Yu Hou
ID: 6587492109

1.

If we define a unit vector $\vec{u} = (u_x, u_y, u_z)$ where $u_x^2+u_y^2+u_z^2=1$. a rotation by an angle θ about an axis in the direction $\vec{u}$ is

$$R = \begin{bmatrix} \cos\theta + u_x^2\cdot(1-\cos\theta), & u_x u_y(1-\cos\theta) - u_z\cdot\sin\theta, & u_x u_z(1-\cos\theta) + u_y\sin\theta \\ u_y\cdot u_x(1-\cos\theta) + u_z\sin\theta, & \cos\theta + u_y^2\cdot(1-\cos\theta), & u_y u_z(1-\cos\theta) - u_x\cdot\sin\theta \\ u_z u_x(1-\cos\theta) - u_y\cdot\sin\theta, & u_z u_y(1-\cos\theta) + u_x\cdot\sin\theta, & \cos\theta + u_z^2\cdot(1-\cos\theta) \end{bmatrix}$$

so. if it is z-axis $\vec{u}=(0.0.1)$

$$R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

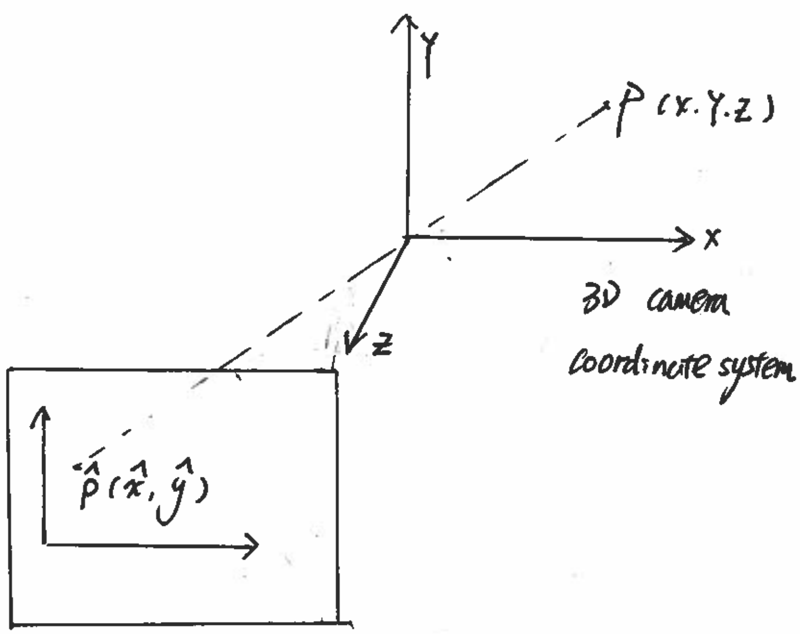
if it is y-axis $\vec{u}=(0,1,0)$

$$R = \begin{bmatrix} \cos\varphi & 0 & \sin\varphi \\ 0 & 1 & 0 \\ -\sin\varphi & 0 & \cos\varphi \end{bmatrix}$$

if it is z-axis $\vec{u}=(1,0$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin\psi & -\sin\psi \\ 0 & \sin\psi & \cos\psi \end{bmatrix}$$

2. a)



normalized image plane

We can know that

$$\begin{cases} \hat{x} = \dfrac{X}{Z} \\ \hat{y} = \dfrac{Y}{Z} \end{cases} \Rightarrow \begin{matrix} X = \hat{x}\cdot Z \\ Y = \hat{y}\cdot Z \end{matrix}$$

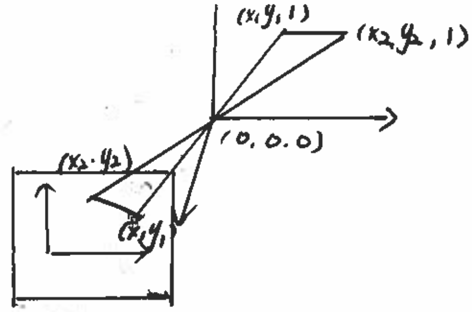
So we know that the line will go through two points $(0,0,0)^T$ $(\hat{x}\cdot Z, \hat{y}\cdot Z, Z$

so the line will be

$$\frac{x}{\hat{x}} = \frac{y}{\hat{y}} = \frac{Z}{1}$$

Yu Hou
ID: 6587492109

2. b).



We know that the line in image is (a, b, c). we assume ~~there~~ there are ~~three~~ two points are on this line. These two points are (x_1, y_1) and (x_2, y_2) so in the camera cooridinates they are $(x_1, y_1, 1)$, $(x_2, y_2, 1)$

we also know that these two points can fit in the line equation so,

$a x_1 + b y_1 + c = 0$ and $a x_2 + b y_2 + c = 0$.

we can say
$$\begin{cases} x_1 = \dfrac{-b y_1 - c}{a} \\ x_2 = \dfrac{-b \cdot y_2 - c}{a} \end{cases}$$

we know that this plane will go three points
$$\begin{cases} (x_1 \quad y_1 \quad , 1) \\ (x_2 \quad y_2 \quad . 1) \\ (0, 0, 0) \end{cases}$$

we need to calculate parameters first,

if we have three points $(x_1 y_1 z_1)$ $(x_2 \cdot y_2 \cdot z_2)$, $(x_3 y_3 \cdot z_3)$

$$A = (y_2 - y_1) \cdot (z_3 - z_1) - (y_3 - y_1) \cdot (z_2 - z_1)$$
$$B = (z_2 - z_1) \cdot (x_3 - x_1) - (z_3 - z_1) * (x_2 - x_1)$$
$$C = (x_2 - x_1) \cdot (y_3 - y_1) - (x_3 - x_1) \cdot (y_2 - y_1)$$

in this case

$$A = (y_2 - y_1)(0 - 1) = y_1 - y_2$$
$$B = x_2 - x_1 = \frac{-b y_2 - c}{a} - \frac{-b y_1 - c}{a} = \frac{b}{a}(y_1 - y_2)$$
$$C = (x_2 - x_1) \cdot (-1) - (-x_1) \cdot (y_2 - y_1) = -\frac{c}{a}(y_2 - y_1)$$

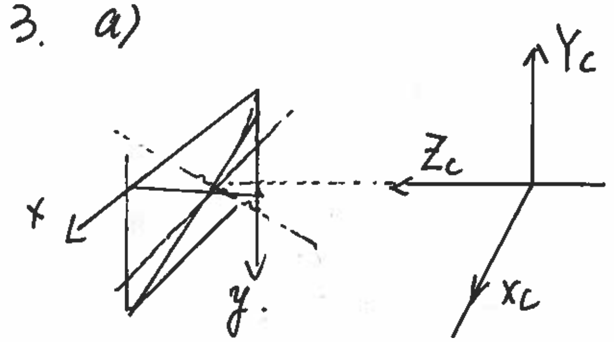
The plane equation will be

$$A \cdot (x - 0) + B(y - 0) + C \cdot (z - 0) = 0$$
$$(y_1 - y_2) \cdot x + \frac{b}{a}(y_1 - y_2) \cdot y + \frac{c}{a}(y_1 - y_2) \cdot z = 0$$
$$a x + b y + c z = 0$$

So the plane is → So the plane is (a, b, c, 0)

Yu Hou
ID: 6587492109

3. a)



we can know that

$$\begin{cases} x = k \cdot f \cdot \frac{X}{Z} \\ y = l \cdot f \cdot \frac{Y}{Z} \end{cases}$$

In this problem, Image center and original are different

$$\begin{cases} x = k \cdot f \frac{X}{Z} + x_0 \\ y = l \cdot f \cdot \frac{Y}{Z} + y_0 \end{cases}$$

we define $\begin{matrix} \alpha = k f \\ \beta = l \cdot f \end{matrix}$

so

$$K \stackrel{def}{=} \begin{pmatrix} \alpha & -\alpha \cdot \cot\theta & x_0 \\ 0 & \frac{\beta}{\sin\theta} & y_0 \\ 0 & 0 & 1 \end{pmatrix}$$

In this case

$k = l = \frac{1}{0.05}$ $\quad f = 50$

$x_0 = 500$ $\quad y_0 = 500$

$\theta = -90°$

$$K \stackrel{def}{=} \begin{pmatrix} 1000 & 0 & 500 \\ 0 & -1000 & 500 \\ 0 & 0 & 1 \end{pmatrix}$$

3 b) i)

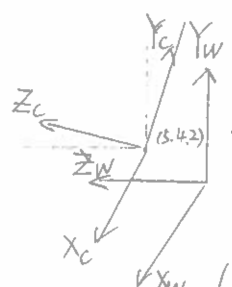
From this a) question, we can know that

$$K \stackrel{def}{=} \begin{pmatrix} 1000 & 0 & 500 \\ 0 & -1000 & 500 \\ 0 & 0 & 1 \end{pmatrix}$$

In this question, it will rotate 30° about x axis

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(30°) & -\sin 30° \\ 0 & \sin(30°) & \cos(30°) \end{bmatrix}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}$$



$$M = K \cdot (R, t)$$

P_c

$$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & t_1 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & t_2 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & t_3 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 5 \\ 4 \\ 2 \\ 1 \end{pmatrix}$$

$t_1 = -5000$ $\quad t_2 = 1000 - 2000\sqrt{3}$ $\quad t_3 = -2000 - 1000\sqrt{3}$

$$M = \begin{pmatrix} 1000 & 0 & 500 \\ 0 & -1000 & 500 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & -5 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 1 - 2\sqrt{3} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -2 - \sqrt{3} \end{pmatrix}$$

$$= \begin{pmatrix} 1000 & 250 & 250\sqrt{3} & -6000 - 500\sqrt{3} \\ 0 & 250 - 500\sqrt{3} & 500 + 250\sqrt{3} & 1500\sqrt{3} - 2000 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & -2 - \sqrt{3} \end{pmatrix}$$

Yu Hou
ID: 6587492109

3 b) ii)

vanishing point of vertical lines. because this line is paralled to Y-axis. infinit point we can definie as $(0, 1, 0, 0)$

$p = K \cdot d$ d is in camera coordinates

$= M \cdot p^w$ p^w is in the world coordinates.

So $p = M \cdot (0, 1, 0, 0)^T$

$= (250, \; 250 - 500\sqrt{3}, \; \frac{1}{2})^T$ this is in the homogenous system.

So the point is $(500, \; 500 - 1000\sqrt{3})$

iii)

Because in the horizontal plane, we define the direction as $(x_0, 0, y_0, 0)$

So $p = M \cdot (x_0, 0, \cancel{z_0}\, 0)^T$

$= ((1000x_0 + 250\sqrt{3}z_0, \; (500 + 250\sqrt{3}) \cdot z_0 \;\; \frac{\sqrt{3}}{2} z_0)^T$

point is $(\frac{2000\sqrt{3}}{3} \cdot \frac{x_0}{z_0} + 500, \; 500 + \frac{1000\sqrt{3}}{3})$

iV)

The line should be $y = 500 + \frac{1000\sqrt{3}}{3}$

cuz x_0, z_0 can be everything