What Practitioners Need to Know . . .

by Mark P. Kritzman
Windham Capital Management

. . . About Uncertainty

The primary challenge to the financial analyst is to determine how to proceed in the face of uncertainty. Uncertainty arises from imperfect knowledge and from incomplete data. Methods for interpreting limited information may thus help analysts measure and control uncertainty.

Long ago, natural scientists noticed the widespread presence of random variation in nature. This led to the development of laws of probability, which help predict outcomes. As it turns out, many of the laws that seem to explain the behavior of random variables in nature apply as well to the behavior of financial variables such as corporate earnings, interest rates, and asset prices and returns.

Table I S&P 500 Annual Returns

1951	24.0%	1961	26.9%	1971	14.3%	1981	−4.9%
1952	18.4%	1962	−8.7%	1972	19.0%	1982	21.4%
1953	−1.0%	1963	22.8%	1973	−14.7%	1983	22.5%
1954	52.6%	1964	16.5%	1974	−26.5%	1984	6.3%
1955	31.6%	1965	12.5%	1975	37.2%	1985	32.2%
1956	6.6%	1966	−10.1%	1976	23.8%	1986	18.8%
1957	−10.8%	1967	24.0%	1977	−7.2%	1987	5.3%
1958	43.4%	1968	11.1%	1978	6.6%	1988	16.6%
1959	12.0%	1969	−8.5%	1979	18.4%	1989	31.8%
1960	0.5%	1970	4.0%	1980	32.4%	1990	−3.1%

Source: Data through 1981 from R. Ibbotson and R. Sinquefield, *Stocks, Bonds, Bills and Inflation: The Past and the Future* (Charlottesville, VA: The Financial Analysts Research Foundation, 1982).

Relative Frequency

A random variable can be thought of as an event whose outcome in a given situation depends on chance factors. For example, the toss of a coin is an event whose outcome is governed by chance, as is next year's closing price for the S&P 500. Because an outcome is influenced by chance does not mean that we are completely ignorant about its possible values. We may, for example, be able to garner some insights from prior experiences.

Suppose we are interested in predicting the return of the S&P 500 over the next 12 months. Should we be more confident in predicting that it will be between 0 and 10 per cent than between 10 and 20 per cent? The past history of returns on the index can tell us how often returns within specified ranges have occurred. Table I shows the annual returns over the last 40 years.

We can simply count the number of returns between 0 and 10 per cent and the number of returns between 10 and 20 per cent. Dividing each figure by 40 gives us the *relative frequency* of returns within each range. Six returns fall within the range of 0 to 10 per cent, while 10 returns fall within the range of 10 to 20 per cent. The relative frequencies of these observations are 15 and 25 per cent, respectively, as Table II shows.

Figure A depicts this information graphically in what is called a *discrete probability distribution.* (It is discrete because it covers a finite number of observations.) The values along the vertical axis represent the probability (equal here to the relative frequency) of observing a return within the ranges indicated along the horizontal axis.

The information we have is limited. For one thing, the return ranges (which we set) are fairly wide. For another, the sample is confined to annual returns, and covers only the past 40 years, a period which excludes two world wars and the Great Depression.

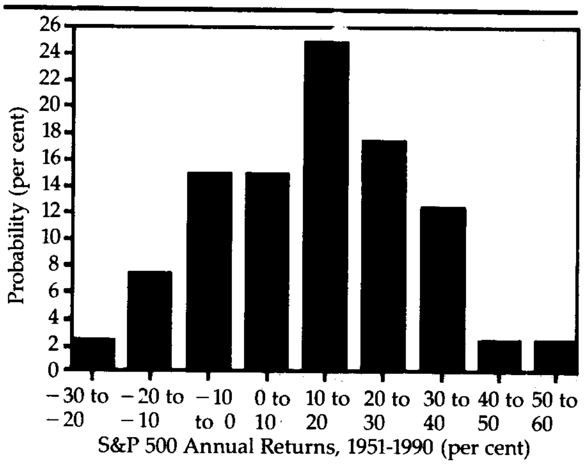
We can nonetheless draw several inferences from this limited information. For example, we may assume that we are about two-thirds more likely to observe a return within the range of 10 to 20 per cent than a return within the range of 0 to 10 per cent. Furthermore, by summing the relative frequencies for the three ranges below 0 per cent, we can also assume that there is a 25 per cent chance of experiencing a negative return.

If we wanted to make more precise inferences, we would have to augment our sample by extending the

Table II Frequency Distribution

Range of Return	*Frequency*	*Relative Frequency*
−30% to −20%	1	2.5%
−20% to −10%	3	7.5%
−10% to 0%	6	15.0%
0% to 10%	6	15.0%
10% to 20%	10	25.0%
20% to 30%	7	17.5%
30% to 40%	5	12.5%
40% to 50%	1	2.5%
50% to 60%	1	2.5%

Figure A Discrete Probability Distribution



measurement period or by partitioning the data into narrower ranges. If we proceed along these lines, the distribution of returns should eventually conform to the familiar pattern known as the bell-shaped curve, or *normal distribution*.

Normal Distribution

The normal distribution is a *continuous probability distribution*; it assumes there are an infinite number of observations covering all possible values along a continuous scale. Time, for example, can be thought of as being distributed along a continuous scale. Stocks, however, trade in units that are multiples of one-eighth, so technically stock returns cannot be distributed continuously. Nonetheless, for purposes of financial analysis, the normal distribution is usually a reasonable approximation of the distribution of stock ranges, as well as the returns of other financial assets.

The formula that gives rise to the normal distribution was first published by Abraham de Moivre in 1733. Its properties were investigated by Carl Gauss in the 18th and 19th centuries. In recognition of Gauss' contributions, the normal distribution is often

sum of the observed returns times their probabilities of occurrence:

$$\bar{R} = R_1 \cdot P_1 + R_2 \cdot P_2 + \ldots + R_n \cdot P_n.$$

$\bar{R}$ equals the mean return. $R_1, R_2, \ldots R_n$ equal the observed returns in years one through n. $P_1, P_2, \ldots P_n$ equal the probabilities of occurrence (or relative frequencies) of the returns in years one through n. This computation yields the arithmetic mean. (The arithmetic mean ignores the effects of compounding; we will discuss later how to modify this calculation to account for compounding.)

The *variance* of returns is computed as the average squared difference from the mean. To compute the variance, we subtract each annual return from the mean return, square this value, sum these squared values, and then divide by the number of observations (or n, which in our example equals 40).[2] The formula for variance, V, is:

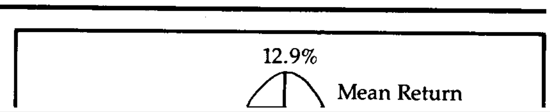
$$V = \frac{(R_1 - \bar{R})^2 + (R_2 - \bar{R})^2 + \ldots + (R_n - \bar{R})^2}{n}.$$

The square root of the variance, which is called the *standard deviation*, is commonly used as a measure of dispersion.

If we apply these formulas to the annual returns in Table I, we find that the mean return for the sample equals 12.9 per cent, the variance of returns equals 2.9 per cent, and the standard deviation of returns equals 16.9 per cent. These values, together with the assumption that the returns of the S&P 500 are normally distributed, enable us to infer a normal probability distribution of S&P 500 returns. This is shown in Figure B.

The normal distribution has several important characteristics. First, it is symmetric around its mean;

Figure B Normal Probability Distribution



Footnotes

1. Although no set of measurements conforms exactly to the specifications of the normal distribution, such diverse phenomenon as noise in electromagnetic systems, the dynamics of star clustering and the evolution of ecological systems behave in accordance with the predictions of a normal distribution.
2. To be precise, we should divide by the number of observations less one, because we lose one degree of freedom by using the same data to calculate the mean. This correction yields a so-called unbiased estimate of the variance, which typically is of little practical consequence.
3. For a more detailed discussion of the lognormal distribution, see S. Brown and M. Kritzman, *Quantitative Methods for Financial Analysis*, Second Edition (Homewood, IL: Dow Jones-Irwin, 1990), pp. 235–238.
4. The value of an option depends on the price of the underlying asset, the exercise price, the time to expiration, the risk-free return and the standard deviation of the underlying asset. All these values except the standard deviation are known, and the standard deviation can be inferred from the price at which the option trades. The implied value for the standard deviation is solved for iteratively. For a review of this technique, see M. Kritzman, *Asset Allocation for Institutional Portfolios* (Homewood, IL: Business One Irwin, 1990), pp. 184–185.
5. For an excellent discussion of this issue, see R. Bookstaber and R. Clarke, "Problems in Evaluating the Performance of Portfolios with Options," *Financial Analysts Journal*, January/February 1985.