Counting Problems

① unique: $\dfrac{7!}{3!} = 7 \cdot 6 \cdot 5 \cdot 4 = \boxed{840}$

all possibles: $\dfrac{7!}{2!} = \boxed{2520}$

② two values: $\dfrac{13!}{2! \, 11!} = 78$

two suits: $\left(\dfrac{4!}{2! \, 2!}\right)^2 = 36$

final card: $\dfrac{11!}{1! \, 10!} = 11$

final card suit: $\dfrac{4!}{3!} = 4$

$78 \cdot 36 \cdot 11 \cdot 4 = \boxed{123,552}$

③ $\dfrac{16!}{1! \, 15!} \cdot \dfrac{15!}{6! \, 9!} = \boxed{80080}$

④

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            3
          /   \
       1,2      9
              /   \
       4,5,6,7,8   10,11,12
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$\dfrac{\frac{4!}{2! \, 2!}}{3} \circ \dfrac{\frac{10!}{5! \, 5!}}{6} \circ \dfrac{\frac{6!}{3! \, 3!}}{4} = \boxed{420}$

⑤ $\dfrac{10!}{7! \, 3!} + \dfrac{10!}{4! \, 6!} = 330$