7.1

2、总体：所有含个不合格的产品数
样本：n个中不合格数

$(X_1,\cdots,X_n)$满足 $\prod_{i=1}^{n} C_m^{x_i} p^{x_i}(1-p)^{m-x_i}$

4、$(X_1,X_2,\cdots,X_n) \sim \dfrac{\lambda^{\sum_{i=1}^{n} x_i} e^{-n\lambda}}{k_1!k_2!\cdots k_n!}$

5、只需证 $\mathrm{cov}(X_1+X_2, X_1-X_2)=0$.

$\mathrm{cov}(X_1+X_2, X_1-X_2) = E(X_1^2-X_2^2) - E(X_1+X_2)\cdot E(X_1-X_2)$

$= E(X_1^2) - E(X_2^2) - E(X_1)^2 + E(X_2)^2$

$= -D(X_1) + D(X_2)$

$= 0$

即得证

7.2.

1. $\overline{X}$ $\cancel{E(X)} = 3$

S^2 $\cancel{D(X)} = \frac{34}{9} \approx 3.78$

$\cancel{\sqrt{DX}}$

$S = \sqrt{\frac{34}{9}} \approx 1.94$

2、$S^2 = \frac{1}{n-1}\sum_{i=1}^{2}(X_i-\overline{X})^2$

$\overline{X} = \frac{X_1+X_2}{2}$

$S^2 = \frac{1}{2}\left(\frac{X_1-X_2}{2}\right)^2 = \frac{(X_1-X_2)^2}{2}$

5.(1) $\overline{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$

$E\overline{X} = \frac{2}{3}$

$D\overline{X} = \frac{1}{n}DX = \frac{2}{9n}$

(2) $E(S^2) = \frac{1}{n-1}\sum E(X_i-\overline{X})^2 = DX = \frac{2}{9}$

(3) $C_n^x \left(\frac{2}{3}\right)^x \left(\frac{1}{3}\right)^{n-x}$

7.3.

3.(1) $X_i - \overline{X} \sim N(0, \frac{9}{10}\sigma^2)$

$\dfrac{X_i-\overline{X}}{\sqrt{\frac{9}{10}\sigma^2}} \sim N(0,1)$

∴ $\cancel{\frac{1}{10}\sum_{i=1}^{n}(X_i-\overline{X})} \sim$

$P\{0.27\sigma^2 \le \frac{1}{10}\sum_{i=1}^{10}(X_i-\overline{X})^2 \le 2.36\sigma^2\}$

$= P\{3 \le \chi^2(10) \le 26\}$

$= 0.97$

(2) $\sum_{i=1}^{n}\dfrac{(X_i-\mu)^2}{\sigma^2} \sim \chi^2(10)$

∴ $\frac{1}{10}\sum_{i=1}^{n}(X_i-\mu)^2 \sim \frac{\sigma^2}{10}\chi^2(10)$

∴ $P\{0.27\sigma^2 \le \frac{1}{10}\sum_{i=1}^{n}(X_i-\mu)^2 \le 2.36\} = 0.98$.

4.(1) ∵ $2X_i \sim N(0,1)$

∴ $\sum_{i=1}^{7}(2X_i)^2 \sim \chi^2(7)$

∴ $\sum_{i=1}^{7}(X_i)^2 \sim \frac{\chi^2(7)}{4}$

∴ $P\{\sum_{i=1}^{7} X_i^2 > 4\} = P\{\chi^2(7) > 16\} = 0.025$

(2) ∵ $X_i - \overline{X} \sim N(0, \frac{3}{14})$

∴ $\dfrac{X_i-\overline{X}}{\sqrt{\frac{3}{14}}} \sim N(0,1)$

∴ $\sum_{i=1}^{7}(X_i-\overline{X}) \sim \frac{3}{14}\chi^2(7)$

∴ $P\{\sum_{i=1}^{7}(X_i-\overline{X})^2 > 4\} = P\{\chi^2(7) > \frac{56}{3}\} = 0.01$

6. (1) $\sum_{i=1}^{m} X_i \sim N(0,m)$

(2) $\frac{1}{\sqrt{n-m}}\sum_{i=m+1}^{n} X_i \sim N(0,1)$

(3) $Y \sim \chi^2(2)$.

10. $\sum_{i=1}^{9} X_i \sim N(0, 9\sigma^2)$

$\frac{\sum_{i=1}^{9} X_i}{3\sigma} \sim N(0,1)$

$\frac{\sum_{i=1}^{9} Y_i}{3\sigma} \sim N(0,1)$

$\therefore \sum_{i=1}^{9} \frac{Y_i^2}{9\sigma^2} \sim \chi^2(9)$

$\therefore \frac{\frac{\sum_{i=1}^{9} X_i}{3\sigma}}{\sqrt{\sum \frac{Y_i^2}{9\sigma^2}/9}} \sim t(n)$

$\therefore U \sim \frac{t(n)}{3}$

故 $U^2 = \frac{\chi(1)}{\chi(9)} \sim 9F(1,9)$

11. 由7.1已证

X_1+X_2 与 X_1-X_2 互相独立

$X_1+X_2 \sim (0,2\sigma^2)$

$(X_1+X_2)^2 \sim 2\sigma^2\chi(1)$

$X_1-X_2 \sim (0,2\sigma^2)$

$(X_1-X_2)^2 \sim 2\sigma^2\chi(1)$

$\therefore \frac{(X_1+X_2)^2}{(X_1-X_2)^2} \sim F(1,1)$

13. $\because \bar{X} \sim N(0,\frac{\sigma^2}{n})$

$\therefore \frac{(\bar{X})^2}{\frac{\sigma^2}{n}} \sim \chi^2(1)$

$\therefore \bar{X}^2 \sim \frac{\sigma^2}{n}\chi^2(1)$

$\because S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i-\bar{X})^2$

$X_i-\bar{X} \sim N(0,\frac{n-1}{n}\sigma^2)$

$\therefore \frac{\sum_{i=1}^{n}(X_i-\bar{X})^2}{\sqrt{\frac{n-1}{n}\sigma^2}} \sim \chi(n)$

$\therefore S^2 \sim \frac{\sigma^2}{n}\chi^2(n)$

$\therefore \frac{n\bar{X}^2}{S^2} \sim F(1,n)$