

## Task 2:

```
clear all
clc
```

```
%system nb 1
```

```
A1=[1 4 3; 0 2 16; 0 -25 -20]
```

```
A1 = 3x3
     1     4     3
     0     2    16
     0    -25   -20
```

```
B1=[-1;0;0]
```

```
B1 = 3x1
    -1
     0
     0
```

```
C1=[-1 3 0]
```

```
C1 = 1x3
    -1     3     0
```

```
D1=[0]
```

```
D1 = 0
```

```
%system nb 2
```

```
A2=[1 0 0; 0 0 0; -2 -4 -3]
```

```
A2 = 3x3
     1     0     0
     0     0     0
    -2    -4    -3
```

```
B2=[-1;0;-1]
```

```
B2 = 3x1
    -1
     0
    -1
```

```
C2=[1 0 0]
```

```
C2 = 1x3
     1     0     0
```

```
D2=[0]
```

```
D2 = 0
```

```
%checking controllability of the 1st system
```

```
poles1 = eig(A1) %one unstable pole at p=1
```

```
poles1 = 3x1 complex
    1.0000 + 0.0000i
   -9.0000 +16.7033i
```

```
-9.0000 -16.7033i
```

```
%finding controllability staircase form
```

```
[Af1, Bf1, Cf1, T1, k1] = ctrbf(A1, B1, C1)
```

```
Af1 = 3x3
    -20    -25     0
     16     2     0
     -3    -4     1
```

```
Bf1 = 3x1
     0
     0
     1
```

```
Cf1 = 1x3
     0     3     1
```

```
T1 = 3x3
     0     0     1
     0     1     0
    -1     0     0
```

```
k1 = 1x3
     1     0     0
```

```
% the poles of the transformed system are the same as the
% ones of the original system.
poles1_new = eig(Af1)
```

```
poles1_new = 3x1 complex
    1.0000 + 0.0000i
   -9.0000 +16.7033i
   -9.0000 -16.7033i
```

```
controllable_poles = eig(Af1(3,3))
```

```
controllable_poles = 1
```

```
uncontrollable_poles = eig(Af1(1:2, 1:2))
```

```
uncontrollable_poles = 2x1 complex
   -9.0000 +16.7033i
   -9.0000 -16.7033i
```

```
system_order = length(A1)
```

```
system_order = 3
```

```
M = ctrb(A1, B1);
rank_of_M = rank(M)
```

```
rank_of_M = 1
```

```
%checking observability of the 1st system
```

```
N = obsv(A1, C1);
rank_of_N = rank(N)
```

```
rank_of_N = 3
```

The 1st system is observable, and because the one unstable pole is controllable we can design a variable feedback controller to stabilize it. This system has one unstable pole at  $p=1$ , the system is therefore unstable.

```
%checking controllability of the 2nd system
```

```
poles2 = eig(A2) %one unstable pole at p=1 and one pole at the origin p=0
```

```
poles2 = 3×1
-3
1
0
```

```
%finding controllability staircase form
```

```
[Af2, Bf2, Cf2, T2, k2] = ctrbf(A2, B2, C2)
```

```
Af2 = 3×3
    0    0    0
-2.8284 -0.0000 3.0000
 2.8284 1.0000 -2.0000
Bf2 = 3×1
    0
-0.0000
 1.4142
Cf2 = 1×3
    0 -0.7071 -0.7071
T2 = 3×3
    0 1.0000    0
-0.7071    0 0.7071
-0.7071    0 -0.7071
k2 = 1×3
    1    1    0
```

```
% the poles of the transformed system are the same as the
% ones of the original system.
```

```
poles2_new = eig(Af2)
```

```
poles2_new = 3×1
-3.0000
 1.0000
    0
```

```
controllable_poles = eig(Af2(2:3,2:3))
```

```
controllable_poles = 2×1
 1.0000
-3.0000
```

```
uncontrollable_poles = eig(Af2(1, 1))
```

```
uncontrollable_poles = 0
```

```
system_order = length(A2)
```

```
system_order = 3
```

```
M = ctrb(A2, B2);
rank_of_M = rank(M)
```

```
rank_of_M = 2
```

```
%checking observability of the 2nd system
```

```
N = obsv(A2, C2);
```

$$\text{rank\_of\_N} = \text{rank}(N)$$

$$\text{rank\_of\_N} = 1$$

In the case of 2nd system the pole at  $p=0$  is uncontrollable, therefore we cannot move it and as a result we are unable to create variable feedback controller for this system. Similarly to the system nb 1 it has an unstable pole at  $p=1$ , therefore it is also unstable.