1.5. (1) $\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(u,v)\, du\, dv = 1$

$a \int_0^2 \int_0^1 (x+y)\, dx\, dy = 1$

∴ $3a = 1$

∴ $a = \frac{1}{3}$

(2) $P\{X \le \frac{1}{2}, Y \le 1\}$

$= \int_{-\infty}^{\frac{1}{2}} \int_{-\infty}^{1} f(x,y)\, dx\, dy$

$= \frac{7}{24}$

$P\{X \ge Y\}$

$= \int_{-\infty}^{x} \int_{-\infty}^{+\infty} f(x,y)\, dx\, dy$

$= \int_0^1 \int_0^x \frac{1}{3}(x+y)\, dy\, dx$

$= \frac{1}{6}$

3.2.

1. (1) $F_X(x) = F(x, +\infty)$

$= \lim_{y \to +\infty} F(x,y)$

$= \begin{cases} 0, & x < -1 \\ \lim_{y \to +\infty} 0.5(x+1), & -1 \le x < 1 \\ \lim_{y \to +\infty} 1, & x \ge 1 \end{cases}$

$= \begin{cases} 0, & x < -1 \\ 0.5(x+1), & -1 \le x < 1 \\ 1, & x \ge 1 \end{cases}$

$F_Y(y) = F(+\infty, y)$

$= \begin{cases} 0, & y < 0 \\ \lim_{x \to +\infty} 0.5y, & 0 \le y < 2 \\ 1, & y \ge 2 \end{cases}$

$= \begin{cases} 0, & y < 0 \\ 0.5y, & 0 \le y < 2 \\ 1, & y \ge 2 \end{cases}$

(2) $P\{-1.5 \le X < 2.5, -0.5 \le Y < 1.5\}$

$= F(2.5, 1.5) - F(1.5, 1.5)$

$= 0.75 - 0$

$= 0.75$

(3) $P\{X < 0.5, Y > 0.5\}$

$= 0.25 \times 1.5 \times 2 - 0.25 \times 1.5 \times 0.5 + 0.5 \times 1.5$

$= 0.5625$

3.3.

2. (1)

X＼Y	0	1	2
0	0.24	0.24	0.12
1	0.12	0.12	0.16

3. (2) $P\{Y=1\} = P\{X=0, Y=1\} + P\{X=1, Y=1\}$

$= 0.24 + 0.12$

$= 0.36$

$P\{X=0 \mid Y=1\} = \dfrac{P\{X=0, Y=1\}}{P\{Y=1\}} = \dfrac{0.24}{0.36} = \dfrac{2}{3}$

$P\{X=1 \mid Y=1\} = \dfrac{P\{X=1, Y=1\}}{P\{Y=1\}} = \dfrac{1}{3}$

X	0	1
$P\{X \mid Y=1\}$	$\frac{2}{3}$	$\frac{1}{3}$

3.4.

1. (1) $S_D = \int_0^1 (\sqrt{x} - x^2)\, dx = \frac{1}{3}$

$f(x,y) = \frac{1}{S_D} = 3$, $f(x,y) = \begin{cases} 3, & x^2 \le y \le \sqrt{x} \\ 0, & \text{其他} \end{cases}$

(2) $f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\, dy = \begin{cases} 3(\sqrt{x} - x^2), & 0 \le x \le 1 \\ 0, & \text{其他} \end{cases}$

∴ $f_X(x) = \begin{cases} 3(\sqrt{x} - x^2), & 0 \le x \le 1 \\ 0, & \text{其他} \end{cases}$

$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\, dx = \begin{cases} 3(\sqrt{y} - y^2), & 0 \le y \le 1 \\ 0, & \text{其他} \end{cases}$

3. (1) $\int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x,y)\, dx\, dy$

$= \int_0^{+\infty} \int_{\frac{y}{2}}^{+\infty} a e^{-(x+y)}\, dy\, dx$

$= \int_0^{+\infty} \int_0^{\frac{y}{2}} a e^{-(x+y)}\, dx\, dy$

$= \frac{a}{3}$

∴ $a = 3$

(2) $f_x(x,y)$

$f_X(x) = \int_{-\infty}^{+\infty} f(x,y)\,dy$

$= \begin{cases} 3e^{-3x}, & x>0 \\ 0, & x\le 0 \end{cases}$

$f_Y(y) = \int_{-\infty}^{+\infty} f(x,y)\,dx$

$= \begin{cases} 3e^{-y}, & y>0 \\ 0, & y\le 0 \end{cases}$ $\quad 3e^{-y}(1-e^{-\frac{y}{2}}), y>0$, $\quad 0, y\le 0$

(3) $P\{x>1, y>2\}$

$= \int_1^{+\infty}\int_{2x}^{+\infty} f(x,y)\,dy\,dx$

$= \frac{1}{e^3}$

4. (1) $S_D = \frac{1}{2}\times 2 = 1$

$f(x,y) = \frac{1}{S_D} = 1$

$f_Y(y) = \int_{\frac{y}{2}}^1 1\cdot dx = 1 - \frac{y}{2}$

$f_X(x) = \int_0^{2x} 1\,dy = 2x$

$f_{X|Y}(x|y) = \frac{f(x,y)}{f_Y(y)} = \begin{cases} \frac{2}{2-y}, & 0\le y\le 2 \\ 0, & \text{其他} \end{cases}$

$f_{Y|X}(y|x) = \frac{f(x,y)}{f_X(x)} = \begin{cases} \frac{1}{2x}, & 0\le x\le 1 \\ 0, & \text{其他} \end{cases}$

(2) $P\{X=\frac{3}{4}\}$

$P\{Y>1 | X=\frac{3}{4}\} = \int_1^{\frac{3}{2}} f_{Y|X}(y|x)\,dy = \frac{1}{3}$

3.5

2. (1) $f_X(x) = \int_{|x|}^1 1\,dy = 1-|x|, \quad -1<x<1$

$f_Y(y) = \int_{-y}^y 1\,dx = 2y, \quad 0<y<1$

(2) $f_X(x)\cdot f_Y(y) = \begin{cases} (1-|x|)2y, & |x|<1, 0<y<1 \\ 0, & \text{其他} \end{cases}$

∴ 不互相独立

Y \ X	y_1	y_2	y_3	$P\{X=x\}$
x_1	$\frac{1}{6}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{3}{8}$
x_2	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{1}{8}$	$\frac{5}{8}$
$P\{Y=y\}$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	1

5. $|x+y|=1$

$\int_0^2\int_0^{\frac{x}{2}} a(x+y)\,dy\,dx$

$= \frac{5}{3}a$

∴ $a = \frac{3}{5}$

(2) $f_X(x) = \int_0^{\frac{x}{2}} \frac{3}{5}(x+y)\,dy = \begin{cases} \frac{3}{8}x^2, & 0\le x\le 2 \\ 0, & \text{其他} \end{cases}$

$f_Y(y) = \int_{2y}^2 \frac{3}{5}(x+y)\,dx = \begin{cases} \frac{6}{5}+\frac{6}{5}y-\frac{18}{5}y^2, & 0\le y\le \frac{x}{2} \\ 0, & \text{其他} \end{cases}$

$f_X(x)\cdot f_Y(y) = \begin{cases} \frac{3}{8}x^2(2+2y-4y^2), & 0\le x\le 2, 0\le y\le\frac{x}{2} \\ 0, & \text{其他} \end{cases}$

不互相独立

(3) $P\{Y>\frac{3}{4} | X\ge 1\}$

$P\{X\ge 1\} = \int_1^2 f_X(x)\,dx = \frac{7}{8}$

$P\{Y>\frac{3}{4}, X\ge 1\} = \int_{\frac{1}{4}}^1\int_{2y}^2 f(x,y)\,dx\,dy$

$= \frac{1}{10}$

$P\{Y>\frac{3}{4} | X\ge 1\} = \frac{\frac{1}{10}}{\frac{7}{8}} = \frac{4}{35}$