概统
第八章

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8.1、

(1) $EX=\int_{-\infty}^{+\infty} f(x;\theta)x\,dx$

$=\int_{0}^{\theta}\frac{2}{\theta^2}(\theta-x)x\,dx$

$=\frac{1}{3}\theta$

$\therefore \frac{1}{3}\hat{\theta}=\frac{1}{n}\sum_{i=1}^{n}X_i$

$\therefore \hat{\theta}=\frac{3}{n}\sum_{i=1}^{n}X_i$

3、$EX=1(1-p)^{1-1}p+\cdots+n(1-p)^{n-1}p$

$=p\left[\frac{1-p}{1-(1-p)}\right]'$ $p\frac{d\frac{1-p}{1-(1-p)}}{d(1-p)}$

$=\frac{1}{p}$

$\therefore \frac{1}{p}=\frac{1}{n}\sum_{i=1}^{n}X_i$

$\therefore \hat{p}=\frac{n}{\sum_{i=1}^{n}X_i}$

7. ~~$L(\theta)=\prod_{i=1}^{n}\frac{x_i^3}{\theta^2}e^{-\frac{x_i^2}{2\theta^2}}=$~~

$L(\theta)=\prod_{i=1}^{n}\frac{2x_i}{\theta^2}\exp(-\frac{x_i^2}{\theta^2})$

$=\frac{2^n\prod_{i=1}^{n}x_i}{\theta^{2n}}\exp(-\frac{\sum_{i=1}^{n}x_i^2}{\theta^2})$

$\ln L(\theta)=n\ln 2+\sum_{i=1}^{n}\ln x_i-2n\ln\theta+\frac{\sum_{i=1}^{n}x_i^2}{\theta^2}$

$\frac{d\ln L(\theta)}{d\theta}=-\frac{2n}{\theta}-\frac{2\sum_{i=1}^{n}x_i}{\theta^3}$

令 $-\frac{2n}{\theta}-\frac{2\sum_{i=1}^{n}x_i}{\theta^3}=0$

得 $\hat{\theta}^2=\frac{\sum_{i=1}^{n}x_i}{n}$

8.(2) ~~$L(\theta)=\prod$~~

$L(\theta)=\prod_{i=1}^{n}\frac{1}{\theta_2-\theta_1}f(x_i;\theta_1,\theta_2)$

由

$L(\hat{\theta})=\max_{\theta\in\Theta}L(\theta)$

$\therefore \theta_2=\max\{x_1,\cdots,x_n\}$

$\theta_1=\min\{x_1,\cdots,x_n\}$

$L(\hat{\theta})=\left(\frac{1}{\max\{x_1,\cdots,x_n\}-\min\{x_1,\cdots,x_n\}}\right)^n$

11、$L(p)=\prod_{i=1}^{100}C_{10}^{j}p^{j}(1-p)^{10-j}=\prod_{i=1}^{100}C_{10}^{j}p^{\sum_{i=1}^{100}j}(1-p)^{1000-\sum_{i=1}^{100}j}$

$\frac{d\ln L(p)}{dp}=\frac{\sum_{i=1}^{100}j}{p}-\frac{1000-\sum_{i=1}^{100}j}{(1-p)}$

令 $\frac{d\ln L(p)}{dp}=0$ 得

$p=0.499$

习8.2

2. (1) $EX=\int_{-\infty}^{+\infty} f(x;\theta)x\,dx$

$=\int_0^\theta \frac{3x^3}{\theta^3}dx$

$=\frac{3}{4}\theta$

$E(T_1)=\frac{2}{3}(EX_1+EX_2)=\theta$

~~$E(T_2)=\frac{7}{6}E(\max\{X_1,X_2\})=\frac{7}{6}\times\frac{6}{7}\theta=\theta$~~

~~∴T_1,T_2均为无偏估计~~

(2) 令 $Z=\max\{X_1,X_2\}$

$F(z)=\int_{-\infty}^{z}\int_{-\infty}^{z}\frac{3x^2}{\theta^3}\cdot\frac{3y^2}{\theta^3}dxdy=\frac{z^6}{\theta^6}$

$f(z)=\frac{dF(z)}{dz}=\frac{6z^5}{\theta^6}$

$EZ=\int_{0}^{\theta} z f(z)=\frac{6}{7}\theta$

$ET_2=\frac{7}{6}EZ=\theta$

∴T_1 T_2均为无偏估计

(2) $E(T_1^2)=\frac{4}{9}[E(X_1^2)+E(X_2^2)+2E(X_1X_2)]$

$=\frac{4}{9}(\frac{6}{5}\theta^2+\frac{9}{8}\theta^2)$

$=\frac{31}{30}\theta^2$

$E(T_2^2)=\frac{49}{36}\int_0^\theta z^2 f(z)$

$=\frac{3}{4}\theta^2\cdot\frac{49}{36}$

$=\frac{49}{48}\theta^2$

$D(T_1)=E(T_1^2)-(ET_1)^2=\frac{1}{30}\theta^2$

$D(T_2)=E(T_2^2)-(ET_2)^2=\frac{1}{48}\theta^2$

$DT_1>DT_2$

4. $E(\hat\theta)=\theta$

$E(\hat\theta^2)=D(\hat\theta)+E(\hat\theta)^2=\theta^2+D(\hat\theta)$

$>\theta^2$

∴不是无偏估计

6.(1)

① $E(\hat\sigma^2)=\frac{1}{n}\sum_{i=1}^{n}E(X_i-\mu_0)^2=\frac{1}{n}\sum_{i=1}^{n}\sigma^2=\sigma^2$

∴是无偏估计

② $P\{|\frac{1}{n}\sum_{i=1}^{n}E(X_i-\mu_0)^2-\sigma^2|<\varepsilon\}$

$=P\{|S^2-ES^2|<\varepsilon\}$

$\geq 1-\frac{DS^2}{\varepsilon^2}$

$DS^2=\frac{\sigma^2}{(n-1)^2}\cdot D[\frac{(n-1)S^2}{\sigma^2}]=\frac{2\sigma^4}{n-1}$

∴$1\geq P\geq 1-\frac{DS^2}{\varepsilon^2}$

∴ $P\xrightarrow{P}1$

∴是一致性估计

(2) $X_i-\bar X\sim(0,\frac{n-1}{n}\sigma^2)$

$E(\hat\sigma_1^2)=\frac{1}{n-1}\sum_{i=1}^{n}\frac{n-1}{n}\sigma^2=\sigma^2$

$E(\hat\sigma_2^2)=\frac{1}{n}\sum_{i=1}^{n}\frac{n-1}{n}\sigma^2=\frac{n-1}{n}\sigma^2$

$E(\hat\sigma_3^2)=\frac{1}{n-1}\sum_{i=1}^{n}\sigma^2=\frac{n}{n-1}\sigma^2$

$E(\hat\sigma_4^2)=\frac{1}{n}\sum_{i=1}^{n}\sigma^2=\sigma^2$

∴ $\hat\sigma_1^2,\hat\sigma_4^2$为无偏估计量

$D(\hat\sigma_1^2)=D(S^2)=\frac{2\sigma^4}{n-1}$

$D(\hat\sigma_4^2)=(\frac{\sigma}{n})^2\cdot\frac{\sum_{i=1}^{n}(X_i-\mu_0)^2}{(\sigma^2)}=\frac{2\sigma^4}{n}$

∴$D(\hat\sigma_1^2)>D(\hat\sigma_4^2)$

∴ $\hat\sigma_4^2$优良性更佳.

概统(A)
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8.4

3、$P\{\bar{X}-z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\le\mu\le\bar{X}+z_{1-\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\}=1-\alpha=99\%$

∴ α 取 0.01

查表得 $-1.17\le\mu\le6.57$

∴置信区间为 $[-1.17, 6.57]$

(2) $P\{\bar{X}-t_{1-\frac{\alpha}{2}}(3)\cdot\frac{S}{2}\le\mu\le\bar{X}+t(3)\cdot\frac{S}{2}\}=0.95$

$S^2=2.277$

∴置信区间为 $[-0.923, 6.324]$

6. 两侧分位点分别为 $-z_{1-x}$, z_{1-y}

$(x+y=\alpha)$

即 $\varphi(b)-\varphi(a)=1-\alpha$

求 $b-a$ 最小

令 $g(a,b)=b-a+\lambda[\varphi(b)-\varphi(a)-1+\alpha]$

$\frac{dg}{da}=-1+\lambda(-\frac{e^{-\frac{a^2}{2}}}{\sqrt{2\pi}})=0$

$\frac{dg}{db}=1+\lambda\frac{e^{-\frac{b^2}{2}}}{\sqrt{2\pi}}=0$

得 $b=-a$

即 $|z_{1-x}|=z_{1-y}|$

∴ $x=\frac{\alpha}{2}=y$

∴ $t\in[z_{1-\frac{\alpha}{2}}, z_{1-\frac{\alpha}{2}}]$ 为最短区间