~~习题~~ 概统

第六章

6.1.

1.(1) $EX = 0.75\times1000 = 750$

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$DX = 0.1875\times1000 = 187.5$

$DX = EX^2 - (EX)^2 = 187500$

$P\{|X-750|<50\} \geq 1 - \frac{187.5}{50^2} = 0.925$

(2) $EX = \frac{3}{4}n$

$DX = EX^2 - (EX)^2 = 0.1875n$

$P\{|X-0.75n|<0.01n\} \geq 1 - \frac{0.1875n}{(0.01n)^2}$

$\geq 1 - 1875\frac{1}{n}$

$\therefore 1 - 1875\frac{1}{n} = 0.9$

$\therefore n = 18750$

2、 $P\{|X-\mu| \geq 3\sigma\} \leq \frac{\sigma^2}{9\sigma^2} = \frac{1}{9}$

3. $E(\xi) = \int_{-\infty}^{+\infty} x p(x)\,dx$

$= \int_0^{+\infty} x^3 \cdot \frac{1}{2} e^{-x}\,dx$

$= 3$

$D(\xi) = \int_{-\infty}^{+\infty} x^2 p(x)\,dx - [E(\xi)]^2 = 3$

$\therefore P\{0<\xi<6\} \geq 1 - \frac{3}{9}$

$= \frac{2}{3}$

6.2.

1. $E(X_i) = -\frac{a}{2i} + \frac{a}{2i} = 0$

$E(X_i^2) = a^2$

$D(X_i) = a^2$

对 $\forall i \in [1,3]$

$D(X_i) = a^2$

$\therefore P\{\lim_{n\to\infty} |\frac{1}{n}\sum_{i=1}^{n} X_i| < \varepsilon\} = 1$

$\therefore \lim_{n\to\infty} P\{|\frac{1}{n}\sum_{i=1}^{n} X_i| \geq \varepsilon\} = 0$

2. $E(\xi_n) = 0$

$E(\xi_n^2) = \ln n$

$D(\xi_n) = \ln n$

$Y_n = \frac{1}{n}\sum_{i=1}^{n} X_i$

$D(Y_n) = \frac{\ln n}{n}$

$E(Y_n) = 0$

$\therefore \{X_n\}$ 服从大数定律

$\lim_{n\to\infty} P\{|\frac{1}{n}\sum \xi_i - E(\frac{1}{n}\sum \xi_i)|\} = \lim_{n\to\infty}(1 - \frac{\frac{\ln n}{n}}{\varepsilon^2}) = 1$

4. $E(X_n) = \sqrt{n}$

$D(X_n) = \sqrt{n}$

$\lim_{n\to\infty} P\{|\frac{1}{n}\sum_{i=1}^{n} X_i - E(\frac{1}{n}\sum_{i=1}^{n} X_i)| < \varepsilon\} = \lim_{n\to\infty}(1 - \frac{n\sqrt{n}}{n^2\varepsilon^2})$

$= 1$

$\therefore \{X_n\}$ 满足大数定律

6.3.

1. (1) $P\{0.4n < \mu_n < 0.6n\} \approx \Phi\left(\frac{0.1n}{\sqrt{\frac{n}{4}}}\right) - \Phi\left(\frac{-0.1n}{\sqrt{\frac{n}{4}}}\right) \geq 90\%$.

即 $\Phi(0.2\sqrt{n}) - \Phi(-0.2\sqrt{n}) \geq 0.9$

$\therefore \Phi(0.2\sqrt{n}) \geq 0.95$

$\therefore n \geq 68$.

(2) 由切比雪夫不等式

$E(X)=\frac{n}{2}$ $D(X)=\frac{n}{4}$

$P\{|\mu_n - E(\mu_n)| < 0.1n\} \geq 1 - \frac{\frac{n}{4}}{(0.1n)^2} \geq 0.9$

即 $\frac{25}{n} \leq 0.1$

即 $n \geq 250$

综上，棣莫弗-拉普拉斯定理更精确

6.6.

$P\{0.76n < X < 0.84n\} \approx \Phi\left(\frac{0.04n}{\sqrt{0.8 \times 0.2n}}\right) - \Phi\left(\frac{-0.04n}{\sqrt{0.16n}}\right) \geq 0.9$

解得 $n \geq 271$

$\therefore$ 至少 271 次