概统

习题 2.2

2.(1) ∵ $\lim_{x\to-\infty} F(x)=0$

$\lim_{x\to+\infty} F(x)=1$

∴ $\begin{cases} a-\frac{\pi}{2}b=0 \\ a+\frac{\pi}{2}b=1 \end{cases}$

∴ $\begin{cases} a=\frac{1}{2} \\ b=\frac{1}{\pi} \end{cases}$

(2) $P\{-1<X\le\sqrt{3}\}$

$=F(\sqrt{3})-F(-1)$

$=\frac{1}{2}+\frac{1}{\pi}\arctan\sqrt{3}-\frac{1}{2}-\frac{1}{\pi}\arctan(-1)$

$=\frac{1}{\pi}\cdot\frac{\pi}{3}-\frac{1}{\pi}\cdot(-\frac{\pi}{4})$

$=\frac{1}{3}+\frac{1}{4}$

$=\frac{7}{12}$

(3) ∵ $P\{X>c\}$

$=1-P\{X\le c\}$

∴ $P\{X\le c\}=1-\frac{1}{4}=\frac{3}{4}$

∴ $\frac{1}{2}+\frac{1}{\pi}\arctan c=\frac{3}{4}$

∴ $c=1$

5. ① ~~取值~~ 单调不减

∵ $F_1(x)$, $F_2(x)$ 单调不减

$a,b>0$

∴ $F(x)=aF_1(x)+bF_2(x)$

单调不减

② 取值范围

$\lim_{x\to-\infty} F_1(x)+\lim_{x\to-\infty} F_2(x)=0$

∴ $\lim_{x\to-\infty} F(x)=0$

$\lim_{x\to+\infty} F_1(x)=\lim_{x\to+\infty} F_2(x)=1$

$a+b=1$

∴ $\lim_{x\to+\infty} F(x)=1$

③ 连续性

∵ $F_1(x)$, $F_2(x)$ 右连续

∴ $F(x)$ 右连续

综上, $F(x)$ 为某随机变量的分布函数.

习题 2.3

3 $P\{X=1\}=0.2$

$P\{X=2\}=0.5$

~~$P\{X=3\}=0$~~

$P\{X=4\}=0.3$

$P\{X<4 \mid X\neq 3\}=0.7-0=0.7$

4 甲: $P\{X=k\}=(0.6\times0.4)^{k-1}\times(0.4+0.6\times0.6)$

$=0.24^{k-1}\times0.76$

乙: $P\{X=k\}=(0.6\times0.4)^{k-1}\times0.6\times(0.6+0.4\times0.4)$

$=0.24^{k-1}\times0.6\times0.76$

$=0.24^{k-1}\times0.456$

5. $P\{X=1\}=\frac{C_3^1}{C_8^1}=\frac{3}{8}$

$P\{X=2\}=\frac{C_5^1C_3^1}{A_8^2}=\frac{15}{56}$

$P\{X=3\}=\frac{A_5^2C_3^1}{A_8^3}=\frac{5}{56}$

$P\{X=4\}=\frac{A_5^3C_3^1}{A_8^4}=\frac{1}{56}$

$P\{1<X\le3\}=\frac{15}{56}+\frac{5}{56}=\frac{5}{14}$

习题2.4

2. $P_1 = 3(1-(1-0.01)^{30} - C_{30}^1(0.01)(1-0.01)^{29})$

$= 3(1-0.99^{30} - 0.3\times 0.99^{29})$

$P_2 = 1 - C_{10}^0(1-0.01)^{10} - C_{10}^1(0.01)(1-0.01)^{9}$

$- C_{10}^2(0.01)^2\times(1-0.01)^{8} - C_{10}^3(0.01)^3(1-0.01)^{7}$

$= 1 - 0.99^{10} - 10\times 0.01\times 0.99^{10} -$

一人包修30台可及时维修的概率为

$P = C_{30}^0 0.99^{30} + C_{30}^1(0.01)(0.99)^{29}$

$= 1.3\times 0.99^{29}$

三人包修90台不可及时维修

$P_1 = 1 - P^3 = 1 - (1.3\times 0.99^{29})^3$

三人包修90台不可及时维修

$P_2 = 1 - C$

记发生故障而为A_1, A_2

一人包修30台为A_1

为B_1, B_2

$P(A_1B_1) = 1-(0.99)^{30} - C_{30}^1(0.01)(0.99)^{29}$

$= 1 - 1.3\times 0.99^{29}$

$P(A_1) = 1-(0.99)^{30}$

$P(B_1|A_1) = \frac{1-1.3\times 0.99^{29}}{1-(0.99)^{30}}$

一人包修30台可及时维修

$P_0 = 0.99^{30} + C_{30}^1(0.01)(0.99)^{29}$

$= 1.3\times 0.99^{29}$

三人包修90台不可及时维修

$P_1 = 1 - (1.3\times 0.99^{29})^{30}$

三人包修90台不可及时维修

$1 - C_{90}^0 0.99^{90} - C_{90}^1 0.99^{89}(0.01) - C_{90}^2 0.99^{88}(0.01)^2 - C_{90}^3 0.99^{87}(0.01)^3$

3. $k = np = 4$

$P\{X\le 10\} = \sum_{k=0}^{10} C_{800}^k (0.005)^k (1-0.005)^{800-k}$

$\approx 1 - \sum_{k=11}^{100} \frac{4^k}{k!}$

$C_{800}^k (0.005)^k (0.995)^{800-k} \approx \frac{4^k}{k!}$

5. (1) $P = C_5^2(0.1)^2(0.9)^3 = 0.0729$

(2) $P_2 = 1 - 0.9^5 - C_5^1$

$C_5^3 0.9^2(0.1)^3 + C_5^4(0.9)(0.1)^4 + C_5^5(0.1)^5$

$= 0.00856$

(3) $1 - C_5^4(0.9)(0.1)^4 - C_5^5(0.1)^5$

$= 0.99954$

(4) $1-(0.9)^5 = 0.40951$

习题2.5

1. $\int_0^2 (a+bx)dx=1$

$\left[x+\frac{1}{2}x^2+ax\right]_0^2 = 2b+2a=1$

$\therefore a+b=\frac{1}{2}$

$P\{X\geq 1\} = \int_1^2 (a+bx)dx = \frac{1}{2}(a+bx)\Big|_1^2$

$= 2a+2b-(a+\frac{1}{2}b)$

$= a+\frac{3}{2}b$

$\begin{cases} a+b=\frac{1}{2} \\ a+\frac{3}{2}b=\frac{1}{4} \end{cases}$

$\therefore \begin{cases} a=1 \\ b=-\frac{1}{2} \end{cases}$

(2) $F(x)=\begin{cases} 0, & x<0 \\ x-\frac{1}{2}x^2+0x, & 0\leq x\leq 2 \\ 1, & x>2 \end{cases}$

5. (1) $\begin{cases} \lim\limits_{x\to\infty} A+Be^{-\frac{x^2}{2}} = A=1 \\ A+Be^0 = A+B=0 \end{cases}$

$\therefore \begin{cases} A=1 \\ B=-1 \end{cases}$

(2) $f(x)=\begin{cases} 0, & x\leq 0 \\ 1+xe^{-\frac{x^2}{2}}, & x>0 \end{cases}$

(3) $P\{\sqrt{\ln 4}<X<\sqrt{\ln 8}\} = 1-e^{-\frac{\ln 8}{2}}-1+e^{-\frac{\ln 4}{2}} = \frac{1}{8}$

6. $F(x)=\begin{cases} 0, & x\leq 0 \\ x^2, & 0<x<1 \\ 1, & x\geq 1 \end{cases}$

$P\{X\leq\frac{1}{2}\} = F(\frac{1}{2})=\frac{1}{4}$

$P\{Y=2\} = C_3^2(\frac{1}{4})^2(\frac{3}{4}) = \frac{9}{64}$

习题2.6

(2) $F(x)$

$$F(x)=\begin{cases}0, & x\le 0\\ 2\times10^{5}(x+5\times10^{-5}), & 0<x\le 5\times10^{-5}\\ 1, & x>5\times10^{-5}\end{cases}$$

$$F(6\times10^{-5})-F(5\times10^{-5})=0.2$$

$$F(x)=\begin{cases}0, & x\le 0\\ 2\times10^{5}x, & 0<x\le 5\times10^{-5}\\ 1, & x>5\times10^{-5}\end{cases}$$

$$F(6\times10^{-5})-F(3\times10^{-5})=1-0.6=0.4$$

5. X 概率分布函数：

$$F(x)=\begin{cases}0, & x\le 200\\ 1-\frac{200}{x}, & x>200\end{cases}$$

$$P\{X>400\}=1-P\{X\le 400\}$$

$$=1-\left(1-\frac{1}{2}\right)$$

$$=\frac{1}{2}$$

$$1-C_3^3\left(1-\frac{1}{2}\right)^3=\frac{7}{8}$$

习题八

3. 由对

$\because P\{X\ge 3\}=P\{X\le -1\}$

$\therefore \mu=\frac{3-1}{2}=1$

$\because P\{X\le -1\}=\Phi(-1)$

$\therefore \frac{-1-1}{\sigma}=-1$

$\therefore \sigma=2$

5(1) $P_i=P_{ii}=\Phi(0.2)=0.2119$

$P_{ii}=0.5762$

记取出产品A

则 $=0.2119\times0.1+0.5762\times0.001+0.2119\times0.2$

$=0.06415$

(2) 设

$$P(i|A)=\frac{0.2119\times0.1}{0.06415}=0.33$$

$$P(ii|A)=\frac{0.5762\times0.001}{0.06415}=0.01$$

$$P(iii|A)=\frac{0.2119\times0.2}{0.06415}=0.66$$

6. $Z_{0.85}\approx 1.04$

$$\frac{m-76}{15}=1.04$$

$$m=91.6$$

$$Z_{0.1}=-1.28$$

$$\frac{m-76}{15}=-1.28$$

$$m=56.8$$