 Problem 4: Inverting Amplifier with Input Offset Voltage

Assume an inverting amplifier (with $R_{\text{in}} = 10\,\text{k}\Omega$ and $R_f = 100\,\text{k}\Omega$) experiences an op amp input offset voltage of $2\,\text{mV}$ at the inverting terminal.

1. Derive the expression for the output offset V_{offset} due to this input offset voltage.
2. Calculate V_{offset}.
3. Write Python code that simulates the offset and compares it with the ideal transfer function for inputs from $-1\,\text{V}$ to $1\,\text{V}$.

3.2.5 Problem 5: Gain-Bandwidth Product for a Non-Inverting Amplifier

A non-inverting amplifier has a desired low-frequency gain of $A_{\text{cl}} = 20$. The op amp has a gain-bandwidth product (GBW) of 1MHz.

1. Estimate the -3dB bandwidth of the closed-loop amplifier.
2. Create a Python script that plots the approximate magnitude of the closed-loop gain from 10Hz to 1MHz.

3.2.6 Problem 6: AC Analysis of an Inverting Amplifier with a Capacitor in Parallel

Consider an inverting amplifier with $R_{\text{in}} = 5\,\text{k}\Omega$, $R_f = 50\,\text{k}\Omega$, and a capacitor $C_f = 100\,\text{pF}$ in parallel with R_f.

1. Determine the approximate cutoff frequency above which the capacitor significantly reduces the closed-loop gain.
2. Plot the magnitude of the closed-loop gain vs. frequency from 100Hz to 1MHz.

3.2.7 Problem 7: Resistor Tolerances in a Non-Inverting Amplifier

A non-inverting amplifier uses $R_1 = 10\,\mathrm{k}\Omega$ and $R_f = 90\,\mathrm{k}\Omega$, giving a nominal gain of 10. Both resistors have a tolerance of $\pm 5\%$.

1. Derive the expression for the actual gain if R_1 and R_f vary within $\pm 5\%$.
2. Use Python to randomly generate 10,000 samples of R_1 and R_f within their respective tolerance ranges and plot a histogram of the resulting gains.
3. Report the mean and standard deviation of the simulated gain distribution.

3.2.8 Problem 8: Input Bias Current – Offset in Inverting Amplifier

An inverting amplifier has $R_{\text{in}} = 20\,\mathrm{k}\Omega$ and $R_f = 200\,\mathrm{k}\Omega$. The op amp's input bias current is $I_b = 100\,\mathrm{nA}$.

1. Estimate the output offset voltage due to this bias current if no compensation resistor is used.
2. Write Python code to compute and plot the output offset as I_b varies from 10nA to 200nA.

3.2.9 Problem 9: Output Clipping with Limited Supply Rails

An inverting amplifier (gain = -5) is powered with ± 12V supplies.

1. Find the maximum undistorted sinusoidal input amplitude before the output clips.
2. Generate a Python plot showing the ideal (no saturation) output versus input, and then the actual output that saturates at ± 12V.

3.2.10 Problem 10: Small-Signal AC Simulation of an Inverting Amplifier

An inverting amplifier has a desired low-frequency gain of -20. The op amp's open-loop gain $A_{\text{ol}}(f)$ falls off at 20dB/decade starting at 10Hz, where $A_{\text{ol}}(10\,\text{Hz}) = 10^5$.

1. Estimate the unity-gain frequency for the op amp.
2. Use Python to approximate the closed-loop magnitude response from 1Hz to 1MHz and plot it on a Bode plot (magnitude in dB vs. frequency).

3.2.11 Problem 11: Comparing Inverting and Non-Inverting Amplifiers

Explain how the input impedance, phase relationship, and gain formula differ between inverting and non-inverting op amp configurations. Discuss typical use cases for each.

3.2.12 Problem 12: Virtual Ground Concept

Describe the concept of a virtual ground in an inverting amplifier. Why does the inverting terminal effectively behave like a ground node, even though it is not directly connected to ground? How does this simplify circuit analysis?

3.2.13 Problem 13: Summing Amplifier vs. Differential Amplifier

Discuss the difference between a summing amplifier configuration (which adds multiple input voltages) and a differential amplifier (which amplifies the difference between two inputs). In what scenarios would you choose one configuration over the other?

3.2.14 Problem 14: Effects of Finite Input Impedance

Analyze why the inverting amplifier has a well-defined input impedance of R_{in}, whereas the non-inverting amplifier can exhibit a much higher input impedance. How does this affect circuit design choices (e.g., when dealing with sensors or signals with high source impedance)?

3.2.15 Problem 15: Negative Feedback and Stability

Explain how negative feedback controls the operating point of the op amp and stabilizes the gain to a value determined by external components. Also, briefly address what can happen if the feedback loop is broken or if the circuit phase shift leads to oscillation.

3.2.16 Problem 16: Input Bias Current and Offset Voltage

Discuss how non-zero input bias currents and offset voltages arise in real op amps. How do these factors affect precision amplifier circuits, and what design strategies are used to mitigate their impact (e.g., offset trimming, resistor matching)?

3.2.17 Problem 17: Frequency Response Limitations

Describe how an op amp's open-loop gain decreases with frequency and how this impacts closed-loop performance in high-gain applications. Include a brief mention of the gain-bandwidth product (GBW) and slew rate.

3.2.18 Problem 18: Using a Buffer (Voltage Follower)

Explain the role of a buffer amplifier (voltage follower), which is a non-inverting amplifier with unity gain. Why might a buffer be placed between a high-impedance sensor and subsequent circuitry?

3.2.19 Problem 19: Implementing a DC Offset Adjust

Describe how a small offset can be intentionally introduced at the input or via the feedback network to shift the output DC level of an amplifier. Give an example scenario where this might be necessary (e.g., level shifting for single-supply operation).

3.2.20 Problem 20: Practical Power Supply Constraints

Discuss the impact of real op amp supply rails on the output voltage swing and linear operation. Include considerations such as rail-to-rail op amps versus standard op amps, and how headroom limitations can cause distortion when the input or output signal nears the supply voltage limits.

4 Solutions to problems

4.1 Amplifiers: Solution Set 1

4.1.1 Solution 1: Inverting Amplifier Design

For an inverting amplifier the closed-loop gain is given by

$$\frac{V_{\text{out}}}{V_{\text{in}}} = -\frac{R_f}{R_{\text{in}}}$$

Setting $-\frac{R_f}{10\,\text{k}\Omega} = -10$ gives

$$R_f = 10 \times 10\,\text{k}\Omega = 100\,\text{k}\Omega.$$

For a range of input voltages, the ideal output is

$$V_{\text{out}} = -10\,V_{\text{in}}.$$

Solution plot is shown in Fig. 1.

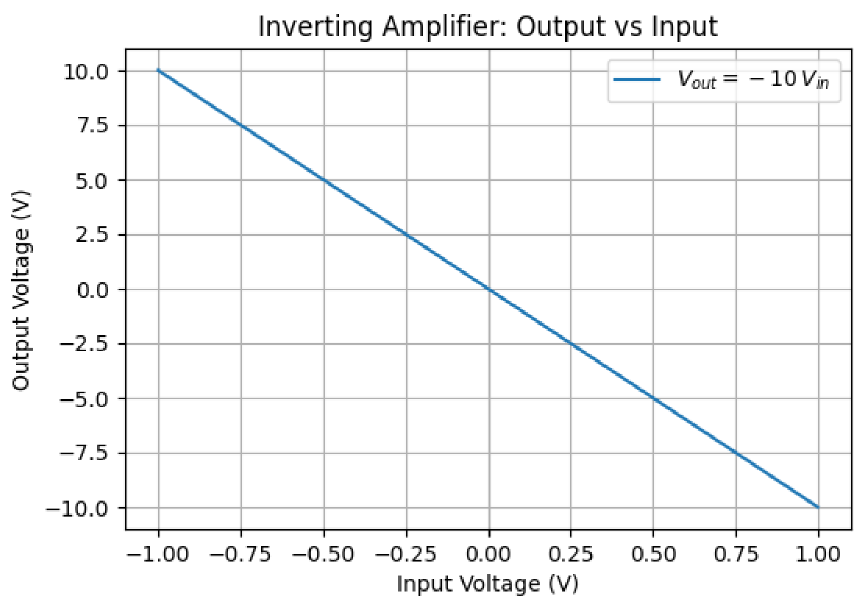

Figure 1: Solution for problem 1. See script `set1_prob1.py` for solution details.

4.1.2 Solution 2: Non-inverting Amplifier Design

For a non-inverting amplifier, the gain is given by

$$\text{Gain} = 1 + \frac{R_f}{R_1}.$$

Setting $1 + \frac{R_f}{10\,\text{k}\Omega} = 5$ gives

$$\frac{R_f}{10\,\text{k}\Omega} = 4 \quad \Rightarrow \quad R_f = 4 \times 10\,\text{k}\Omega = 40\,\text{k}\Omega.$$

Thus, the output is

$$V_{\text{out}} = 5\,V_{\text{in}}.$$

Solution plot is shown in Fig. 2.

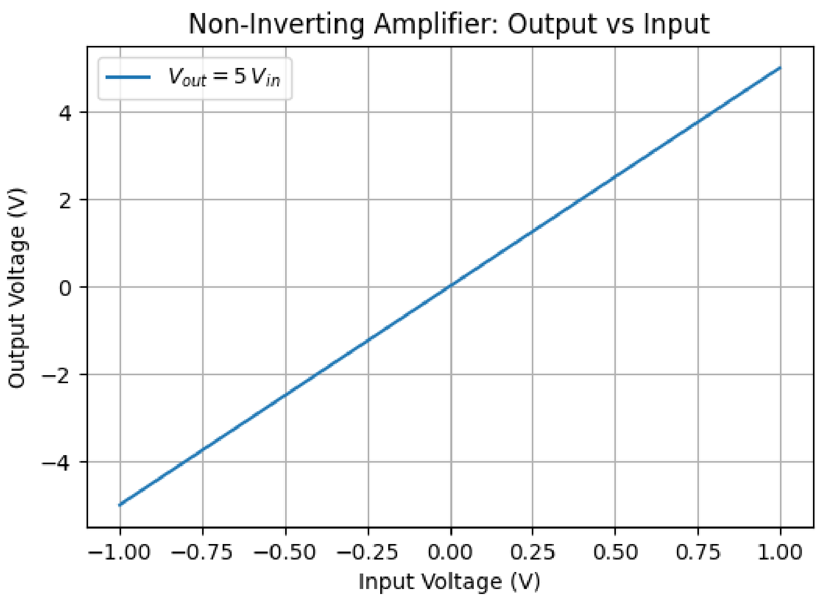

Figure 2: Solution for problem 2. See script `set1_prob2.py` for solution details.

4.1.3 Solution 3: Frequency Response of an Inverting Amplifier with Finite Bandwidth

For an inverting amplifier with finite open-loop gain, a common expression for the closed-loop gain is

$$A_{\text{cl}}(f) = \frac{-R_f/R_{\text{in}}}{1 + \frac{1+R_f/R_{\text{in}}}{A_{\text{ol}}(f)}}$$

Here, $R_f/R_{\text{in}} = 20$ and the feedback factor is

$$\beta = \frac{R_{\text{in}}}{R_f + R_{\text{in}}} = \frac{1}{21}.$$

For large A_{ol}, the ideal gain is –20; however, as frequency increases, $A_{\text{ol}}(f)$ decreases. A simplified approach is to approximate the closed-loop bandwidth (–3dB frequency) as

$$f_{-3dB} \approx \frac{\text{GBW}}{1 + |A_{\text{cl}}|} = \frac{1\,\text{MHz}}{21} \approx 47.6\,\text{kHz}.$$

A simplified frequency-dependent model is

$$|A_{\text{cl}}(f)| \approx \frac{20}{\sqrt{1 + \left(\frac{f}{f_{-3dB}}\right)^2}}.$$

Solution plot is shown in Fig. 3.

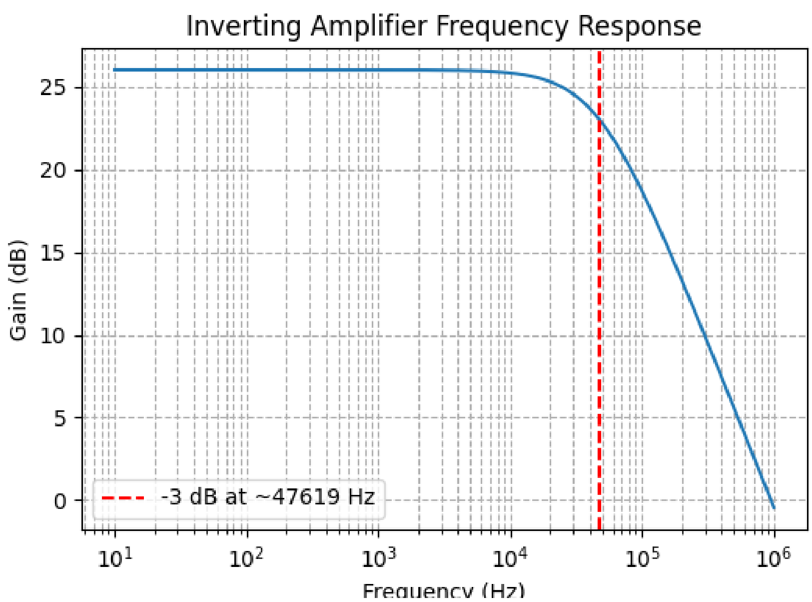

Figure 3: Solution for problem 3. See script `set1_prob3.py` for solution details.

4.1.4 Solution 4: Non-inverting Amplifier with a Low-Pass Filter

1. For a non-inverting amplifier,

$$\text{Gain} = 1 + \frac{R_f}{R_1} = 10 \quad \Rightarrow \quad R_f = 9\,R_1 = 90\,\text{k}\Omega.$$

2. With the capacitor in parallel with R_f, at high frequencies the impedance of the capacitor

$$Z_C = \frac{1}{j2\pi f C}$$

becomes small. The cutoff frequency (where $|Z_C| = R_f$) is

$$f_c = \frac{1}{2\pi R_f C} = \frac{1}{2\pi \cdot 90 \times 10^3 \cdot 100 \times 10^{-12}} \approx 17.7\,\text{kHz}.$$

3. The closed-loop gain is frequency dependent: at low frequencies,

$$A_{cl} \approx 1 + \frac{R_f}{R_1} = 10,$$

and at high frequencies the capacitor shorts R_f so that

$$A_{cl} \approx 1.$$

Solution plot is shown in Fig. 4.

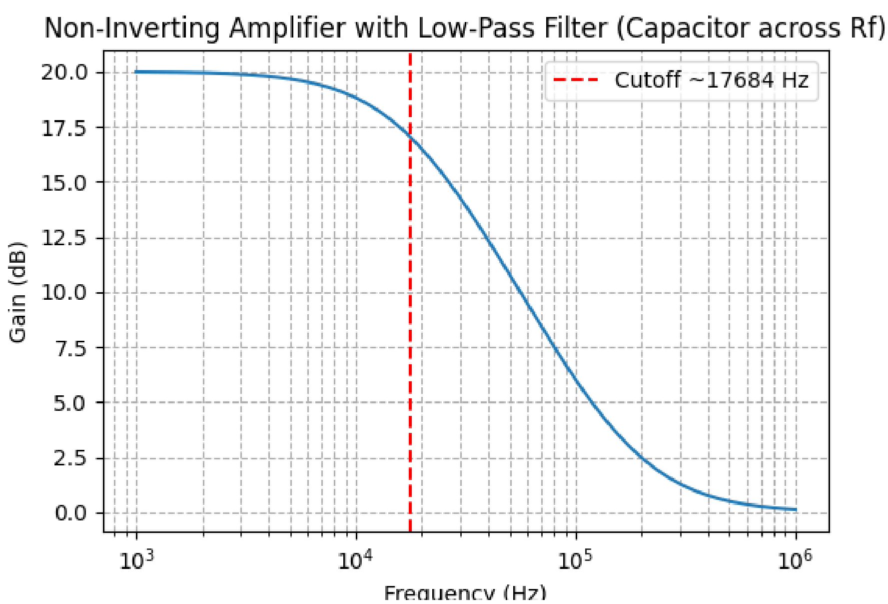

Figure 4: Solution for problem 4. See script `set1_prob4.py` for solution details.

4.1.5 Solution 5: Summing Amplifier

For a summing amplifier, the output voltage is given by

$$V_{\text{out}} = -R_f \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} \right).$$

Using the given values:

$$V_{\text{out}} = -20\,\text{k}\Omega \left(\frac{1}{10\,\text{k}\Omega} + \frac{2}{10\,\text{k}\Omega} \right) = -20(0.1 + 0.2) = -20 \times 0.3 = -6\,\text{V}.$$

Solution plot is shown in Fig. 5.

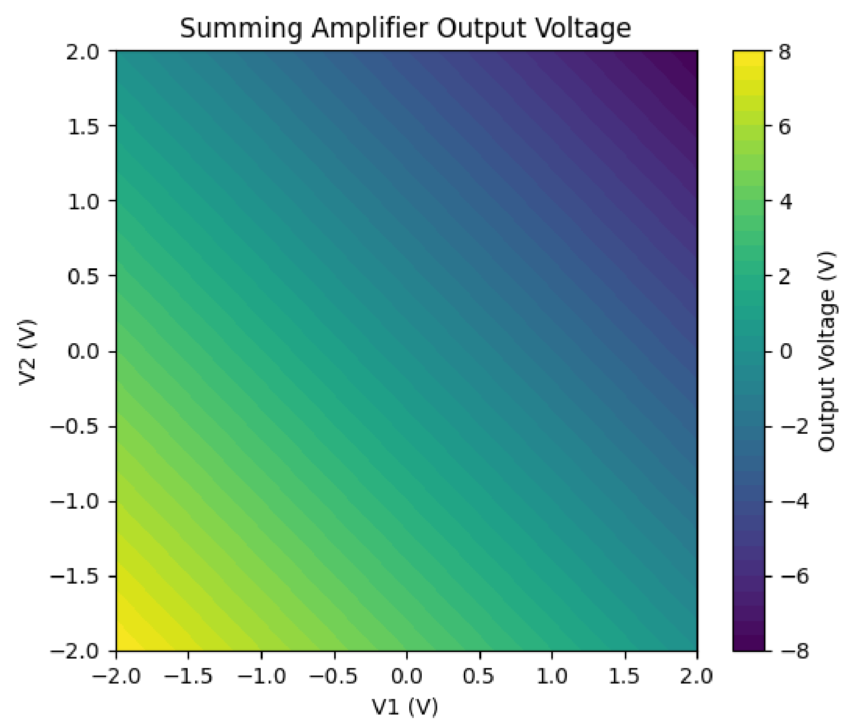

Figure 5: Solution for problem 5. See script `set1_prob5.py` for solution details.

4.1.6 Solution 6: Differential Amplifier

For the classic differential amplifier, if we choose

$$V_{\text{out}} = \frac{R_2}{R_1}(V_2 - V_1),$$

to have a gain of 5, set

$$\frac{R_2}{R_1} = 5 \quad \Rightarrow \quad R_2 = 5 \times 10\,\text{k}\Omega = 50\,\text{k}\Omega.$$

Also, choose $R_4 = R_2 = 50\,\text{k}\Omega$ (and $R_3 = R_1 = 10\,\text{k}\Omega$) for proper matching.
For $V_2 = 3\,\text{V}$ and $V_1 = 1\,\text{V}$:

$$V_{\text{out}} = 5 \times (3 - 1) = 10\,\text{V}.$$

Solution plot is shown in Fig. 6.

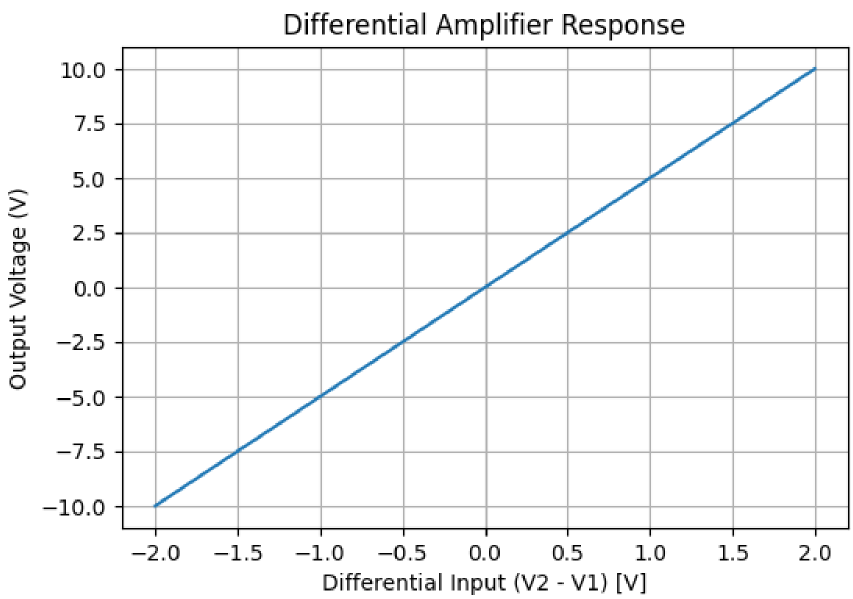

Figure 6: Solution for problem 6. See script `set1_prob6.py` for solution details.

4.1.7 Solution 7: Resistor Tolerance Effects on Gain

If each resistor varies by ±5%, then the actual values are

$$R_{\text{in,actual}} = R_{\text{in}} \times (1 + \epsilon_1), \quad R_{f,\text{actual}} = R_f \times (1 + \epsilon_2),$$

with ϵ_1, ϵ_2 uniformly distributed in $[-0.05, 0.05]$. The actual gain becomes

$$\text{Gain}_{\text{actual}} = -\frac{R_{f,\text{actual}}}{R_{\text{in,actual}}}.$$

Solution plot is shown in Fig. 7.

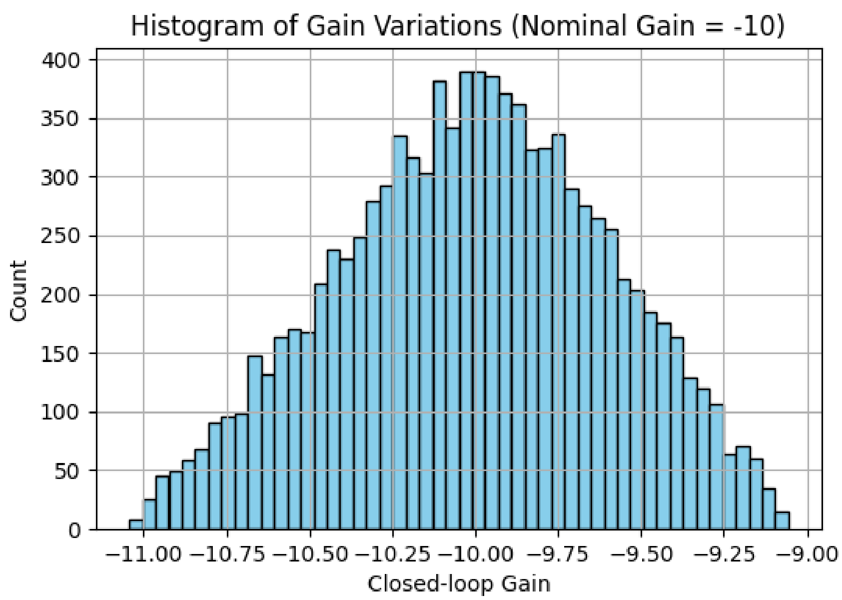

Figure 7: Solution for problem 7. See script `set1_prob7.py` for solution details.

4.1.8 Solution 8: Effect of Input Bias Currents

In an inverting configuration without bias current compensation, the bias current flowing into the inverting node produces a voltage drop across the input resistor. An approximate offset at the output is given by

$$V_{\text{offset}} \approx I_b \times R_{\text{in}} \times \left(1 + \frac{R_f}{R_{\text{in}}}\right).$$

Here, $1 + \frac{R_f}{R_{\text{in}}} = 1 + 10 = 11$. Thus, for $I_b = 50\,\text{nA}$:

$$V_{\text{offset}} \approx 50 \times 10^{-9} \times 10\,\text{k}\Omega \times 11 \approx 50 \times 10^{-9} \times 110e3 \approx 0.0055\,\text{V} \quad (5.5\,\text{mV}).$$

Solution plot is shown in Fig. 8.

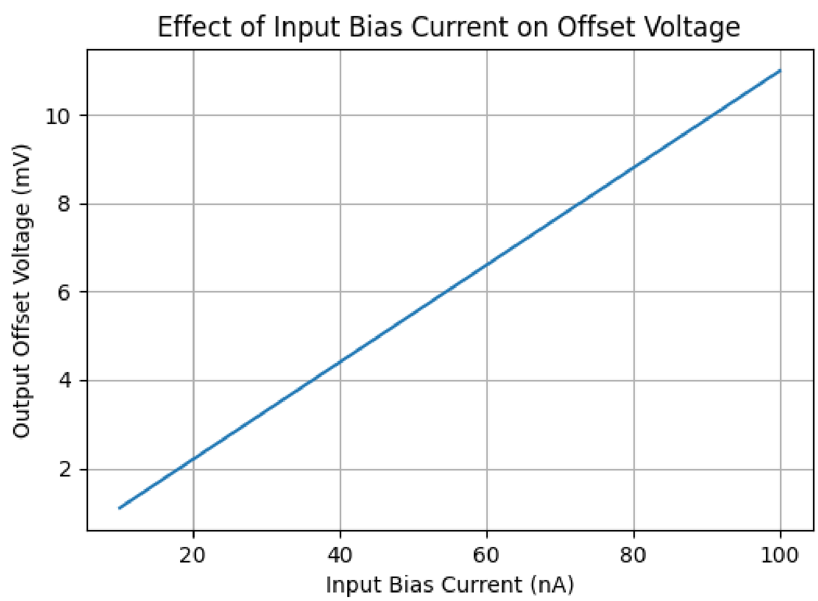

Figure 8: Solution for problem 8. See script `set1_prob8.py` for solution details.

4.1.9 Solution 9: Output Voltage Swing and Clipping

Since the amplifier multiplies the input by −10, the output is

$$V_{\text{out}} = -10\, V_{\text{in}}.$$

For an undistorted output, $|V_{\text{out}}|$ must be less than the supply rails (here ±15V). Therefore,

$$10\,|V_{\text{in}}| \leq 15 \quad \Rightarrow \quad |V_{\text{in}}| \leq 1.5\,\text{V}.$$

Solution plot is shown in Fig. 9.

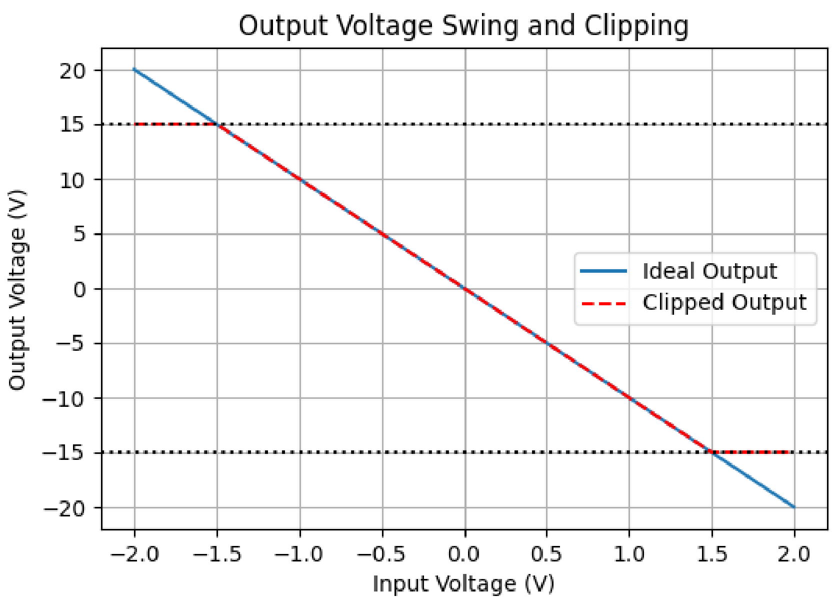

Figure 9: Solution for problem 9. See script `set1_prob9.py` for solution details.

4.1.10 Solution 10: Noise Gain Analysis

At low frequencies, the ideal noise gain is

$$1 + \frac{18\,\mathrm{k}\Omega}{2\,\mathrm{k}\Omega} = 10.$$

Due to the finite open-loop gain (which falls off as $A_{\mathrm{ol}}(f) = \frac{\mathrm{GBW}}{f}$), the effective closed-loop gain may begin to deviate as frequency increases. A simplified model is:

$$|A_{\mathrm{cl}}(f)| \approx \frac{10}{\sqrt{1 + \left(\frac{f}{f_{-3dB}}\right)^2}},$$

where $f_{-3dB} \approx \frac{\mathrm{GBW}}{1+10} = \frac{2\,\mathrm{MHz}}{11} \approx 182\,\mathrm{kHz}$.

Solution plot is shown in Fig. 10.

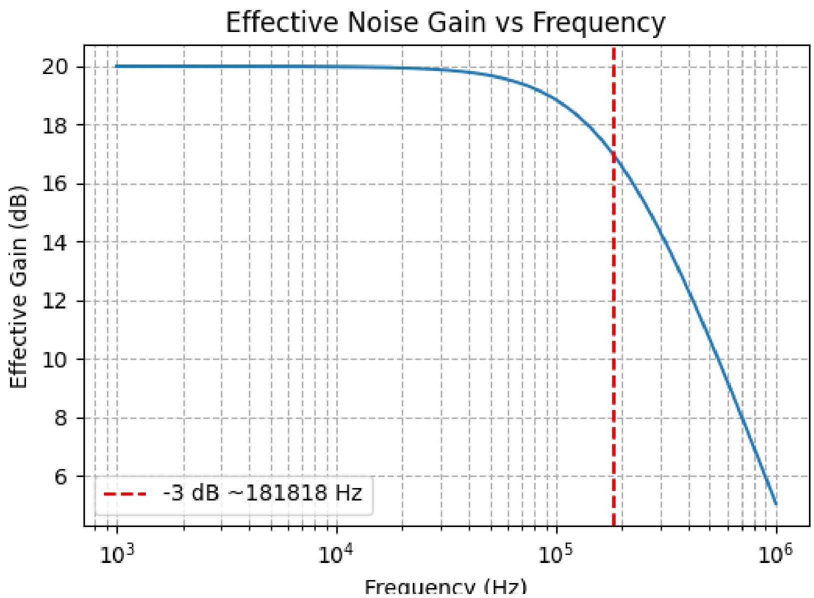

Figure 10: Solution for problem 10. See script `set1_prob10.py` for solution details.

4.1.11 Solution 11: Inverting vs. Non-inverting Configurations

- **Gain Formula**:
  - **Inverting**: $V_{\text{out}}/V_{\text{in}} = -R_f/R_{\text{in}}$. The gain is determined solely by resistor ratios.
  - **Non-inverting**: $V_{\text{out}}/V_{\text{in}} = 1 + R_f/R_1$.
- **Input Impedance**:
  - **Inverting**: The inverting input is held at a virtual ground, so the input impedance is approximately R_{in}.
  - **Non-inverting**: The input is applied directly to the non-inverting terminal, resulting in a very high input impedance (typically Megaohms or more).
- **Phase Relationship**:
  - **Inverting**: The output is 180° out of phase with the input.
  - **Non-inverting**: The output is in phase with the input.

These differences affect how each configuration is used in circuits (e.g., impedance matching and phase requirements).

4.1.12 Solution 12: Virtual Ground in the Inverting Amplifier

In an inverting amplifier, the non-inverting input is usually connected to ground. Because of the high open-loop gain and negative feedback, the inverting input is forced to be at nearly the same potential as the non-inverting input—even though it is not physically connected to ground. This condition is known as a *virtual ground*.

Simplification: The virtual ground allows one to assume that the voltage at the inverting node is 0V when writing nodal equations. This greatly simplifies the analysis since the input voltage is then seen entirely across R_{in}, and the gain depends only on resistor ratios.

4.1.13 Solution 13: Finite Open-Loop Gain Effects

- **Ideal vs. Real**: In an ideal op amp, the open-loop gain is infinite, so the closed-loop gain is exactly set by the resistor network.
- **Finite Gain Effects**: With finite open-loop gain, the actual closed-loop gain deviates slightly from the ideal value. For example, in an inverting amplifier, the closed-loop gain becomes

$$\frac{-R_f/R_{\text{in}}}{1 + \frac{1+R_f/R_{\text{in}}}{A_{\text{ol}}}},$$

  meaning that if A_{ol} is not very high, the gain error can be significant. This deviation is especially important at higher frequencies where the open-loop gain drops.

4.1.14 Solution 14: Negative Feedback Stabilization

- **Negative Feedback**: In negative feedback, a portion of the output is fed back to the inverting input, which counteracts the input signal.
- **Stabilization**: This feedback forces the op amp to adjust its output so that the difference between its inputs is very small, thereby "locking" the closed-loop gain to a value determined by the external resistor network.
- **Improved Linearity**: Negative feedback reduces distortion, increases bandwidth, and improves the linearity of the amplifier because errors and nonlinearities inside the op amp are greatly diminished by the feedback action.

4.1.15 Solution 15: Common-Mode Rejection Ratio (CMRR)

- **Definition**: CMRR measures an op amp's ability to reject common-mode signals—those signals that appear simultaneously and in phase on both inputs.
- **Importance**: A high CMRR is crucial for differential amplifiers because it ensures that only the difference between the two inputs is amplified while any noise or interference common to both inputs is minimized.

- **Applications**: This property is vital in sensor applications and environments with significant electrical noise, where accurate differential measurement is required.

4.1.16 Solution 16: Frequency Compensation Techniques

- **Compensation Capacitors**: Adding capacitors (often in parallel with feedback resistors or across the op amp's internal stages) can create dominant poles that set the bandwidth and ensure phase margin.
- **Miller Compensation**: A capacitor connected between the output and an internal node can be used to shift the dominant pole to a lower frequency, thereby stabilizing the amplifier.
- **Snubber Networks**: RC networks may be used to damp oscillations and improve transient response. Overall, compensation techniques help manage the phase shift introduced by internal poles and ensure stable closed-loop operation.

4.1.17 Solution 17: Slew Rate

- **Slew Rate**: The slew rate is the maximum rate at which an op amp's output can change, typically expressed in volts per microsecond (V/µs).
- **Impact**: If the input signal changes faster than the slew rate allows, the output cannot follow the input accurately, leading to distortion (especially in high-frequency or large-amplitude signals).
- **Design Consideration**: In high-speed or high-frequency applications, the slew rate must be sufficiently high to avoid "slew-induced" distortion.

4.1.18 Solution 18: Input Bias Current and Offset Voltage

- **Input Bias Current**: Small currents flowing into the op amp's inputs can create voltage drops across resistors in the input network, leading to output offset errors.
- **Input Offset Voltage**: This is a small differential voltage required between the inputs to zero the output. It also contributes to error in precision circuits.
- **Mitigation**:
  - Use resistor matching and add compensation resistors (e.g., placing a resistor equal to the parallel combination of the feedback network at the non-inverting input) to balance the bias currents.
  - Use op amps with low bias currents and low offset voltages for precision applications.
  - Employ trimming and calibration techniques in the design.

4.1.19 Solution 19: Inverting vs. Non-inverting: Noise and Linearity

- **Inverting Configuration**:
  - **Advantages**:
    * Gain is set solely by resistor ratios.
    * Lower noise at the input if a low-value resistor is used.
  - **Disadvantages**:
    * Input appears at a virtual ground, so the input impedance is equal to R_{in}, which may be low.
    * The 180° phase shift might require additional considerations in certain applications.
- **Non-inverting Configuration**:
  - **Advantages**:
    * Very high input impedance, beneficial for sensor interfacing.
    * No phase inversion.
  - **Disadvantages**:
    * The noise gain equals the closed-loop gain, which can amplify noise if high gains are needed.
    * The design may be more sensitive to bias current effects if the resistor values are large.

In many designs, the choice depends on the source impedance and the importance of phase versus noise and linearity.

4.1.20 Solution 20: Limitations of the Ideal Op Amp Model

- **Ideal Assumptions**:
  - Infinite open-loop gain, infinite input impedance, zero output impedance, zero bias currents, infinite bandwidth, and no noise.
- **Practical Limitations**:
  - Real op amps have finite open-loop gain and bandwidth, non-zero input bias currents and offset voltages, limited slew rates, and finite input/output impedances.
- **Modeling Non-idealities**:
  - Finite gain and bandwidth are modeled with a dominant pole (gain-bandwidth product).
  - Input bias and offset errors are included in simulation models.
  - Slew rate limits are specified, and output impedance is taken into account in high-current or high-frequency applications.

Understanding these non-ideal factors is essential for accurate circuit simulation, proper compensation design, and achieving the desired performance in practical applications.

4.2 Amplifiers: Solution Set 2

4.2.1 Solution 1: Basic Inverting Amplifier – Gain and Output Plot

1. The gain of an inverting amplifier is

$$A_{\mathrm{cl}} = -\frac{R_f}{R_{\mathrm{in}}} = -\frac{50\,000}{5\,000} = -10.$$

2. Over the range $-1\,\mathrm{V} \leq V_{\mathrm{in}} \leq 1\,\mathrm{V}$,

$$V_{\mathrm{out}} = -10 \times V_{\mathrm{in}}.$$

Solution plot is shown in Fig. 11.

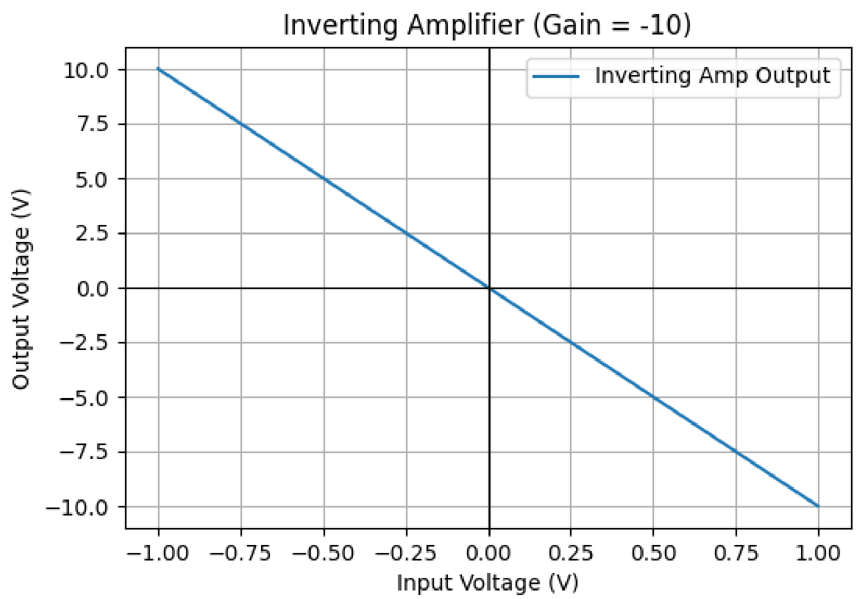

Figure 11: Solution for problem 1. See script `set2_prob1.py` for solution details.

4.2.2 Solution 2: Non-Inverting Amplifier – Gain Design

For a non-inverting amplifier,

$$A_{\text{cl}} = 1 + \frac{R_f}{R_1} = 8 \quad \Rightarrow \quad \frac{R_f}{R_1} = 7 \quad \Rightarrow \quad R_f = 7 \cdot 10\,\text{k}\Omega = 70\,\text{k}\Omega.$$

Hence, the amplifier's overall behavior is $V_{\text{out}} = 8\,V_{\text{in}}$.

Solution plot is shown in Fig. 12.

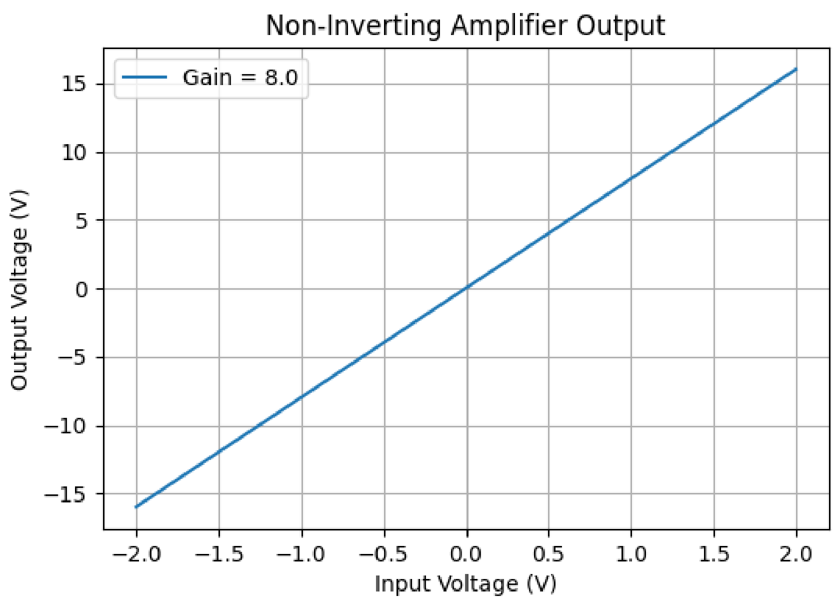

Figure 12: Solution for problem 2. See script `set2_prob2.py` for solution details.

4.2.3 Solution 3: Three-Input Summing Amplifier

1. For an inverting summing amplifier with three inputs and equal R_{in}, the output is

$$V_{\text{out}} = -R_f\left(\frac{V_1}{R_{\text{in}}} + \frac{V_2}{R_{\text{in}}} + \frac{V_3}{R_{\text{in}}}\right) = -\frac{R_f}{R_{\text{in}}}(V_1 + V_2 + V_3).$$

   With $R_f = 30\,\text{k}\Omega$ and $R_{\text{in}} = 10\,\text{k}\Omega$, the coefficient becomes -3.

2. Substituting $V_1 = 1.0\,\text{V}, V_2 = 0.5\,\text{V}, V_3 = -0.5\,\text{V}$:

$$V_{\text{out}} = -3(1.0 + 0.5 - 0.5) = -3 \times 1.0 = -3.0\,\text{V}.$$

Solution plot is shown in Fig. 13.

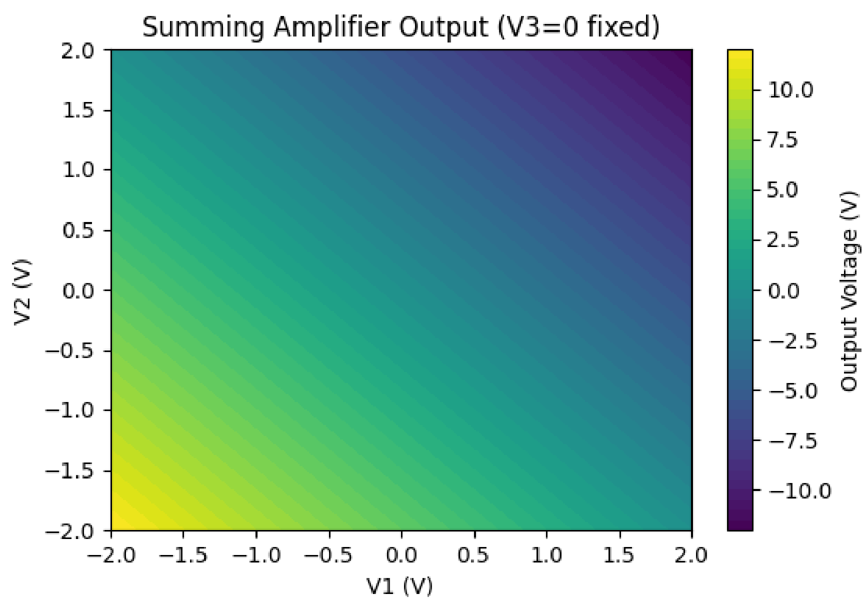

Figure 13: Solution for problem 3. See script `set2_prob3.py` for solution details.

4.2.4 Solution 4: Inverting Amplifier with Input Offset Voltage

1. For an inverting amplifier with an input offset V_{os} effectively appearing at the inverting input, the output offset is approximately

$$V_{\text{offset}} = -\frac{R_f}{R_{\text{in}}} V_{\text{os}} = -\frac{100\,\text{k}\Omega}{10\,\text{k}\Omega} \times V_{\text{os}}.$$

2. Substituting $V_{\text{os}} = 2\,\text{mV}$:

$$V_{\text{offset}} = -10 \times 2\,\text{mV} = -20\,\text{mV}.$$

Solution plot is shown in Fig. 14.

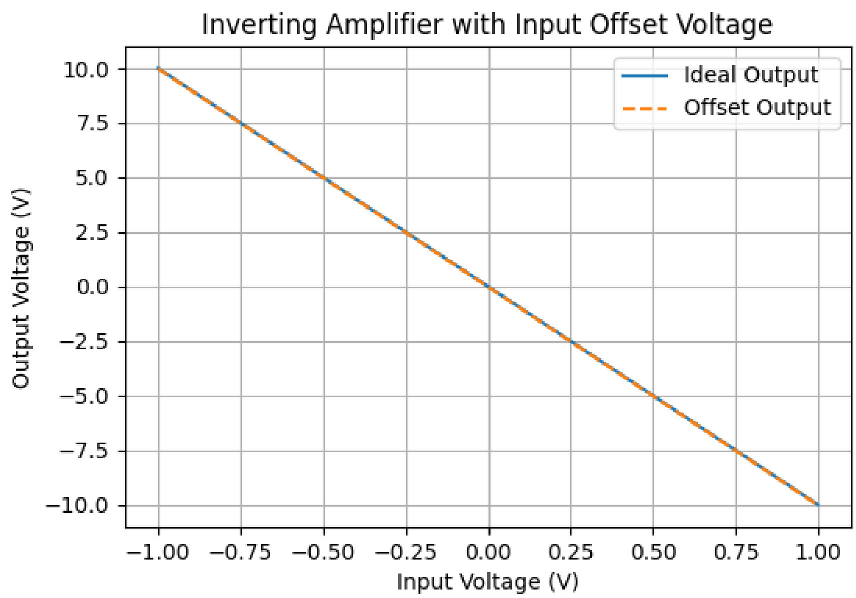

Figure 14: Solution for problem 4. See script `set2_prob4.py` for solution details.

4.2.5 Solution 5: Gain-Bandwidth Product for a Non-Inverting Amplifier

1. For an op amp characterized by a gain-bandwidth product (GBW), the simplified relationship for a non-inverting amplifier is

$$A_{\mathrm{cl}}(f) \times f \approx \mathrm{GBW}.$$

   Thus, the -3dB bandwidth $f_{-3\mathrm{dB}}$ is roughly

$$f_{-3\mathrm{dB}} = \frac{\mathrm{GBW}}{A_{\mathrm{cl}}} = \frac{1\,\mathrm{MHz}}{20} = 50\,\mathrm{kHz}.$$

2. Solution plot is shown in Fig. 15.

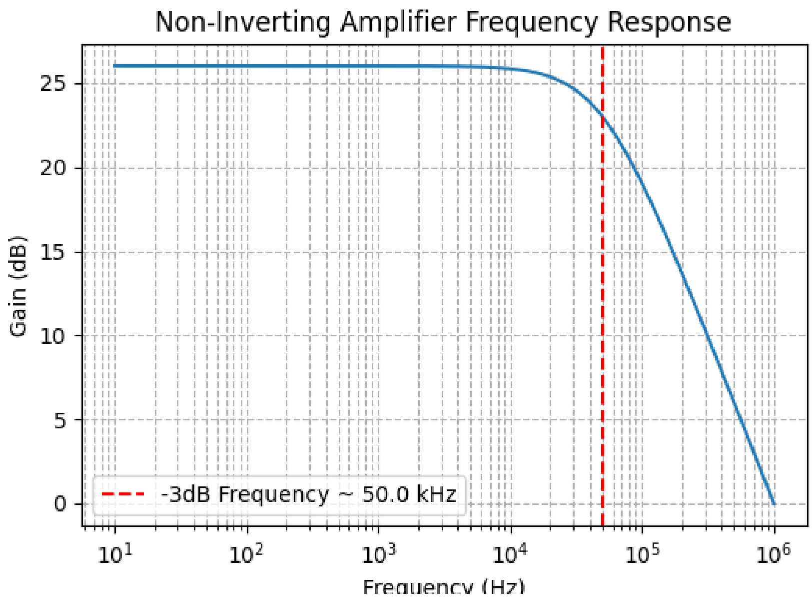

Figure 15: Solution for problem 5. See script `set2_prob5.py` for solution details.

4.2.6 Solution 6: AC Analysis of an Inverting Amplifier with a Capacitor in Parallel

1. At higher frequencies, the capacitor C_f in parallel with R_f creates a feedback impedance $Z_f(\omega) = \left[\frac{1}{R_f} + j\,\omega\,C_f\right]^{-1}$. The cutoff occurs approximately where $|X_C| \approx R_f$, i.e. $\frac{1}{\omega_c C_f} \approx R_f$, giving

$$f_c = \frac{1}{2\pi R_f C_f} = \frac{1}{2\pi \times 50\,000 \times 100 \times 10^{-12}} \approx 31.8\,\text{kHz}.$$

2. Solution plot is shown in Fig. 16.

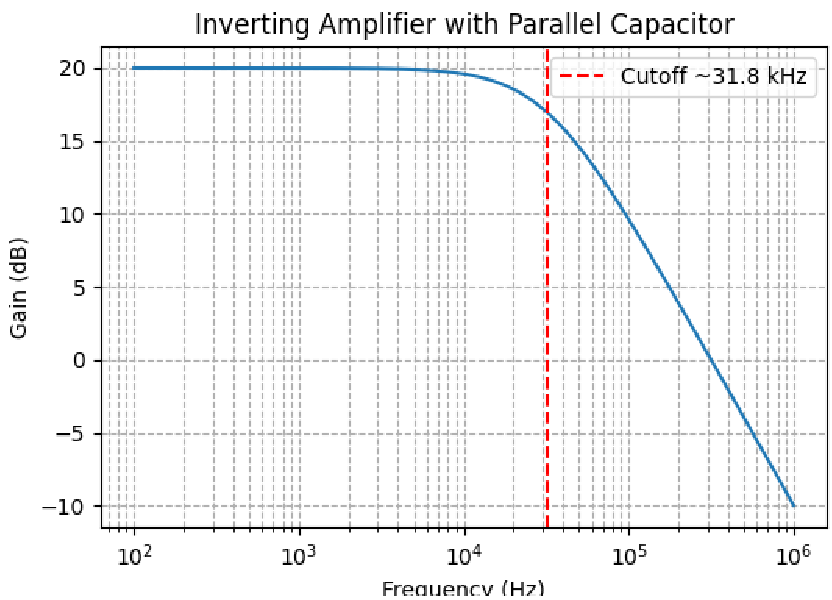

Figure 16: Solution for problem 6. See script `set2_prob6.py` for solution details.

4.2.7 Solution 7: Resistor Tolerances in a Non-Inverting Amplifier

1. Nominal gain:

$$A_{\text{nominal}} = 1 + \frac{R_f}{R_1} = 1 + \frac{90\,\text{k}\Omega}{10\,\text{k}\Omega} = 10.$$

With tolerances, let $R_1' = R_1(1+\delta_1)$ and $R_f' = R_f(1+\delta_2)$ where $\delta_1, \delta_2 \in [-0.05, +0.05]$. The actual gain is

$$A_{\text{actual}} = 1 + \frac{R_f'}{R_1'}.$$

2. Solution plot is shown in Fig. 17.

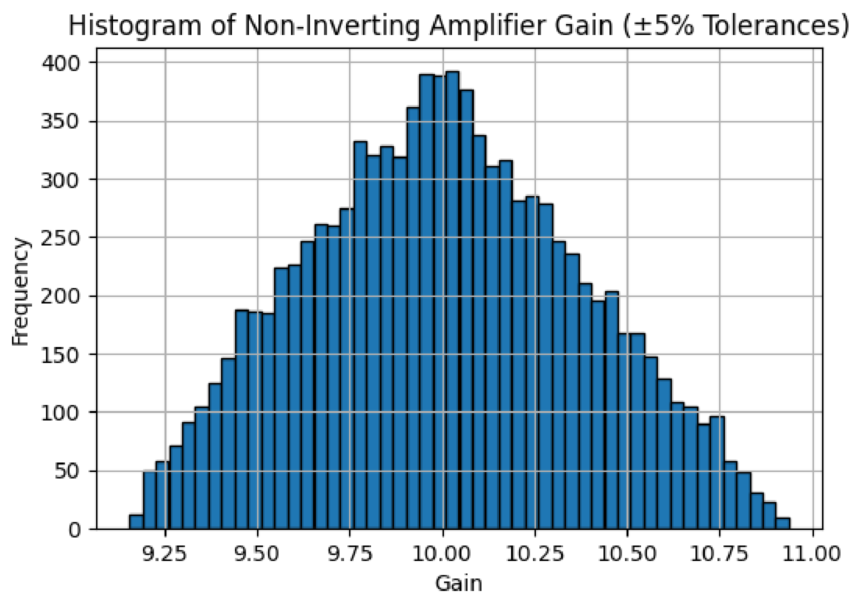

Figure 17: Solution for problem 7. See script `set2_prob7.py` for solution details.

4.2.8 Solution 8: Input Bias Current – Offset in Inverting Amplifier

1. In an inverting amplifier, the input bias current through R_{in} forces a voltage at the inverting terminal (ideally at virtual ground), which is then multiplied by the closed-loop factor $\left(1+\frac{R_f}{R_{\text{in}}}\right)$. Approximate offset at the output:

$$V_{\text{offset}} \approx I_b \times R_{\text{in}} \times \left(1+\frac{R_f}{R_{\text{in}}}\right) = I_b\, R_{\text{in}}\,(1+10).$$

   Since $R_{\text{in}} = 20\,\text{k}\Omega$ and $R_f/R_{\text{in}} = 10$,

$$V_{\text{offset}} = I_b \times 20\,000 \times 11.$$

   For $I_b = 100\,\text{nA}$:

$$V_{\text{offset}} = 100 \times 10^{-9} \times 220\,000 = 22 \times 10^{-3} = 22\,\text{mV}.$$

2. Solution plot is shown in Fig. 18.

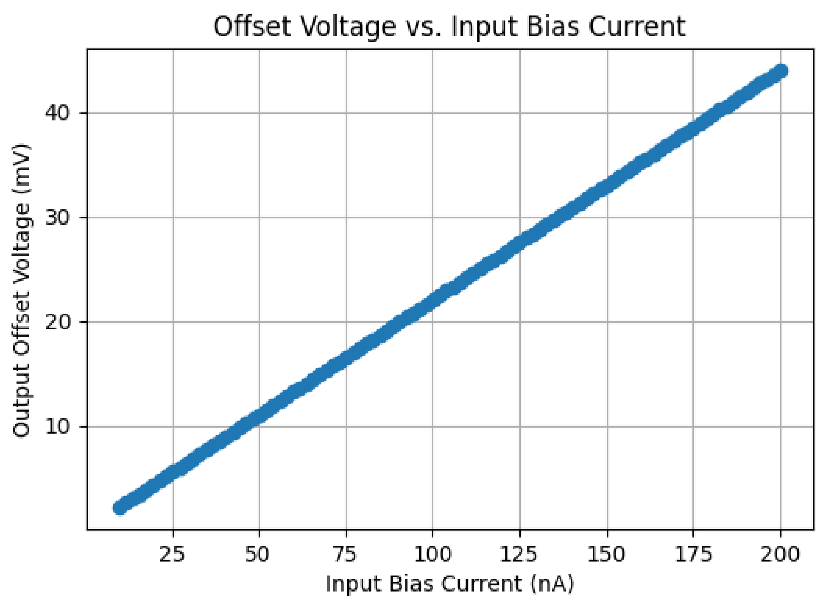

Figure 18: Solution for problem 8. See script `set2_prob8.py` for solution details.

4.2.9 Solution 9: Output Clipping with Limited Supply Rails

1. The output is

$$V_{\text{out}} = -5\,V_{\text{in}}.$$

   To avoid clipping, $|V_{\text{out}}| \leq 12\,\text{V}$. Thus,

$$5\,|V_{\text{in}}| \leq 12 \quad \Rightarrow \quad |V_{\text{in}}| \leq \frac{12}{5} = 2.4\,\text{V}.$$

2. Solution plot is shown in Fig. 19.

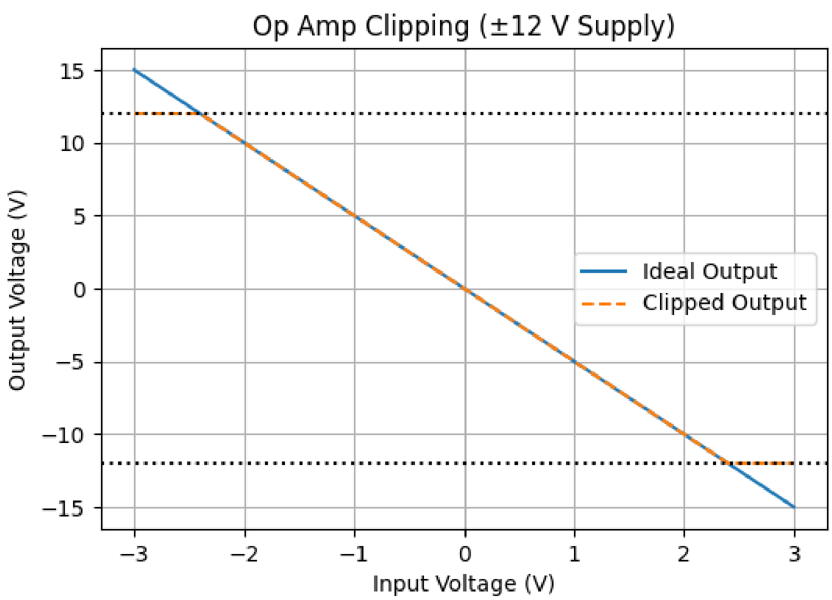

Figure 19: Solution for problem 9. See script `set2_prob9.py` for solution details.

4.2.10 Solution 10: Small-Signal AC Simulation of an Inverting Amplifier

1. At 10Hz, the open-loop gain is 10^5 (which is 100dB). As frequency increases, the gain falls at 20dB/decade. The gain hits unity (0dB) when it has dropped 100dB from its value at 10Hz. That is 5 decades above 10Hz, i.e. at $10 \times 10^5 = 10^6\,\text{Hz} = 1\,\text{MHz}$.
2. The closed-loop response can be approximated by combining the amplifier's open-loop roll-off with the desired feedback factor. At low frequencies, the gain is 20. After a certain breakpoint, the op amp's limited open-loop gain causes roll-off of the closed-loop gain.
3. Solution plot is shown in Fig. 20.

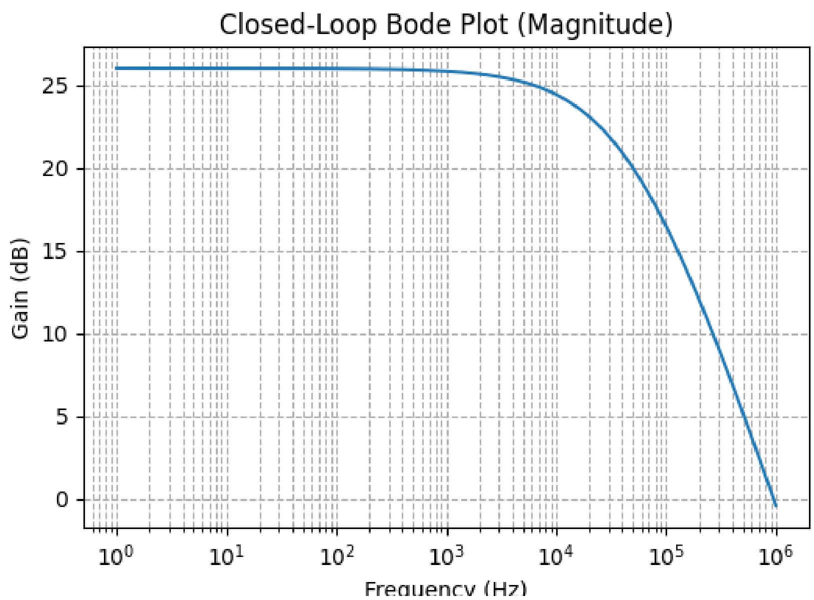

Figure 20: Solution for problem 10. See script `set2_prob10.py` for solution details.

4.2.11 Solution 11: Comparing Inverting and Non-Inverting Amplifiers

1. **Input Impedance**
   - **Inverting Amplifier**: The input is applied through an input resistor R_{in} to the op amp's inverting terminal, which is at a "virtual ground." As a result, the effective input impedance is R_{in}, which is relatively low.
   - **Non-Inverting Amplifier**: The input is applied directly to the non-inverting terminal of the op amp (which ideally draws negligible current). Therefore, the input impedance is very high, making it suitable for high-impedance sources.
2. **Phase Relationship**
   - **Inverting Amplifier**: The output signal is inverted with respect to the input, meaning there is a 180° phase shift. Mathematically, the gain is negative (e.g., $-\frac{R_f}{R_{\text{in}}}$).
   - **Non-Inverting Amplifier**: The output is in phase with the input (0° phase shift). The gain is positive (e.g., $1 + \frac{R_f}{R_1}$).
3. **Gain Formula**
   - **Inverting Amplifier**:
   $$V_{\text{out}} = -\frac{R_f}{R_{\text{in}}} V_{\text{in}}$$
   The magnitude of the gain is set by the resistor ratio $\frac{R_f}{R_{\text{in}}}$.
   - **Non-Inverting Amplifier**:
   $$V_{\text{out}} = \left(1 + \frac{R_f}{R_1}\right) V_{\text{in}}$$
   The "+1" in the formula reflects the direct connection of the input to the non-inverting terminal.
4. **Use Cases**
   - **Inverting Amplifier**:
     - Sums multiple inputs easily (making it ideal for summing amplifiers).
     - Provides precise gain control by resistor ratios.
     - Often used where the input source impedance is low and where phase inversion does not pose a problem.
   - **Non-Inverting Amplifier**:
     - High input impedance → suitable for buffering high-impedance sensors or signals.
     - Retains the input's phase.
     - Used in many general-purpose amplification scenarios, especially where the source cannot drive a low impedance directly.

In summary, the *inverting* amplifier offers easy summation and a precise gain set by resistor ratios but inverts the signal and has a lower input impedance. The *non-inverting amplifier* maintains the signal phase and has a much higher input impedance, which is beneficial in many buffer or sensor-related applications.

4.2.12 Solution 12: Virtual Ground Concept

1. **Definition of Virtual Ground**
   - In an *inverting amplifier*, the non-inverting terminal is typically connected to ground (0V).
   - Due to the op amp's very high open-loop gain and negative feedback, the voltage at the inverting terminal is forced to be extremely close to the non-inverting terminal's voltage—i.e., close to 0V.
   - This is called a "virtual ground," because the inverting input remains at nearly 0V potential, even though it is not physically connected to ground.
2. **Why It Occurs**
   - The op amp tries to keep its input differential voltage ($V^+ - V^-$) very close to zero when in a negative feedback configuration.

- Because V^+ is 0V (true ground), the op amp output adjusts itself so that V^- "virtually" matches 0V.

3. **Simplification in Analysis**
   - Because the inverting node is effectively at 0V, it allows us to treat that node as if it were ground in circuit equations. This means:
     - The voltage drop across R_{in} is simply $V_{\text{in}} - 0 = V_{\text{in}}$.
     - The current through R_{in} is $\frac{V_{\text{in}}}{R_{\text{in}}}$.
     - The same current (neglecting input bias current) flows through the feedback resistor R_f because virtually no current enters the op amp input.
   - This straightforward analysis leads directly to the inverting amplifier gain formula $-\frac{R_f}{R_{\text{in}}}$.

Hence, the concept of a virtual ground is a cornerstone of analyzing and understanding *inverting op amp* circuits. Without physically tying the inverting input to ground, it *behaves* as though it is grounded, simplifying the circuit equations considerably.

4.2.13 Solution 13: Summing Amplifier vs. Differential Amplifier

1. **Summing Amplifier**
   - **Purpose**: Adds multiple inputs into one output. For n inputs $V_1, V_2, \ldots, V_n$ each applied through its own input resistor R_{in} to the inverting node, the output becomes (for equal resistors):

$$V_{\text{out}} = -R_f \sum_{k=1}^{n} \frac{V_k}{R_{\text{in}}}.$$

   - **Configuration**: Usually an *inverting* op amp with multiple input resistors leading to the inverting terminal and a single feedback resistor.
   - **Use Cases**: Audio mixers, analog summation, weighted summations in analog computing or control systems, generating combined waveforms.

2. **Differential Amplifier**
   - **Purpose**: Amplifies the voltage difference between two inputs V_1 and V_2. The ideal differential amplifier outputs

$$V_{\text{out}} = \left(\frac{R_2}{R_1}\right)(V_2 - V_1),$$

     assuming a classic 4-resistor differential amplifier with matched pairs.
   - **Use Cases**: Situations where common-mode signals (noise present on both inputs) need to be rejected; measuring signals in environments where both lines are subject to interference or offset (e.g., sensor bridge circuits, instrumentation).

When to Choose:

- **Summing Amplifier**: When you need to combine or mix multiple voltages into one output (like adding signals or creating weighted sums).
- **Differential Amplifier**: When measuring the difference between two signals is more important than their absolute voltages, particularly if you want to reject any voltage that is common to both inputs (common-mode noise).

4.2.14 Solution 14: Effects of Finite Input Impedance

1. **Inverting Amplifier**
   - The input signal enters through R_{in} directly into the inverting input node (virtual ground). Since that node is kept near 0V, the op amp input does *not* raise the impedance; effectively, the circuit input impedance is R_{in}.
   - **Result**: Designers know exactly that if they pick a $R_{\text{in}} = 10\,\text{k}\Omega$, the amplifier sees about 10kΩ as input impedance. For some applications, this may be too low (especially if the signal source is high impedance).

2. **Non-Inverting Amplifier**
   - The signal is applied to the non-inverting input of the op amp, which ideally draws negligible current. This results in *very high* input impedance—often Megaohms to Gigaohms in real op amps.
   - **Result**: The circuit does not significantly load the source. This is ideal for sensors or circuits that cannot drive low-impedance loads.
3. **Design Implications**
   - **Low Source Impedance**: Inverting amplifiers are acceptable when the source impedance is low or well-defined, ensuring minimal distortion or loading effects.
   - **High Source Impedance**: If you have a high-impedance source (e.g., a photodiode, a sensor with large output resistance), a non-inverting amplifier (or voltage follower buffer) is often necessary to avoid loading the source.
   - **Precision vs. Simplicity**: Inverting amplifiers can be simpler for summing multiple signals and guaranteeing a known input impedance. Non-inverting amplifiers are more straightforward for high input impedance applications.

Hence, the *inverting* amplifier sets a fixed, relatively low input impedance by design, whereas the *non-inverting* amplifier provides very high input impedance, which is critical for many measurement or sensing applications.

4.2.15 Solution 15: Negative Feedback and Stability

1. **Negative Feedback Mechanism**
   - In an op amp circuit, negative feedback means feeding back a portion of the output signal to the inverting input. The op amp continuously adjusts its output to keep the inverting and non-inverting inputs at nearly the same voltage.
   - This action "locks" the closed-loop gain to a stable value determined by resistor ratios (in linear circuits). For example, in an inverting amplifier, $-R_f/R_{\text{in}}$ sets the gain, *not* the huge open-loop gain of the op amp.
2. **Stabilization**
   - Because the open-loop gain of the op amp is very large, even tiny differences between the inputs cause large output swings. The negative feedback reduces this difference to a negligible level, which yields a predictable and stable overall gain.
   - This forced balance at the inputs also drastically improves linearity and reduces distortion.
3. **If Feedback is Broken**
   - If you remove or degrade the feedback path, the op amp is left with its huge open-loop gain. Even microvolts of differential input can drive the output to a rail (saturation). The amplifier output becomes unstable, effectively acting like a comparator.
4. **Phase Shift and Oscillation**
   - At higher frequencies, if the amplifier or feedback network introduces excessive phase shift (approaching or exceeding 180°) and the loop gain is ≥ 1, the circuit can oscillate (this relates to the Barkhausen criterion for oscillation).
   - Designers use compensation techniques (internal or external capacitors) to ensure the op amp has sufficient phase margin, preventing sustained oscillations.

In practice, *negative* feedback is what makes op amps so useful: it *stabilizes* the gain and linearizes the amplifier's performance. Yet care must be taken with frequency response and phase margins to avoid oscillatory behaviors.

4.2.16 Solution 16: Input Bias Current and Offset Voltage

1. **Origins of Bias Currents**
   - Inside an op amp's input stage (often bipolar or FET transistors), small currents must flow into or out of the input terminals to bias the transistor junctions. These are the *input bias currents* I_b.
   - Even in a FET-input op amp, the bias current, while small, is non-zero.

2. **Offset Voltage**
   - Imperfections in transistor matching and internal circuit design cause a small *input offset voltage* V_{os}, meaning the op amp requires a tiny differential voltage between its inputs to force the output to zero.
   - This offset can be in the microvolt to millivolt range depending on the op amp quality.
3. **Effects on Precision Circuits**
   - **Bias Currents**: When these currents flow through resistors in the input path, they create unwanted offset voltages (Ohm's law). For high-value resistors, even a small I_b can yield a significant offset.
   - **Offset Voltage**: Any inherent offset is multiplied by the circuit gain, leading to output offset. This can degrade accuracy, particularly in high-gain or high-precision applications.
4. **Mitigation Strategies**
   - **Resistor Matching/Compensation**: Adding a resistor in series with the non-inverting terminal to match the resistor seen by the inverting input, thus balancing bias current effects.
   - **Offset Trimming**: Many op amps have offset null pins or can be externally trimmed to reduce offset voltage.
   - **Choosing Low Offset / Low Bias Current Op Amps**: Specialty op amps (e.g., chopper stabilized, precision FET inputs) have very low offset and bias currents.
   - **Calibration**: In digital control systems, software calibration or offset subtraction can mitigate offsets after measurement.

Hence, designers of *precision* amplifiers carefully address *input bias current* and *offset voltage* through resistor choice, layout strategies, device selection, and calibration.

4.2.17 Solution 17: Frequency Response Limitations

1. **Open-Loop Gain Roll-Off**
   - Real op amps have an internal dominant pole causing the open-loop gain $A_{\mathrm{ol}}(f)$ to roll off at about 20dB/decade beyond a certain corner frequency.
   - As frequency increases, the effective open-loop gain becomes insufficient to maintain the ideal closed-loop gain.
2. **Closed-Loop Bandwidth**
   - Because of negative feedback, the closed-loop gain remains approximately constant up to a point. Beyond that, the available open-loop gain drops too much to sustain the desired closed-loop gain. The amplifier's output amplitude begins to fall, defining its bandwidth.
   - **Gain-Bandwidth Product (GBW)**: A common figure of merit for op amps. For a single-pole approximation, $\mathrm{GBW} = A_{\mathrm{cl}} \times f_{\mathrm{BW}}$. For instance, if $\mathrm{GBW} = 1\,\mathrm{MHz}$ and the closed-loop gain is 10, the bandwidth is 100kHz.
3. **Slew Rate**
   - **Definition**: The *slew rate* (SR) is the maximum rate (V/µs) at which the op amp output can change.
   - Even if the bandwidth suggests the amplifier can handle a certain frequency, if the amplitude is high, the required dV/dt may exceed the slew rate. This results in *slew-limited distortion*.

Thus, at high gain or high frequency, *both* the finite open-loop gain roll-off (affecting bandwidth) and the *slew rate* limit how fast the output can swing without distortion. Designers must consider these effects in fast or high-gain applications.

4.2.18 Solution 18: Using a Buffer (Voltage Follower)

1. **What is a Buffer or Voltage Follower?**
   - A *voltage follower* is a non-inverting amplifier with a gain of 1, achieved by directly connecting the output to the inverting input and connecting the input signal to the non-inverting input.
   - The output "follows" the input voltage ($V_{\mathrm{out}} = V_{\mathrm{in}}$).

2. **Purpose of a Buffer**
   - **High Input Impedance**: Almost no current is drawn from the source.
   - **Low Output Impedance**: The buffer can drive the next stage (e.g., another amplifier or ADC) without significant voltage drop.
   - This is crucial when the source (such as a sensor) has a high output impedance and cannot drive low-resistance loads directly.
3. **Why Place a Buffer After a High-Impedance Sensor?**
   - Sensors (like photodiodes, certain transducers) may produce small currents or voltages that are affected by loading if connected to a lower-impedance circuit.
   - The buffer provides *isolation*: the high-impedance sensor sees virtually no load, and the following stages receive a stable, low-impedance signal from the op amp output.

In short, a *voltage follower* is a critical piece of many analog signal chains, ensuring accurate signal transfer from a high-impedance source to lower-impedance stages downstream.

4.2.19 Solution 19: Implementing a DC Offset Adjust

1. **Why Introduce an Offset?**
   - Some applications require the amplifier's output to rest at a certain DC voltage, rather than 0V. This is common in single-supply circuits where the output cannot swing negative or in signal-processing scenarios needing a reference offset.
   - For example, you might want the amplifier's idle output to be +2.5V in a 0–5V system to handle AC signals without clipping the negative side.
2. **Methods of Adding Offset**
   - **At the Input**: A reference voltage can be added in series with the input or via a resistor divider that shifts the op amp input.
   - **Via the Feedback Network**: A resistor network or a potentiometer can be placed in the feedback loop to inject a DC offset from a stable reference source. This effectively shifts the operating point.
   - **Dedicated Offset Pin**: Many precision op amps have an offset null or offset adjust pin that, when connected to an external trim potentiometer, can adjust the internal offset to a desired level.
3. **Example Scenario**
   - **Single-Supply Op Amp**: Suppose you only have a +5V rail and ground. You might use a resistive divider to create a 2.5V reference. By adding this reference to the non-inverting terminal (and suitably configuring the feedback network), the amplifier's output can idle around 2.5V. An AC signal riding on this offset avoids hitting 0V or saturating.

Overall, adding or controlling the DC offset is *integral* in designing many practical amplifier systems where the signal range or supply constraints prohibit a purely symmetric ± voltage swing.

4.2.20 Solution 20: Practical Power Supply Constraints

1. **Supply Rails and Output Swing**
   - Traditional op amps powered at ±15V can swing their output close to ±15V, but not exactly to the rails; they often need some headroom (e.g., ±13V or ±14V maximum).
   - If the circuit attempts to drive the output beyond these limits, the signal clips or saturates, distorting the waveform.
2. **Rail-to-Rail Op Amps**
   - Rail-to-rail *output* op amps can swing their output voltage closer to the supply rails, sometimes within millivolts of the rails.
   - Rail-to-rail *input* op amps can also handle input signals that extend to or even slightly beyond the supply rails.
   - Such devices are crucial for low-voltage single-supply applications (e.g., 0–5V or 0–3.3V systems).

3. **Headroom Limitations**
   - Even rail-to-rail amplifiers typically require some small margin.
   - Non-rail-to-rail amplifiers can require significant margins (e.g., 1–2V away from each rail) to operate linearly.
   - If the signal range is not accounted for, the op amp saturates or "railtos," producing a clipped or distorted output.

Design Considerations:

- Choose op amps that can handle the required input and output voltage ranges given the available supply voltages.
- In low-voltage designs, rail-to-rail devices or suitable offsetting strategies are often necessary to fully utilize the limited voltage swing.

Hence, *practical power supply constraints* directly affect an op amp's ability to faithfully reproduce signals. Understanding these limitations is essential in selecting the right op amp and ensuring the design meets the required signal amplitude specifications.