Kinematic Car Model

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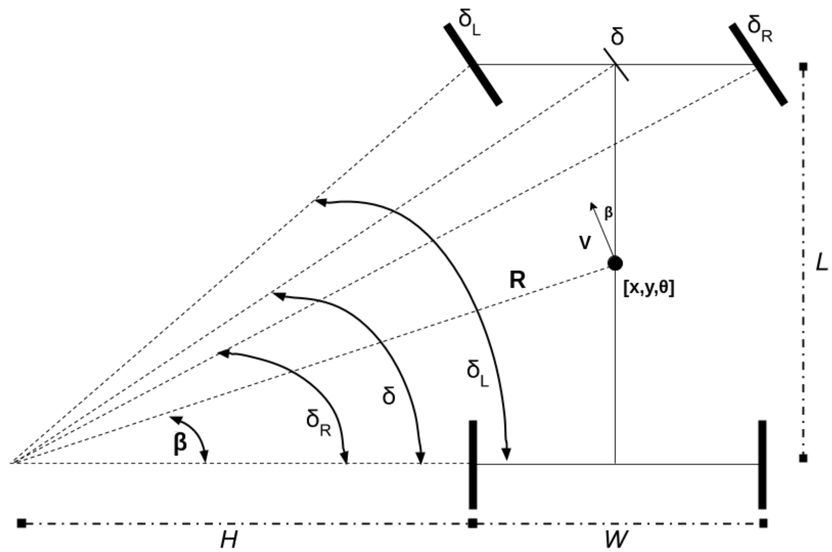

Figure 1: Visual representation of the kinematic car model

The Kinematic Model of a robot car. State consists of position and heading angle $\mathbf{x} = (x, y, \theta)$. Controls consist of velocity and steering angle $\mathbf{u} = (v, \delta)$. This model is also called the bicycle model because we model the car as a bicycle where the wheels are centered on the vehicle. Without any control the state changes like so:

$$\dot{x} = V cos(\theta + \beta)$$
$$\dot{y} = V sin(\theta + \beta)$$
$$\dot{\theta} = \omega$$

ω is the angular velocity from the center of rotation to the center of mass (modeled as center of vehicle) and can further be defined as follows.

$$\omega = \frac{V}{R}$$

And

$$R = \frac{L}{2sin(\beta)}$$

Where V in this case is the tangential velocity. And we know by definition, $\omega = \frac{V}{r}$. Figure 1 shows the tangential velocity V. β is also defined as follows:

$$\beta = tan^{-1}\left(\frac{1}{2}tan(\delta)\right)$$

And $\dot{\theta}$

$$\dot{\theta} = \frac{V}{R}$$

$$= \frac{V}{\frac{L}{2sin(\beta)}}$$

$$= \frac{2V}{L}sin(\beta)$$

Now if we apply a control u_t that affects the state at $t+1$. We can integrate over the time step to see how the state changes as theta changes.

$$\frac{\partial x}{\partial t} = Vcos(\theta + \beta)$$
$$\frac{\partial y}{\partial t} = Vsin(\theta + \beta)$$
$$\frac{\partial \theta}{\partial t} = \frac{2V}{L}sin(\beta)$$

Change in θ:

$$\int_{\theta_t}^{\theta_{t+1}} d\theta = \int_{t}^{t+\Delta t} \frac{2V}{L}sin(\beta)dt$$
$$\theta_{t+1} - \theta_t = \frac{2V}{L}sin(\beta)[t + \Delta t - t] = \frac{2V}{L}sin(\beta)\Delta t$$

Change in x:

$$\int_{x_t}^{x_{t+1}} dx = \int_{t}^{t+\Delta t} Vcos(\theta+\beta)dt = \int_{t}^{t+\Delta t} Vcos(\theta+\beta)\frac{d\theta}{\frac{2V}{L}sin(\beta)} = \frac{L}{2sin(\beta)}\int_{\theta_t}^{\theta_{t+1}} cos(\theta+\beta)d\theta$$

$$x_{t+1} - x_t = \frac{L}{2sin(\beta)}[sin(\theta_{t+1} + \beta) - sin(\theta_t + \beta)]$$

Change in y:

$$\int_{y_t}^{y_{t+1}} dy = \int_{t}^{t+\Delta t} V sin(\theta+\beta) dt = \int_{\theta_t}^{\theta_{t+1}} V sin(\theta+\beta) \frac{d\theta}{\frac{2V}{L} sin(\beta)} = \frac{L}{2sin(\beta)} \int_{\theta_t}^{\theta_{t+1}} sin(\theta+\beta) d\theta$$

$$y_{t+1} - y_t = \frac{L}{2sin(\beta)}[-cos(\theta_{t+1}+\beta) + cos(\theta_t+\beta)]$$

Putting it all together we have:

$$x_{t+1} = x_t + \frac{L}{2sin(\beta)}[sin(\theta_{t+1}+\beta) - sin(\theta_t+\beta)]$$
$$y_{t+1} = y_t + \frac{L}{2sin(\beta)}[-cos(\theta_{t+1}+\beta) + cos(\theta_t+\beta)]$$
$$\theta_{t+1} = \theta_t + \frac{2V}{L} sin(\beta) \Delta t$$