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ANALYZING CROP SALT TOLERANCE DATA: MODEL DESCRIPTION AND USER'S MANUAL

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ABSTRACT

M. Th. van Genuchten.[1] 1983. Analyzing Crop Salt Tolerance Data: Model Description and User's Manual. Research Report No. 120, U.S. Salinity Laboratory, USDA/ARS, California, 50 p.

This report describes a computer program that can be used to analyze experimentally derived crop salt tolerance data. The program uses a non-linear least squares inversion method to find the unknown parameters in several salt tolerance response functions. One of three models included in the program is the familiar piecewise linear response function. Application of this function leads to estimates for the salinity threshold and the slope of the response curve. Two alternative types of salinity response functions are also considered. The report gives a detailed description of the computer model and the required input data. Application of the program is illustrated with several examples. A listing of the program is given in an appendix.

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CONTENTS

1. INTRODUCTION

The presence or accumulation of excess soluble salts in the soil root zone and its negative effect on crop productivity is a widespread problem, especially in the arid and semiarid regions of the world. Although in some cases soil salinity can be controlled effectively by applying suitable water management schemes, high soil salinities often are difficult to prevent because of a lack of good quality irrigation water. In that case, an effective use of available soil and water resources dictates the production of agricultural (or other) crops that are relatively tolerant to high soil salinities. For this purpose, numerous field and laboratory experiments have been carried out to determine the salt tolerance of various crops. Results of these experiments are best analyzed in terms of an appropriate salt tolerance response function.

One popular way to express the relative salt tolerance of crops is by means of a piecewise linear response function (Maas and Hoffman, 1977). This function contains two independent parameters: the salinity threshold (c_t), being the maximum salinity without yield reduction as compared to the yield under nonsaline control conditions, and the slope (s) of the curve determining the fractional yield decline per unit increase in salinity beyond the threshold. In mathematical form:

$$Y_r = \begin{cases} 1 & 0 \leqslant c \leqslant c_t \\ 1 - s(c - c_t) & c_t < c \leqslant c_o \\ 0 & c > c_o \end{cases} \qquad [1]$$

where Y_r is the relative yield, c is the average rootzone salinity during the growing season, c_t is the threshold concentration, c_o is the concentration beyond which the yield is zero, and s is the absolute value of the slope of the reponse function between c_t and c_o. Soil salinity can be expressed in terms of concentration, osmotic potential, or in terms of electrical conductivities, either of soil water (EC_{sw}) or of the soil saturation extract (EC_e).

Equation [1] is formulated in relative terms; the absolute yield curve is given by

$$Y = \begin{cases} Y_m & 0 < c < c_t \\ Y_m - Y_m s(c - c_t) & c_t < c < c_o \\ 0 & c > c_o \end{cases} \qquad [2]$$

where Y_m is the absolute yield under nonsaline conditions, and

$$Y = Y_r Y_m. \qquad [3]$$

Figure 1 gives a schematic representation of Eq. [2]. Note that the response function is continuous and consists of three piecewise linear curves. Equation [2] contains several parameters, three of which are independent: the non-saline control yield (Y_m), the salinity threshold (c_t) and the slope (s). Consequently, the salinity beyond which the yield becomes zero (c_o, see Fig. 1) is given by

$$c_o = c_t + \frac{1}{s} \qquad [4]$$

Over the years, many experiments have been carried out to quantify these type of linear or other response functions. This has resulted in extensive tables as published by this laboratory (U.S. Salinity Laboratory Staff, 1954; Bernstein, 1974; Maas and Hoffman, 1977) and other institutions (van den Berg, 1950; De Forges, 1970). Unfortunately,

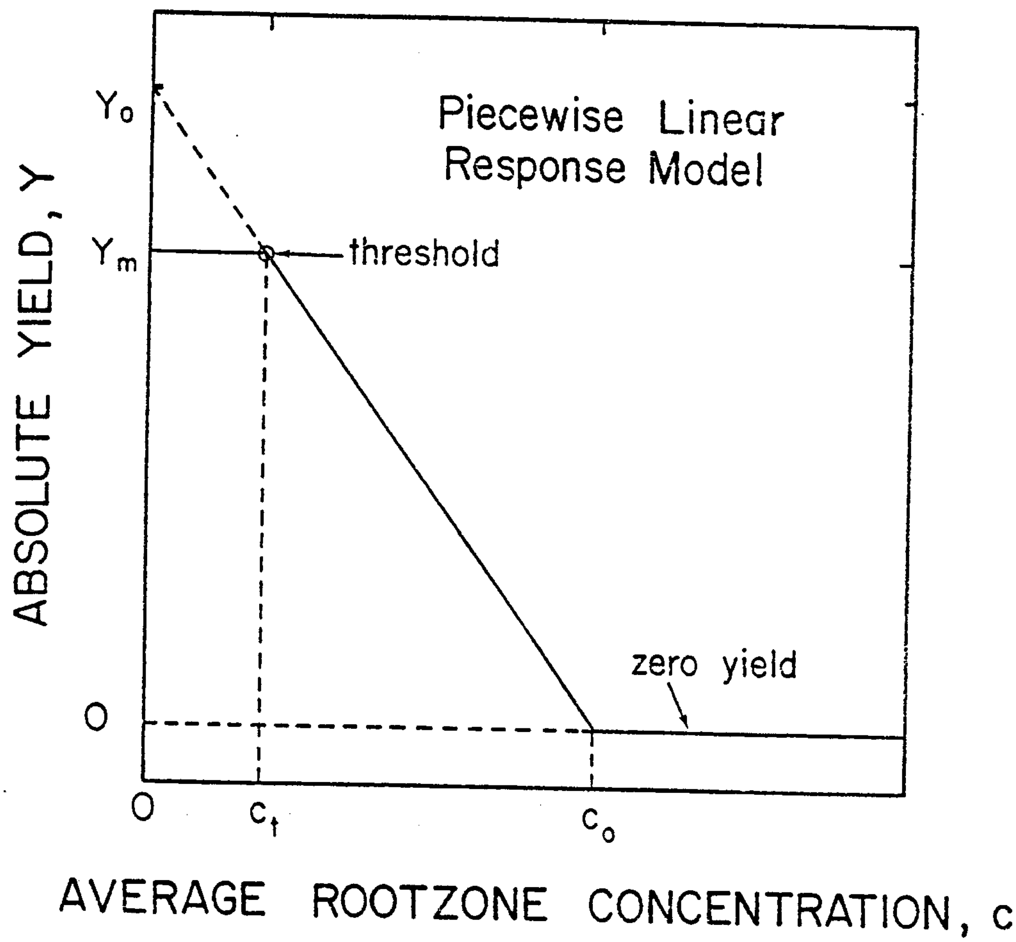

Figure 1. Graphical representation of the piecewise linear crop salt tolerance response function (Eq. 2).

accurate and reliable salt tolerance data are not easily obtained; they generally require elaborate and time-consuming field trials. Also, the salt tolerance parameters often are variety-specific and may depend, among other factors, on the prevailing soil and environmental conditions of the experiment. Because of the time, labor and money involved, the number of experimental data points available to construct a salt tolerance response function generally is limited. Moreover, the observed data frequently reflect some type of experimental variability. This makes it important to have an accurate method that can be used to determine the threshold and the slope from a limited number of data points.

This report describes a computer program that can be used to statistically fit the unknown coefficients in Eqs. [1] or [2] to experimental data. Several options are possible with the program, such as a simple linear regression fit to estimate the slope of a response curve that is already normalized into relative yield fractions (Y_m is known), or a more complete nonlinear least squares fit to simultaneously estimate the three unknown coefficients in Eq. [2]. Other and more complicated options are discussed also. Examples will be given to illustrate the various options.

2. THE INVERSE PROBLEM

Until recently, a common practice for estimating c_t and s from experimental data has been to either eye-fit these coefficients to the data, or to use linear regression techniques (Shalhevet and Bernstein, 1968; Maas and Hoffman, 1977). Feinerman et al. (1982) recently proposed a more accurate switching regression method to estimate the coefficients that appear in a two-piece linear response model. Unfortunately, their method is restricted to those data sets that have at least two data points to the left and at least three points to the right of the fitted threshold value. This makes the method less suitable for experiments with a limited number of data points. In this report we will use a more general non-linear least squares method. Appendix A gives a detailed description of the computer program (called "SALT"); the program itself is listed in Appendix D.

To allow for flexibility in analyzing different types of data sets, 20 different options have been included in the program. These options relate to the choice of a particular salt tolerance response function (Eq. [2] or alternative models), and to the type and number of model parameters that are fitted to the data. The different options are discussed briefly below. Specific examples are given in the next section.

Table 1 gives a list of the available options. A particular option in the program is chosen by specifying the input variable NOPT ("Option Number", see Table 1). When NOPT = 1, a simple linear regression analysis of the type

$$Y = Y_o - s_1 c \qquad [5]$$

with two unknown parameters (Y_o, s_1) will be carried out. Application of this method assumes that an independent estimate for Y_m is available, and hence, that the data already are normalized into relative yield fractions. It is important to realize that this method can be applied only to data points that are located between c_t and c_o (see Eq. 2). Once the regression based on Eq. [5] is carried out, the salinity threshold and slope can be calculated with the expressions

$$c_t = (Y_o - Y_m)/s_1 \qquad [6a]$$

Table 1. Values of NOPT (option number), NP (number of unknown parameters) and associated model type.

NOPT	NP	Equation	Model Description
1	2	5	Linear regression with two unknowns (c_t and s).
2	1	7	Linear fit of s; Y_m and c_t are fixed.
3	2	2	Nonlinear fit of s and Y_m; c_t is fixed.
4	2	2	Nonlinear fit of c_t and s; Y_m is fixed.
5	3	2	Nonlinear fit of c_t and s and Y_m.
6	4	2	Nonlinear fit of c_t, s and Y_m^1, Y_m^2.
7	5	2	Nonlinear fit of c_t, s and Y_m^1, Y_m^2, Y_m^3.
8	6	2	Nonlinear fit of c_t, s and $Y_m^1,\ldots, Y_m^4$.
9	7	2	Nonlinear fit of c_t, s and $Y_m^1,\ldots, Y_m^5$.
10	8	2	Nonlinear fit of c_t, s and $Y_m^1,\ldots, Y_m^6$.
11	2	8	Nonlinear fit of c_{50} and p; Y_m is fixed.
12	3	8	Nonlinear fit of c_{50}, p and Y_m.
13	4	8	Nonlinear fit of c_{50}, p, Y_m^1, and Y_m^2.
14	5	8	Nonlinear fit of c_{50}, p, Y_m^1, Y_m^2 and Y_m^3.
15	6	8	Nonlinear fit of c_{50}, p and $Y_m^1,\ldots, Y_m^4$.
16	7	8	Nonlinear fit of c_{50}, p and $Y_m^1,\ldots, Y_m^5$.
17	8	8	Nonlinear fit of c_{50}. p and $Y_m^1,\ldots, Y_m^6$.
18	2	9	Nonlinear fit of α and β; Y_m is fixed.
19	3	9	Nonlinear fit of α, β and Y_m.
20	2	9	Nonlinear fit of β and Y_m ($\alpha = 0$).

and

$$s = s_1/Y_m. \qquad [6b]$$

When NOPT = 2, it is assumed that both Y_m and c_t are already known, thus leaving only the slope s to be calculated from the experimental data. In this study, s is obtained with the simple equation

$$s = \sum_{i=1}^{n} (Y_m - Y_i) / \sum_{i=1}^{n} (c_i - c_t) \qquad [7]$$

where (c_i, Y_i) represents the i-th data point $(1 \leq i \leq n)$, and n is the number of observed data points used in the analysis. An iterative procedure was built into the program such that only data points between c_t and c_o are considered. As will be shown later, Eq. [7] is especially useful when other methods based on Eq. [2] lead to salinity threshold values that are located to the left of the first measured data point (usually the non-saline control). As an alternative to Eq. [7], least squares techniques could have been used also to calculate s once Y_m and c_t are known. Because least squares techniques were found to give relatively more weight to data points that are far away from the threshold value (i.e., to data points associated with relatively low yields and high salinity levels), and because salt tolerance studies generally are concerned more with the region close to the threshold (i.e., with the higher yield values), it was decided to use only Eq. [7].

Nonlinear least squares techniques are used whenever NOPT $\geq$ 3. When NOPT = 3, the threshold c_t is assumed to be known beforehand and only s and Y_m are fitted to the data. When NOPT = 4, both c_t and s are calculated (Y_m is fixed), whereas for NOPT = 5 all three unknowns (Y_m, c_t and s) in Eq. [2] are fitted. The nonlinear least squares model used for this purpose is a simplification of a more general program described by Daniel and Wood (1971). The technique uses the maximum neighborhood method of Marquardt (1963), which is based on an optimum interpolation between the Taylor series method and the method of steepest descent. Several examples based on the nonlinear least squares method are given later.

Salt tolerance studies for the same crop (variety) often are carried out over a period of several years. One could analyze these data on a year-to year basis by fitting the unknown parameters to the experimental data for each year separately. Even though the maximum yield (Y_m) may vary from year to year because of varying soil, environmental or management conditions, the assumption sometimes is made that the threshold and the slope should remain constant from year to year. "Average" thresholds and/or slopes could then be derived by simply averaging the yearly value of the two parameters.

An alternative and more accurate procedure for this problem would be to fit the time-independent values of c_t and s directly to all data, while at the same time allowing Y_m to vary from year to year. This can be done when $6 < NOPT < 10$ (see Table 1). For example, suppose that experimental data for a certain variety are available for two consecutive years. If c_t and s are assumed to remain constant over these two years, then a total of 4 unknown coefficients must be fitted to the data: c_t, s, and the maximum yields (Y_m^1 and Y_m^2) for the two years. This case is carried out when NOPT = 6 (Table 1). The program can consider analogous problems for up to 6 years (up to 8 unknown parameters). Of course, similar situations with fixed values of c_t and s and varying Y_m-values are also possible when salt tolerance studies are carried out within a fixed time period, but with different management schemes (e.g., with varying leaching fractions or irrigation methods). One example of this type is considered in section 3.6.

Although Eq. [2] has been the more popular model for quantifying the salt tolerance of crops, two alternative formulations are also considered in this report. One expression is of the form

$$Y = \frac{Y_m}{1 + (c/c_{50})^p} \tag{8}$$

where c_{50} is the salinity at which the yield is reduced by 50%, and where p is an empirical constant. Figure 2 gives a dimensionless plot of Y_r versus c/c_{50}. Equation [8] is used in the program for NOPT-values that run from 11 through 17 (see Table 1). As with Eq. [2], choice of a particular option depends on the number of unknown parameters in Eq. [8], and on the number of multiple Y_m-values available for different years or treatments. Examples based on Eq. [8] are shown in sections 3.3. and 3.5.

A second alternative salt tolerance model used in the computer program assumes an exponential relation between the yield and the average rootzone salinity:

$$Y = Y_m \exp(\alpha c - \beta c^2) \tag{9}$$

where α and β are empirical constants. Figure 3 shows relative salt tolerance curves based on this equation, using three different combinations of α and β. Note that the curve for $\alpha > 0$ reaches a maximum at some positive value of the concentration; this maximum is located at $c = \alpha/2\beta$. When $\alpha = 0$, the initial slope of the response function is zero, and the curve is similar in shape to the curves shown in Fig. 2. Response

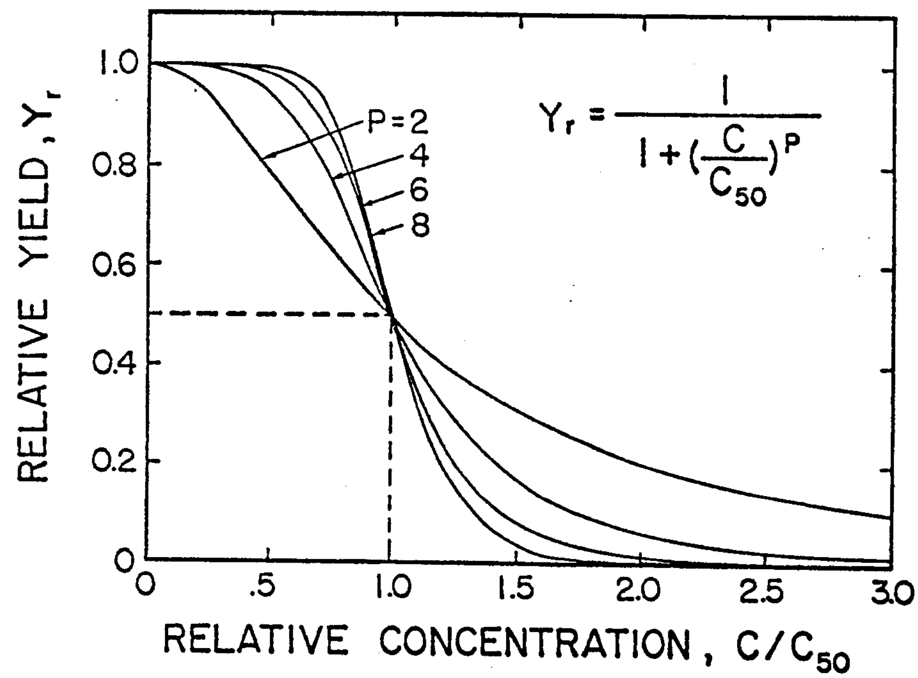

Figure 2. Dimensionless plot of Eq. [8] for various values of p.

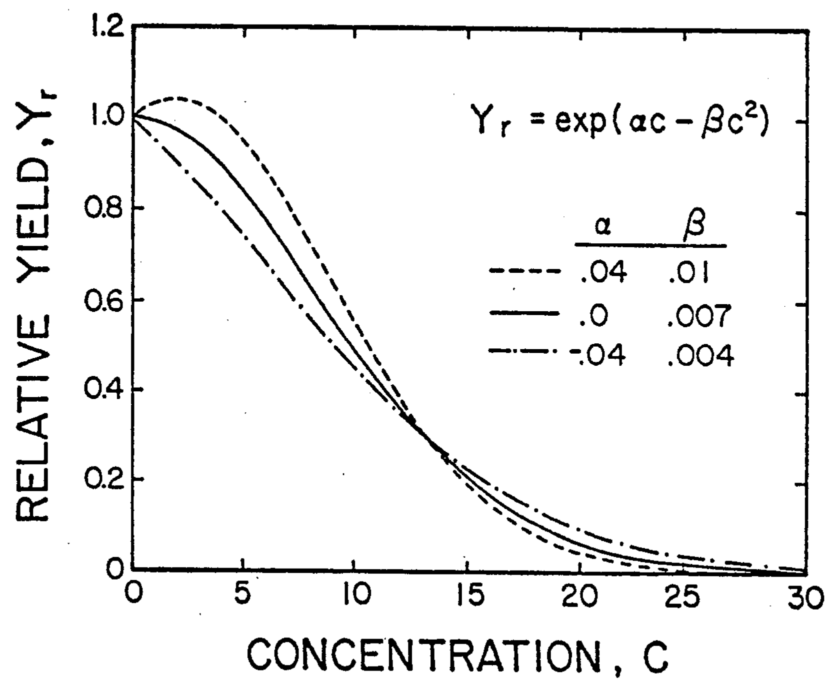

Figure 3. Plot of the exponential salt tolerance response function (Eq. 9; $Y_r = Y/Y_m$).

functions based on Eq. [9] are used whenever $18 < \text{NOPT} < 20$ (Table 1). An example is given in section 3.2.

3. EXAMPLES

This section presents several examples illustrating the type of results that can be obtained with the optimization method. The examples, taken from the literature, were chosen such that various program options are clearly demonstrated. The concentration units for each example are the same as those used in the original publication. Appendix B lists the input data that were used in the calculations; computed results for these same input data are given in Appendix C.

3.1. Tall Fescue

This example considers the salt tolerance of tall fescue (Brown and Bernstein, 1953). Figure 4 compares two fitted curves with the observed data. Results for the solid line were obtained with NOPT = 5 (see Table 1), indicating that all three unknowns (c_t, s and Y_m) in Eq. [2] were fitted to the data. Note that only one point is located to the left of the threshold. This shows that the curve could have been calculated also with NOPT = 4, i.e., by fixing Y_m equal to the maximum observed yield and carrying out a two-parameter fit for c_t and s. However, in general it is impossible to know beforehand whether or not only one data point appears to the left of c_t, and hence it is always better to calculate all three unknowns simultaneously using NOPT = 5. Because the required computer time on an IBM 360/91 is in the order of a few seconds (or less), there is also no reason to limit the number of unknowns in the program by artificially fixing Y_m.

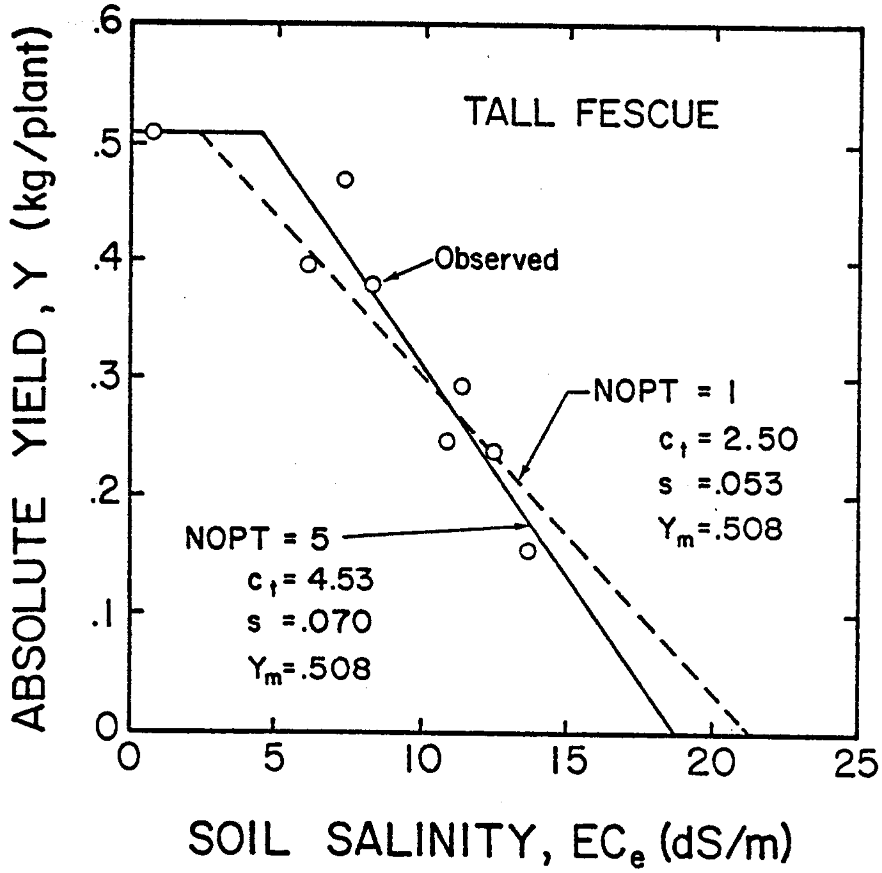

Figure 4. Observed and fitted salt tolerance response functions for tall fescue (data from Brown and Bernstein, 1953).

The dashed line in Fig. 4 is based on a linear regression fit of all data (NOPT = 1). Using this method and assuming that Y_m is equal to the control yield of the first data point, a drastically different threshold value is obtained: 2.50 for NOPT = 1 as compared to 4.53 for NOPT = 5. On the other hand, if the first data point was deleted from the data set, linear regression in this case would have generated exactly the same results as the complete three-parameter fit using NOPT = 5.

3.2. Perennial Rye

Results for the salt tolerance of perennial rye (Brown and Bernstein, 1953), shown in Fig. 5, are very similar to those of the previous example. Again, only one data point appears to the left of the threshold value, indicating that NOPT = 4 and NOPT = 5 would have produced exactly the same results. Also the use of linear regression techniques (NOPT = 1) would have lead to the same results, again provided that the first data point is deleted from the data set.

Figure 6 shows results for the same perennial rye data when Eq. [9], rather than Eq. [2], is fitted to the data. Note that the parameter α was found to be positive, causing the curve to acquire a maximum at $c = \alpha/2\beta = 2.5$ dS/m. Judging from Figs. 5 and 6, the exponential curve failed to produce better results than the piecewise linear model. This conclusion also follows from a comparison of the sum of the squared deviations of the observed (Y_i) versus the fitted (Y_i') yield values (SSQ):

$$SSQ = \sum_{i=1}^{n} (Y_i - Y_i')^2 \qquad [10]$$

Nearly identical values of SSQ were obtained for the two models: .0214 for

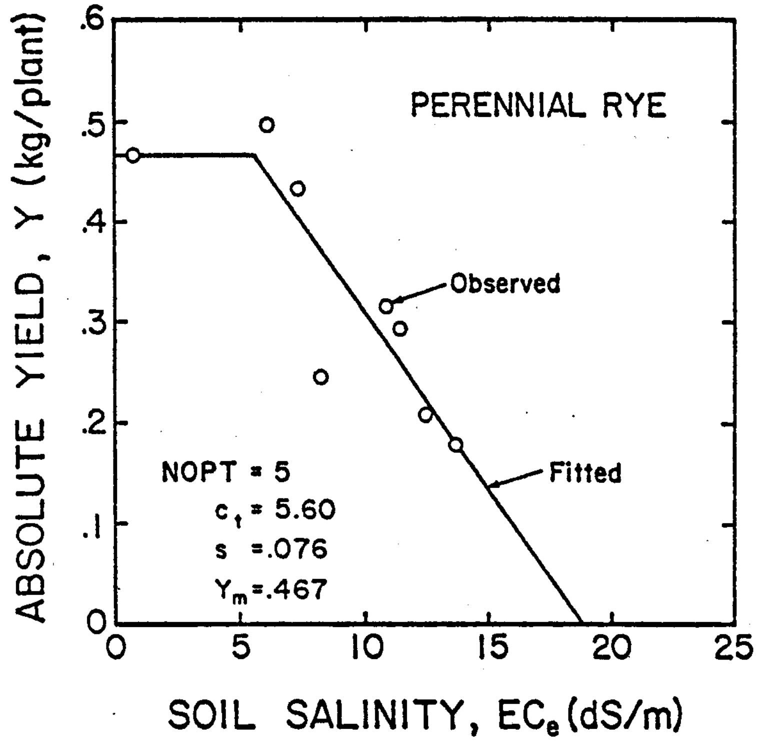

Figure 5. Observed and fitted salt tolerance response functions for perennial rye (data from Brown and Bernstein, 1953). The fitted curve was based on Eq. [2].

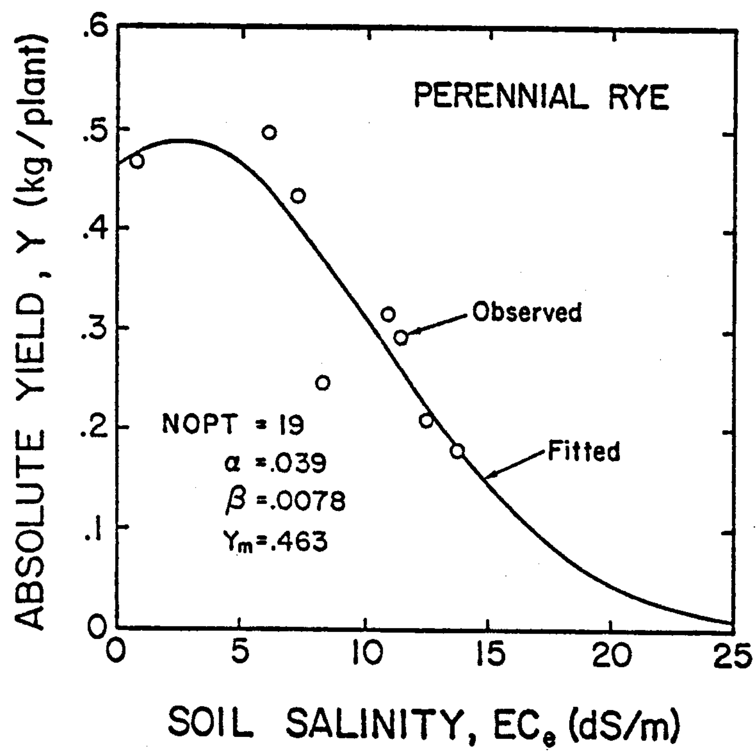

Figure 6. Observed and fitted salt tolerance response functions for perennial rye (data from Brown and Bernstein, 1953). The fitted curve was based on Eq. [9].

the piecewise linear model and .0231 for the exponential model. Hence, both models are about equally successful in describing the salt tolerance data of perennial rye.

3.3. Tomato

Figure 7 shows salt tolerance data for tomato (Osawa, 1965). The dashed line represents the complete three-parameter fit based on Eq. [2] (NOPT = 5). Note that the threshold concentration appears to the left of the first data point. This situation leads to a unique (well-defined) value for the absolute slope ($Y_m s$). However, the fitted values of c_t and Y_m in this case are meaningless since no data points at the lower salinity values are available to fix these parameters. In fact, different initial estimates of the coefficients in the nonlinear least squares procedure would lead to different fitted values for c_t and s. There are two ways to resolve this problem. One method would be to assume that either c_t is known beforehand and equal to the salinity of the first data point, or that Y_m is known and coincides with the yield of that first point. Either assumption will fix the endpoint of the dashed curve in the upper left part of Fig. 7. One can accomplish this in the program by using either NOPT= 3 (c_t is fixed) or NOPT = 4 (Y_m is fixed), respectively. Unfortunately, this method still results in either a Y_m-value that is less than the yield associated with the first data point (NOPT = 3), or in a threshold salinity value that lies to the left of this point (NOPT = 4).

An alternative and more realistic approach would be to fix both c_t and Y_m by their values at the first data point in Fig. 7. In the program this is accomplished with a one-parameter fit for s based on Eq. [7] (NOPT = 2). Actually, the program switches automatically from NOPT = 5 to NOPT

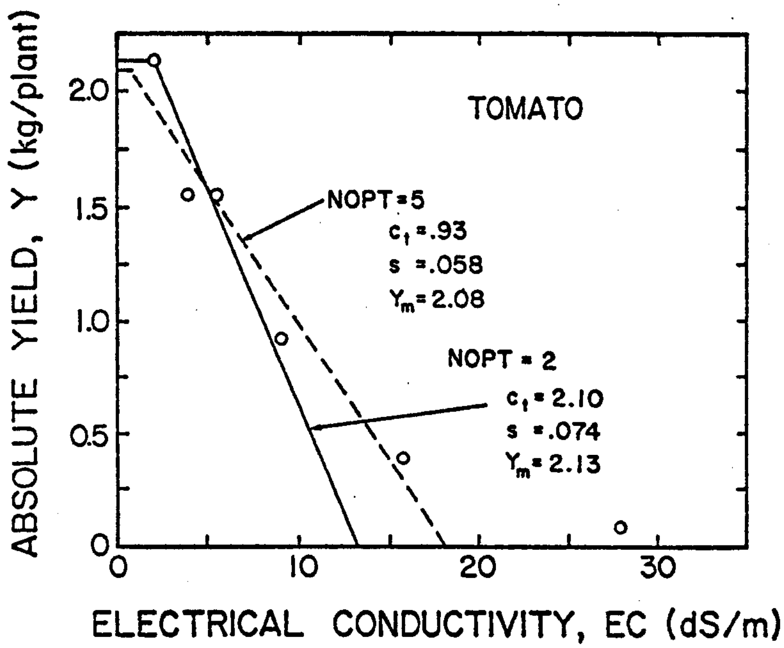

Figure 7. Observed and fitted salt tolerance response functions for tomato (data from Osawa, 1965)

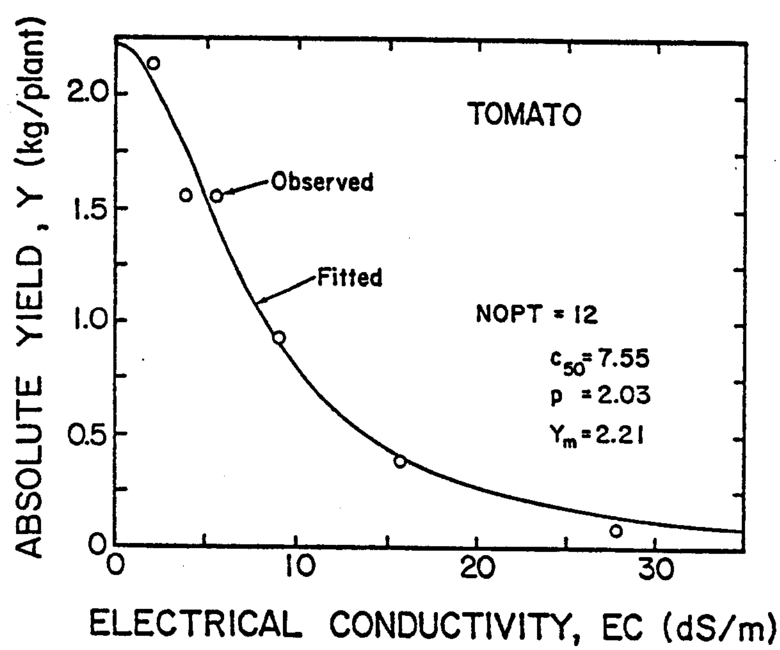

Figure 8. Observed and fitted salt tolerance response functions for tomato (data from Osawa, 1965). The fitted curve was based on Eq. [8].

= 2 whenever all observed data points are to the right of the fitted threshold value. The solid line in Fig. 7 was obtained with NOPT = 2. Note also that two data points appear to the right of c_o, the intersection between the fitted line and the concentration axis. The program uses an iterative procedure such that all points to the right of c_o are automatically discarded from the data set. In other words, no data points are included in the analysis whenever those points produce negative yield values as calculated with the fitted curve (see the computer output for this example in Appendix C).

For illustrative purposes, the same tomato data were analyzed also with Eq. [8]. Results are presented in Fig. 8. Clearly, Eq. [8] leads to a much better fit of the data than the piecewise linear response model, especially at higher salinity levels.

3.4. Grapefruit

Example 4 analyzes the salt tolerance of grapefruit using the same data as listed in a recent study by Feinerman et al. (1982). The data first were analyzed with NOPT = 5, i.e., for the three unknown parameters Y_m, c_t and s in Eq. 2. Figure 9 compares the fitted curve with the observed data points. Note that all data are located in the upper left part of the figure close to the threshold value. Because of a lack of observed data at the higher soil salinities, both the threshold and the slope of the curve have extremely large standard errors (see Appendix C). Actually, this was the only example that exhibited uniqueness problems during the inversion process. Uniqueness problems become apparent when different initial estimates in the computer program generate different values for the fitted parameters. The least squares method is

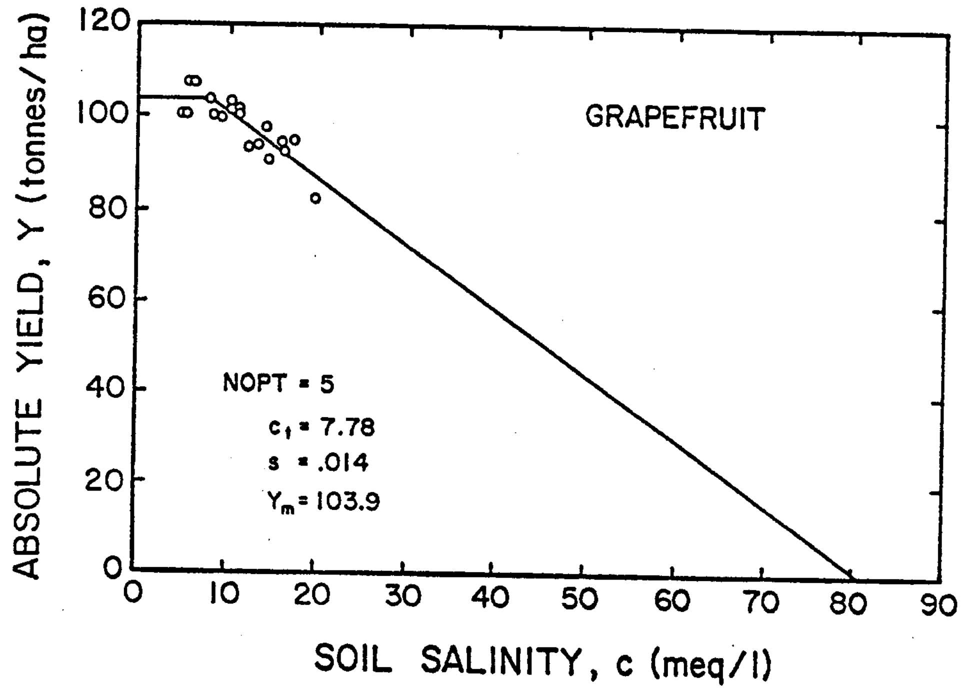

Figure 9. Observed and fitted salt tolerance response functions for grapefruit (data from Feinerman et al., 1982). The fitted curve was based on Eq. [2].

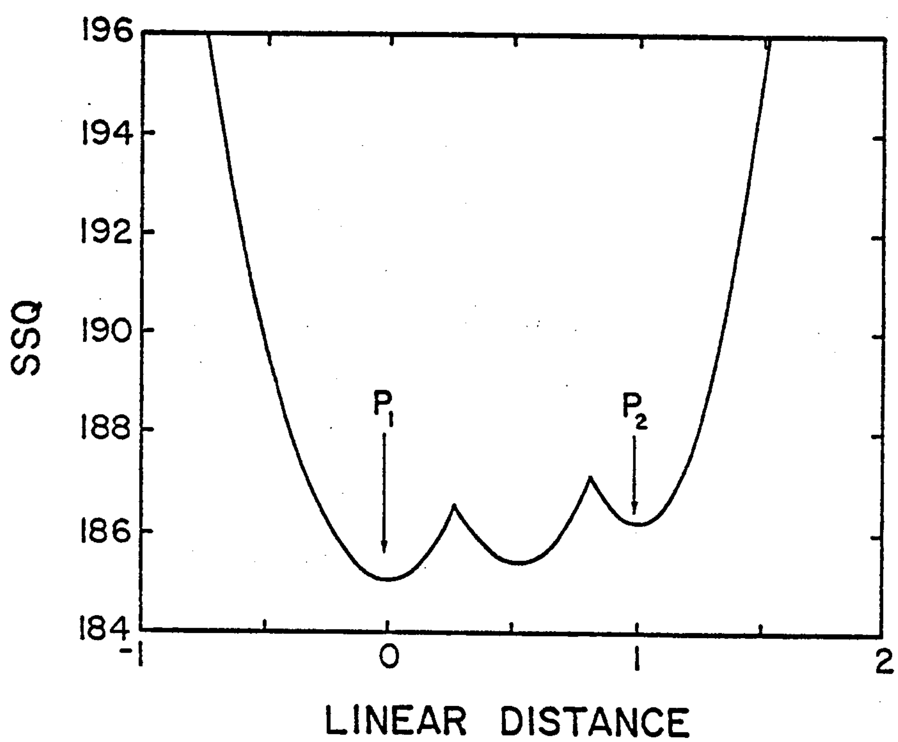

Figure 10. Plot of SSQ evaluated along a linear line through the points P_1 and P_2 (see sext for explanation).

based on the principle that the sum of squares (SSQ) of the deviations between the observed (Y_i) and calculated yields (Y_i') is minimized (see Eq. 10). In general, SSQ can be viewed as a three-dimensional function of the unknown parameters Y_m, c_t and s. In some cases, this function may manifest multiple minima to which the inversion method can converge. For the present example, several minima of SSQ were observed, one of which was located at $P_1 \equiv (Y_m,c_t,s)_1 = (103.95,\ 7.78,\ 0.0137)$, and one at $P_2 \equiv (Y_m,c_t,s)_2 = (102.76,\ 9.72,\ 0.0165)$. Figure 10 shows graphically the variation of SSQ along a straight line through these two points. Note that in actuality three minima with nearly identical SSQ's are present. The fitted line in Fig. 9 uses the parameter values associated with the lowest SSQ (P_1 in Fig. 10). From this figure, it must be clear that in this case little confidence can be attached to the accuracy of the fitted values. Example 1 was the only case encountered that exhibited this type of uniqueness problem. Nevertheless, it is recommended that the least squares inversion method be carried out with at least two different sets of initial estimates whenever the observed data are clustered around the threshold value such as was the case in this example. If the inversion results obtained with widely different initial estimates are identical, then it is probably safe to assume that the inversion is unique.

Results obtained here for grapefruit differ slightly from those obtained by Feinerman et al. (1982). This is because their regression technique differs somewhat from the nonlinear least squares method used in this study. In essence, the technique used by Feinerman et al. (1982) assumes unequal variances for the two line segments on either side of the threshold salinity, while the least squares technique used here assumes that the variances for the two lines are the same. For comparison, the fitted response function obtained by Feinerman et al. (1982) is determined by $(Y_m, c_t, s) \equiv (102.7, 10.28, 0.0181)$, a result that is very close to the point P_2 in Fig. 10.

3.5. Bromegrass

Data on the salt tolerance of bromegrass were taken from a study by McElgunn and Lawrence (1973). These authors published their results in the form of a continuous graph; to obtain a discrete set of data, points at equal soil salinity intervals were taken directly from their Fig. 3. Figure 11 compares the regular three-parameter fit of Eq. [2] (NOPT = 5) with the "observed" data. Note that the data are plotted on a relative yield scale. This is because all data in the original publication were plotted in percentages of the control yield, whereas no information was given about the control yield itself. The fitted value of Y_m (.985; see Fig. 11) was found to be slightly less than the assumed control yield, set here at exactly 1.000. The new relative yield scale hence should run from 0 to 0.985, with s being defined in terms of this new scale.

As an alternative to the three-parameter fit, the same data in Fig. 11 could have been analyzed also by fixing the maximum yield at 1.00 (using NOPT = 4), thereby assuming that the control yield was determined more accurately than the other points. This would be a reasonable assumption if the control yield value were based on several replicates. No information of this kind was available, and hence there was also no reason to put more weight on this control point than on any other point of the observed data set.

Because of the smooth and sigmoidal shape of the observed curve, the same grapefruit data were analyzed also with Eq. [8]. Figure 12 shows

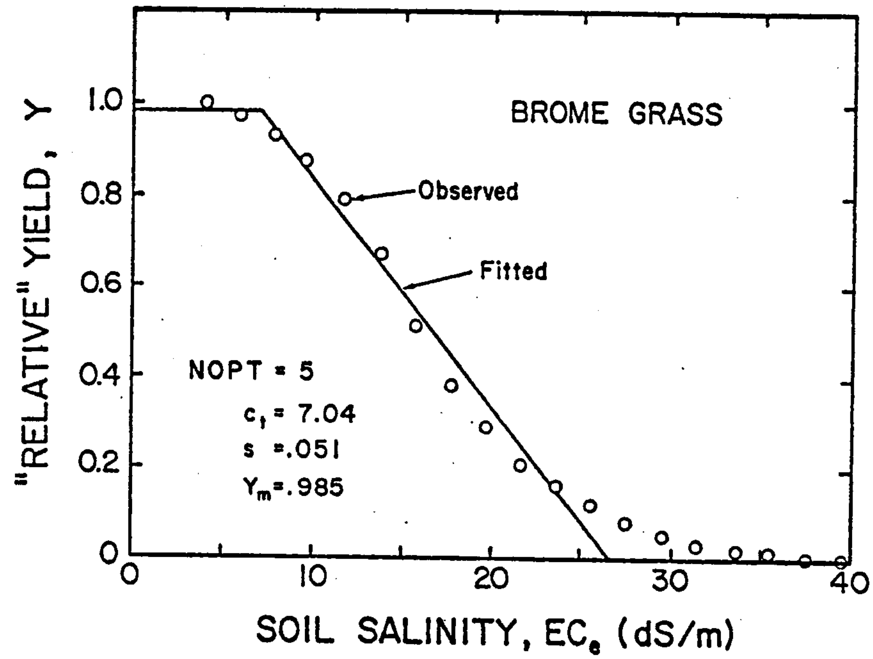

Figure 11. Observed and fitted salt tolerance response functions for bromegrass (data from McElgunn and Lawrence, 1973). The fitted curve was based on Eq. [2].

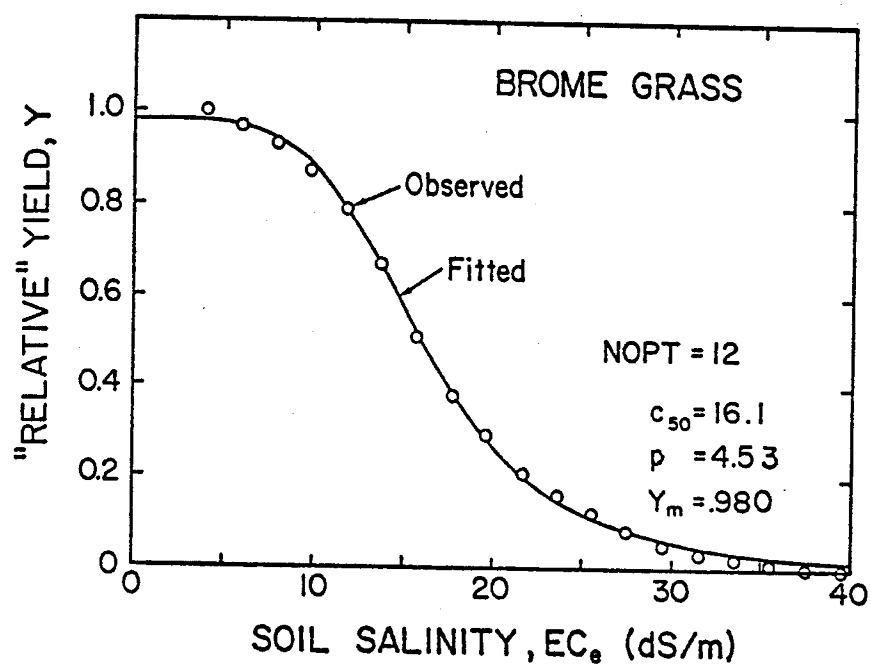

Figure 12. Observed and fitted salt tolerance response functions for bromegrass (data from McElgunn and Lawrence, 1973). The fitted curve was based on Eq. [8].

that this particular response model leads to an excellent fit of the data. A similar fit with the exponential model based on Eq. [9] with $\alpha < 0$ (NOPT = 19) was found to be somewhat less successful than the fit based on Eq. [8] (results not shown here).

3.6. Corn

Hoffman et al. (1983) recently conducted a series of field experiments to establish the salt tolerance of corn grown on the organic soils of the Sacramento - San Joaquin Delta in California. The experiments were carried out over a period of three years on both sprinkler-irrigated and subsurface-irrigated plots. The experimental data for the different years and irrigation treatments are shown in Figure 13. Assuming that the threshold salinity (c_t) and the relative slope (s) not only are time-independent, but also independent of the irrigation method, an 8-parameter fit based on the piecewise linear reponse model (Eq. 2; NOPT = 10) was carried out. Hence, this approach assumes that c_t and s are constant, but that the control yield Y_m^i (i = 1,6) can vary from year to year and also as a function of the irrigation method. Figure 13 shows that the fitted curves indeed are different only with respect to the absolute yield axis. In particular, note that the values of c_t and c_o are identical for the six fitted curves. By dividing the absolute yields of the different treatments with the appropriate Y_m^i-value, the experimental data can be normalized into relative yield fractions as shown in Fig. 14 (see also the computer output in Appendix C). In conclusion, the solid line in Fig. 14 expresses the relative salt tolerance of corn grown on the organic soils of the Sacramento - San Joaquin Delta from 1979 until 1981 as determined by two irrigation methods.

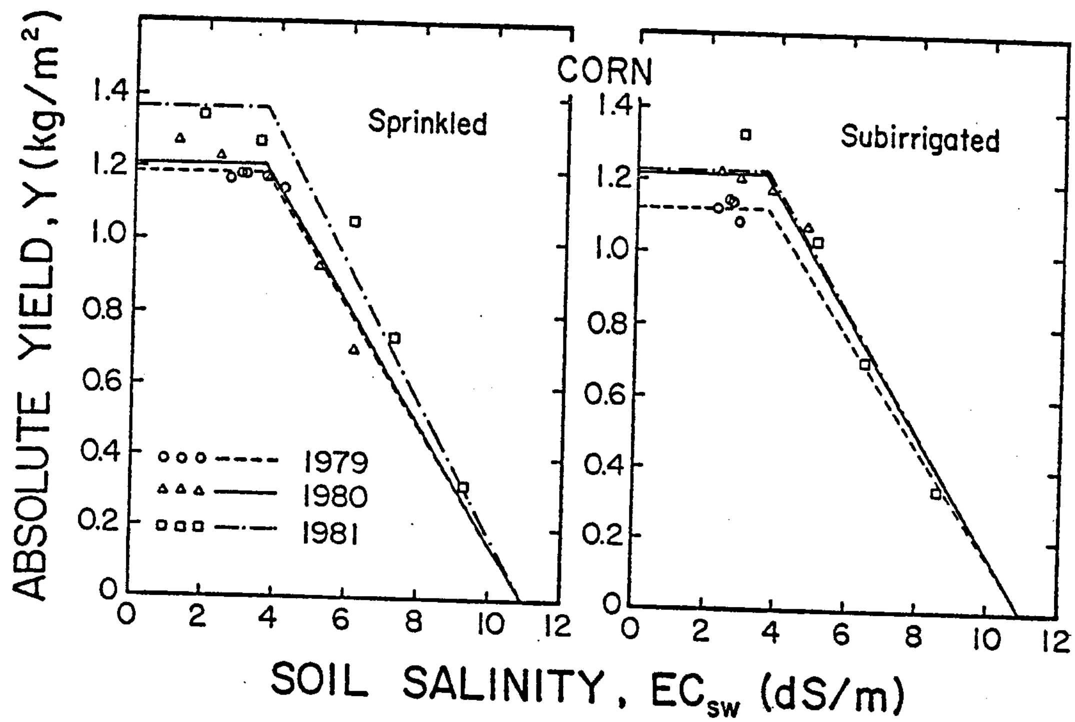

Figure 13. Observed and fitted salt tolerance response functions for corn (data from Hoffman et al., 1983).

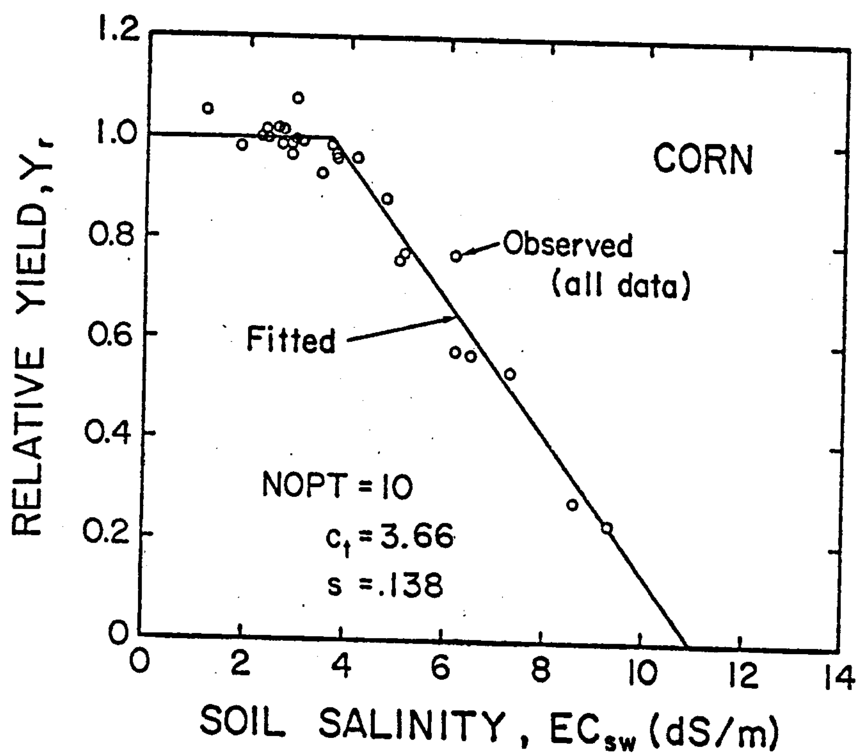

Figure 14. Plot of the relative salt tolerance function for corn as determined from the six fitted curves in Fig. 12.

It is also possible to analyze the same corn data by considering the experimental data separately for the two irrigation methods. This can be done by twice carrying out a 5-parameter fit (NOPT = 7; see Table 1) on both the sprinkler and the subirrigated data. Effects of the irrigation method on the fitted values of the threshold and the slope were found to be relatively small; results of this type are discussed in more detail by Hoffman et al. (1983).

4. SUMMARY AND CONCLUSIONS

This study shows that salt tolerance data can be analyzed conveniently by coupling an appropriate salt tolerance model with a least squares optimization method. A piecewise linear response function historically has been the most popular model. As shown here, other and equally useful response functions can be defined also. The computer model described in this report provides an efficient and accurate tool for quantifying the unknown parameters that appear in three different response functions. To allow for flexibility in analyzing different types of data sets, 20 options were included in the program. These options relate to the choice of a particular salt tolerance model, and to the type and number of unknown parameters that appear in that model. A particularly useful feature of the program is its ability to consider salt tolerance data that have been collected over a period of several years, or that pertain to different management conditions (e.g., leaching fractions or irrigation methods). For the linear response model, this approach assumes that the salinity threshold and the relative slope remain constant, whereas the control yield is allowed to vary from year to year or among different treatments.

In general, few uniqueness problems were observed when applying the nonlinear least squares inversion method. In one example, the observed data were found to be clustered in a relatively small portion of the salinity response curve. Data of this type can lead to large standard errors of the unknown coefficients. It is recommended that salt tolerance trials be carried out over a relatively broad range of salinity values with concomitant broad variations in observed crop yields. Such data lead to a better definition of the response function and produce smaller standard errors of the coefficients.

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6. APPENDIX A. Description of Computer Program

This appendix gives a brief description of SALT, a computer program that can be used to calculate the unknown parameters in three different salt tolerance response models. The parameters are obtained by means of a least squares fit of Eqs. [2], [8] or [9] to the observed data. Table 1 in the text lists the various options that are available. These options relate to the type of response model adopted in the program, and to the number and type of parameters assumed to be unknown in that model. Choice of a particular option is governed by the input parameter NOPT.

The computer program, listed in Appendix D, consists of a main program (MAIN) and two subroutines (MODEL and MATINV). Most of the calculations for the least squares analysis are carried out in MAIN, including the input and output instructions, calculation of a correlation matrix between the unknown coefficients, and calculation of a 95% confidence interval for each unknown coefficient (only for NOPT > 3). Subroutine MODEL calculates for each reponse model the absolute yield as a function of the root zone salinity, whereas subroutine MATINV describes a matrix inversion scheme that is needed for the least squares analysis in MAIN.

Table A1 defines the most significant variables that appear in SALT, while Table A2 gives instructions of how to set up the input data file. Some additional comments regarding these input data are given below. Appendix B lists the actual input file used for the examples in this study. The computer printout for these same input data is shown in Appendix C.

The first input parameter NC (Table A2) determines the number of examples that are listed in the data input file. Hence, when NC > 1, the input file contains information for more than one example. Each particular example starts with a title card, followed by a card specifying the values of KNOB, NOPT, KBI and KB. If KNOB is positive, its value equals NOB, the number of observed data points. However, when KNOB is negative (or zero), the remaining information of that particular example still is read in, but the example is not executed. This feature allows one to set up a large file of observed data without having to execute again every case whenever some of the input parameters (e.g, NOPT) are modified. The input parameter KBI indicates whether new names of the unknown coefficients are read in for the example in question. For the first example (NCASE = 1), a card with the appropriate coefficient names must always be supplied. However, when NCASE > 1, the card with the coefficient names can be skipped if so desired. This is done by setting KBI = 0 (card 3, Table A2). In that case, the coefficient names of the previous example will be used. Finally, the input parameter KB specifies whether the initial estimates B(I) are specified in the input file (KB = 1), or are generated internally in the program (KB = 0). The option KB = 0 can be used only in connection with the piecewise linear response model (Eq. 2) and for NOPT $\leq$ 5. For all other NOPT-values, KB must be set equal to one, indicating than initial estimates of the various coefficients will be read in from the data file.

The fourth data card (Table A3) specifies the coefficient names and, as mentioned above, is needed only when either NCASE = 1 or KBI = 1 (or both). For NOPT $\leq$ 10, the first three coefficient names relate to the slope (s), the salinity threshold (c_t) and the control yield (Y_m or Y_m^1), respectively. For 11 $\leq$ NOPT $\leq$ 17, the first three coefficient names are related to c_{50}, p and Y_m, in that order. When NOPT = 18 or 19, the coefficient names are related to α, β and Y_m, again in that order. Finally, when NOPT = 20, the coefficient names are associated with β and Y_m, respectively.

The fifth data card specifies the initial estimates of the unknown coefficients, unless these estimates are generated internally and not needed in the input file (KB = 0). The initial estimates are related to the unknown coefficients in exactly the same way as explained in the previous paragraph for the coefficient names. The program requires that the initial estimates be inputted in exactly that same sequence. Also, if one of the coefficients (e.g., Y_m) is known independently and fixed in the program, the value of that parameter still must be specified at its proper location on the fifth data card.

Table A1. List of the most significant variables in SALT.

VARIABLE	DEFINITION
B(I)	Vector containing the values of the unknown coefficients.
BI(I)	Vector of coefficient names.
C(I)	Vector of observed concentrations.
CMAX	Maximum of observed concentrations.
CMIN	Minimum of observed concentrations.
CZERO	Calculated value of c_o.
IND(I)	Index for each data point indicating if only one value of Y_m (INDEX = 1) or multiple Y_m-values (INDEX > 1) are fitted to the data (see input file for an example).
KNOB	Input parameter indicating the number of observed data points that will be read in; if KNOB < 0, the example is not executed.
MIT	Maximum number of iterations allowed in the least squares analysis (arbitrarily set at 20).
NB	Input parameter indicating whether the initial estimates of the coefficients are given in the input file (NB = 1), or

	whether they are generated internally in the program (NB = 0). NB is set to one in the program for all NOPT-values exceeding 10.
NBI	Input parameter indicating whether the coefficient names are given in the input file (NBI = 1) or whether they are assumed to be the same as for the previous example. NBI is set to one in the program for all NOPT-values exceeding 10.
NC	Number of cases to be executed.
NIT	Iteration number during least squares analysis.
NOB	Number of data points (equal to the absolute value of KNOB).
NOPT	Option number indicating the type of response model and the number of unknown coefficients in that model (see Table 1).
NP	Number of unknown coefficients; generated internally as a function of the input parameter NOPT.
NPA(I)	Vector used to generate the value of NP as a function of NOPT.
SLOPE	Assumed or fitted value of the slope.
SR	Calculated correlation coefficient for the linear fit (NOPT = 1).
SREL	Value of the relative slope, s (NOPT = 1,2).
SSQ	Residual sum of squares.
STDA	Standard error of the fitted value of Y_o in Eq. [5] (NOPT = 1).
STDB	Standard error of the fitted value of s_1 in Eq. [5] (NOPT = 1).
STOPCR	Stop criterion. The iterative curve-fitting process is terminated when the relative change in the ratio of all coefficients becomes less than STOPCR. The value of STOPCR is arbitrarily set at .00001.
TITLE(I)	Vector containing the information of the title card (input label).
Y(I)	Vector of observed yields.
YMAX	Maximum of observed yields.
YMIN	Minimum of observed yields.
YZERO	Calculated value of Y_o.

Table A2. Data Input instructions.

CARD	COLUMNS	VARIABLE	FORMAT	COMMENTS
1	1-5	NC	I5	Number of cases considered. The cards below are read in for each example.
2	1-80	TITLE	20A4	Title card.
3	1-5	KNOB	I5	Number of observation if KNOB > 0; if KNOB < 0, the remaining cards for this case still are read in, but the example is not executed.
3	6-10	NOPT	I5	Option number (see Table 1).
3	11-15	KBI	I5	Input code for the coefficient names. If KBI = 1, the names (card 4) are read in; if KBI = 0, the names are known from an earlier example (see text for explanation).
3	16-20	KB	I5	Input code for initial coefficient estimates. If KB = 1, the values are read in; if KB = 0, the values are generated internally and card 5 is skipped (see text for explanation).
4	1-80	BI(I)	8(A4,A2,4X)	Coefficient names (only needed if KBI = 1)
5	1-80	B(I)	8F10.0	Initial values of the coefficients (only needed in KB = 1).
6	1-10	C(I)	F10.0	Value of observed concentration.
6	11-20	Y(I)	F10.0	Value of observed yield.
6	21-30	IND(I)	I10	Value of IND(I) associated with the observed data point on this card.
				Card 6 is repeated for all NOB observations.

7. APPENDIX B. Listing of Input Data

```
                   1         2         3         4         5         6         7         8
Column:   12345678901234567890123456789012345678901234567890123456789012345678901234567890
Card
  1           6
  2                    EXAMPLE 1: TALL FESCUE    (CR53)
  3           8    1    1    0
  4       SLOPE        THRESH     YM
  5                0.8       .508
  6                6.1       .397
  7                7.3       .470
  8                8.3       .380
  9               10.9       .248
 10               11.4       .293
 11               12.5       .238
 12               13.7       .157
 13                    EXAMPLE 2: PERENNIAL RYE    (CR53)
 14           8    2    0    0
 15                0.8       .467
 16                6.1       .496
 17                7.3       .433
 18                8.3       .245
 19               10.9       .317
 20               11.4       .293
 21               12.5       .208
 22               13.7       .179
 23                    EXAMPLE 3: TOMATO   (OSAWA, 1965)
 24           6    3    0    0
 25                2.1      2.133
 26                3.9      1.557
 27                5.6      1.557
 28                9.1       .917
 29               15.8       .384
 30               28.0       .085
 31                    EXAMPLE 4: GRAPEFRUIT (FEINERMAN ET AL., 1982)
 32          19    5    0    0
 33              5.069      100.7
 34              5.469      100.4
 35              5.476      107.4
 36              6.410      107.3
 37              7.980      103.9
 38              8.306      100.2
 39              9.346       99.4
 40             10.325      103.0
 41             10.358      101.9
 42             11.381      101.7
 43             11.467      100.7
 44             12.414       93.1
 45             13.411       93.8
 46             14.399       97.8
 47             14.461       90.3
 48             16.172       94.6
 49             16.457       92.6
 50             17.346       94.8
 51             19.822       81.9
```

```
                 1         2         3         4         5         6         7         8
Column: 12345678901234567890123456789012345678901234567890123456789012345678901234567890
Card
 52               EXAMPLE 5: BROME GRASS   (MCELGUNN AND LAWRENCE, 1973)
 53        19   12    1    1
 54      C50      P-EXP      YM
 55           15.0       2.0       1.00
 56            4.0      1.00
 57            5.9       .97
 58            7.9       .93
 59            9.85      .87
 60           11.8       .79
 61           13.8       .67
 62           15.75      .51
 63           17.75      .38
 64           19.7       .29
 65           21.65      .21
 66           23.65      .16
 67           25.6       .12
 68           27.5       .08
 69           29.5       .05
 70           31.5       .03
 71           33.5       .02
 72           35.5       .01
 73           37.5       .00
 74           39.5       .00
 75               EXAMPLE 6: CORN    (GRAIN)  (HOFFMAN ET AL., 1983)
 76        27   10    1    0
 77     SLOPE     THRESH    SPR-79    SPR-80    SPR-81    SUB-79    SUB-80    SUB-81
 78            2.7      1.17          1         SPRINKLER-IRRIGATED PLOTS, 1979
 79            3.0      1.18          1
 80            3.1      1.18          1
 81            3.7      1.17          1
 82            4.2      1.14          1
 83            1.2      1.27          2
 84            2.4      1.23          2         SPRINKLER-IRRIGATED PLOTS, 1980
 85            3.8      1.17          2
 86            5.2      0.93          2
 87            6.2      0.70          2
 88            1.9      1.34          3
 89            3.5      1.27          3         SPRINKLER-IRRIGATED PLOTS, 1981
 90            6.1      1.05          3
 91            7.3      0.73          3
 92            9.3      0.32          3
 93            2.4      1.12          4
 94            2.6      1.14          4         SUBSURFACE IRRIGATED PLOTS, 1979
 95            2.7      1.14          4
 96            2.9      1.08          4
 97            2.3      1.22          5
 98            2.9      1.20          5         SUBSURFACE IRRIGATED PLOTS, 1980
 99            3.8      1.17          5
100            4.8      1.07          5
101            3.0      1.32          6
102            5.1      0.93          6         SUBSURFACE IRRIGATED PLOTS, 1981
103            6.5      0.70          6
104            8.6      0.34          6
```

8. APPENDIX C. Listing of Computer Output

```
********************************************************************************
*                                                                              *
*          LEAST SQUARES ANALYSIS OF SALINITY RESPONSE CURVE          SALT    *
*                                                                              *
*          EXAMPLE 1: TALL FESCUE    (CR53)                                    *
*          NOPT =  1; NP =  2                                                  *
*                                                                              *
********************************************************************************

LINEAR REGRESSION RESULTS FOR Y=YZERO-SLOPE*C
============================================

YZERO =      0.5752 WITH STANDARD ERROR OF      0.0447
SLOPE =      0.0269 WITH STANDARD ERROR OF      0.0046
CORRELATION COEFFICIENT =      0.9222


CONTROL YIELD (YM) =      0.5080
THRESHOLD (CT) =      2.4985

SLOPE FOR ORIGINAL DATA (S*YM) =  0.026915
SLOPE FOR RELATIVE YIELD DATA (S) =  0.052983
INTERSECTION AT ZERO SALINITY (YZERO) =    0.57525
SALINITY EXTRAPOLATED TO ZERO YIELD (CZERO)=  21.37256
```

CONC	Y-OBS	Y-FITTED	DEVIATION	REL YIELD	INDEX
0.8000	0.5080	0.5537	-0.0457	1.0000	1
6.1000	0.3970	0.4111	-0.0141	0.7815	1
7.3000	0.4700	0.3788	0.0912	0.9252	1
8.3000	0.3800	0.3519	0.0281	0.7480	1
10.9000	0.2480	0.2819	-0.0339	0.4882	1
11.4000	0.2930	0.2684	0.0246	0.5768	1
12.5000	0.2380	0.2388	-0.0008	0.4685	1
13.7000	0.1570	0.2065	-0.0495	0.3091	1

```
*******************************************************************************
*                                                                             *
*          LEAST SQUARES ANALYSIS OF SALINITY RESPONSE CURVE          SALT    *
*                                                                             *
*          EXAMPLE 2: PERENNIAL RYE    (CR53)                                 *
*          NOPT =  2; NP =  1                                                 *
*                                                                             *
*******************************************************************************
```

CONTROL YIELD (YM) = 0.4670
THRESHOLD (CT) = 0.8000

SLOPE FOR ORIGINAL DATA (S*YM) = 0.016997
SLOPE FOR RELATIVE YIELD DATA (S) = 0.036396
INTERSECTION AT ZERO SALINITY (YZERO) = 0.48060
SALINITY EXTRAPOLATED TO ZERO YIELD (CZERO)= 28.27557

CONC	Y-OBS	Y-FITTED	DEVIATION	REL YIELD	INDEX
0.8000	0.4670	0.4670	0.0000	1.0000	1
6.1000	0.4960	0.3769	0.1191	1.0621	1
7.3000	0.4330	0.3565	0.0765	0.9272	1
8.3000	0.2450	0.3395	-0.0945	0.5246	1
10.9000	0.3170	0.2953	0.0217	0.6788	1
11.4000	0.2930	0.2868	0.0062	0.6274	1
12.5000	0.2080	0.2681	-0.0601	0.4454	1
13.7000	0.1790	0.2477	-0.0687	0.3833	1

```
*******************************************************************************
*                                                                             *
*     LEAST SQUARES ANALYSIS OF SALINITY RESPONSE CURVE          SALT         *
*                                                                             *
*     EXAMPLE 3: TOMATO  (OSAWA, 1965)                                        *
*     NOPT =  3; NP =  2                                                      *
*                                                                             *
*******************************************************************************
```

NIT	SSQ	SLOPE	YM	THRESH
0	2.78435	0.03707	2.09034	7.00000
1	0.83345	0.04798	1.53612	7.00000
2	0.43518	0.09659	1.64509	7.00000
3	0.43342	0.09383	1.64622	7.00000
4	0.43342	0.09380	1.64623	7.00000
5	0.43342	0.09380	1.64623	7.00000

CORRELATION MATRIX
==================

	1	2
1	1.0000	
2	0.1858	1.0000

VARIABLE	VALUE	S.E.COEFF	T-VALUE	95% CONFIDENCE LIMITS LOWER	UPPER
SLOPE	0.093800	0.02249	4.170	0.0313	0.1563
YM	1.646228	0.17477	9.420	1.1610	2.1314

SLOPE FOR ORIGINAL DATA (S*YM) = 0.154416
SLOPE FOR RELATIVE YIELD DATA (S) = 0.093800
INTERSECTION AT ZERO SALINITY (YZERO) = 3.46285
SALINITY EXTRAPOLATED TO ZERO YIELD (CZERO)= 17.66096

CONC	Y-OBS	Y-FITTED	DEVIATION	REL YIELD	INDEX
2.1000	2.1330	1.6462	0.4868	1.2957	1
3.9000	1.5570	1.6462	-0.0892	0.9458	1
5.6000	1.5570	1.6462	-0.0892	0.9458	1
9.1000	0.9170	1.3220	-0.4050	0.5570	1
15.8000	0.3840	0.2874	0.0966	0.2333	1
28.0000	0.0850	0.0	0.0850	0.0516	1

```
**********************************************************************************
*                                                                                *
*         LEAST SQUARES ANALYSIS OF SALINITY RESPONSE CURVE               SALT   *
*                                                                                *
*         EXAMPLE 4: GRAPEFRUIT (FEINERMAN ET AL., 1982)                         *
*         NOPT =  5; NP =  3                                                     *
*                                                                                *
**********************************************************************************
```

NIT	SSQ	SLOPE	THRESH	YM
0	430.74512	0.01609	5.57590	105.25198
1	195.60670	0.01426	8.21826	102.92697
2	185.06572	0.01361	7.76376	103.92889
3	185.05125	0.01369	7.77908	103.94966
4	185.05118	0.01369	7.77895	103.94998
5	185.05118	0.01369	7.77860	103.95018
6	185.05124	0.01369	7.77860	103.95018

CORRELATION MATRIX
=================

	1	2	3
1	1.0000		
2	0.6341	1.0000	
3	-0.0893	-0.7293	1.0000

VARIABLE	VALUE	S.E.COEFF	T-VALUE	95% CONFIDENCE LIMITS LOWER	UPPER
SLOPE	0.013693	0.00251	5.461	0.0084	0.0190
THRESH	7.778600	1.63822	4.748	4.3056	11.2516
YM	103.950180	1.70056	61.127	100.3450	107.5554

SLOPE FOR ORIGINAL DATA (S*YM) = 1.423402
SLOPE FOR RELATIVE YIELD DATA (S) = 0.013693
INTERSECTION AT ZERO SALINITY (YZERO) = 115.02222
SALINITY EXTRAPOLATED TO ZERO YIELD (CZERO)= 80.80794

CONC	Y-OBS	Y-FITTED	DEVIATION	REL YIELD	INDEX
5.0690	100.7000	103.9502	-3.2502	0.9687	1
5.4690	100.4000	103.9502	-3.5502	0.9658	1
5.4760	107.4000	103.9502	3.4498	1.0332	1
6.4100	107.3000	103.9502	3.3498	1.0322	1
7.9800	103.9000	103.6635	0.2365	0.9995	1
8.3060	100.2000	103.1995	-2.9995	0.9639	1
9.3460	99.4000	101.7191	-2.3191	0.9562	1
10.3250	103.0000	100.3256	2.6744	0.9909	1
10.3580	101.9000	100.2787	1.6213	0.9803	1
11.3810	101.7000	98.8225	2.8775	0.9784	1
11.4670	100.7000	98.7001	1.9999	0.9687	1
12.4140	93.1000	97.3521	-4.2522	0.8956	1
13.4110	93.8000	95.9330	-2.1330	0.9024	1
14.3990	97.8000	94.5267	3.2733	0.9408	1
14.4610	90.3000	94.4384	-4.1384	0.8687	1
16.1720	94.6000	92.0030	2.5970	0.9101	1
16.4570	92.6000	91.5973	1.0027	0.8908	1
17.3460	94.8000	90.3319	4.4681	0.9120	1
19.8220	81.9000	86.8076	-4.9076	0.7879	1

```
******************************************************************************
*                                                                            *
*         LEAST SQUARES ANALYSIS OF SALINITY RESPONSE CURVE          SALT    *
*                                                                            *
*         EXAMPLE 5: BROME GRASS    (MCELGUNN AND LAWRENCE, 1973)            *
*         NOPT = 12; NP =  3                                                 *
*                                                                            *
******************************************************************************
```

NIT	SSQ	C50	P-EXP	YM
0	0.31535	15.00000	2.00000	1.00000
1	0.11802	18.45212	3.77748	0.87089
2	0.01553	15.31569	4.18366	0.98748
3	0.00312	16.13985	4.48685	0.97950
4	0.00307	16.13974	4.53310	0.98035
5	0.00307	16.14027	4.53221	0.98036
6	0.00307	16.14027	4.53221	0.98036

CORRELATION MATRIX
=================

	1	2	3
1	1.0000		
2	0.5217	1.0000	
3	-0.6993	-0.5296	1.0000

VARIABLE	VALUE	S.E.COEFF	T-VALUE	95% CONFIDENCE LIMITS LOWER	UPPER
C50	16.140274	0.13079	123.408	15.8630	16.4175
P-EXP	4.532212	0.12099	37.459	4.2757	4.7887
YM	0.980359	0.00864	113.499	0.9620	0.9987

CONC	Y-OBS	Y-FITTED	DEVIATION	REL YIELD	INDEX
4.0000	1.0000	0.9786	0.0214	1.0200	1
5.9000	0.9700	0.9702	-0.0002	0.9894	1
7.9000	0.9300	0.9433	-0.0133	0.9486	1
9.8500	0.8700	0.8859	-0.0159	0.8874	1
11.8000	0.7900	0.7895	0.0005	0.8058	1
13.8000	0.6700	0.6572	0.0128	0.6834	1
15.7500	0.5100	0.5173	-0.0073	0.5202	1
17.7500	0.3800	0.3862	-0.0062	0.3876	1
19.7000	0.2900	0.2827	0.0073	0.2958	1
21.6500	0.2100	0.2049	0.0051	0.2142	1
23.6500	0.1600	0.1474	0.0126	0.1632	1
25.6000	0.1200	0.1079	0.0121	0.1224	1
27.5000	0.0800	0.0804	-0.0004	0.0816	1
29.5000	0.0500	0.0598	-0.0098	0.0510	1
31.5000	0.0300	0.0452	-0.0152	0.0306	1
33.5000	0.0200	0.0346	-0.0146	0.0204	1
35.5000	0.0100	0.0268	-0.0168	0.0102	1
37.5000	0.0000	0.0210	-0.0210	0.0000	1
39.5000	0.0000	0.0167	-0.0167	0.0000	1

```
**************************************************************************
*                                                                        *
*        LEAST SQUARES ANALYSIS OF SALINITY RESPONSE CURVE        SALT   *
*                                                                        *
*        EXAMPLE 6: CORN   (GRAIN)  (HOFFMAN ET AL., 1983)               *
*        NOPT = 10; NP =  8                                              *
*                                                                        *
**************************************************************************
```

NIT	SSQ	SLOPE	THRESH	SPR-79	SPR-80	SPR-81	SUB-79	SUB-80	SUB-81
0	0.28631	0.09397	2.32500	1.31320	1.31320	1.31320	1.31320	1.31320	1.31320
1	0.10895	0.11864	3.28729	1.19264	1.21002	1.39224	1.04557	1.23992	1.23515
2	0.07137	0.13077	3.46939	1.19769	1.22066	1.37362	1.11925	1.23074	1.23719
3	0.07124	0.13171	3.47784	1.19663	1.21959	1.37004	1.11999	1.22926	1.23793
4	0.07071	0.13316	3.52701	1.19389	1.21636	1.36683	1.12000	1.22600	1.23559
5	0.06912	0.13750	3.66275	1.18626	1.20735	1.36554	1.12000	1.21696	1.22907
6	0.06912	0.13758	3.66185	1.18621	1.20688	1.36580	1.12000	1.21682	1.22849
7	0.06912	0.13758	3.66189	1.18620	1.20688	1.36577	1.12000	1.21681	1.22849
8	0.06912	0.13758	3.66189	1.18620	1.20688	1.36577	1.12000	1.21681	1.22849

CORRELATION MATRIX
=================

	1	2	3	4	5	6	7	8
1	1.0000							
2	0.7756	1.0000						
3	-0.2956	-0.4001	1.0000					
4	-0.2331	-0.4168	0.1701	1.0000				
5	0.1309	-0.0929	0.0447	0.0846	1.0000			
6	0.0000	0.0000	0.0000	0.0000	0.0000	1.0000		
7	-0.2817	-0.4086	0.1648	0.1782	0.0559	0.0000	1.0000	
8	0.0117	-0.2341	0.1008	0.1413	0.1202	0.0000	0.1127	1.0000

VARIABLE	VALUE	S.E.COEFF	T-VALUE	95% CONFIDENCE LIMITS LOWER	UPPER
SLOPE	0.137576	0.00840	16.371	0.1200	0.1552
THRESH	3.661889	0.19820	18.476	3.2470	4.0767
SPR-79	1.186201	0.02990	39.669	1.1236	1.2488
SPR-80	1.206878	0.03356	35.960	1.1366	1.2771
SPR-81	1.365769	0.03866	35.331	1.2849	1.4467
SUB-79	1.120000	0.03016	37.139	1.0569	1.1831
SUB-80	1.216812	0.03454	35.227	1.1445	1.2891
SUB-81	1.228492	0.04492	27.351	1.1345	1.3225

SLOPE FOR ORIGINAL DATA (S*YM) = 0.163193
SLOPE FOR RELATIVE YIELD DATA (S) = 0.137576
INTERSECTION AT ZERO SALINITY (YZERO) = 1.78379
SALINITY EXTRAPOLATED TO ZERO YIELD (CZERO)= 10.93060

CONC	Y-OBS	Y-FITTED	DEVIATION	REL YIELD	INDEX
2.7000	1.1700	1.1862	-0.0162	0.9863	1
3.0000	1.1800	1.1862	-0.0062	0.9948	1
3.1000	1.1800	1.1862	-0.0062	0.9948	1
3.7000	1.1700	1.1800	-0.0100	0.9863	1
4.2000	1.1400	1.0984	0.0416	0.9611	1
1.2000	1.2700	1.2069	0.0631	1.0523	2
2.4000	1.2300	1.2069	0.0231	1.0192	2
3.8000	1.1700	1.1839	-0.0139	0.9694	2
5.2000	0.9300	0.9515	-0.0215	0.7706	2
6.2000	0.7000	0.7855	-0.0855	0.5800	2
1.9000	1.3400	1.3658	-0.0258	0.9811	3
3.5000	1.2700	1.3658	-0.0958	0.9299	3
6.1000	1.0500	0.9077	0.1423	0.7688	3
7.3000	0.7300	0.6822	0.0478	0.5345	3
9.3000	0.3200	0.3064	0.0136	0.2343	3
2.4000	1.1200	1.1200	0.0000	1.0000	4
2.6000	1.1400	1.1200	0.0200	1.0179	4
2.7000	1.1400	1.1200	0.0200	1.0179	4
2.9000	1.0800	1.1200	-0.0400	0.9643	4
2.3000	1.2200	1.2168	0.0032	1.0026	5
2.9000	1.2000	1.2168	-0.0168	0.9862	5
3.8000	1.1700	1.1937	-0.0237	0.9615	5
4.8000	1.0700	1.0263	0.0437	0.8793	5
3.0000	1.3200	1.2285	0.0915	1.0745	6
5.1000	0.9300	0.9854	-0.0554	0.7570	6
6.5000	0.7000	0.7488	-0.0488	0.5698	6
8.6000	0.3400	0.3939	-0.0539	0.2768	6

9. Listing of Computer Program

```
                    MAIN
C
C     ***********************************************************************
C     *
C     *         NON-LINEAR LEAST-SQUARES ANALYSIS              SALT          *
C     *         OF SALINITY-RESPONSE CURVES                                  *
C     *                                                                      *
C     *         JANUARY 20, 1983                                             *
C     *                                                                      *
C     ***********************************************************************
C
C
      DIMENSION C(50),Y(50),F(50),R(50),DELZ(50,8),B(8),E(8),TH(8),P(8)
     1,PHI(8),Q(8),TB(8),A(8,8),BI(32),D(8,8),TITLE(20),IND(50),NPA(13)
      DATA MIT/20/,STOPCR/0.00001/,NPA/2,1,2,2,3,4,5,6,7,8,2,3,2/
C
C     ----- READ NUMBER OF CASES -----
      READ(5,1002) NC
      DO 120 NCASE=1,NC
C
C     ----- READ TITLE AND INPUT PARAMETERS -----
      READ(5,1000) TITLE
      READ(5,1002) KNOB,NOPT,KBI,KB
      IF(NOPT.GE.18) KBI=1
      IF(NOPT.GE.11) KB=1
      I=NOPT
      IF(I.GT.10) I=I-7
      NP=NPA(I)
      IF(KNOB.GT.0) WRITE(6,1003) TITLE,NOPT,NP
      NOB=IABS(KNOB)
C
C     ----- READ INITIAL ESTIMATES -----
      IF(NCASE.EQ.1.OR.KBI.EQ.1) READ(5,1006) (BI(I),I=17,32)
      IF(KB.EQ.1) READ(5,1008) (B(I),I=1,8)
      CMAX=0.0
      CMIN=10000.0
      YMAX=0.0
      YMIN=10000.0
C
C     ----- READ INPUT DATA -----
      DO 2 I=1,NOB
      READ(5,1010) C(I),Y(I),IND(I)
      IF(NP.LE.3) IND(I)=1
      CMIN=AMIN1(CMIN,C(I))
      YMIN=AMIN1(YMIN,Y(I))
      IF(Y(I).LT.0.001) GO TO 2
      CMAX=AMAX1(CMAX,C(I))
    2 YMAX=AMAX1(YMAX,Y(I))
      IF(KNOB.LT.0) GO TO 120
```

MAIN

```
      IF(NOPT-2) 4,10,22
C
C     ----- NOPT = 1: LINEAR REGRESSION -----
    4 SX=0.0
      SY=0.0
      SXY=0.0
      SXX=0.0
      SYY=0.0
      ROB=FLOAT(NOB)
      DO 6 I=1,NOB
      SX=SX+C(I)
      SY=SY+Y(I)
      SXY=SXY+C(I)*Y(I)
      SXX=SXX+C(I)*C(I)
    6 SYY=SYY+Y(I)*Y(I)
      SD=ROB*SXX-SX*SX
      YZERO=(SXX*SY-SX*SXY)/SD
      SLOPE=(SX*SY-ROB*SXY)/SD
      SR=ABS(SLOPE)*SQRT(ABS(SD/(ROB*SYY-SY*SY)))
      SIGMA=(SYY-YZERO*SY+SLOPE*SXY)/(ROB-2.)
      STDA=SQRT(SIGMA*SXX/SD)
      STDB=SQRT(ROB*SIGMA/SD)
      WRITE(6,1012) YZERO,STDA,SLOPE,STDB,SR
      YM=Y(1)
      IF(KB.EQ.1) YM=B(3)
      CT=(YZERO-YM)/SLOPE
      WRITE(6,1014) YM,CT
      SREL=SLOPE/YM
      CZERO=YZERO/SLOPE
      WRITE(6,1034) SLOPE,SREL,YZERO,CZERO
      TH(3)=YM
      DO 8 I=1,NOB
      F(I)=YZERO-SLOPE*C(I)
    8 R(I)=Y(I)-F(I)
      GO TO 109
C
C     ----- NOPT = 2: SLOPE IS FITTED ONLY -----
   10 YM=Y(1)
      CT=C(1)
      IF(NOPT.EQ.2) GO TO 12
      NOPT=2
      IF(MODE.NE.4) KB=0
      WRITE(6,1003) TITLE,NOPT,NP
   12 IF(KB.EQ.0) GO TO 14
      YM=B(3)
      CT=B(2)
   14 SA=0.0
      SB=0.0
```

MAIN

```
      DO 16 I=1,NOB
      SA=SA+Y(I)-YM
   16 SB=SB+C(I)-CT
      SLOPE=-SA/SB
      SREL=SLOPE/YM
      YZERO=YM+SLOPE*CT
      CZERO=CT+YM/SLOPE
      WRITE(6,1014) YM,CT
      WRITE(6,1034) SLOPE,SREL,YZERO,CZERO
      TH(3)=YM
      DO 18 I=1,NOB
      F(I)=YM-SLOPE*(C(I)-CT)
   18 R(I)=Y(I)-F(I)
      IR=0
      DO 20 I=1,NOB
      IF(F(I).GE.0.) GO TO 20
      IR=1
      C(I)=CT
      Y(I)=YM
   20 CONTINUE
      IF(IR.GT.0) GO TO 14
      GO TO 109
C
C     ----- NONLINEAR LEAST-SQUARES ANALYSIS -----
   22 IF(KB.EQ.1) GO TO 26
      B(1)=(1.-YMIN/YMAX)/(CMAX-CMIN)
      B(2)=AMAX1(1.1*CMIN,0.25*CMAX)
      DO 24 I=3,8
   24 B(I)=0.98*YMAX
      IF(NOPT.EQ.4.OR.NOPT.EQ.11) B(3)=Y(1)
   26 IP=MAX0(NP,3)
      IF(NOPT.EQ.22) IP=2
      NP2=2*NP
      IP2=2*IP
      DO 28 I=1,16
   28 BI(I)=BI(I+16)
      NT=0
      B1=B(1)
      B2=B(2)
      B3=B(3)
      IF(NOPT.NE.3) GO TO 30
      B2=B(3)
      B(3)=B(2)
      B(2)=B2
      BI(3)=BI(21)
      BI(4)=BI(22)
      BI(5)=BI(19)
      BI(6)=BI(20)
```

MAIN

```
   30 CONTINUE
      DO 32 I=1,8
      TB(I)=B(I)
   32 TH(I)=B(I)
C
C     ----------
      GA=0.02
      NIT=0
      CALL MODEL(TH,F,NOB,C,IND,NOPT)
      SSQ=0.
      DO 34 I=1,NOB
      R(I)=Y(I)-F(I)
   34 SSQ=SSQ+R(I)*R(I)
      WRITE(6,1016) (BI(J),BI(J+1),J=1,IP2,2)
      WRITE(6,1018) NIT,SSQ,(B(I),I=1,IP)
C
C     ----- BEGIN OF ITERATION -----
   36 NIT=NIT+1
      GA=0.1*GA
      DO 40 J=1,NP
      TEMP=TH(J)
      TH(J)=1.01*TH(J)
      Q(J)=0.0
      CALL MODEL(TH,DELZ(1,J),NOB,C,IND,NOPT)
      DO 38 I=1,NOB
      DELZ(I,J)=DELZ(I,J)-F(I)
   38 Q(J)=Q(J)+DELZ(I,J)*R(I)
      Q(J)=100.*Q(J)/TH(J)
   40 TH(J)=TEMP
      DO 46 I=1,NP
      DO 44 J=1,I
      SUM=0.0
      DO 42 K=1,NOB
   42 SUM=SUM+DELZ(K,I)*DELZ(K,J)
      D(I,J)=10000.*SUM/(TH(I)*TH(J))
   44 D(J,I)=D(I,J)
   46 E(I)=SQRT(D(I,I))
   50 DO 52 I=1,NP
      DO 52 J=1,NP
   52 A(I,J)=D(I,J)/(E(I)*E(J))
      DO 54 I=1,NP
      P(I)=Q(I)/E(I)
      PHI(I)=P(I)
   54 A(I,I)=A(I,I)+GA
      CALL MATINV(A,NP,P)
      STEP=1.0
   56 DO 58 I=1,NP
   58 TB(I)=P(I)*STEP/E(I)+TH(I)
```

MAIN

```
      DO 62 I=1,NP
      IF(TH(I)*TB(I)) 66,66,62
   62 CONTINUE
      SUMB=0
      CALL MODEL(TB,F,NOB,C,IND,NOPT)
      DO 64 I=1,NOB
      R(I)=Y(I)-F(I)
   64 SUMB=SUMB+R(I)*R(I)
   66 SUM1=0.0
      SUM2=0.0
      SUM3=0.0
      DO 68 I=1,NP
      SUM1=SUM1+P(I)*PHI(I)
      SUM2=SUM2+P(I)*P(I)
   68 SUM3=SUM3+PHI(I)*PHI(I)
      ANGLE=89.959-57.29578*ATAN(SUM1/SQRT(SUM2*SUM3-SUM1**2))
      DO 72 I=1,NP
      IF(TH(I)*TB(I)) 74,74,72
   72 CONTINUE
      IF(SUMB/SSQ-1.0) 80,80,74
   74 IF(ANGLE-30.0) 76,76,78
   76 STEP=0.5*STEP
      GO TO 56
   78 GA=10.*GA
      GO TO 50
C
C     ----- PRINT COEFFICIENTS AFTER EACH ITERATION -----
   80 CONTINUE
      DO 82 I=1,NP
   82 TH(I)=TB(I)
      WRITE(6,1018) NIT,SUMB,(TH(I),I=1,IP)
      IF(NOPT.GT.10.AND.NOPT.LT.18) GO TO 88
      IF(NOPT.EQ.3.OR.NOPT.EQ.20) GO TO 88
      IF(NOPT.LT.11) GO TO 85
      IF(ABS(TH(I)).GT.1.0E-06) GO TO 88
      IF(NT.EQ.1) GO TO 83
      NT=1
      B(1)=-B1
      B(2)=B2
      B(3)=B3
      GO TO 30
   83 B(1)=B(2)
      B(2)=B(3)
      DO 84 I=1,6
   84 BI(I)=BI(I+2)
      NOPT=20
      NP=2
      WRITE(1,1020)
```

MAIN

```
      WRITE(1,1003) TITLE,NOPT,NP
      GO TO 30
   85 IF(TH(2).GT.0.001) GO TO 88
      IF(NOPT.EQ.5) GO TO 86
      WRITE(6,1022)
      GO TO 119
   86 NP=2
      WRITE(6,1024)
      GO TO 10
   88 DO 90 I=1,NP
      IF(ABS(P(I)*STEP/E(I))/(1.0E-20+ABS(TH(I)))-STOPCR) 90,90,92
   90 CONTINUE
      GO TO 94
   92 SSQ=SUMB
      IF(NIT.LE.MIT) GO TO 36
   94 CONTINUE
      IF(NOPT.LT.5.OR.NOPT.GT.10) GO TO 96
      IF(TH(2).GT.CMIN) GO TO 96
      WRITE(6,1025)
      GO TO 110
   96 CALL MATINV(D,NP,P)
C
C     ----- WRITE CORRELATION MATRIX -----
      DO 98 I=1,NP
   98 E(I)=SQRT(AMAX1(D(I,I),1.E-20))
      WRITE(6,1026) (I,I=1,NP)
      DO 102 I=1,NP
      DO 100 J=1,I
  100 A(J,I)=D(J,I)/(E(I)*E(J))
  102 WRITE(6,1028) I,(A(J,I),J=1,I)
C
C     ----- CALCULATE 95% CONFIDENCE INTERVAL -----
      Z=1./FLOAT(NOB-NP)
      SDEV=SQRT(Z*SUMB)
      TVAR=1.96+Z*(2.3779+Z*(2.7135+Z*(3.187936+2.466666*Z**2)))
      WRITE(6,1030)
      DO 108 I=1,NP
      SEC=E(I)*SDEV
      TVALUE=TH(I)/SEC
      TSEC=TVAR*SEC
      TMC=TH(I)-TSEC
      TPC=TH(I)+TSEC
      J=2*I-1
  108 WRITE(6,1032) BI(J),BI(J+1),TH(I),SEC,TVALUE,TMC,TPC
C
C     ----- CALCULATE VARIOUS PARAMETERS -----
      IF(NOPT.GT.10) GO TO 109
  110 SLOPE=TH(1)*TH(3)
```

MAIN

```
      IF(NOPT.EQ.3) SLOPE=TH(1)*TH(2)
      YZERO=TH(3)*(1.+TH(1)*TH(2))
      IF(NOPT.EQ.3) YZERO=B(2)*(1.+TH(1)*TH(3))
      CZERO=TH(2)+1./TH(1)
      IF(NOPT.EQ.3) CZERO=TH(3)+1./TH(1)
      WRITE(6,1034) SLOPE,TH(1),YZERO,CZERO
C
C     ----- PREPARE FINAL OUTPUT -----
  109 WRITE(6,1036)
      DO 118 I=1,NOB
      K=2+IND(I)
      IF(NOPT.EQ.3.OR.NOPT.EQ.20) K=2
      RY=Y(I)/TH(K)
      WRITE(6,1038) C(I),Y(I),F(I),R(I),RY,IND(I)
  118 CONTINUE
  119 IF(NOPT.NE.5) GO TO 120
      IF(TH(2).GT.CMIN) GO TO 120
      WRITE(6,1024)
      NP=1
      GO TO 10
  120 CONTINUE
C
C     ----- END OF PROBLEM -----
 1000 FORMAT(20A4)
 1002 FORMAT(6I5)
 1003 FORMAT(1H1,10X,82(1H*)/11X,1H*,80X,1H*/11X,1H*,10X,'LEAST SQUARES
     1ANALYSIS OF SALINITY RESPONSE CURVE',11X,'SALT',6X,1H*/11X,1H*,80X
     2,1H*/11X,1H*,20A4,1H*/11X,1H*,10X,'NOPT =',I3,': NP =',I3,52X,1H*/
     311X,1H*,80X,1H*/11X,82(1H*))
 1006 FORMAT(8(A4,A2,4X))
 1008 FORMAT(8F10.0)
 1010 FORMAT(2F10.0,I10)
 1012 FORMAT(//11X,'LINEAR REGRESSION RESULTS FOR Y=YZERO-SLOPE*C'/11X,4
     15(1H=)//11X,'YZERO =',F10.4,' WITH STANDARD ERROR OF',F10.4/11X,'S
     2LOPE =',F10.4,' WITH STANDARD ERROR OF',F10.4/11X,'CORRELATION COE
     3FFICIENT =',F10.4)
 1014 FORMAT(//11X,'CONTROL YIELD (YM) =',F10.4/11X,'THRESHOLD (CT) =',F
     110.4)
 1016 FORMAT(///11X,'NIT',6X,'SSQ',3X,10(4X,A4,A2))
 1018 FORMAT(10X,I3,F12.5,1X,10F10.5)
 1020 FORMAT(//11X,'ALPHA IS TOO SMALL, NEW NOPT =20'/11X,32(1H=))
 1022 FORMAT(//11X,'THRESHOLD IS TOO SMALL, USE ANOTHER NOPT'/11X,40(1H=
     1))
 1024 FORMAT(//11X,'CHANGED TO OPTION NUMBER 2')
 1025 FORMAT(//11X,'WARNING: THRESHOLD IS LESS THAN CMIN, USE ANOTHER NO
     1PT'/11X,55(1H=))
 1026 FORMAT(//13X,'CORRELATION MATRIX'/13X,18(1H=)/11X,10(3X,I2,4X))
 1028 FORMAT(9X,I4,10(1X,F7.4,1X))
```

MAIN

```
1030 FORMAT(//62X,'95% CONFIDENCE LIMITS'/12X,'VARIABLE',5X,'VALUE',6X,
    1'S.E.COEFF',6X,'T-VALUE',5X,'LOWER',10X,'UPPER')
1032 FORMAT(12X,A4,A2,1X,F12.6,3X,F10.5,3X,F10.3,3X,F9.4,5X,F9.4)
1034 FORMAT(/11X,'SLOPE FOR ORIGINAL DATA (S*YM) =',F10.6/11X,'SLOPE FO
    1R RELATIVE YIELD DATA (S) =',F10.6/11X,'INTERSECTION AT ZERO SALIN
    2ITY (YZERO) =',F10.5/11X,'SALINITY EXTRAPOLATED TO ZERO YIELD (CZE
    3RO)=',F10.5)
1036 FORMAT(/13X,'CONC',7X,'Y-OBS',6X,'Y-FITTED',3X,'DEVIATION',3X,'REL
    1 YIELD',5X,'INDEX')
1038 FORMAT(6X,5F12.4,7X,I2)
     STOP
     END
```

MODEL

```
      SUBROUTINE MODEL(B,Y,NOB,C,IND,NOPT)
C
C     PURPOSE: TO CALCULATE Y(C)
C
      DIMENSION B(8),Y(50),C(50),IND(50)
C
C     ---------------
      IF(NOPT.GT.10) GO TO 10
    2 B2=B(2)
      IF(NOPT.EQ.3) B2=B(3)
      IF(NOPT.EQ.3) B3=B(2)
      DO 8 I=1,NOB
      IF(NOPT.NE.3) B3=B(2+IND(I))
      IF(C(I)-B2) 4,4,6
    4 Y(I)=B3
      GO TO 8
    6 Y(I)=B3-B(1)*B3*(C(I)-B2)
      Y(I)=AMAX1(Y(I),0.)
    8 CONTINUE
      RETURN
   10 IF(NOPT.GT.17) GO TO 14
      DO 12 I=1,NOB
   12 Y(I)=B(2+IND(I))/(1.+(C(I)/B(1))**B(2))
      RETURN
   14 IF(NOPT.EQ.20) GO TO 18
      DO 16 I=1,NOB
   16 Y(I)=B(3)*EXP(B(1)*C(I)-B(2)*C(I)**2)
      RETURN
   18 DO 20 I=1,NOB
   20 Y(I)=B(2)*EXP(-B(1)*C(I)**2)
      RETURN
      END
```

MATINV

```
      SUBROUTINE MATINV(A,NP,B)
      DIMENSION A(8,8),B(8),INDEX(8,2)
      DO 2 J=1,NP
    2 INDEX(J,1)=0
      I=0
    4 AMAX=-1.0
      DO 12 J=1,NP
      IF(INDEX(J,1)) 12,6,12
    6 DO 10 K=1,NP
      IF(INDEX(K,1)) 10,8,10
    8 P=ABS(A(J,K))
      IF(P.LE.AMAX) GO TO 10
      IR=J
      IC=K
      AMAX=P
   10 CONTINUE
   12 CONTINUE
      IF(AMAX) 30,30,14
   14 INDEX(IC,1)=IR
      IF(IR.EQ.IC) GO TO 18
      DO 16 L=1,NP
      P=A(IR,L)
      A(IR,L)=A(IC,L)
   16 A(IC,L)=P
      P=B(IR)
      B(IR)=B(IC)
      B(IC)=P
      I=I+1
      INDEX(I,2)=IC
   18 P=1./A(IC,IC)
      A(IC,IC)=1.0
      DO 20 L=1,NP
   20 A(IC,L)=A(IC,L)*P
      B(IC)=B(IC)*P
      DO 24 K=1,NP
      IF(K.EQ.IC) GO TO 24
      P=A(K,IC)
      A(K,IC)=0.0
      DO 22 L=1,NP
   22 A(K,L)=A(K,L)-A(IC,L)*P
      B(K)=B(K)-B(IC)*P
   24 CONTINUE
      GO TO 4
   26 IC=INDEX(I,2)
      IR=INDEX(IC,1)
      DO 28 K=1,NP
      P=A(K,IR)
      A(K,IR)=A(K,IC)
   28 A(K,IC)=P
      I=I-1
   30 IF(I) 26,32,26
   32 RETURN
      END
```