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Multi-objective optimization in spatial planning: Improving the effectiveness of multi-objective evolutionary algorithms (non-dominated sorting genetic algorithm II)

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Multi-objective optimization in spatial planning: Improving the effectiveness of multi-objective evolutionary algorithms (non-dominated sorting genetic algorithm II)

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The multi-objective nature of most spatial planning initiatives and the numerous constraints that are introduced in the planning process by decision makers, stakeholders, *etc*., synthesize a complex spatial planning context in which the concept of solid and meaningful optimization is a unique challenge. This article investigates new approaches to enhance the effectiveness of multi-objective evolutionary algorithms (MOEAs) via the adoption of a well-known metaheuristic: the non-dominated sorting genetic algorithm II (NSGA-II). In particular, the contribution of a sophisticated crossover operator coupled with an enhanced initialization heuristic is evaluated against a series of metrics measuring the effectiveness of MOEAs. Encouraging results emerge for both the convergence rate of the evolutionary optimization process and the occupation of valuable regions of the objective space by non-dominated solutions, facilitating the work of spatial planners and decision makers. Based on the promising behaviour of both heuristics, topics for further research are proposed to improve their effectiveness.

Keywords: multi-criteria analysis; genetic algorithms; spatial planning; initialization; crossover operator

1. Introduction

Spatial planning initiatives constitute a rather complex type of multi-objective optimization problem (MOOP), owing to the conflicting nature of most design criteria and the presence of many spatial constraints governing the allocation of entities in space. The ultimate goal of an effective spatial optimizer is the derivation of applicable results in which all spatial entities (land uses) to be allocated are optimized in terms of their location, shape and size. Such an optimization framework requires the introduction of robust evolutionary algorithms, distinguished by their effectiveness in a number of engineering optimization tasks. The non-dominated sorting genetic algorithm II (NSGA-II) metaheuristic (Deb *et al.* 2000) has been introduced numerous times in a variety of engineering optimization assignments, including land use allocation problems (Datta *et al.* 2007; Datta, Fonseca, and Deb 2008; Fotakis and Sidiropoulos 2011; Cao *et al.* 2011; Fotakis *et al.* 2012; Karakostas and Economou 2014), producing valuable results, either when compared with the true Pareto front of solutions or when tested against other metaheuristics. However, practice has more than once indicated that each optimization problem favours the introduction of different

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evolutionary processes to optimize its convergence to the true Pareto front while providing a uniform spread of solution along its k-dimensional surface, where k indicates the number of objective functions. Within this research context, the present study addresses the case of spatial planning optimization and focuses, first, on the introduction of a sophisticated initialization heuristic to evaluate its impact on the effectiveness of NSGA-II. This subject has already been referenced as a valid topic for further research in spatial planning optimization studies (Karakostas and Economou 2014; Cao *et al.* 2011).

Most articles on multi-objective evolutionary algorithms (MOEAs) adopt random initializations, where the initial parents comprise both feasible and infeasible solutions. This approach is likely to exhibit the robustness of an investigated MOEA if its convergence to the true Pareto optimal front is maintained with different random initializations. Moreover, when such an optimal front is known *a priori*, one could suggest an enhanced random initialization heuristic producing solutions away from the front to evaluate the algorithm's effectiveness in both exploring the entire feasible search space and exploiting areas with promising offspring (Deb and Padhye 2010). However, this heuristic cannot be incorporated in complex engineering problems with no prior knowledge of their true optimal Pareto set. This is also the case with spatial planning optimization studies in which the planner is unlikely to be provided with any reliable hint regarding the true optimal Pareto front of maps. As a consequence, simple random initialization schemes are adopted.

This study evaluates the impact of a sophisticated initialization scheme in a real-scale spatial planning optimization assignment in terms of convergence efficiency and diversity metrics. In particular, the optimum spatial development of wind farms is investigated by the application of the NSGA-II metaheuristic, in which algorithmic modifications are eventually proposed for the crossover operator to exploit the new initialization scheme. The resulting Pareto-optimal maps provide useful guidelines for both optimization researchers and land use planners, by exhibiting the practical implications of such methodologies.

2. Overview of multi-objective optimization algorithms

Genetic algorithms have been extensively used for MOOPs. John Holland introduced them in 1975 and set the foundations for their promising development in engineering-related topics (Goldberg 1989; Michalewicz 1992; Mitchell 1996; Gen and Cheng 1997; Vose 1999). Their algorithmic structure was extensively treated and challenged in a variety of optimization problems, giving birth to alternative processes of evaluating and selecting 'parent' solutions (Goldberg and Deb 1991; Baker 1985) or enhanced genetic operators for the development of efficient 'offspring' aiming to provide better convergence characteristics (Spears 1998). In parallel, special attention was given to developing a robust optimization framework able to incorporate multiple and conflicting design criteria (MOOPs). David Schaffer (Schaffer 1985) was a pioneer in treating such problems by developing the vector evaluated genetic algorithm (VEGA) approach . This algorithm was used to separate the population into clusters; each subpopulation was evaluated against a single criterion to provide a portion of the parents required for the creation of the next generation. Numerous versions of VEGA followed (Kursawe 1990; Hajela and Lin 1993), striving to enhance its effectiveness. However, one of the most important breakthroughs in genetic algorithms was the comparison of candidate solutions on a Pareto basis. This new scheme of evaluating competing solutions, without the need to assume relative weights of importance for each criterion, established the foundations for many well-documented MOEAs, such as the NSGA (Srinivas and Deb 1994) and niched-Pareto genetic algorithms (NPGAs) (Horn, Nafpliotis, and Goldberg 1994).

Going one step further, the concept of elitism was introduced in the algorithmic structure of MOEAs, enhancing their effectiveness. NSGA-II (Deb *et al.* 2000) constitutes a robust genetic solver that incorporates elitism by jointly evaluating parents and offspring in each generation, to select only the best of them to serve as parents for the next generation. SPEA (Zitzler and Thiele 1999) introduced elitism by explicitly maintaining non-dominated solutions via an external population of variable size. When the size of this external archive tends to exceed a predefined limit, solutions in less crowded regions of the non-dominated front are preserved by the application of a clustering method. SPEA2 (Zitzler, Laumanns, and Thiele 2002) proposed a fixed size for this external population; whenever the number of non-dominated solutions is less than the predefined external population size, dominated 'chromosomes' are utilized until the external archive is full. PAES (Knowles and Corne 2000) applies a local search evolution strategy to find and maintain non-dominated solutions in an archive. The algorithm is initiated by a random solution; at iteration t a new solution (B) is generated by mutating the current solution (A). Between these two solutions (A and B), the non-dominated solution is accepted. However, if neither (A) nor (B) dominates the other, the new solution (B) is compared to previously archived solutions, and dominated solutions in the archive set are replaced. If none of the archive members dominates (B) and the external archive is not full, then the new solution is added to the archive. If the archive is full, the solution in the most crowded region of the space is removed and the new one is added. Inspired by the algorithmic structure of PAES, an enhanced MOEA utilizing the concept of ε-dominance was proposed: ε-MOEA (Laumanns *et al.* 2002). This sophisticated metaheuristic addressed the potential deterioration that could take place within an evolutionary algorithm and provided a good distribution of solutions along the true optimal Pareto front with less computational overhead.

Beyond the aforementioned MOEAs, a substantially differentiated optimization framework was introduced by particle swarm optimization (PSO) methodology (Kennedy and Eberhart 1995), inspired by the behaviour of organisms such as in flocks of birds or schools of fish. The PSO approach, or MOPSO for multi-objective problems, gained immense popularity in the research community, mainly owing to its simplistic implementation and documented success in benchmark test problems and real applications. Nevertheless, the similarity of the PSO's child update rule to a genetic algorithm's crossover operator has been recognized since the algorithm's inception. This resemblance has been thoroughly discussed, leading to the consideration of PSO as an evolutionary algorithm (Eberhart and Shi 1998) and the improvement of PSO's effectiveness by incorporating procedural elements from genetic algorithms (Deb and Padhye 2010).

3. Optimum spatial development of wind farms

A critical concern in the development of an efficient, and thus profitable, wind farm project is the selection of its location. However, this decision-making process should reflect the conflicting nature of major design criteria: locations with attractive development costs, or even suitable regions meeting all spatial rules imposed by existing spatial planning frameworks, are likely to exhibit low wind potential scorings. The opposite also holds: windy spots are often situated in restricted areas. The present study introduces the popular NSGA-II metaheuristic to derive the optimal spatial allocation, and proposes, in parallel, the most appropriate size and shape of each candidate site. In essence, the motivation is to enhance an existing stand-alone decision support system (DSS), serving optimal site selection purposes, urban/rural spatial planning projects, allocation of renewable energy sources (RES), *etc*. Having defined the most appropriate development sites for a wind farm, the exact location of wind turbines within each site can be derived by carefully formulating the corresponding 'microscale' MOOP, aiming to maximize the farm's power output via the most cost-effective layout (Geem and Hong 2013).

4. Methodology

4.1. *Overview*

The primary concern of this article is to enhance the effectiveness of established MOEAs by introducing a sophisticated initialization scheme and, eventually, a differentiated algorithmic context for the crossover operator. This impact-assessment analysis comprises two stages.

Stage I: Evaluation of a initialization heuristic for NSGA-II

The original NSGA-II algorithm is applied to compare the optimal Pareto front emerging from a random initialization against the one derived from an enhanced initial population. Both approaches are evaluated against metrics measuring their convergence efficiency, the percentage of non-dominated solutions provided by each initialization scheme and the distribution of containing solutions in the objective space.

Stage II: Evaluation of a modified NSGA-II metaheuristic against the exploitation of enhanced initializations

Having as a starting point the NSGA-II metaheuristic coupled with the initialization heuristic of Stage I, a modified algorithmic process controlling the crossover operator was proposed, aiming to improve the spatial distribution of non-dominated solutions within the three-dimensional objective space.

4.2. *Spatial model representation*

This study adopts the spatially explicit representation approach, according to which a map is segmented into finite map cells by creating a grid of m columns and n rows (Datta *et al.* 2007; Datta, Fonseca, and Deb 2008; Cao *et al.* 2011; Karakostas and Economou 2014). Despite the substantial computational effort that such representation could entail, owing to its enormous search phenotype (genotype) space, the practical significance of exploring any possible spatial configuration of a land use, in terms of its size, location(s) and shape(s), prevails. Finite or implicit representation schemes are likely to ignore the true Pareto front of optimal maps since all allocations are constrained by fixed land parcels, in terms of their shape and size, heuristics for building maps based on sequential allocation algorithms, *etc*.

However, one should be mindful when incorporating a spatial explicit representation in a MOEA. For example, its application in a PSO, or MOPSO, optimization scheme (Shifa, Jianhua, and Feng 2011; Masoomi, Mesgari, and Hamrah 2013) could significantly distort the process of 'flying' by assuming as a 'particle' each map cell carrying a specific land use. More specifically, if a particle 'lands' on a location (map cell) already occupied by a previously flying particle, the 'flight' must be either repeated or extended until an available map cell is found; such heuristic is likely to deteriorate the underlying essence of PSOs. On the contrary, the NSGA-II framework welcomes explicit spatial representations by adjusting its genetic operators and not its primary algorithm structure.

4.3. *Genetic operators*

4.3.1. *Initial crossover operator*

The dynamic crossover operator proposed in the work of Karakostas and Economou (2014) is incorporated, in which the size and dimensions of rectangular map patches to be exchanged are randomly defined each time the operator is executed. More specifically, on an $m \times n$ map grid each side is randomly divided into R_m (> 2) and R_n (> 2) regions, where m' and n' denote the number of cells for each R_m and R_n region, respectively; every application of the crossover operator is performed on a new random rectangular pattern. Moreover, the parent for each rectangular patch is randomly selected.

In order to confine the exploration of the algorithm within the feasible genotype space, a slightly modified procedure is applied (Step 1):

(1) Both feasible and infeasible solutions are selected to participate in the crossover operator, provided that at least one parent satisfies all constraints.
(2) The offspring preserves map cells containing fixed land uses.
(3) The crossover operator is repeated until a feasible solution is produced.

4.3.2. *Mutation operator*

As in the case of the crossover operator, a modified mutation operator is adopted (Karakostas and Economou 2014) to effectively treat land uses with small occupancy levels. More specifically, if a map cell is selected for mutation, the existing land use is replaced if all occupancy constraints, for both the existing and the new land use, are satisfied. If, at least, an occupancy limit is violated, for at least one land use, a random map cell carrying the new (random) land use is selected. The procedure concludes with the exchange of land uses between the two map cells. This approach increases the number of mutations by one each time a land use swap is performed, since two map cells are mutated instead of one.

4.4. *Design criteria and constraints*

The underlying objective function (F) consists of three design criteria, all to be maximized: suitability, contiguity and wind potential. However, since all criteria are treated on a Pareto basis, F cannot be explicitly expressed (*i.e.* via a summation equation). Given the adoption of explicit spatial representations, all objectives and constraints are formulated on a cellular basis.

4.4.1. *Suitability*

$S^k_{i,j}$ denotes the overall suitability score for allocating land use k at map cell (i,j). Provided that all spatial constraints are satisfied, $S^k_{i,j}$ is calculated according to the following equation:

$$S^k_{ij} = \sum_{r=1}^{R} w_r \, SR^k_r(ij) \tag{1}$$

where k is the proposed land use (from 1 to K) in map cell (i,j), r is the suitability spatial rule (from 1 to R), $SR^k_r(ij)$ is the score of spatial rule r applied for k land use in map cell (i,j), and w_r is the relative weight of importance for suitability spatial rule r, imposed by the legal framework controlling the allocation of each land use.

If a spatial constraint is violated for land use k in map cell (i,j), $S^k_{i,j} = 0$. The overall suitability score of a proposed map (SSc) is derived via the summation of all $S^k_{i,j}$.

4.4.2. *Contiguity*

Contiguity can be treated as a separate objective function or a design constraint, or promoted via customized spatial operators (Fotakis and Sidiropoulos 2011). Ligmann-Zielinska and colleagues (Ligmann-Zielinska, Church, and Jankowski 2005) proposed a density-based design constraint

(DBDC) to control the contiguity of a candidate map. According to this heuristic, the neighbourhood of each available map cell (i,j) is scanned to count the number of cells carrying similar land uses. In particular, if cell (i,j) proposes land use m, the algorithm counts how many times this land use is found in adjacent map cells. The number of map cells containing the same land use as cell (i,j) should be greater than $(p+1)^2$, where p denotes the number of surrounding grid zones whose land use is being compared against land use m.

Based on DBDC, the objective function measuring the contiguity of a proposed map is formulated. In particular, if the aforementioned threshold of $(p+1)^2$ is satisfied for a given map cell (i,j), $CoS_{ij} = 1.0$; otherwise, $CoS_{ij} = 0.0$. Having defined CoS for all available map cells, the provided map is given the following overall scoring:

$$CSc = \sum_{M}^{ij} CoS_{ij} \tag{2}$$

4.4.3. *Wind potential*

WPS_{ij} denotes the wind potential at map cell (i,j). Hence, the total score of the proposed map, in terms of how such potential is exploited, is given by the following equation:

$$WPSc = \sum_{ij}^{M} WPS_{ij}\, pl_{ij}^{wp} \tag{3}$$

where M is all available map cells, WPS_{ij} is the wind potential at map cell (i,j), ij is a map cell within M, and pl_{ij}^{wp} equals 1.0 if the allocation of the proposed land use at map cell (i,j) is affected by wind potential and 0.0 otherwise.

4.5. *Multi-objective metaheuristic: NSGA-II*

The elitist version of NSGA (NSGA-II), in which both parents and offspring of generation t are jointly evaluated to derive the parents for next generation $(t+1)$, has already been applied in producing optimal land use allocations (Datta *et al.* 2007; Cao *et al.* 2011). The following notation is adopted for each element of the genetic optimization process.

NSGA-II notation:

$\boldsymbol{P}_t$	parent of current (t) generation
$\boldsymbol{P}_{t+1}$	parent of next generation $(t+1)$
$\boldsymbol{Q}_t$	offspring of current (t) generation
$\boldsymbol{R}_t$	parents and offspring of current (t) generation
$\boldsymbol{F}_i$	Pareto front i
N_j	population of j Pareto front
N	population of each generation
$2N$	combined population of parents and offspring

The N parents of each generation ($\boldsymbol{P}_t$) produce N offspring ($\boldsymbol{Q}_t$) via a tournament-selection operator and the application of the above-mentioned genetic operators. Then, the Pareto fronts of the combined population of $\boldsymbol{P}_t$ and $\boldsymbol{Q}_t$ are derived; $\boldsymbol{P}_{t+1}$ is derived by copying solutions starting from the highest rank Pareto front and proceeding to the subsequent fronts until N solutions are selected. If the subpopulation of a Pareto front is larger than the number of remaining solutions to be selected, the solutions with the largest crowding distances are selected to promote diversity.

4.6. *Initialization heuristic*

If K land uses must be allocated according to J criteria (objective functions), a new heuristic is proposed to derive N enhanced initial maps based on a random sequential order of allocation and the adoption of variable occupancy percentages.

Terminology:

(1) Let **SV** represent the vector containing the random sequence of allocating all K land uses:

$$\mathbf{SV}^{\mathrm{T}} = [lu_1, lu_2, \ldots, lu_K]$$

where lu_1 represents the fist land use to be allocated and lu_K is the last one. When **SV** is initialized, every lu_k element takes a random integer value from 1 to K that is not already selected in the first $k-1$ slots of **SV**.

(2) Let **SVA** be the archive matrix of **SV** vectors; every successive column is occupied by a newly defined and utilized **SV**.

(3) Let **RO** constitute the vector of random occupancy percentages for all K land uses; such random selection is performed within the predefined occupancy limits declared for each land use:

$$\mathbf{RO}^{\mathrm{T}} = [oc_1, oc_2, \ldots, oc_K]$$

where oc_k represent the random occupancy percentage of land use k.

Having defined the **SV** and **RO** vectors, the initialization heuristic applies the following procedural steps for the development of a single initial map.

(1) Vector **SV** is randomly initialized and compared against **SVA**; if **SV** has already been utilized in the initialization process of creating N initial maps, a new **SV** is created until a new sequential order is found (not found in **SVA**).
(2) Vector **RO** is randomly initialized.
(3) Real scalar w_1 is randomly chosen between 0.0 and 1.0.
(4) Vector $\mathbf{RO}_1$ is derived by multiplying each element of **RO** with w_1.
(5) The 'remaining' occupancy percentages are stored in $\mathbf{RO}_2 = \mathbf{Iv} - \mathbf{RO}_1$, where $\mathbf{Iv}^{\mathrm{T}} = [1, 1, \ldots, 1]$, for all K land uses.
(6) First part of the sequential allocation algorithm (iterates land use k from 0 to K):
(6.1) Land use lu_k is retrieved from the newly defined **SV**.
(6.2) Random selection of the jth criterion out of J objective functions whose suitability maps are known *a priori* (*i.e.* before optimization commences) and remain unchanged throughout the entire evolutionary process. Out of the three design criteria presented in this study, the random selection can take place only between *suitability* and *wind potential*. In contrast, contiguity can be evaluated only on a 'chromosome' basis (*i.e.* for a derived map, acting either as a parent or as an offspring within the evolutionary optimization context).
(6.3) Available map cells are assigned land use lu_k until the corresponding occupancy percentage declared in $\mathbf{RO}_1$ is reached, starting from the best map cells in terms of the jth objective; for instance, if *suitability* is randomly selected as the control criterion and $lu_k = 3$, all available map cells are scanned, starting with the ones having the highest suitability scores, until oc_3 is reached.
(7) Second part of the sequential allocation algorithm (again iterates land use k from 0 to K):
(7.1) Land use lu_k is retrieved from the same **SV**.
(7.2) The same jth criterion is utilized with the first sequential allocation algorithm: loop (4).
(7.3) Available map cells are assigned land use lu_k until the corresponding occupancy percentage declared in $\mathbf{RO}_2$ is reached, starting from the best map cells in terms of the jth objective.

To generate an additional initial map, the same **SV** can be adopted *I* times by generating, for the *i*th iteration, a new **RO** vector. This process corresponds to the consecutive application of steps (2) to (7) until *I* map cells have been generated. Upon their completion, the algorithm either goes back to step (1), to adopt a new **SV**, or exits if all available **SV**s are utilized.

5. Stage I: Application of initialization heuristic

5.1. *Paradigm on wind farms*

To assess the impact of the proposed initialization heuristic in practice, the already published results of a real-scale planning paradigm are adopted as a benchmark (Karakostas and Economou 2014), aiming to derive the optimum spatial allocation of wind farms on a Greek island (Lesvos).

5.1.1. *Project outline*

In line with the aforementioned study (Karakostas and Economou 2014), the spatial model for Lesvos consists of 62,120 map cells while each cell corresponds to a land patch of 150 × 150 m.

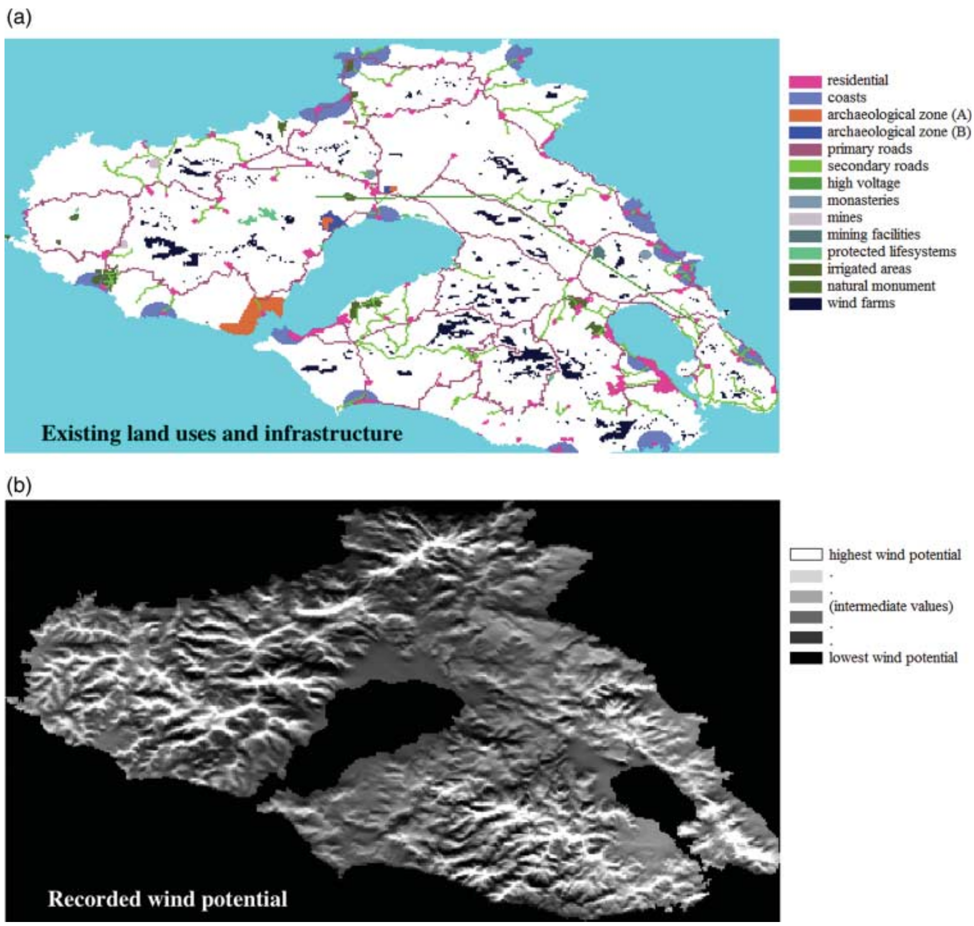

Figure 1. Lesvos' spatial model: (a) existing land uses and infrastructure; (b) recorded wind potential data (highest wind potential values are denoted with white colour).

Table 1. Spatial rules and constraints imposed by the renewable energy sources national framework.

	Distance constraints (m)		Visibility zones (distances in m)					
			A		B		C	
Existing land uses and infrastructure	Min. distance	Max. distance	From	To	From	To	From	To
Archaeological zone A	595	NA	0	595	596	3,000	3,001	6,000
Declared natural monuments	300	NA	0	300	301	1,000	NA	NA
High-voltage network	128	20,000	NA	NA	NA	NA	NA	NA
Irrigated areas	128	NA	NA	NA	NA	NA	NA	NA
Mining facilities	1, 500	NA	NA	NA	NA	NA	NA	NA
Mining zones	500	NA	NA	NA	NA	NA	NA	NA
Monasteries	500	NA	NA	NA	NA	NA	NA	NA
Primary road network	128	10,000	NA	NA	NA	NA	NA	NA
Priority habitats (wildlife)	300	NA	0	300	301	1,000	NA	NA
Protected coastline	1, 500	NA	NA	NA	NA	NA	NA	NA
Residential boundaries	1, 000	NA	0	1,000	1,001	3,000	NA	NA
Secondary road network	128	10,000	NA	NA	NA	NA	NA	NA

The available surface for allocating wind farms ($K = 1$) comprises 52,914 map cells (Figure 1). There are three objective functions to be maximized: suitability, exploited wind potential and the contiguity of proposed sites. The calculation of the suitability score (SSc) of a given map takes into account the Greek legal framework for allocating wind farms on Greek islands by introducing different distance scorings against existing infrastructure, residential areas, industrial sites, *etc*., coupled with visibility constraints between wind farms and residential areas, archaeological sites, *etc*. Table 1 summarizes the relevant spatial rules.

In contrast to the original study (Karakostas and Economou 2014), the typical NSGA-II meta-heuristic is introduced instead of the controlled NSGA-II (CNSGA-II) to evaluate the proposed initialization algorithm against a popular evolutionary algorithm with numerous applications in engineering MOOPs. The parameters of the genetic optimization process remain the same, as listed below.

5.1.2. *Optimization set-up*

- Population of maps per generation: 200
- Number of generations: 70
- Mutation rate: 5% (up to 10%); crossover rate: 80%
- Planning criteria: suitability (derived from existing legal framework), wind potential, contiguity of proposed areas
- Occupancy constraints for wind farms: minimum spatial occupancy 3.5% (user defined), maximum spatial occupancy 4.0% (as imposed by the national RES framework)
- Computational requirements (for a single run): 25 h (on a 2.26 GHz Intel Core Duo personal computer, 3 GB RAM), 600 MB of hard disk space.

5.2. *Metrics for comparing random and enhanced initializations*

The comparison between the two distinct initialization schemes focuses on different performance metrics, each measuring unique attributes of the derived non-dominated solutions (maps):

(1) Metric (a): convergence diagrams monitoring the evolution of the best scoring (sum of all raw objective scores) achieved by a chromosome within each generation.
(2) Metric (b): percentage of non-dominated solutions provided by each approach.

(3) Spacing indicator (S): a common metric measuring the uniform distribution of Pareto optimal solutions in the objective space:

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\overline{d_{\min}} - d_{\min}^{i}\right)^{2}} \tag{4}$$

$$\overline{d_{\min}} = \frac{\sum_{i=1}^{n} d_{\min}^{i}}{n} \tag{5}$$

where n is the population of Pareto optimal solutions ($\boldsymbol{F}_1$ front), $d_{\min}^{i}$ is the distance of solution i from its nearest neighbour within $\boldsymbol{F}_1$, and $\overline{d_{\min}}$is the average distance from the nearest neighbour in $\boldsymbol{F}_1$.

Ideally, a uniform distribution of Pareto optimal maps in the objective space implies that $S = 0$.

(4) Distance from centroid (D_C): a simple metric measuring the average distance, in the three-dimensional objective space, of all (normalized) non-dominated solutions from point [0.5, 0.5, 0.5]$^{\mathrm{T}}$. A small value of D_C denotes a concentrated Pareto front around such centroid, comprised of solutions favouring simultaneously all design criteria. However, the proposed metric assumes only a specific point of the normalized objective space; such an approach is both problem dependent and inappropriate when evaluating the algorithm's explorative performance against broader regions of the Pareto front. Moreover, in cases of symmetric Pareto fronts, the effectiveness of D_C is questioned.

(5) Chromosome balance metric (C_{BM}): corresponds to the average standard deviation of the three normalized scores, one for each objective function, derived for each chromosome:

$$C_{BM} = \frac{\sum_{i=1}^{n} StDev_{\mathrm{obj}}^{i}}{n} \tag{6}$$

where n is the population of Pareto optimal solutions ($\boldsymbol{F}_1$ front), and $StDev_{\mathrm{obj}}^{i}$ is the standard deviation of normalized scores of solution i within $\boldsymbol{F}_1$.

According to Equation (6), a small value of C_{BM} denotes a population of non-dominated solutions having balanced normalized scores for all (three) design criteria. To avoid (or limit) potential implications emerging from symmetric allocations of solutions in the objective space, or from chromosomes residing in $\boldsymbol{F}_1$'s boundaries, C_{BM} can be applied for a portion of non-dominated solutions (*i.e.* for maps having simultaneously all normalized scores greater than a certain threshold).

Owing to the stochastic nature of all evolutionary optimization processes, 10 trial runs were executed for each initialization scheme. As a consequence, Metric (a) monitors the average (raw) score of all 10 best chromosomes for each generation, while providing the evolving standard deviation of this mean value. Accordingly, metrics S, D_C and C_{BM} are averaged across all trial runs; the resulting standard deviation of each metric provides valuable insight into the statistical significance of any recorded differences among alternative optimization methodologies.

5.3. *Results*

The application of the proposed initialization heuristic, in all 10 trial runs, exhibited an obvious concentration of solutions in specific areas of the objective space, deteriorating the uniform distribution of Pareto optimal solutions (Figure 2b). In contrast, random initializations led to relatively uniform distribution schemes, apart from an elliptical surface around the centroid of $\boldsymbol{F}_1$'s surface (Figure 2a).

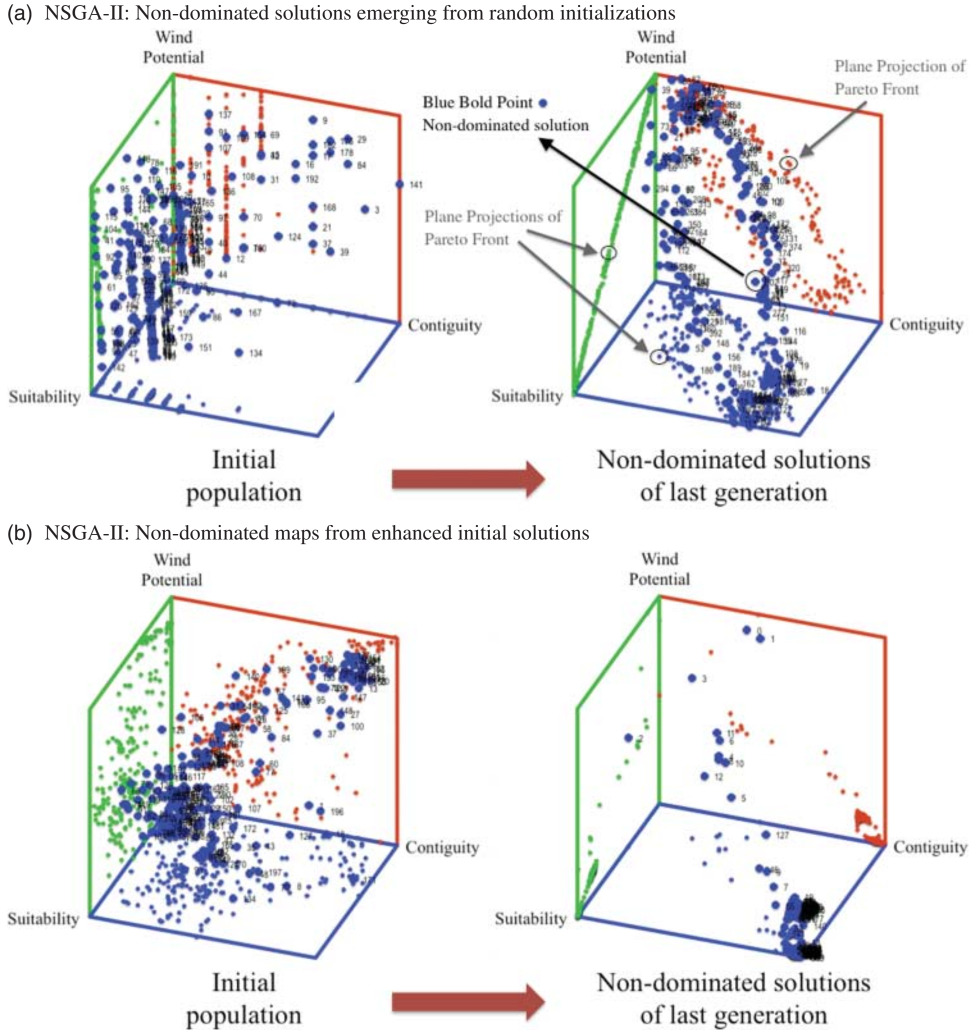

Figure 2. Distribution of non-dominated solutions: NSGA-II (denoted by numbered bold points); the remaining solutions constitute the projections of non-dominated maps on each two-dimensional plane of the objective space.

Figure 2(a, b) displays the trial run with the closest Spacing (S) indicator to its mean value. The same rule applies for all figures depicting the spatial distribution of non-dominated solutions of the last generation (Figures 2, 7 and 8). Figure 3 shows the evolution of the average value of Metric (a) for both initialization schemes. Surprisingly, the best scoring emerging from the application of the initialization heuristic was higher than the one obtained from the randomly initiated NSGA-II by 73 points; according to the evolution of standard deviations of Figure 6 (Section I) for both initialization schemes, this difference is statistically significant at the 99% confidence level. Moreover, the blue curve of Figure 3 (enhanced initialization) remains relatively flat for almost the entire evolutionary process, exhibiting a minor improvement after the 60th generation.

Hence, when the initialization heuristic is applied, the contribution of NSGA-II is, primarily, confined to populating the optimal Pareto front with more non-dominated solutions instead of generating maps with higher raw scorings.

Regarding the spatial distribution of non-dominated solutions, a common finding in both initialization schemes is the lack of candidate maps close to and around the centre of $\boldsymbol{F}_1$'s

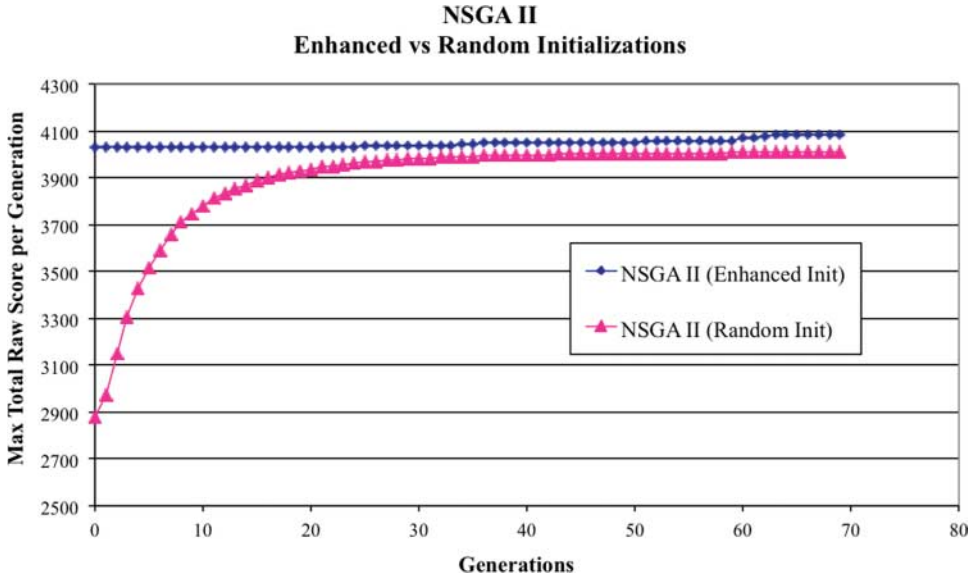

Figure 3. Evolution of Metric (a): NSGA-II, random versus enhanced initializations.

three-dimensional surface. This area is highly important for decision makers and spatial planning experts, as it includes alternative solutions balancing all objective functions; solutions away from this area tend to neglect at least one design criterion.

6. Stage II: Controlled crossover operator

6.1. *Controlled crossover operator heuristic*

The intriguing results of Section 5.2, being in line with previous findings addressing the effectiveness of NSGA-II (Ghiasi *et al.* 2011), triggered a reassessment of the adopted evolutionary process. In particular, the extent to which a crossover operator is likely to affect the distribution of Pareto optimal solutions along the three-dimensional surface of $\boldsymbol{F}_1$ is investigated. Within this context, a modified crossover operator is proposed, enriched with two simple heuristics:

(1) In a typical MOEA the crossover operator is not always applied, according to its predefined probability of occurrence (P_{co}). When the evolutionary process skips its application, one of the preselected parents is randomly chosen to produce a new solution via mutation.

Alternatively, instead of selecting a random parent when crossover is bypassed, the best parent in terms of its Pareto rank or, if both parents belong in the same Pareto front, the one residing in a less crowded region of the objective space (greater crowding distance metric) is selected.

(2) When the crossover operator is applied, the selected parents are compared in terms of their Pareto ranks. If they belong in different Pareto fronts, crossover is executed as usual. Otherwise, the solution lying in a less crowed region of the common front is selected for mutation (*i.e.* crossover is not performed), promoting the development of a new map closer to unexploited regions of the objective space.

In essence, the crossover operator is solely applied between maps of different Pareto ranks, aiming to discover new Pareto frontiers and substantially differentiated maps. A similar approach has been considered before (Yijie and Gongzhang 2008) by recognizing the implications of 'heterosis', *i.e.* parents with similar genes are much more likely to have inferior children. In the presented heuristic the 'gene similarity' is evaluated on a Pareto basis, instead of applying alternative distance-related rules to avoid potential drawbacks when the shape of the actual optimal Pareto front is unknown. The selective application of the crossover operator is expected to accelerate the development of improved offspring and promote the diversity of non-dominated maps. In all other cases, either the most differentiated map is mutated to populate neglected areas of the objective space, or solutions with superior Pareto rankings are preferred to accelerate the MOEA's convergence rate. As a direct consequence of such algorithmic structure, the application of the crossover operator is confined ($< P_{co}$), especially if non-dominated solutions constitute a significant portion of the entire parent population.

Alternatively, one could consider the incorporation of project-specific information regarding the kind of solutions that might be preferred; such preferences could address specific regions of the objective space. Branke (2008) provides an interesting overview of relevant attempts that could be meaningful for the present purposes, such as the adoption of reference points to focus on in the objective space. However, such methodologies require the knowledge of such points in the objective space; given the unknown shape of the actual Pareto front in spatial planning problems, an arbitrary selection could challenge their effectiveness since their exploratory power is steered by user-defined preferences.

6.2. *Results*

Figure 4 shows the evolution of the mean value of Metric (a) when both the initialization heuristic and the controlled crossover operator (CCO) are applied, via the consideration of alternative probabilities of occurrence (P_{co}), against the initial NSGA-II curve. The results indicate similar evolutionary patterns for all values of P_{co} and small differences in the achieved best raw scorings ($\pm 1.2\%$) that are statistically insignificant. Figure 6 (Section III) indicates, once again, growing variances in the evolution of best scores via, however, sharper and more abrupt changes.

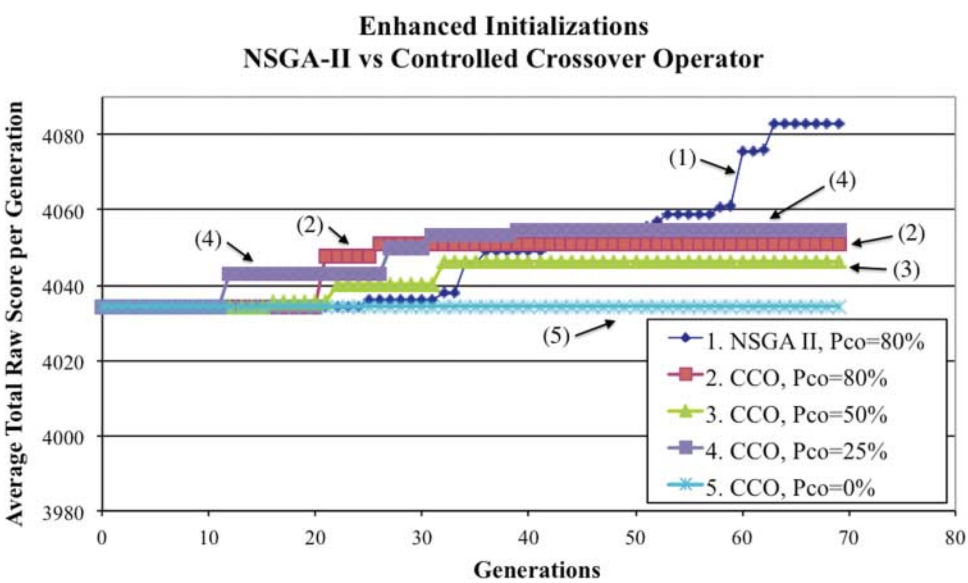

Figure 4. Evolution of Metric (a): combined application of initialization heuristic and controlled crossover operator (CCO).

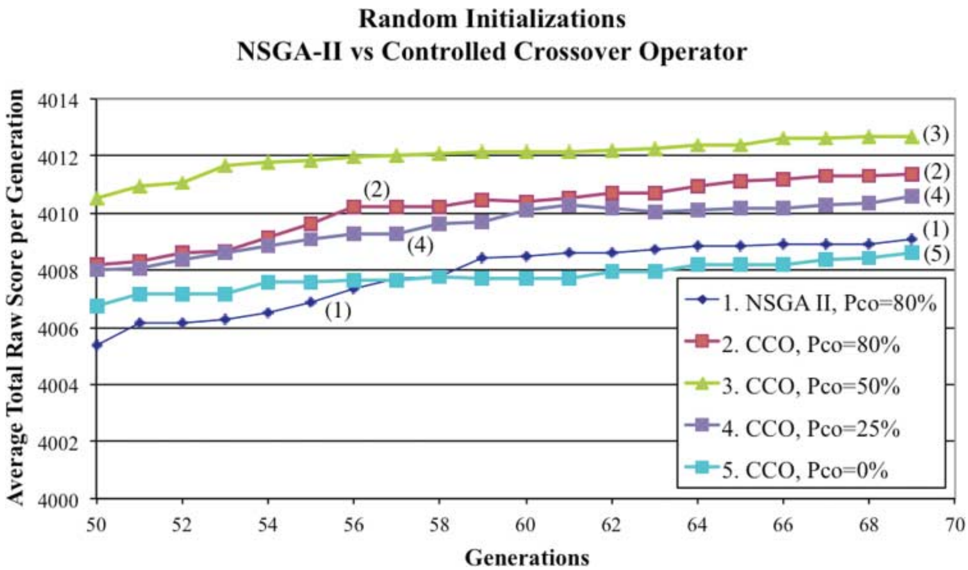

Figure 5. Evolution of Metric (a): application of controlled crossover operator (CCO) on random initializations.

Similarly, when CCO is applied on a random initial population, the evolution of Metric (a) is not significantly affected. Figure 5 provides a magnified view for the last 20 generations of the entire evolutionary process; all five optimization schemes exhibit statistically insignificant differences.

Figure 6 (Section I) depicts the evolution of the standard deviation of Metric (a) without the consideration of CCO. As expected, random initializations exhibit a diminishing trend along the execution of the evolutionary algorithm. In contrast, the introduction of enhanced initial solutions causes lower variations in the beginning of each run, which are significantly magnified as generations grow in number. Despite the fact that a reduction in the variation of generated maps constitutes an important reliability indicator, the opposite (*i.e.* increasing standard deviations of best scorings) could be also attributed to a growing likelihood of spontaneously discovering solutions with higher scorings, or an indication that the evolutionary process has not converged. When CCO is applied, the resulting volatility curves (Figure 6, Section II) are quite similar to the initial curves of NSGA-II (Figure 6, Section I).

In contrast to the results of Figures 4 and 5, the spatial distribution of non-dominated solutions in the objective space is severely affected by the introduction of the CCO. Figure 7 demonstrates such an effect in the case of random initializations.

Regardless of the adopted value for P_{co}, non-dominated solutions have populated the ‘neglected’ area of Figure 2(a) by forming a contiguous Pareto front. Similarly, when enhanced chromosomes are used to initiate the evolutionary algorithm, the introduction of CCO tends to improve the spatial distribution of non-dominated maps (Figure 8) by producing new clusters of solutions closer to the almost vacant centroid of $\boldsymbol{F}_1$.

To quantify the impact of CCO on the spatial distribution of non-dominated solutions, three metrics were investigated: the spacing indicator (S), distance from centroid (D_C) and chromosome balance metric (C_{BM}). Table 2 demonstrates how the spacing indicator (S) is affected by both the initialization procedure and the introduction of CCO. Beyond the obvious superiority of random initializations for all optimization set-ups and despite the modified spatial distribution of

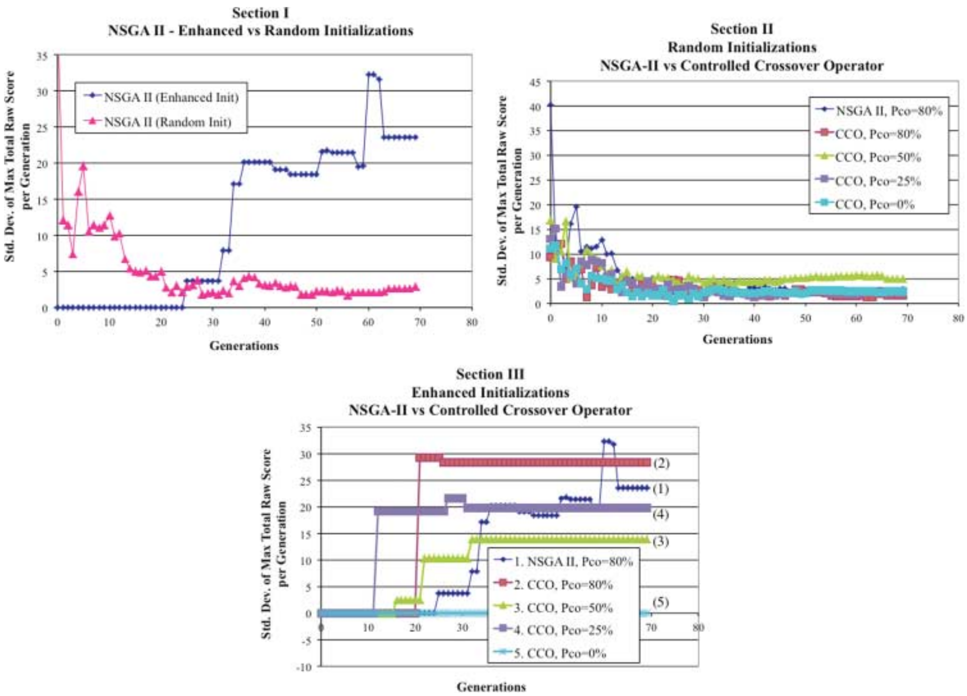

Figure 6. Evolution of standard deviation of Metric (a). CCO = controlled crossover operator.

non-dominated solutions (Figures 7 and 8), the introduction of CCO does not significantly affect the uniformity of the resulting optimal Pareto front; only trials runs adopting $P_{co} = 25\%$ (random initializations) and $P_{co} = 0\%$ (enhanced initial solutions) exhibit a slight reduction in S at the 90% confidence level.

On the other hand, CCO improves the distance from centroid (D_C) metric when the initialization heuristic is applied. As Table 3 demonstrates, D_C is reduced for all values of P_{co} (95% confidence level) when CCO is applied on enhanced initializations.

However, when CCO is executed on random initial maps, D_C is slightly improved for all values of P_{co} (90% confidence level) owing to the geometric features of the Pareto front derived by NSGA-II; despite the gap around its centroid, all solutions are (almost) radially allocated around point $[0.5, 0.5, 0.5]^T$. When CCO is applied, non-dominated maps tend to preserve their radial allocation despite their narrower placement in the objective space. Because of this shortcoming, a chromosome balance metric (C_{BM}) was introduced to verify the increase in chromosomes satisfying simultaneously all design criteria. Moreover, to avoid the (numerical) noise generated by chromosomes close to extreme points of the objective space, C_{BM} concentrates on maps possessing a normalized score greater than 0.4 for all three objective functions (suitability, wind potential and contiguity). Table 4 summarizes the mean values and standard deviations ofC_{BM} for each optimization set-up. The corresponding average population of participating solutions (satisfying the aforementioned constraint simultaneously for all objective functions) is provided in the right-hand columns of Table 4.

When CCO is applied on random initializations, the reduction in C_{BM} for most values of P_{co} (80%, 50% and 25%) remains statistically significant at the 95% confidence level; when $P_{co} = 0\%$, the corresponding reduction in C_{BM} is even more solid (99% confidence level), while the average number of participating chromosomes is maximized.

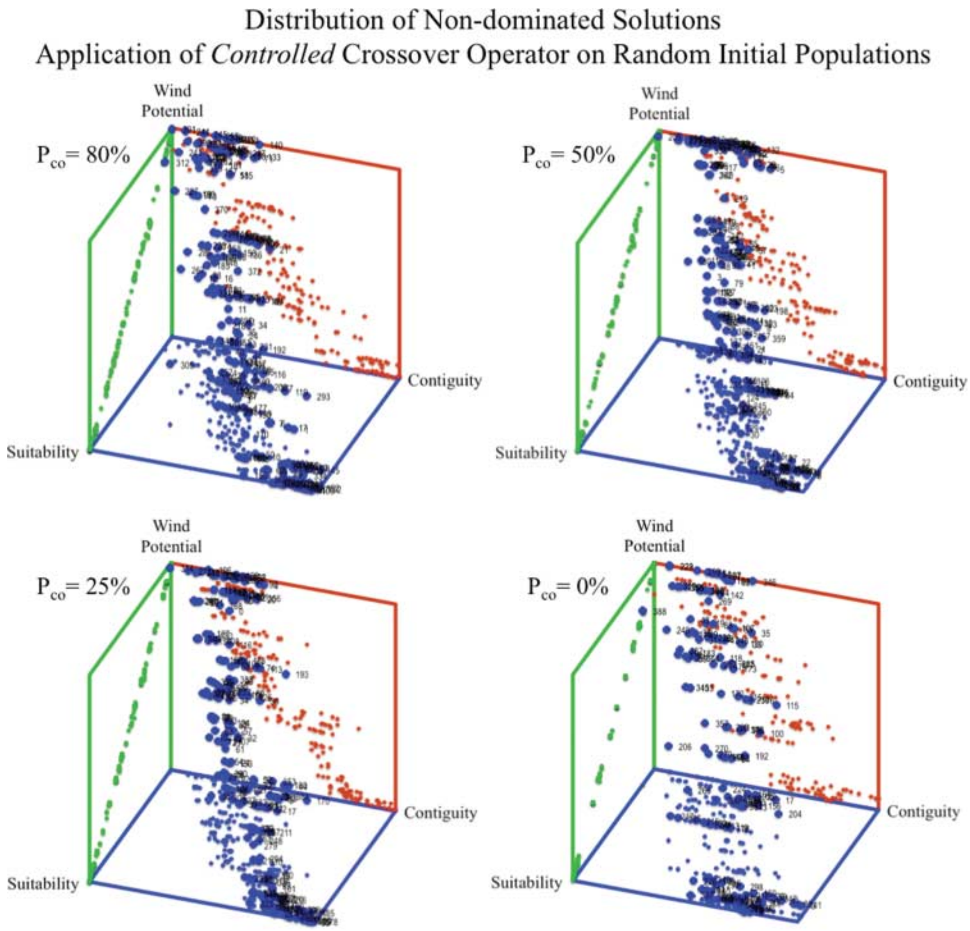

Figure 7. Distribution of non-dominated solutions: controlled crossover operator on random initial populations (denoted by numbered bold points).

The non-dominated solutions derived from the trial runs, having achieved the highest cumulative raw scorings among random and enhanced initializations, were then pooled in a single population of 400 maps, denoted by $\boldsymbol{F}_{1\_C}$. Figure 9 reveals the new set of non-dominated chromosomes when the fourth trial run applying CCO ($P_{co} = 50\%$) on random initial solutions (set A) is combined with the second trial of NSGA-II initiated by an enhanced initial population (set B). Despite the higher cumulative raw scorings derived from enhanced initial maps and the superiority of random initializations in terms of the achieved spatial uniformity of non-dominated maps in the objective space, the new set of non-dominated solutions contains more chromosomes from set A (65.55%), the region around the centroid of $\boldsymbol{F}_{1\_C}$ is solely populated by solutions from set B, and the spacing indicator (S) lies closer to mean values derived by optimization runs with enhanced initializations ($S = 0.032$).

Another valuable interpretation of Figure 9 is the tendency for random initializations to develop solutions having, simultaneously, higher suitability and contiguity scorings than optimization runs initiated by enhanced maps, but at the expense of exploited wind potential. In contrast, only enhanced initializations led to the development of a few non-dominated maps (within set B) residing close to the centroid of $\boldsymbol{F}_{1\_C}$, providing the decision maker with additional valuable spatial planning alternatives. Figure 10 shows the two maps that achieved the highest

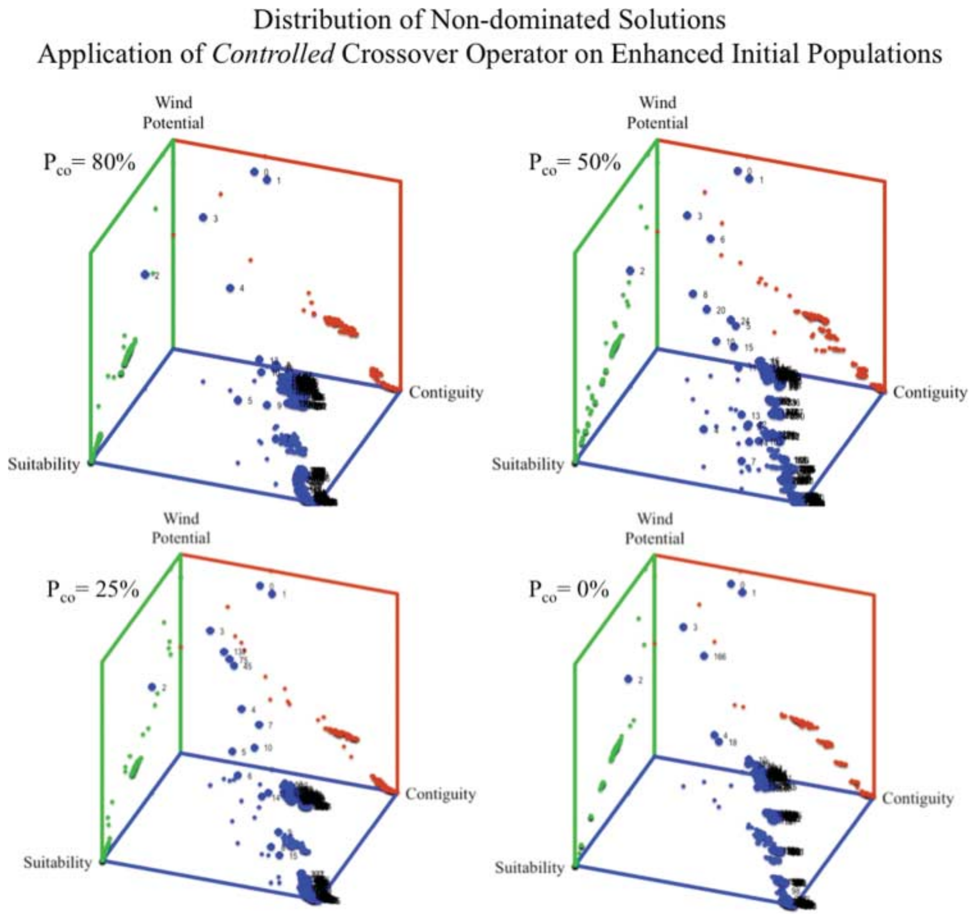

Figure 8. Distribution of non-dominated solutions: controlled crossover operator on enhanced initial populations (denoted by numbered bold points).

Table 2. Spacing indicator.

	Random initialization		Enhanced initialization heuristic	
	S indicator		*S* indicator	
P_{co}	Average	Std. dev.	Average	Std. dev.
NSGA-II				
80%	0.018	0.0019	0.040	0.0019
Controlled crossover operator				
80%	0.016	0.0033	0.038	0.0042
50%	0.016	0.0013	0.039	0.0058
25%	0.015	0.0007	0.038	0.0034
0%	0.019	0.0037	0.033	0.0030

cumulative raw scorings among random and enhanced initializations; their comparison reveals many areas in which the proposed contiguity, and thus spatial allocation, of wind farms differs substantially.

Table 3. Distance from centroid.

	Random initialization		Enhanced initialization heuristic	
	D_C		D_C	
P_{co}	Average	Std. dev.	Average	Std. dev.
NSGA-II				
80%	0.643	0.010	0.795	0.027
Controlled crossover operator				
80%	0.618	0.007	0.666	0.013
50%	0.625	0.013	0.684	0.024
25%	0.627	0.016	0.685	0.013
0%	0.617	0.010	0.668	0.030

Table 4. Chromosome balance.

	Random initialization			
	C_{BM}		Participating chromosomes	
P_{co}	Average	Std. dev.	Average	Std. dev.
NSGA-II				
80%	0.121	0.013	20	2.2
Controlled crossover operator				
80%	0.059	0.016	21	6.6
50%	0.058	0.015	18	7.2
25%	0.070	0.020	17	2.3
0%	0.058	0.008	27	12.8

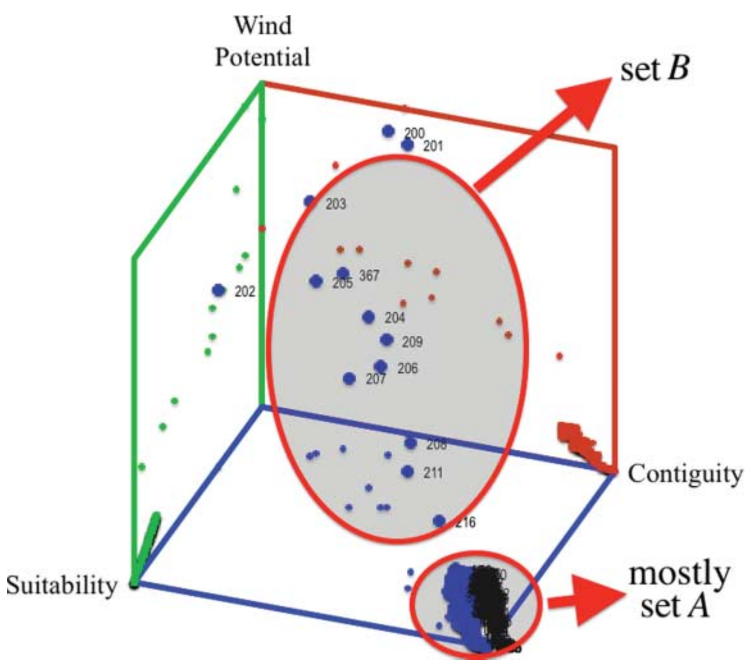

Figure 9. Combined Pareto front of non-dominated solutions emerging from random and enhanced initializations.

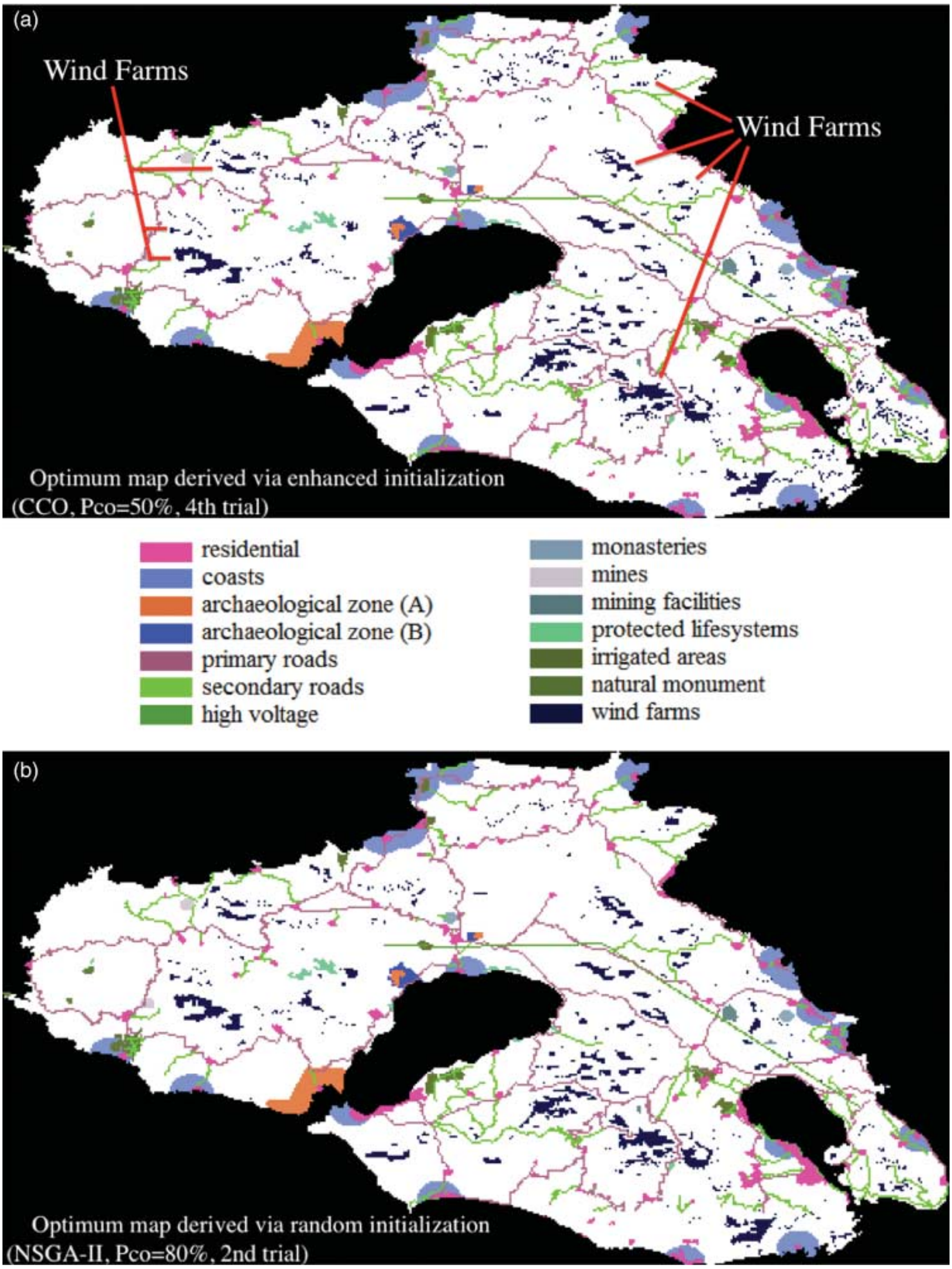

Figure 10. Comparison of optimum spatial solutions derived by random and enhanced initializations. CCO = controlled crossover operator.

7. Conclusions

This study demonstrates the impact of introducing enhanced initial solutions in a well-known evolutionary algorithm (NSGA-II) and proposes a modified algorithmic context for the crossover operator to populate neglected areas of the objective space.

Initially, an initialization heuristic was proposed, aiming to accelerate NSGA-II by improving the initial pool of solutions. Its application led to the development of initial maps with higher raw scores than those produced by the randomly initiated NSGA-II after 70 generations, but at the expense of the uniform distribution of the resulting optimal Pareto front. As a consequence, the contribution of the adopted evolutionary process was confined to populating the optimal Pareto front with heavily concentrated non-dominated maps in the objective space.

Regardless of the initialization process, both approaches failed to produce non-dominated offspring close to the centre of the three-dimensional surface of $\boldsymbol{F}_1$. Given the practical significance of solutions residing close to this region of the objective space, a second heuristic was developed, aiming to populate this 'neglected' area of $\boldsymbol{F}_1$. In particular, the CCO was proposed; this is a modified algorithmic process confining the application of the adopted (dynamic) crossover operator between maps with different Pareto rankings and promoting the exploration of less populated areas of all evolving Pareto fronts, solely via mutation. The resulting non-dominated maps of all optimization runs applying CCO exhibited a clear tendency to occupy neglected regions of $\boldsymbol{F}_1$ for both initialization procedures, without, however, improving their uniform distribution in the objective space. In addition, the proposed initialization heuristic triggered a growing variance in the evolving cumulative best raw scores of non-dominated maps and, at the same time, managed to produce offspring within valuable areas of the objective space that random initializations failed to occupy.

Based on the encouraging findings from the application of both the initialization heuristic and the CCO, future research topics could address methodologies aiming to improve the uniform distribution of non-dominated chromosomes and control (reduce) the growing variance of consecutive optimal spatial allocations when an initialization heuristic is applied. Moreover, the investigation of the optimum structure of hybrid initial populations, comprised of both random and enhanced maps, could provide useful guidelines for spatial planning optimization assignments.

Although both heuristics were thoroughly evaluated against their practical significance in the presented paradigm of wind farms, their effectiveness should be verified in more complex planning projects. For example, the introduction of multiple land uses to be simultaneously allocated or the consideration of additional design criteria could serve as themes for further research towards the verification of their added value in spatial planning optimization.

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