```
clear
syms a1 a2 a3 a4
syms s11 s21 s31 s41 s12 s22 s32 s42 s13 s23 s33 s43 s14 s24 s34 s44
A=[0\ 1\ 0\ 0;\ 42.51\ 0\ 0\ 0;\ 0\ 0\ 0\ 1;\ -2.943\ 0\ 0\ 0]
A = 4 \times 4
       0 1.0000
                          0
                                   0
  42.5100
             0
                          0
                                   0
                 0
                          0 1.0000
  -2.9430
B=[0 -3.333 \ 0 \ 1]'
B = 4 \times 1
        0
  -3.3330
  1.0000
C=[1 0 0 0;0 0 1 0]
C = 2 \times 4
    1
    0
Contr=[B A*B (A^2)*B (A^3)*B]
Contr = 4 \times 4
     0 -3.3330 0 -141.6858
  -3.3330
            0 -141.6858
      0 1.0000 0 9.8090
  1.0000
               0 9.8090
                                   0
a=[a1 a2 a3 a4]
a = (a_1 \ a_2 \ a_3 \ a_4)
Ac=[a;1 0 0 0;0 1 0 0;0 0 1 0]
Ac =
(a_1 \ a_2 \ a_3 \ a_4)
 1
    0 0 0
   1 0 0
(0 \ 0 \ 1 \ 0)
Bc=[1 0 0 0]'
Bc = 4 \times 1
    1
    0
    0
Cc=[Bc Ac*Bc (Ac^2)*Bc (Ac^3)*Bc]
```

Cc =

$$\begin{pmatrix} 1 & a_1 & a_1^2 + a_2 & a_3 + a_1 a_2 + a_1 (a_1^2 + a_2) \\ 0 & 1 & a_1 & a_1^2 + a_2 \\ 0 & 0 & 1 & a_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

det(Cc)

ans = 1

T1=[s11 s21 s31 s41;s12 s22 s32 s42;s13 s23 s33 s43;s14 s24 s34 s44]

T1 =

$$egin{pmatrix} s_{11} & s_{21} & s_{31} & s_{41} \ s_{12} & s_{22} & s_{32} & s_{42} \ s_{13} & s_{23} & s_{33} & s_{43} \ s_{14} & s_{24} & s_{34} & s_{44} \ \end{pmatrix}$$

s4=[s14 s24 s34 s44]

$$s4 = (s_{14} \ s_{24} \ s_{34} \ s_{44})$$

s3=[s13 s23 s33 s43]

 $s3 = (s_{13} \ s_{23} \ s_{33} \ s_{43})$ 

s2=[s12 s22 s32 s42]

 $s2 = (s_{12} \ s_{22} \ s_{32} \ s_{42})$ 

s1=[s11 s21 s31 s41]

 $s1 = (s_{11} \ s_{21} \ s_{31} \ s_{41})$ 

s4==[0 0 0 1]/(Contr)

ans =

$$\left(s_{14} = -\frac{5289001001014353}{576460752303423488} \quad s_{24} = 0 \quad s_{34} = -\frac{8814120168190419}{288230376151711744} \quad s_{44} = 0\right)$$

var = vpa(ans)

 $var = (s_{14} = -0.0091749542009244306611792652006443 \quad s_{24} = 0.0 \quad s_{34} = -0.0305801223516811264413473$ 

s3==s4\*A

ans =

$$\left(s_{13} = \frac{4251 \, s_{24}}{100} - \frac{2943 \, s_{44}}{1000} \, s_{23} = s_{14} \, s_{33} = 0 \, s_{43} = s_{34}\right)$$

var = vpa(ans)

$$var = (s_{13} = 42.51 s_{24} - 2.943 s_{44} s_{23} = s_{14} s_{33} = 0.0 s_{43} = s_{34})$$

s2==s3\*A

ans =

$$\left(s_{12} = \frac{4251 \, s_{23}}{100} - \frac{2943 \, s_{43}}{1000} \, s_{22} = s_{13} \, s_{32} = 0 \, s_{42} = s_{33}\right)$$

% s14 = -0.0092; s24 = 0; s34 = -0.0306; s44 = 0;

% s13 = 0 ; s23 = s14; s33 = 0; s43 = s34;

var2 = subs(ans,[s14,s24,s34,s44,s13,s23,s33,s43],[-0.0092,0,-0.0306,0,0,-0.0092,0,-0.0306])

var2 =

$$\left(s_{12} = -\frac{1505181}{5000000} \quad s_{22} = 0 \quad s_{32} = 0 \quad s_{42} = 0\right)$$

var = vpa(var2)

$$var = (s_{12} = -0.3010362 \quad s_{22} = 0.0 \quad s_{32} = 0.0 \quad s_{42} = 0.0)$$

s1==s2\*A

ans =

$$\left(s_{11} = \frac{4251 \, s_{22}}{100} - \frac{2943 \, s_{42}}{1000} \, s_{21} = s_{12} \, s_{31} = 0 \, s_{41} = s_{32}\right)$$

var3 = subs(ans,[s12,s22,s32,s42],[-0.3 0 0 0])

var3 =

$$\left(s_{11} = 0 \quad s_{21} = -\frac{3}{10} \quad s_{31} = 0 \quad s_{41} = 0\right)$$

var4 = vpa(var3)

var4 = 
$$(s_{11} = 0.0 \ s_{21} = -0.3 \ s_{31} = 0.0 \ s_{41} = 0.0)$$

T1=[s11 s21 s31 s41;s12 s22 s32 s42;s13 s23 s33 s43;s14 s24 s34 s44]

T1 =

$$\begin{pmatrix} s_{11} & s_{21} & s_{31} & s_{41} \\ s_{12} & s_{22} & s_{32} & s_{42} \\ s_{13} & s_{23} & s_{33} & s_{43} \\ s_{14} & s_{24} & s_{34} & s_{44} \end{pmatrix}$$

T1ac = subs(T1,[s14,s24,s34,s44,s13,s23,s33,s43,s12,s22,s32,s42,s11,s21,s31,s41],[-0.0092,0,-0]

T1ac =

$$\begin{pmatrix}
0 & -\frac{3}{10} & 0 & 0 \\
-\frac{3}{10} & 0 & 0 & 0 \\
0 & -\frac{23}{2500} & 0 & -\frac{153}{5000} \\
-\frac{23}{2500} & 0 & -\frac{153}{5000} & 0
\end{pmatrix}$$

### vpa(inv(T1ac))

ans =

### Ac=T1ac\*A\*inv(T1ac)

Ac =

$$\begin{pmatrix}
0 & \frac{4251}{100} & 0 & 0 \\
1 & 0 & 0 & 0 \\
0 & \frac{501727}{500000} & 0 & 0 \\
0 & 0 & 1 & 0
\end{pmatrix}$$

### Bc=vpa(T1ac\*B)

Bc =

## Bc=[1 0 0 0]'

 $Bc = 4 \times 1$ 

1 0

0

٥

$$Cc=([Bc Ac*Bc (Ac^2)*Bc (Ac^3)*Bc])$$

Cc =

$$\begin{pmatrix}
1 & 0 & \frac{4251}{100} & 0 \\
0 & 1 & 0 & \frac{4251}{100} \\
0 & 0 & \frac{501727}{500000} & 0 \\
0 & 0 & 0 & \frac{501727}{500000}
\end{pmatrix}$$

### Cc=C\*inv(T1ac)

Cc =

$$\begin{pmatrix}
0 & -\frac{10}{3} & 0 & 0 \\
0 & \frac{460}{459} & 0 & -\frac{5000}{153}
\end{pmatrix}$$

### vpa(Cc)

ans =

### Dc=[0;0]

aa

$$Fc = (f_1 \ f_2 \ f_3 \ f_4)$$

 $1_m =$ 

$$\begin{pmatrix}
lamb & 0 & 0 & 0 \\
0 & lamb & 0 & 0 \\
0 & 0 & lamb & 0 \\
0 & 0 & 0 & lamb
\end{pmatrix}$$

## P=det(1\_m-Ac)

P =

```
\frac{\text{lamb}^2 (100 \, \text{lamb}^2 - 4251)}{100}
```

```
simplify(P)
```

ans =

$$lamb^4 - \frac{4251 \ lamb^2}{100}$$

```
a1=0;a2=-42.51;a3=0;a4=0;
acl1=13.52;acl2=56.64;acl3=76.72;acl4=32.6;
Fc=[a1-acl1 a2-acl2 a3-acl3 a4-acl4]
```

Fc = 1×4 -13.5200 -99.1500 -76.7200 -32.6000

### F=Fc\*T1ac

F =

$$\left(\frac{751123}{25000} \ \frac{148807}{31250} \ \frac{24939}{25000} \ \frac{146727}{62500}\right)$$

### vpa(F)

ans =  $(30.04492 \ 4.761824 \ 0.99756 \ 2.347632)$ 

### round(vpa(eig(A+B\*F)),4)

ans =

$$-1.0448$$
 $-0.9594$ 

## round(vpa(eig(Ac+Bc\*Fc)),4)

ans =

$$-1.0912$$

(-0.9239)

$$Asf = A+B*F$$

Asf =

```
0
                1
                              0
                                           0
<u>1440742959</u> <u>495973731</u> <u>83121687</u>
                                       489041091
 25000000
              31250000
                           25000000
                                        62500000
                              0
   0
                 0
                                          1
 169387
               148807
                            24939
                                        146727
               31250
  6250
                            25000
                                         62500
```

### vpa(Asf)

ans =

```
\begin{pmatrix} 0 & 1.0 & 0 & 0 \\ -57.62971836 & -15.871159392 & -3.32486748 & -7.824657456 \\ 0 & 0 & 0 & 1.0 \\ 27.10192 & 4.761824 & 0.99756 & 2.347632 \end{pmatrix}
```

Asf=[0 1 0 0;-57.6297 -15.8712 -3.3249 -7.8247;0 0 0 1;27.1019 4.7618 0.9976 2.3476]

Asf = 4×4 0 1.0000 0 0 -57.6297 -15.8712 -3.3249 -7.8247 0 0 0 1.0000 27.1019 4.7618 0.9976 2.3476

Bsf = B

Bsf = 4×1 0 -3.3330 0 1.0000

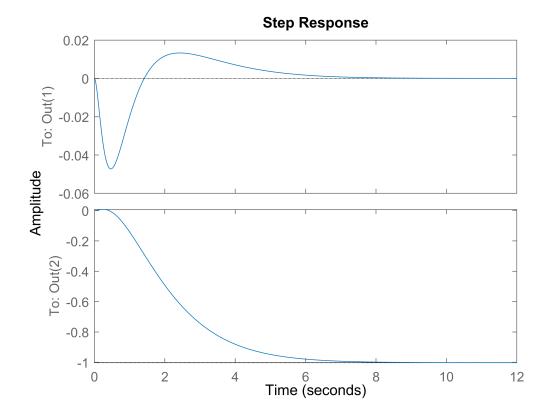
Csf = C

Csf = 2×4 1 0 0 0 0 0 1 0

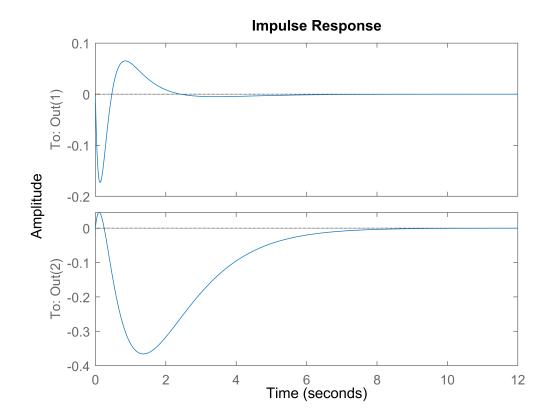
Dsf = Dc

Dsf = 2×1 0 0

sys=ss(Asf,Bsf,Csf,Dsf);
step(sys)



# impulse(sys)



# Is different due to rounding error? Perhaps?

Task4

syms t4

eq1 =

eq1=t4==inv(Ooa)\*[0;0;0;1]

```
syms a1 a2 a3 a4
 Ao=[a1 1 0 0;a2 0 1 0;a3 0 0 1; a4 0 0 0]
 Ao =
  (a_1 \ 1 \ 0 \ 0)
  a_2 \ 0 \ 1 \ 0
  a_3 \ 0 \ 0 \ 1
  a_4 \ 0 \ 0 \ 0
 Co=[1 0 0 0]
 Co = 1 \times 4
                       0
 Ooa=[C(2,:);(C(2,:)*A);(C(2,:)*A^2);(C(2,:)*A^3)]
 0oa = 4 \times 4
                       1.0000
                                 1.0000
    -2.9430
Is it correct to use Ooa (with substituted values from A and C) intead
of Oo (that has symbolic values)
 Oo=[Co;(Co*Ao);(Co*Ao^2);(Co*Ao^3)]
 0o =
                                  0 \quad 0
  (a_3 + a_1 a_2 + a_1 (a_1^2 + a_2) a_1^2 + a_2 a_1 1)
 det(Oo)
 ans = 1
```

$$\begin{pmatrix}
t_4 = 0 \\
t_4 = -\frac{1000}{2943} \\
t_4 = 0 \\
t_4 = 0
\end{pmatrix}$$

### vpa(eq1)

ans =

```
\begin{aligned}
t_4 &= 0.0 \\
t_4 &= -0.33978933061501868841318382602786 \\
t_4 &= 0.0 \\
t_4 &= 0.0
\end{aligned}
```

### t4=[0 -0.3398 0 0]'

t4 = 4×1 0 -0.3398 0

### t3=A\*t4

t3 = 4×1 -0.3398 0

### t2=A\*t3

t2 = 4×1 0 -14.4449 0 1.0000

### t1=A\*t2

t1 = 4×1 -14.4449 0 1.0000 0

### S=[t1 t2 t3 t4]

### Ao=inv(S)\*A\*S

```
Ao = 4 \times 4
               1.0000
                                        0
                              0
    42.5100
                         1.0000
                    0
                                        0
          0
                    0
                              0
                                   1.0000
          0
                    0
                              0
                                        0
  Bo=inv(S)*B
  Bo = 4 \times 1
     1.0000
    -32.7000
  Co=C*S
  Co = 2 \times 4
    -14.4449
                    0
                        -0.3398
                                        0
     1.0000
                                        0
  Do=[0;0];
  syso=ss(Ao,Bo,Co,Do);
  round(vpa(eig(A)),4)
  ans =
     0
     0
    6.52
   -6.52
  round(vpa(eig(Ac)),4)
  ans =
     0
   -6.52
   6.52
  round(vpa(eig(Ao)),4)
  ans =
   6.52
   -6.52
     0
Why inverted?
```

```
syms lamb
det(lamb*eye(4)-Ao)
```

ans =

```
lamb^2 (100 \, lamb^2 - 4251)
  expand(ans)
  ans =
  1 amb^4 - \frac{4251 \ lamb^2}{100}
  a1=0;a2=-42.51;a3=0;a4=0;
  acl1=135.2; acl2=5664; acl3=76720; acl4=326000;
  Lo=[a1-acl1 a2-acl2 a3-acl3 a4-acl4]'
  Lo = 4 \times 1
  10<sup>5</sup> ×
     -0.0014
     -0.0571
     -0.7672
     -3.2600
  L=S*Lo
  L = 4 \times 1
  10<sup>5</sup> ×
      0.2802
     1.9320
     -0.0014
     -0.0571
Is this error due to rounding acceptable?
  eig(A+L*C(2,:))
  ans = 4 \times 1
    -65.2121
    -49.9921
    -10.0091
     -9.9910
  eig(Ao+Lo*Co(2,:))
  ans = 4 \times 1
    -65.2121
    -49.9921
    -10.0091
     -9.9910
  Aso = A+L*C(2,:); Bso = B; Cso = C; Dso = Do;
  sysso=ss(Aso,Bso,Cso,Dso)
```

```
-2.943
x4
                         0
                                -5707
                                                0
B =
         u1
x1
     -3.333
x2
х3
          0
х4
          1
C =
     x1 x2 x3 x4
              0
у1
         0
                  0
      1
      0
          0
              1
                  0
y2
D =
     u1
у1
```

Continuous-time state-space model.

### eig(Aso)

y2

ans = 4×1 -65.2121 -49.9921 -10.0091 -9.9910

# Task5

closed loop state feedback system

### Asf=A+B\*F

Asf =

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ -\frac{1440742959}{25000000} & -\frac{495973731}{31250000} & -\frac{83121687}{25000000} & -\frac{489041091}{62500000} \\ 0 & 0 & 0 & 1 \\ \frac{169387}{6250} & \frac{148807}{31250} & \frac{24939}{25000} & \frac{146727}{62500} \end{pmatrix}$$

### vpa(Asf)

ans =

$$\begin{pmatrix} 0 & 1.0 & 0 & 0 \\ -57.62971836 & -15.871159392 & -3.32486748 & -7.824657456 \\ 0 & 0 & 0 & 1.0 \\ 27.10192 & 4.761824 & 0.99756 & 2.347632 \end{pmatrix}$$

Asf=[0 1 0 0;-57.6297 -15.871 -3.3248 -7.824657;0 0 0 1;27.10192 4.7618 1 2.3476]

Asf = 4×4 0 1.0000 0 0

```
-57.6297 -15.8710 -3.3248 -7.8247
0 0 0 1.0000
27.1019 4.7618 1.0000 2.3476
```

```
Bsf=B;
Csf=C;
Dsf=Do;
```

### Observer base state feedback system

```
Areg = [ (A+B*F) B*F; zeros(size(A)) (A+L*C(2,:)) ]
```

Areg =

(	0	1	0	0	0	0	0
	$-\frac{1440742959}{25000000}$	$-\frac{495973731}{31250000}$	$-\frac{83121687}{25000000}$	$-\frac{489041091}{62500000}$	$-\frac{2503492959}{25000000}$	$-\frac{495973731}{31250000}$	$-\frac{8312}{2500}$
-	0	0	0	1	0	0	0
	$\frac{169387}{6250}$	$\frac{148807}{31250}$	$\frac{24939}{25000}$	$\frac{146727}{62500}$	$\frac{751123}{25000}$	$\frac{148807}{31250}$	$\frac{249}{250}$
	0	0	0	0	0	1	$\frac{192568509}{687194}$
	0	0	0	0	$\frac{4251}{100}$	0	$\frac{331923241}{171798}$
	0	0	0	0	0	0	$-\frac{47570764}{3518437}$
	0	0	0	0	$-\frac{2943}{1000}$	0	$-\frac{62745711}{109951}$

### vpa(Areg)

ans =

Areg=[0 1 0 0 0 0 0;-57.6297 -15.871 -3.3248 -7.8246 -100.139 -15.871 -3.3248 -7.8246;0 0 0 0 0 0 42.51 0 193204.7548 0; 0 0 0 0 0 0 -135.204 1;0 0 0 0 -2.943 0 -5706.69 0]

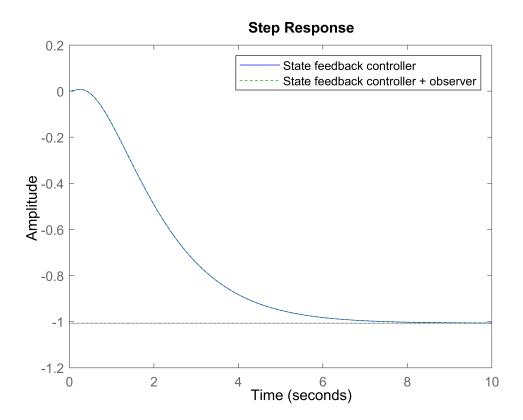
 $Areg = 8 \times 8$ 10<sup>5</sup> × 0.0000 -0.0006 -0.0002 -0.0000 -0.0001 -0.0010 -0.0002 -0.0000 -0.0001 0.0000 0 0.0003 0.0000 0.0000 0.0000 0.0003 0.0000 0.0000 0.0000 0.0000 0.2802 0 0

```
0
                     0
                                0
                                     0.0004
                                                     0
                                                          1.9320
0
          0
                     0
                                                          -0.0014
                                                                     0.0000
                                0
                                                     0
0
                                    -0.0000
                                                          -0.0571
```

```
Breg = [ B; zeros(size(B)) ];
Creg = [ C(2,:) zeros(size(C(2,:))) ];
Dreg = 0;
sys_sf=ss(Asf,Bsf,Csf(2,:),Dsf(2,:));
sys_reg=ss(Areg,Breg,Creg,Dreg);
figure
impulse(sys_sf);hold on
impulse(sys_reg);hold off
legend('State feedback controller','State feedback controller +observer');
xlim([0 10]);
```

# Impulse Response 0.05 State feedback controller State feedback controller +observer -0.05 -0.1 -0.25 -0.3 -0.35 -0.4 0 2 4 6 Time (seconds)

```
figure
step(sys_sf,'b');
hold on
step(sys_reg,'g--');legend('State feedback controller','State feedback controller + observer')
xlim([0 10]);
```



```
A = [0, 1, 0, 0; 42.51, 0, 0, 0; 0, 0, 0, 1; -(2.943), 0, 0, 0];
B = [0; -(3.333); 0; 1];
C = [1, 0, 0, 0; 0, 0, 1, 0];
D = [0; 0];
system_order = length(A)
```

 $system\_order = 4$ 

```
M = ctrb(A,B);
rank(M)
```

ans = 4

```
N = obsv(A,C);
rank(N)
```

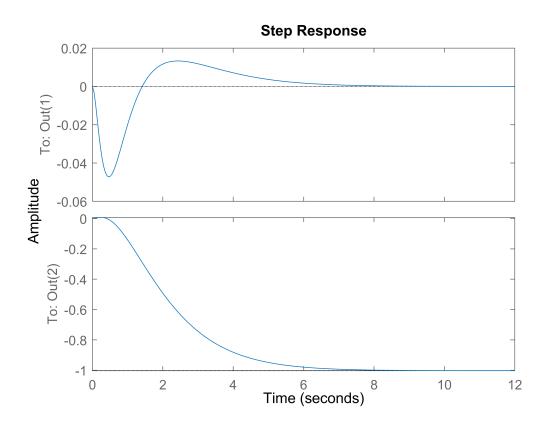
ans = 4

```
desiredPoles = [-1 -1 -5 -6.52];
K = acker(A,B,desiredPoles);
```

Warning: Pole locations are more than 10% in error.

```
% K = place(A,B,desiredPoles);
% closed-loop system
Ac = A-B*K; Bc = B; Cc = C; Dc = D;
sys_closed = ss(Ac,Bc,Cc,Dc);
```

```
tf_closed = tf(sys_closed);
figure
step(sys_closed);
```



### stepinfo(sys\_closed)

ans =  $2 \times 1$  struct

Fields RiseTime TransientTime SettlingTime SettlingMin SettlingMax 9.0206e-17 6.7445 NaN -0.0472 0.0133 2 3.3709 6.1824 6.1911 -1.003 -0.9057

### eig(sys\_closed)

```
ans = 4×1 complex

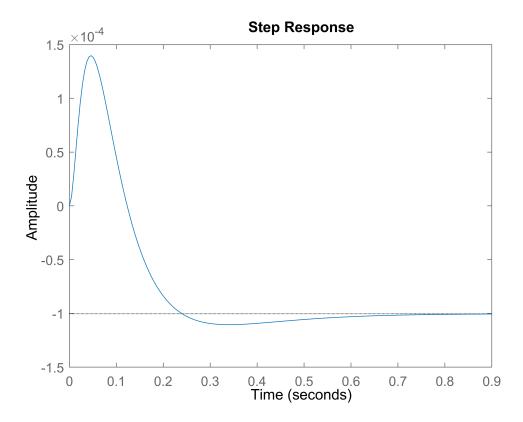
-6.5200 + 0.0000i

-5.0000 + 0.0000i

-1.0000 + 0.0000i

-1.0000 - 0.0000i
```

```
observerPoles = 10 * desiredPoles;
L = acker(A',C(2,:)',observerPoles);
% L = place(A',C(2,:)',observerPoles);
Ao = A - L'*C(2,:); Bo = B; Co = C(2,:); Do = D(2,:);
sys_observer = ss(Ao,Bo,Co,Do);
tf_observer = tf(sys_observer);
figure
```



### stepinfo(sys\_observer)

```
ans = struct with fields:
    RiseTime: 0.0835
TransientTime: 0.5168
SettlingTime: 0.6412
SettlingMan: -1.1045e-04
SettlingMax: -9.2001e-05
    Overshoot: 10.1105
Undershoot: 139.2860
    Peak: 1.3972e-04
PeakTime: 0.0461
```

### eig(sys\_observer)

```
ans = 4×1 complex

-65.2000 + 0.0000i

-50.0000 + 0.0000i

-10.0000 + 0.0000i

-10.0000 - 0.0000i
```

```
Areg = [(A-B*K) B*K;zeros(size(A)) (A+L'*C(2,:))];
Breg = [B; zeros(size(B))];
Creg = [C zeros(size(C))];
Dreg = 0;

sys_reg = ss(Areg,Breg,Creg,Dreg);
tf_reg = tf(sys_reg);
result_eig = eig(sys_reg)
```

```
result_eig = 8×1 complex

10<sup>2</sup> x

-0.0652 + 0.0000i

-0.0500 + 0.0000i

-0.0100 + 0.0000i

-0.0100 - 0.0000i

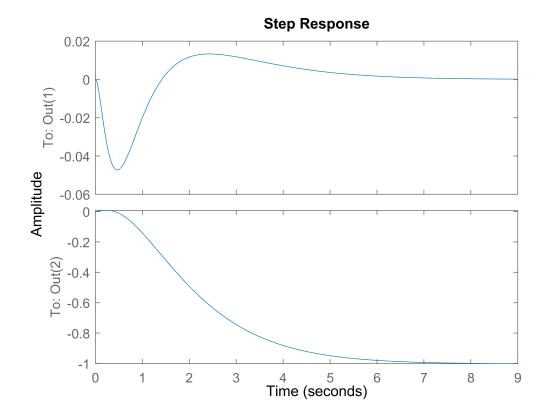
1.7141 + 0.0000i

-0.1381 + 0.0553i

-0.1381 - 0.0553i

-0.0859 + 0.0000i
```

```
figure
step(sys_reg);
```



## stepinfo(sys\_reg)

ans =  $2 \times 1$  struct

Fields RiseTime SettlingMin SettlingMax TransientTime SettlingTime 1 3.4584e-15 6.744 NaN -0.0472 0.0133 2 3.3708 6.1821 6.1909 -1.0026 -0.9029