A Niched Pareto Genetic Algorithm for Multiobjective Optimization

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***Abstract*— Many, if not most, optimization problems have multiple objectives. Historically, multiple objectives have been combined ad hoc to form a scalar objective function, usually through a linear combination (weighted sum) of the multiple attributes, or by turning objectives into constraints. The genetic algorithm (GA), however, is readily modified to deal with multiple objectives by incorporating the concept of Pareto domination in its selection operator, and applying a niching pressure to spread its population out along the Pareto optimal tradeoff surface. We introduce the Niched Pareto GA as an algorithm for finding the Pareto optimal set. We demonstrate its ability to find and maintain a diverse "Pareto optimal population" on two artificial problems and an open problem in hydrosystems.**

I. Introduction

Genetic algorithms (GAs) have been applied almost exclusively to single-attribute[1] problems. But a careful look at many real-world GA applications reveals that the objective functions are really multiattribute. Typically, the GA user finds some ad-hoc function of the multiple attributes to yield a scalar fitness function. Often-seen tools for combining multiple attributes are constraints, with associated thresholds and penalty functions, and weights for linear combinations of attribute values. But penalties and weights have proven to be problematic. The final GA solution is usually very sensitive to small changes in the penalty function coefficients and weighting factors [9].

A few studies have tried a different approach to multicriteria optimization with GAs: using the GA to find all possible tradeoffs among the multiple, conflicting objectives. Such solutions are *non-dominated*, in that there are no other solutions superior in all attributes. In attribute space, the set of non-dominated solutions lie on a surface known as the *Pareto optimal frontier*[2]. The goal of a *Pareto GA* is to find a representative sampling of solutions all along the Pareto front.

II. Previous Work

We assume the reader is familiar with the simple GA [3]. Here we review previous approaches to multiobjective optimization with GAs.

In his 1984 dissertation [10], and later in [11], Schaffer proposed his Vector Evaluated GA (VEGA) for finding multiple solutions to multiobjective (vector valued) problems. He created VEGA to find and maintain multiple classification rules in a set covering problem. VEGA tried to achieve this goal by selecting a fraction of the next generation using one of each of the attributes (e.g., cost, reliability). Although Schaffer reported some success, VEGA seems capable of finding only extreme points on the Pareto front, where one attribute is maximal, since it never selects according to *tradeoffs* among attributes.

In his review of GA history, including Schaffer's VEGA, Goldberg [3] suggested the use of non-domination ranking and selection to move a population toward the Pareto front in a multiobjective problem. He also suggested using some kind of niching to keep the GA from converging to a single point on the front. A niching mechanism, such as sharing [5], would allow the GA to maintain individuals all along the non-dominated frontier.

Fonseca and Fleming [2], and, independently, Horn and Nafpliotis [7], implemented Goldberg's two suggestions, and successfully applied the resulting algorithms to difficult, open problems. Fonseca and

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[1] We use the terms "attribute", "objective", and "criteria" interchangeably to describe a scalar value to be maximized or minimized. "Decision variable" refers to the parameters of the problem encoded in the genome of the genetic algorithm.

[2] We assume familiarity with the concept of Pareto optimality, but note here that the Pareto front often goes by the names Pareto optimal set, non-dominated frontier, *efficient* points, and *admissible* points.

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Fleming found many good tradeoffs in a four attribute gas turbine design problem. Horn and Nafpliotis concentrated on a series of two attribute problems, which we describe later in this paper.

III. The Niched Pareto GA

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sharing to choose a winner, as we explain later. The sample size t_{dom} (size of comparison set) gives us control over selection pressure, or what we call *domination pressure*. The performance of the Niched Pareto GA is somewhat sensitive to the amount of domination versus sharing pressure applied [7].

[redacted]

tournament selection, however, the niched GA exhibits chaotic behaviour [8]. The wild fluctuations in niche subpopulations induced by the "naive" combination of sharing and tournament selection can be avoided. Oei, Goldberg, and Chang [8] suggest the use of tournament selection with ***continuously updated sharing***, in which niche counts are calculated not by using the current population, but rather the partly filled next generation population. This method was used successfully by Goldberg, Deb, and Horn [4] on a "niching-difficult" problem. Also in [4], it was found empirically that sampling the population was sufficient to estimate the niche count and so avoid the $O(N^2)$ comparisons needed to calculate ***exactly*** the m_i. We incorporate both techniques (continously updated sharing and niche count sampling) in the Niched Pareto GA.

In any application of sharing, we can implement genotypic sharing, since we always have a genotype (the encoding). But Deb's work [1] indicated that in general, phenotypic sharing is superior to genotypic sharing. Intuitively, we want to perform sharing in a space we "care more about", that is, some phenotypic space. Since we are interested in maintaining diversity along the phenotypic Pareto optimal front,

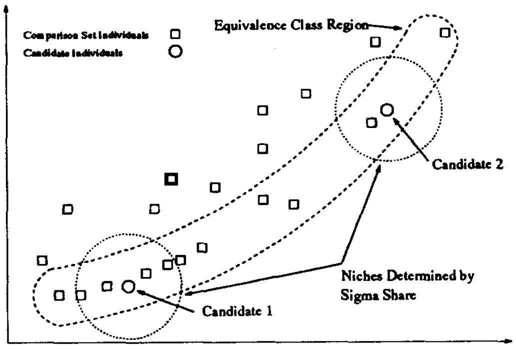

Figure 1: Equivalence class sharing.

want to maintain useful diversity (i.e., a representative sampling of the Pareto frontier), it is apparent that it would be best to choose the candidate that has the smaller niche count. In this case, that is candidate 2.

IV. Application to Three Problems

A. Problem 1: A simple test function

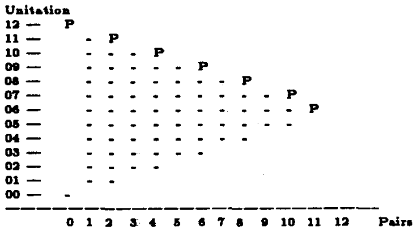

Figure 2: **Problem 1's discrete, two dimensional attribute space, with feasible (-) and Pareto (P) points indicated.**

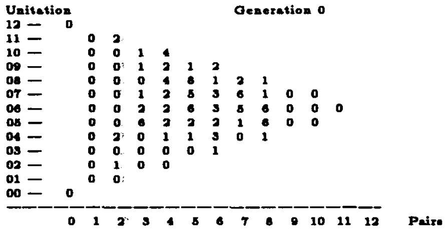

Figure 3: **Distribution of the randomly generated initial population.**

appears to be maintaining substantial subpopulations at each such point. Moreover, there are few (none here) dominated individuals in the current population. Although not shown, we have plotted population distributions over many generations, and noticed that the GA does indeed maintain roughly equal size subpopulations at each Pareto point over many generations. Dominated solutions regularly appear, due to crossover and mutation, but are not maintained.

We have observed similar behaviour over many runs of the GA on different initial population distributions (all random, but using different random seeds). We have also successfully tried larger problems ($l > 12$), with correspondingly larger population sizes N, such as 400 individuals on a 28 bit problem [7].

Finally, we note that this problem is GA-easy (in that it is easy to find points on the front), but not necessarily easy for the Niched Pareto GA. Since there are actually many more solutions at middle points on the front, and only one or two at each end point of the front, it should be harder to maintain equal size subpopulations at the extreme points.

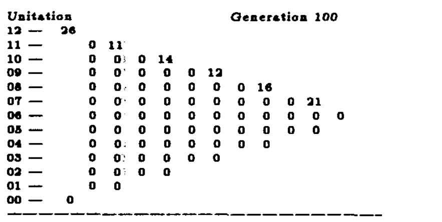

Figure 4: **Stable subpopulations on the Pareto front.**

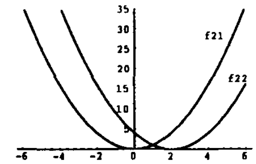

Figure 5: **Schaffer's function F2,** $P = \{x \mid 0 \leq x \leq 2\}$

B. Problem 2: Schaffer's F2

Next we compare our algorithm to Schaffer's VEGA by running it on one of the test functions from Schaffer's dissertation [10]. This is the simple function F2, with a single decision variable, the real-valued x, and two attributes, $f21$ and $f22$ to be minimized:

$$f21(x) = x^2 \qquad f22(x) = (x-2)^2$$

The decision variable is mapped to a 14-bit string as a binary-coded integer. Thus $00000000000000 = x_{min} = -6.00$ and $11111111111111 = x_{max} = 6.00$. We plot $f21$ and $f22$ over this range of x in Figure 5. It is clear that the Pareto front is where the tradeoff exists between the two functions. That is, for $0 \leq x \leq 2.00$, one of the functions is decreasing to its best while the other is increasing away from its best.

Like Schaffer, we use a small population size $N = 30$. Our niche size $\sigma_{share} = 0.1$ and tournament size $t_{dom} = 4$. As Figure 6 illustrates, the Niched Pareto GA is able to maintain a fairly even spread of solutions along the Pareto front. There are a few dominated individuals in the population (to the right of $x = 2.00$), as in the VEGA run above, but most individuals are on the front. Although our population has several gaps in its distribution on the front, it appears more evenly distributed than generation 3 of the VEGA run[8]. Most importantly, the Niched Pareto GA exhibits stability in this population distribution for many more generations than were indicated for VEGA ($200 > 3$).

F2 is an easy problem for the GA: the initial population contains many individuals on the front already. However, this front is much denser than that of problem 1 above, challenging the Niched Pareto GA to maintain N subpopulations of size 1 along the front.

[8]Schaffer (1984) only gave results for generations 0, 1, and 3.

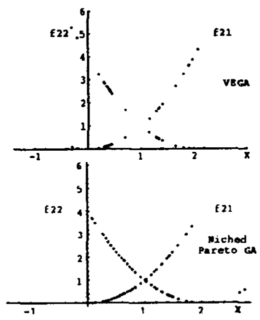

Figure 6: VEGA versus the Niched Pareto GA. Top: VEGA on F2's Pareto frontier, generation 3. Bottom: The Niched Pareto GA's distribution, generation 200.

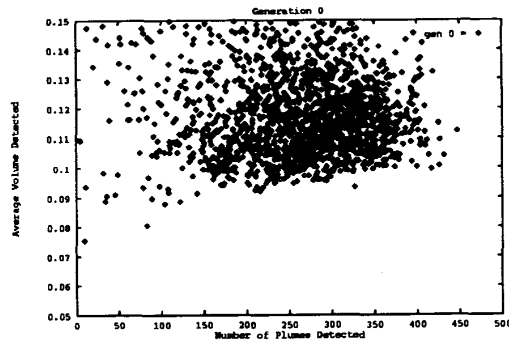

Figure 7: Initial population distribution, problem 3.

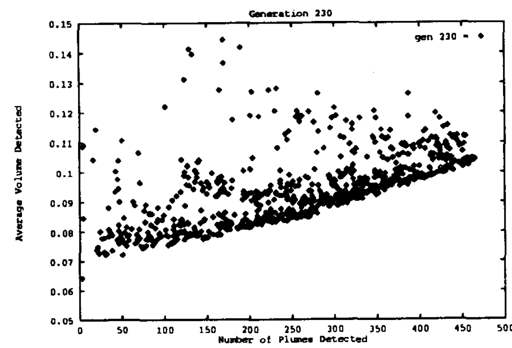

Figure 8: Final distribution, problem 3.

C. Problem 3: Open problem in hydrosystems

To challenge the Niched Pareto GA's ability to *search* for diverse tradeoffs, we chose a larger, real-world (i.e., unsolved) application for our third test problem: optimal well placement for groundwater contaminant monitoring[9]. The problem is to place a set of k out of a possible w wells in order to maximize the number of *detected* leak plumes from a landfill into the surrounding groundwater, and to minimize the volume of cleanup involved. These two objectives conflict. Simply optimizing for the minimum volume of cleanup will give us an answer with attributes $\{0,0\}$, where we detect no plumes and therefore have no volume of contaminant to clean up. If we maximize the number of detected plumes our volume of cleanup increases dramatically.

It is important to note that this problem is intractable. The search space is of size $\binom{w}{k}$. In our specific example we have $w = 396$ and $k = 20$. The whole search space is then $\binom{396}{20}$ which is 2.269×10^{33}. This makes it impossible to know the actual Pareto optimal front from enumeration.

Monte-Carlo simulation was used to develop a set of possible leak plumes, the set of wells that detect each plume, and the volume leaked when each well detected the contaminant plume. Using these data, we constructed a vector-valued fitness function to return the number of plumes and average volume detected by any given set of wells.

In our first few runs, $N = 2000$, $\sigma_{share} = 40$, $t_{dom} = 40$, $p_c = 0.8$, and no mutation. In Figure 7 one can see that the random initial population is distributed throughout the search space. Figure 8 shows that after 230 generations the Niched Pareto GA has found an apparent front that is indeed improved over the initial population. It is promising to see that even after a large number of generations we are maintaining diversity over most of an apparent front. There is definite improvement as to the location of the front and the decrease in the number of dominated individuals in the population.

We do not know yet whether this is the actual Pareto optimal front or a sub-optimal front. But our first few runs indicate that the equivalence-class sharing and dominance tournaments are working together. We have shown that a tradeoff curve better than a random sampling can be developed by the Niched Pareto GA on an open problem.

V. Discussion

These preliminary results on the application of the niched, Pareto technique are encouraging. Fonseca and Fleming [2] have also reported initial success with a similar algorithm. But we have found that the performance of the Niched Pareto GA is sensitive to the settings of several parameters. In particular, it is important to have a large enough popula-

[9] This open problem was developed by Wayland Eheart and his colleagues at the Civil Engineering Department at the University of Illinois at Urbana-Champaign. We are grateful to Dr. Eheart and his students S. Ranjithan, P. Stork, and S. Cieniawski for helping us implement it.

tion to search effectively and to sample the breadth of the Pareto front. Both [7] and [2] discuss the setting of σ_{share} and population size together to yield effective sampling.

But the behaviour of the Niched Pareto GA seems to be most affected by the degree of selection pressure applied. Just as tournament size t_{size} is critical to selection pressure and premature convergence in a regular GA with tournament selection, so t_{dom} directly effects the convergence of the Niched Pareto GA. Horn and Nafpliotis [7] illustrate the effects of too little and too much dominance pressure. Here, we summarize their empirically-derived, order-of-magnitude guidelines:

- $t_{dom} \approx 1\%$ of N; results in too many dominated solutions (a very fuzzy front).
- $t_{dom} \approx 10\%$ of N; yields a tight and complete distribution.
- $t_{dom} \gg 20\%$ of N; causes the algorithm to prematurely converge to a small portion of the front. Alternative tradeoffs were never even found.

We have not yet addressed the critical issue of search, but we have some intuitions. Our intuition in the case of Pareto optimization is that the diver-

Pareto approach to multiobjective problems.

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