5.1

2. $P\{X=k\}=(1-p)^{k-1}p$

$EX=\sum_{k=1}^{+\infty}k(1-p)^{k-1}p$

$=-p\left(\sum_{k=1}^{+\infty}(1-p)^k\right)'$

$=\left(\frac{1-p}{p}\right)'\cdot p$

$=\frac{1}{p}$

5. $P\{X=k\}=\begin{cases}0, & k<n\\ C_{k-1}^{n-1}p^{n-1}(1-p)^{k-n}\cdot p & k\geq n\end{cases}$

$EX=\sum_{k=n}^{+\infty}k\cdot\frac{(k-1)(k-2)\cdots(k-n+1)}{(n-1)!}\cdot p^n(1-p)^{k-n}$

$=\frac{p^n}{(n-1)!}\left[\sum_{k=n}^{+\infty}(1-p)^k\right]^{(n)}\cdot(-1)^n$

$=\frac{p^n}{(n-1)!}\left[\frac{(1-p)^n}{p}\right]^{(n)}(-1)^n$

$=\frac{p^n}{(n-1)!}\left[\frac{1}{p}+C_n^1+C_n^2p+\cdots+C_n^np^{n-1}\right]^{(n)}(-1)^n$

$=\frac{p^n}{(n-1)!}\cdot(-1)^n\cdot p^{-n-1}\cdot(n+1)!\cdot(-1)^n$

$=\frac{n}{p}$

6. $E(X+Y)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}(x+y)f(x,y)\,dxdy$

$=\int_0^1\int_0^x(x+y)\cdot 2\,dy\,dx$

$=\frac{4}{3}$

$E(XY)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}xyf(x,y)\,dxdy$

$=\int_0^1\int_0^x 2xy\,dy\,dx$

$=\frac{1}{4}$

8. 记 $Y_j=\frac{X_j}{\sum_{i=1}^{n}X_i}$

则 $Y_j\ (j=1,\cdots,n)$ 相互独立且服从同一分布，且 $|Y_j|\leq 1$

则 EY_j 存在且相等

$\therefore E(\sum_{j=1}^{n}Y_j)=nEY_j=1$

$\therefore E(\sum_{j=1}^{k}Y_j)=kEY_j=k=\frac{k}{n}$

5.2

3. ~~$EX=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}$~~

$f(x)=\int_{-\infty}^{+\infty}f(x,y)\,dy=\int_0^2\frac{1}{3}(x+y)\,dy=\frac{2}{3}x+\frac{2}{3},\ 0<x<1$

$f(x)=\begin{cases}\frac{2}{3}x+\frac{2}{3}, & 0<x<1\\ 0, & \text{其他}\end{cases}$

~~$EX=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\frac{2}{3}(x+1)\,x\,dx$~~

$=\int_0^1\frac{2}{3}(x+1)x\,dx$

$=\frac{5}{9}$

$EX^2=\int_0^1\frac{2}{3}(x+1)x^2\,dx$

$=\frac{5}{18}$

$DX=EX^2-(EX)^2=\frac{5}{18}-\frac{25}{81}=\frac{13}{162}$

5. 设点与横轴的夹角为 θ

则 θ 均匀分布

$X=R\cos\theta$

~~$EX=\int_0^{2\pi}R\cos\theta\cdot\frac{1}{2\pi}d\theta$~~

~~$=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}R\cos\theta\cdot\frac{1}{2\pi}d\theta+\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}}R\cos\theta\cdot\frac{1}{2\pi}d\theta$~~

~~$=0$~~

$EX=\int_0^{2\pi}R\cos\theta\frac{1}{2\pi}d\theta$

$=\int_0^{\pi}R\cos\theta\cdot\frac{1}{2\pi}d\theta+\int_{\pi}^{2\pi}R\cos\theta\frac{1}{2\pi}d\theta$

$=0$

$EX^2=\int_0^{2\pi}R^2\cos^2\theta\cdot\frac{1}{2\pi}d\theta=\frac{R^2}{2}$

$DX=EX^2-(EX)^2=\frac{R^2}{2}$

5.2

1. $\cancel{E(aX_1-bX_2}$

$DX_1=3^2=9$

$DX_2=3^2=9$

$D(aX_1-bX_2)$

$=9a^2-9b^2$

$\cancel{E(aX_1-bX_2^2)}$

$\Rightarrow E(X)^2=DX+(EX)^2=9+4=13$

$E(aX_1^2-bX_2^2)=13(a-b)$

2. $E(X)=100p$

$D(X)=E(X-100p)^2$

$=\sum_{k=0}^{100}(k-100p)^2\cdot p$

$=$

$\cancel{E(X)=\sum_{k=0}^{100}kC_{100}^{k}p^k(1-p)}$

$\cancel{E(X)=100E(X_0)}$

X_0表示1次实验中成功的次数

X_0服从(0-1)分布

$E(X_0)=p$

$\cancel{E(X)=100E(X_0)=100p}$

$E(X_0^2)=p$

$D(X_0)=p-p^2=p=p(1-p)$

$D(X)=100p(1-p)$

$\sqrt{D(X)}=10\sqrt{p(1-p)}$

$\therefore p=\frac{1}{2}$时 $\sqrt{D(X)}$最大

为 $D(X)=5$

4. 记X_0为1次射击命中目标的次数

X_0服从(0-1)分布

$E(X_0)=p$

$E(X)=10p$

$D(X_0)=p(1-p)$

$D(X)=10p(1-p)$

$EX^2=\cancel{DX+(EX)}^2=10p-10p^2+100p^2=90p^2+10p$

$(EX)^2+DX$

$90p^2+10p=18.4$

5. $\cancel{Z服从}$

$EX=\mu=1$

$\sigma=\sqrt{DX}=\sqrt{2}$

$\therefore X\sim N(1,\sqrt{2})$

$\therefore Z服从\sim N(5,9)$

$\therefore f_Z(z)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(z-\mu)^2}{2\sigma^2}}=\frac{1}{3\sqrt{2\pi}}e^{-\frac{(z-5)^2}{18}}$

5.4

5.4

1. $f_X(x)=\int_{-\infty}^{+\infty} f(x,y)\,dy$

$=\begin{cases}\int_{-x}^{x} dy, & 0<x<1\\ 0, & 其他\end{cases}$

$=\begin{cases}2x, & 0<x<1\\ 0, & 其他\end{cases}$

$f_Y(y)=\int_{-\infty}^{+\infty} f(x,y)\,\cancel{dy}\,dx$

$=\begin{cases}\int_{|y|}^{1} dx, & -1<y<1\\ 0, & 其他\end{cases}$

$=\begin{cases}1-|y|, & -1<y<1\\ 0, & 其他\end{cases}$

$E(X)=\int_{-\infty}^{+\infty} x f_X(x)\,dx$

$=\int_0^1 x\cdot 2x\,dx$

$=\cancel{\frac{2}{3}x^3}\ \frac{2}{3}$

$E(Y)=\int_{-\infty}^{+\infty} y f_Y(y)\,dy$

$=\int_{-1}^{1} y(1-|y|)\,dy$

$=\int_{-1}^{0} y(1+y)\,dy+\int_0^1 y(1-y)\,dy$

$=-\frac{1}{6}+\frac{1}{6}$

$=0$

$E(XY)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(x,y)\cdot xy\,dx\,dy.$

$=\int_0^1\int_{-x}^{x} xy\,dy\,dx$

$=\frac{2}{3}$

$cov(X,Y)=\frac{2}{3}-0=\frac{2}{3}$

5.

X\Y	0	1
0	$\frac{2}{3}$	$\frac{1}{12}$
1	$\frac{1}{6}$	$\frac{1}{12}$

~~$\rho_{XY}=EX=$~~

$DX=\cancel{\frac{3}{3}\times\frac{1}{6}=\frac{1}{9}}\ \frac{1}{4}\times\frac{3}{4}=\frac{3}{16}$

$DY=\frac{5}{6}\times\frac{1}{6}=\frac{5}{36}$

~~$cov(X,Y)=E(XY)$~~

$E(XY)=\frac{1}{12}$

$COV(X,Y)=\frac{1}{12}-\frac{1}{4}\times\frac{1}{6}=\frac{1}{24}$

$\rho_{XY}=\dfrac{\frac{1}{24}}{\frac{\sqrt{3}}{4}\times\frac{\sqrt{5}}{6}}=\frac{\sqrt{15}}{15}$