-15 7 15日

4.1

2、 $P\{Z=0\} = P\{X=0, Y=0\} = \frac{1}{4}$

$P\{Z=1\} = 1 - P\{Z=0\} = \frac{3}{4}$

∴分布律为

Z	0	1
P	$\frac{1}{4}$	$\frac{3}{4}$

4. $P\{Z=1\} = P\{X=0, Y=1\} + P\{X=1, Y=0\} = \frac{1}{2}$

$P\{Z=0\} = P\{X=-1, Y=0\} + P\{X=0, Y=0\} = \frac{1}{2}$

∴分布律为

Z	0	1
P	$\frac{1}{2}$	$\frac{1}{2}$

4.2

3. $h(y) = \arcsin x$

$f_Y(y) = \sin(\arcsin x) \cdot |\arcsin x| = |\arcsin x|$

5. $h(y) = \sqrt{x}$

$f_Y(y) = (\sqrt{x})^2$

3、 $h(y) = \arcsin x$

$$f(x) = \begin{cases} \frac{1}{\pi}, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0, & 其他 \end{cases}$$

$$f_Y(y) = \begin{cases} \frac{1}{\pi} |\arcsin' x|, & -\frac{\pi}{2} < x < \frac{\pi}{2} \\ 0 & 其他 \end{cases}$$

5. $f(x) = \begin{cases} \frac{1}{2} \\ 0 \end{cases}$

$h(y) = \sqrt{x}$

$$f_Y(y) = \begin{cases} \frac{1}{2} \cdot \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{4} x^{-\frac{1}{2}}, & 0 < x < 2 \\ 0, & 其他 \end{cases}$$

4.3

1. ∵ $\int_0^{+\infty} \lambda e^{-\lambda y} dy = 1$

∴ $\lambda = 1$

∴ $F_Z = \int_0^z \int_0^{z-x} 2e^{-y} dy\, dx = 1 - 2e^{-z}(e^{\frac{1}{2}} - 1)$

$f_Z(z) = \frac{dF_Z}{dz} = 2(e^{\frac{1}{2}} - 1)e^{-z}$

9. $F_Z(z) = \int_0^z \int^x dy\, dx = \frac{3}{2} z^3$

$f_Z(z) = \frac{dF_Z(z)}{dz} = \frac{9}{2} z^2$

~~1. $f(x,y) = \{ 2\lambda e^{-\lambda y}, y>0$~~

1. $\int_0^{+\infty} \lambda e^{-\lambda y} dy = 1$

$\lambda = 1$

$$f(x,y) = \begin{cases} 2e^{-y}, & 0<x<\frac{1}{2}, y>0 \\ 0. & 其他 \end{cases}$$

$$F_Z(z) = \begin{cases} \int_0^z \int_0^{z-x} 2e^{-y} dy\, dx = 2z - 2 + 2e^{-z}, & 0<z<\frac{1}{2} \\ \int_0^{\frac{1}{2}} \int_0^{z-x} 2e^{-y} dy\, dx = 1 - e^{-z}(2e^{\frac{1}{2}} - 2), & z>\frac{1}{2} \\ 0. & 其他 \end{cases}$$

$$f_Z(z) = \frac{dF_Z(z)}{dz} = \begin{cases} 2 - 2e^{-z}, & 0<z<\frac{1}{2} \\ 2e^{-z}(e^{\frac{1}{2}} - 1) & z>\frac{1}{2} \\ 0, & 其他 \end{cases}$$

9. 解

$$F_Z(z)=\begin{cases}1, & z\geq 1\\ \int_0^z\int_0^x 3x\,dy\,dx+\int_z^1\int_{x-z}^x 3x\,dy\,dx=\frac{3}{2}z-\frac{z^3}{2}, & 0<z<1\\ 0, & 其他\end{cases}$$

$$f_Z(z)=\frac{dF_Z(z)}{dz}=\begin{cases}\frac{3}{2}-\frac{3}{2}z^2, & 0<z<1\\ 0, & 其他\end{cases}$$

其他： 1. X在$[0,\pi]$上服从均匀分布
求$Y=\sin X$的概率密度

$0\leq y\leq 1$

$$F_Y(y)=P\{Y\leq y\}$$
$$=P\{\sin X\leq y\}$$
$$=P\{0\leq X\leq \arcsin y\}+P\{\pi-\arcsin y\leq X\leq \pi\}$$
$$=2\arcsin y$$
$$f_Y(y)=\frac{dF_Y(y)}{dy}=\frac{2}{\sqrt{1-y^2}}$$