Katedra Robotyki i Mechatroniki
Akademia Górniczo-Hutnicza w Krakowie

Industrial Robots

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1C
Mechanics of manipulators 2
Geometrical model
Forward Kinematics

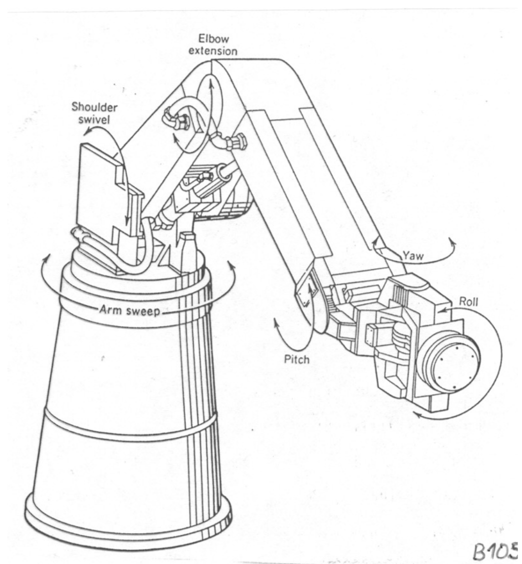
Problems:

- Denavit-Hartenberg notation
- Examples of geometrical models of manipulators

Common features of industrial (manipulating) robot manipulators:

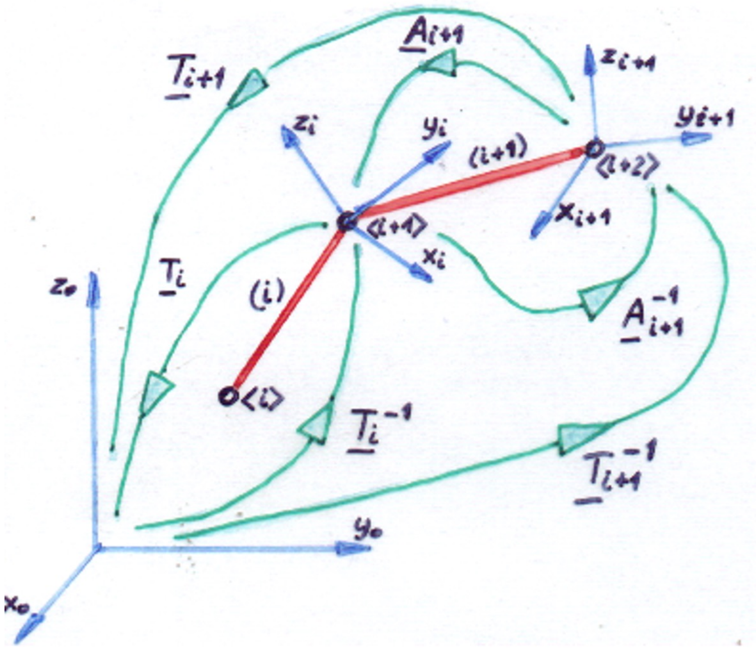
- Joints are of kinematic pairs of class V (1 DOM) – rotary or prismatic ones
- Straight line links
- In each kinematic pair a motion axis of a next link is parallel or perpendicular to an axis of a link located closer to a base

Arm



Wrist

Geometry model of a manipulator



Joint coordinates: $q_1, q_2, \ldots, q_n$

Cartesian coordinates: $x, y, z, \varphi, \theta, \psi$

Position and orientation of link i with respect to the reference co-ordinate frame:

$$^{0}\underline{T}_i = \underline{T}_i = \underline{A}_1 \underline{A}_2 \ldots \underline{A}_{i-1} \underline{A}_i$$

matrix $\underline{A}_i$ describes position and orientation of i-th coordinate frame assigned to link i with respect of coordinate frame assigned to link i-1.

Denavit-Hartenberg notation determines definition of local coordinate frames and of homogeneous transformation $\underline{A}_i$.

Algorithm:

LOCATION OF ORIGINS

Locate an **origin of *i*-th local coordinate frame**:

-at intersection point of axis of motion of link i and axis of motion of link $i+1$ when the axes cross each other

-at axis of motion of link $i+1$ so that the distance d_i is minimum when motion axes of neighbouring links are parallel to each other

-at the intersection point of i+1 motion axis and line perpendicular to the both axes of motion of link i and link i+1 when axes of motion do not cross each other and are not parallel either.

In the case of the last link (wrist flange or end-effector) the origin of the local coordinate frame might be placed: at the wrist motion axes intersection point or at an arbitrarily selected point of the wrist flange or at the end-effector.

ORIENTING OF AXES

Orient axes of the local *i*-th coordinate frame so that:

direction of

- x_i axis is the same as direction of x_0 axis
- z_i axis is an axis of i+1 translation or an axis of i+1 rotation

Note that axis x_i might be also the axis of $i+1$ rotation or translation

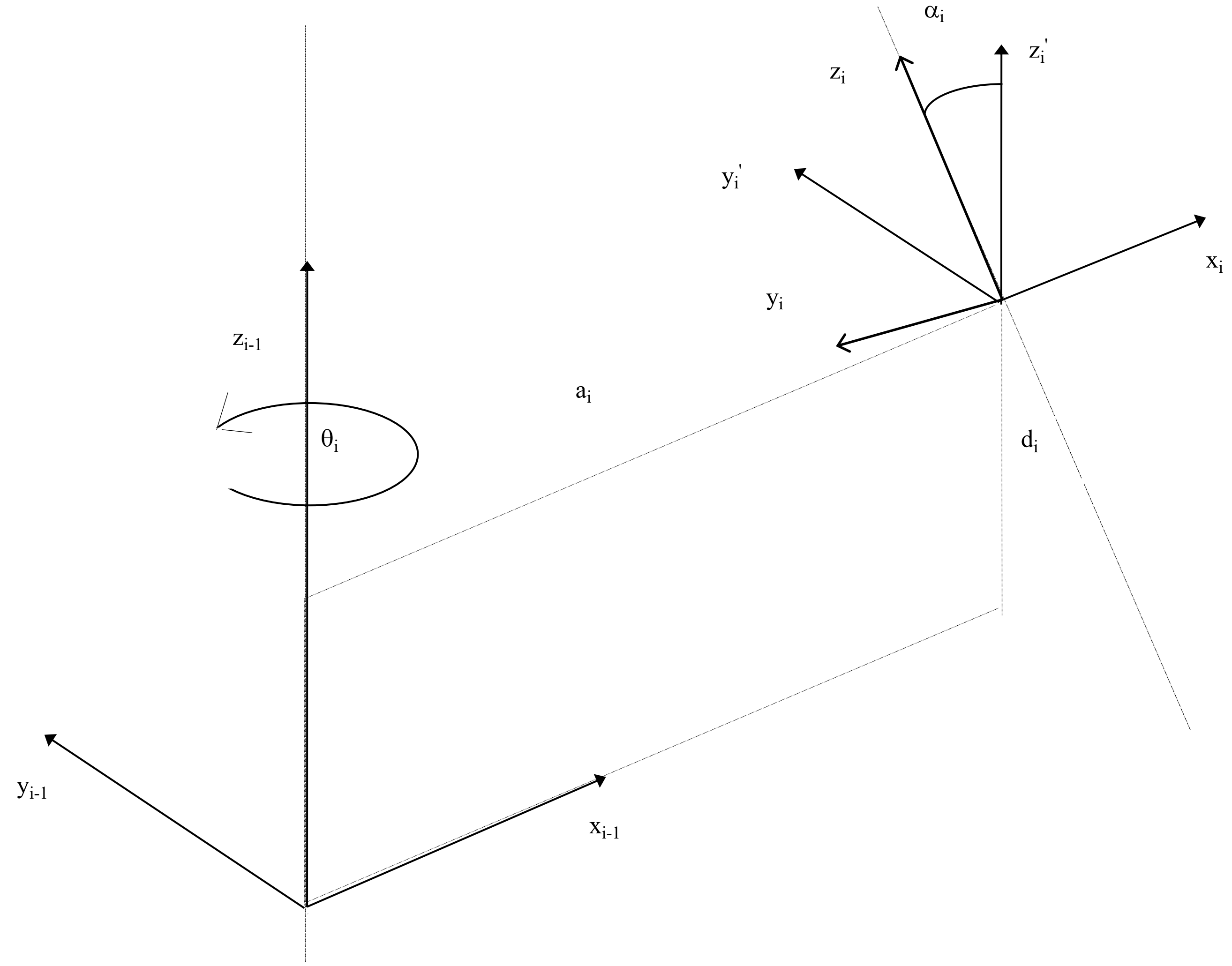
PARAMETERS OF TRANSFORMATIONS

Homogenous transformation used in manipulator geometrical model formulation is composed of 4 basic transformations:

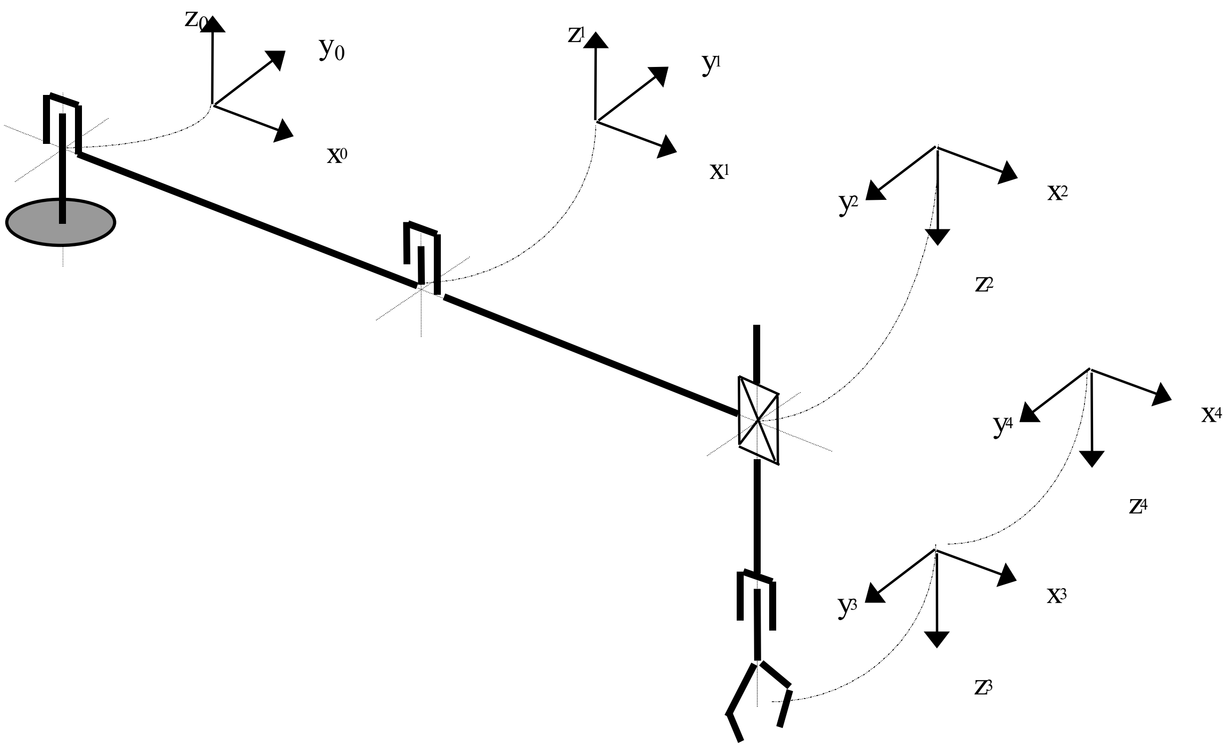
- rotation about axis z_{i-1} by angle θ_i
- translation along axis z_{i-1} by d_i
- translation along axis $\mathbf{x}_{i-1}$ by a_i
- rotation about axis x_i by angle α_i

Interpretation of the geometry model parameters

$$\underline{A}_i = \underline{Rot}(z_{i-1}, \theta_i) \cdot \underline{Tra}(z_{i-1}, d_i) \cdot \underline{Tra}(x_{i-1}, a_i) \cdot \underline{Rot}(x_i, \alpha_i)$$



SCARA RRPR Manipulator



Link No.	θ	d	a	α	Motion range
1	θ_1 v	0	a_1	0	-120º÷120º
2	θ_2 v	0	a_2	π	0º÷150º
3	0	d_3 v	0	0	0.1 m ÷0.3 m
4	θ_4 v	0	0	0	-180º÷180º

Determination of cartesian coordinates of an end-effector of a SCARA manipulator

$$
{}^{o}\underline{T}_{e} = \begin{bmatrix} N_x & O_x & A_x & P_x \\ N_y & O_y & A_y & P_y \\ N_z & O_z & A_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}\underline{T}_{4} = \begin{bmatrix} C_{12}C_4 + S_{12}S_4 & -C_{12}S_4 + S_{12}C_4 & 0 & a_1C_1 + a_2C_{12} \\ S_{12}C_4 - C_{12}S_4 & -C_{12}C_4 - S_{12}S_4 & 0 & a_1S_1 + a_2S_{12} \\ 0 & 0 & -1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
x = a_1C_1 + a_2C_{12}
$$

$$
y = a_1S_1 + a_2S_{12}
$$

$$
z = -d_3
$$

$$
\varphi = arctan\left(\frac{N_y}{N_x}\right)
$$

$$
\theta = arctan\left(\frac{-N_z}{\sqrt{1-N_z^2}}\right) =
$$

$$
= arcsin(-N_z)
$$

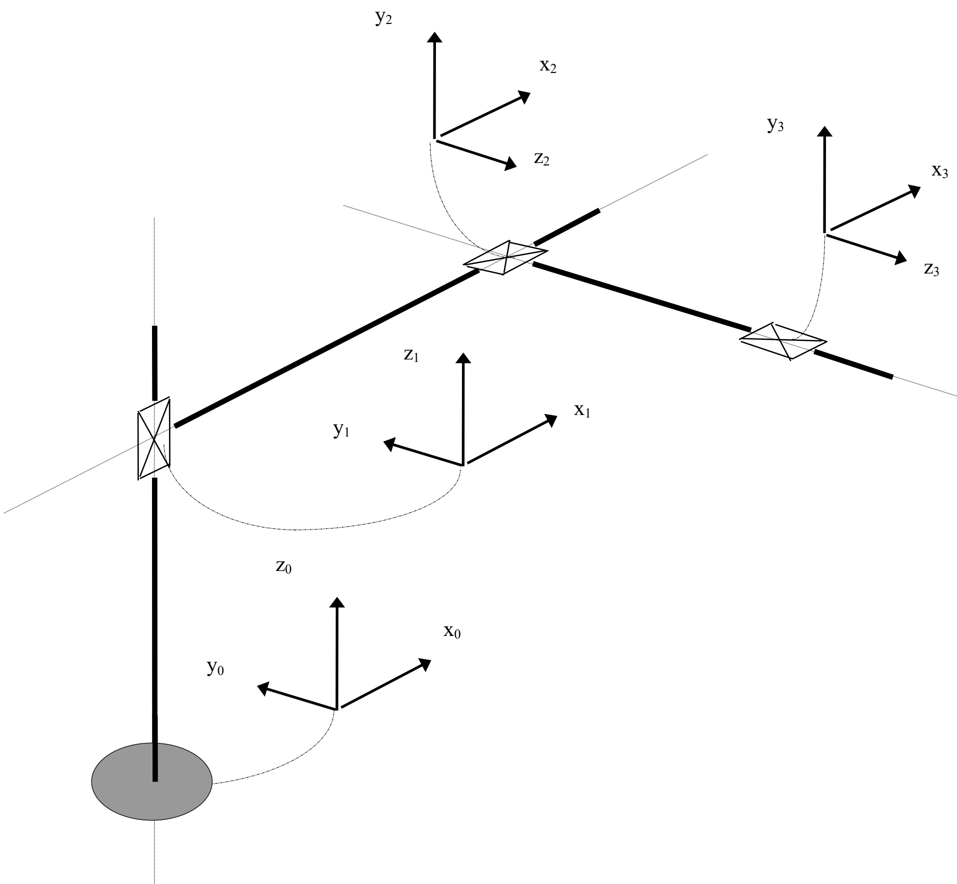
$$
\psi = arctan\left(\frac{O_z}{A_z}\right)
$$

$$
\varphi = arctan\left(\frac{S_{12}C_4 - C_{12}S_4}{C_{12}C_4 - S_{12}S_4}\right) =
$$

$$
= arctan\left(\frac{sin(\theta_1+\theta_2-\theta_4)}{cos(\theta_1+\theta_2-\theta_4)}\right) = \theta_1+\theta_2-\theta_4
$$

$$
\theta = arctan\left(\frac{0}{\sqrt{1}}\right) = 0°
$$

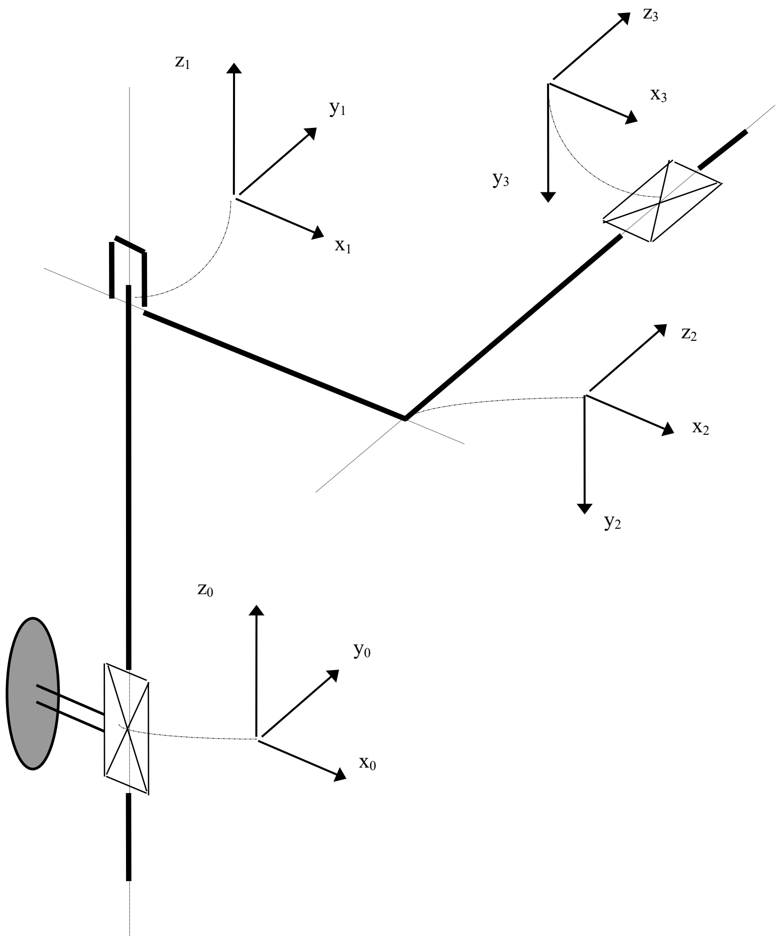
$$
\psi = arctan\left(\frac{0}{-1}\right) = 180°
$$



No. of link	θ	d	a	α
1	0	d_1 v	0	0
2	0	0	a_2 v	90°
3	0	d_3 v	0	0

$$\underline{T}_3 = \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 0 & -1 & -d_3 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

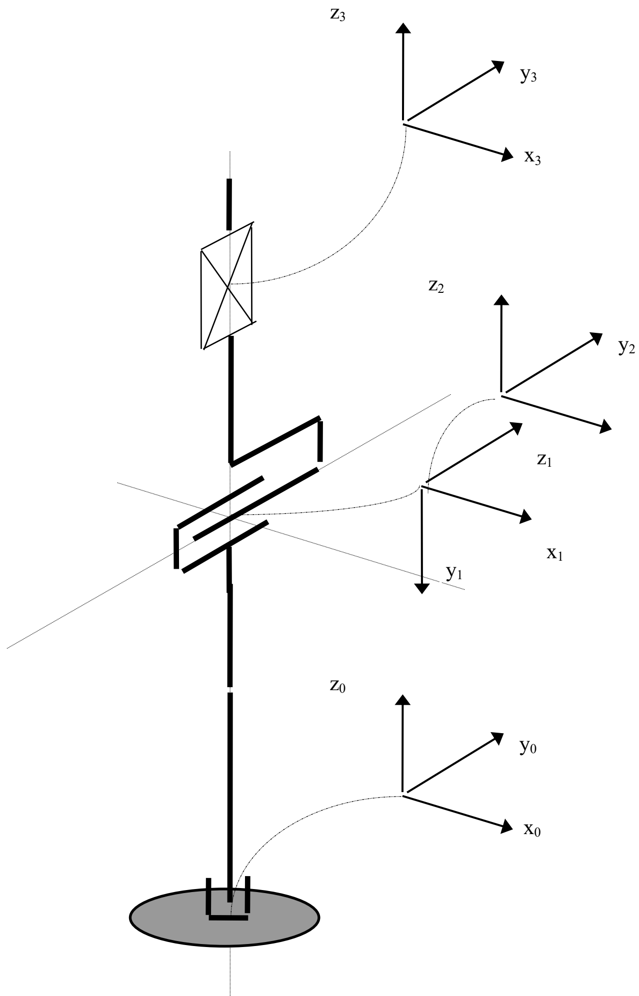
Cartesian PPP arm



No. of link	θ	d	a	α
1	0	d_1 v	0	0
2	θ_2 v	0	a_2	-90°
3	0	d_3 v	0	0

$$\underline{T}_3 = \begin{bmatrix} C_2 & 0 & -S_2 & a_2C_2 - d_3S_2 \\ S_2 & 0 & C_2 & a_2S_2 + d_3C_2 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

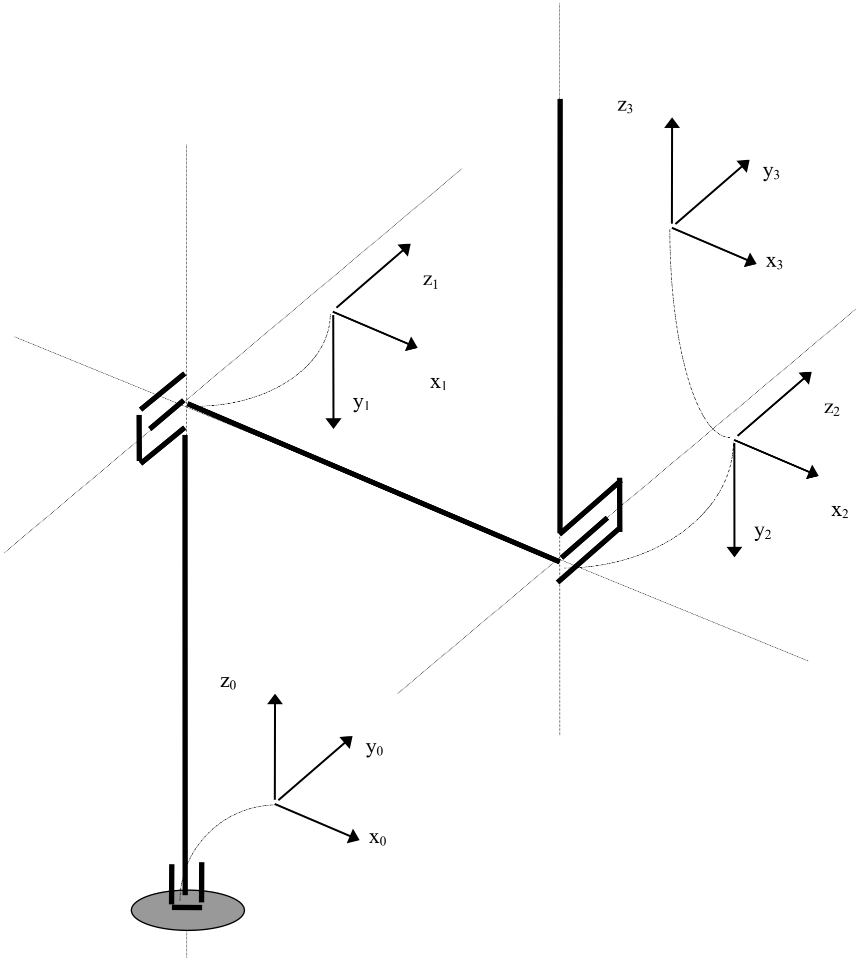
Cylindrical PRP arm



No. of link	θ	d	a	α
1	θ_1 v	d_1	0	-90°
2	θ_2 v	0	0	90°
3	0	d_3 v	0	0

$$\underline{T}_3 = \begin{bmatrix} C_1C_2 & -S_1 & C_1S_2 & d_3C_1S_2 \\ S_1C_2 & C_1 & S_1S_2 & d_3S_1S_2 \\ -S_2 & 0 & C_2 & d_1 + d_3C_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

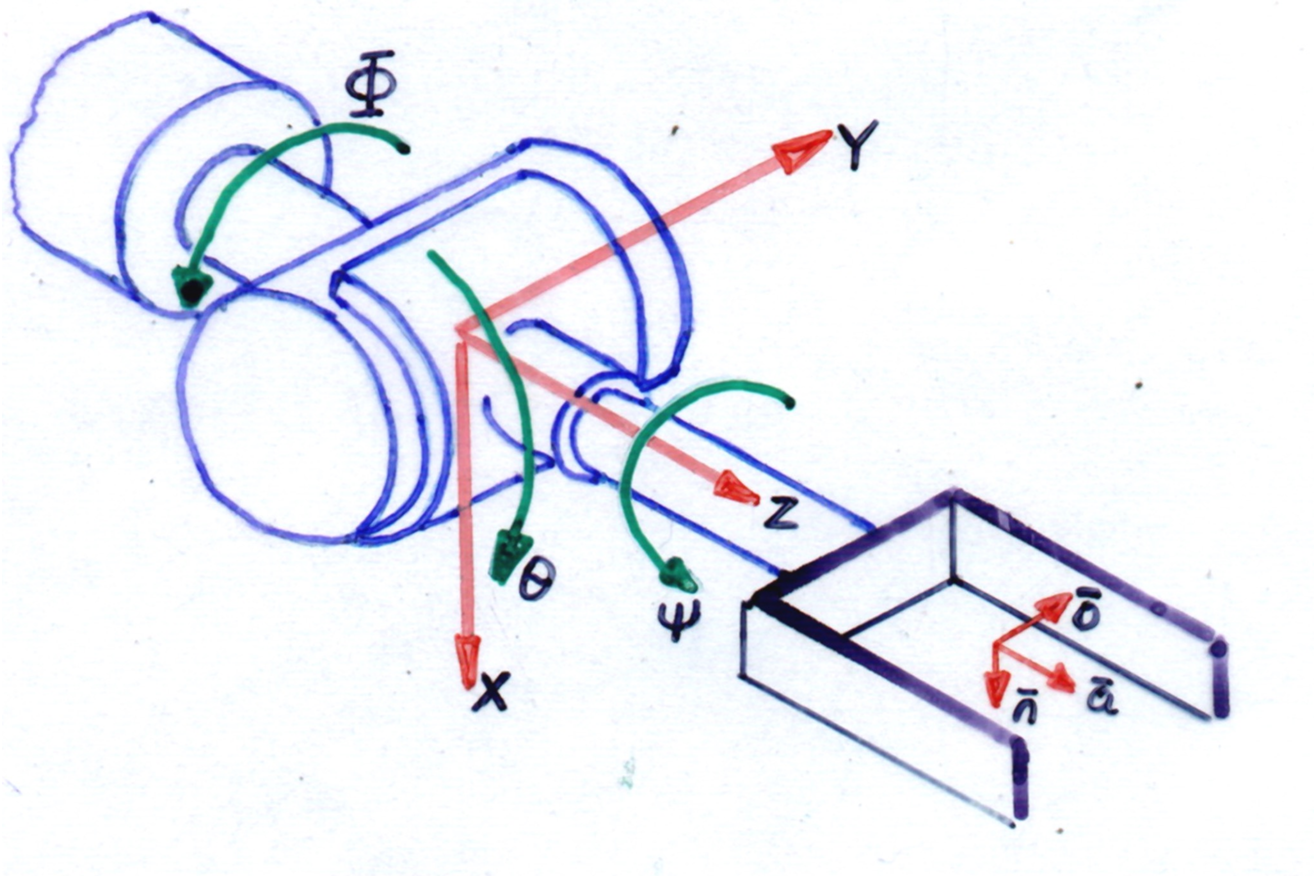
Spherical RRP arm



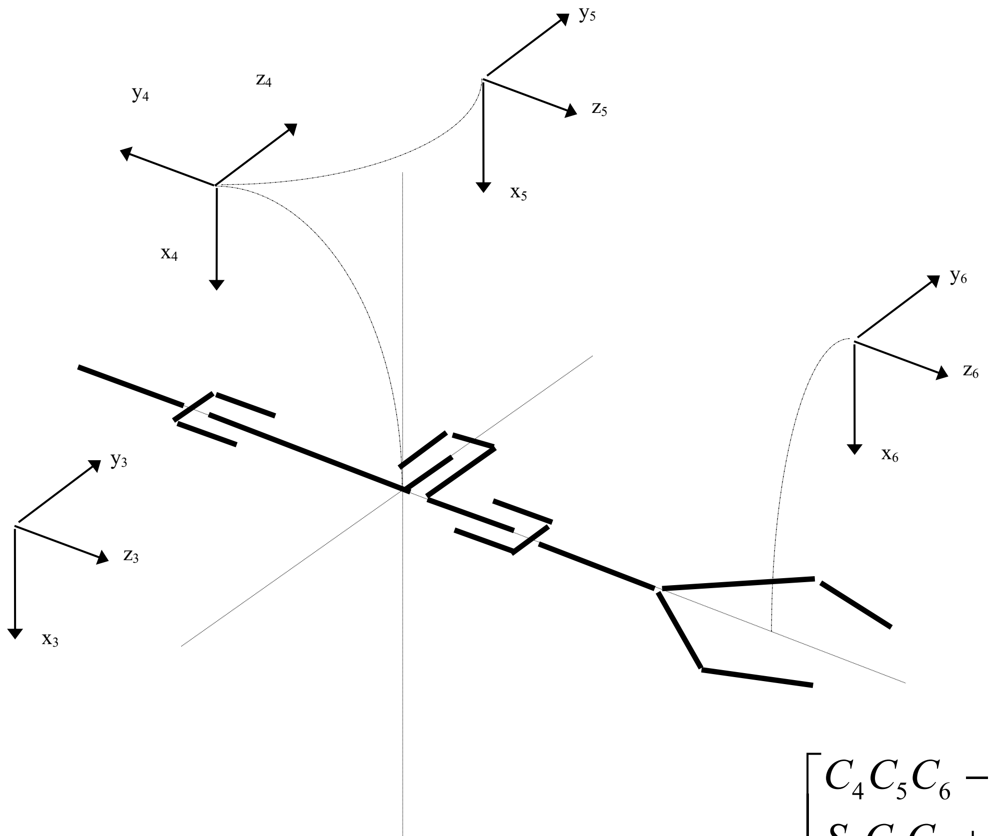
No. of link	θ	d	a	α
1	θ_1 v	d_1	0	-90^o
2	θ_2 v	0	a_2	0
3	θ_3 v	0	0	90^o

$$\underline{T}_3 = \begin{bmatrix} C_1C_{23} & -S_1 & C_1S_{23} & a_2C_1C_2 \\ S_1C_{23} & C_1 & S_1S_{23} & a_2S_1C_2 \\ -S_{23} & 0 & C_{23} & d_1 - a_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Anthropomorphic RRR arm



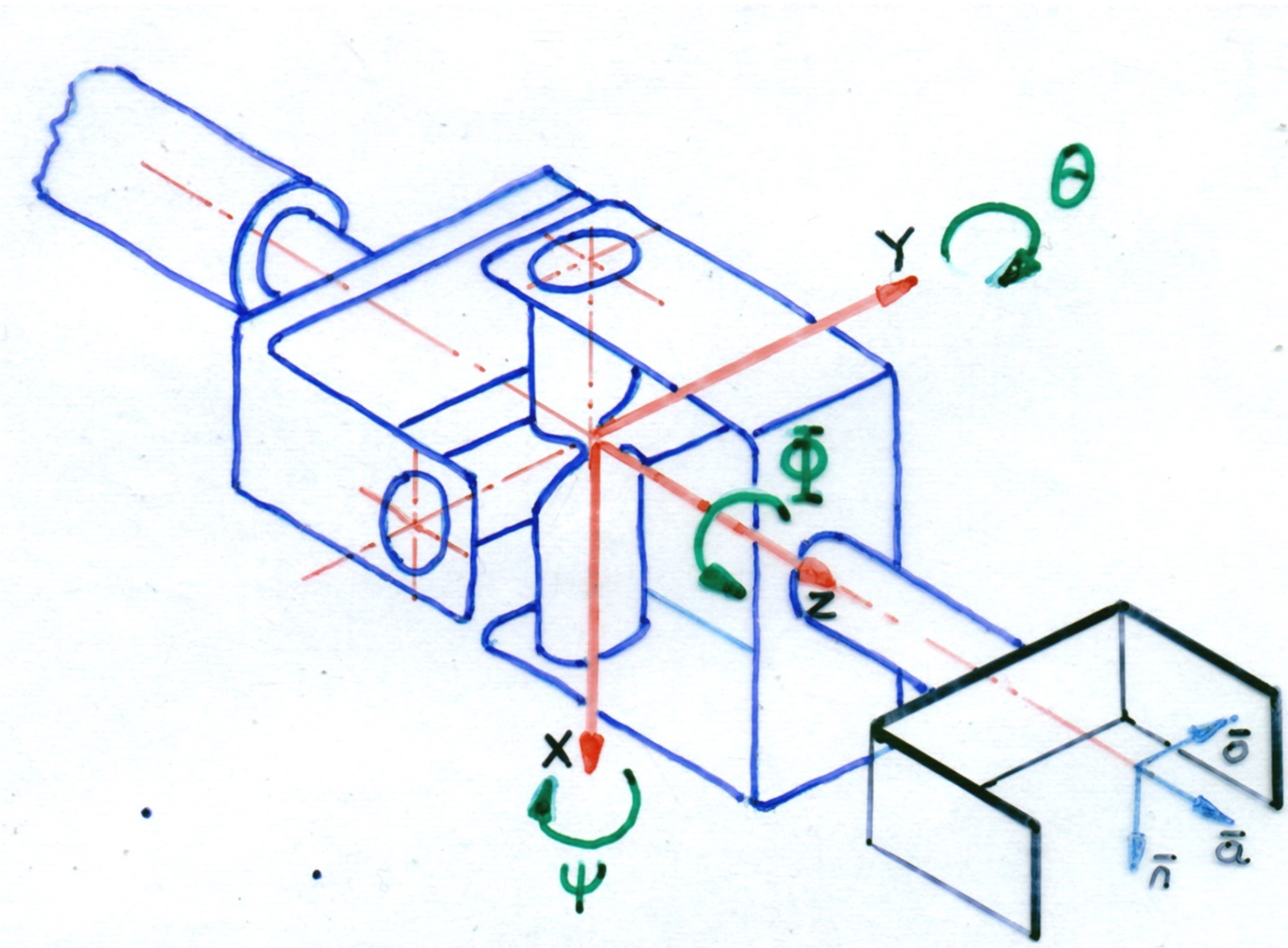
Spherical wrist (Euler - type RBR)



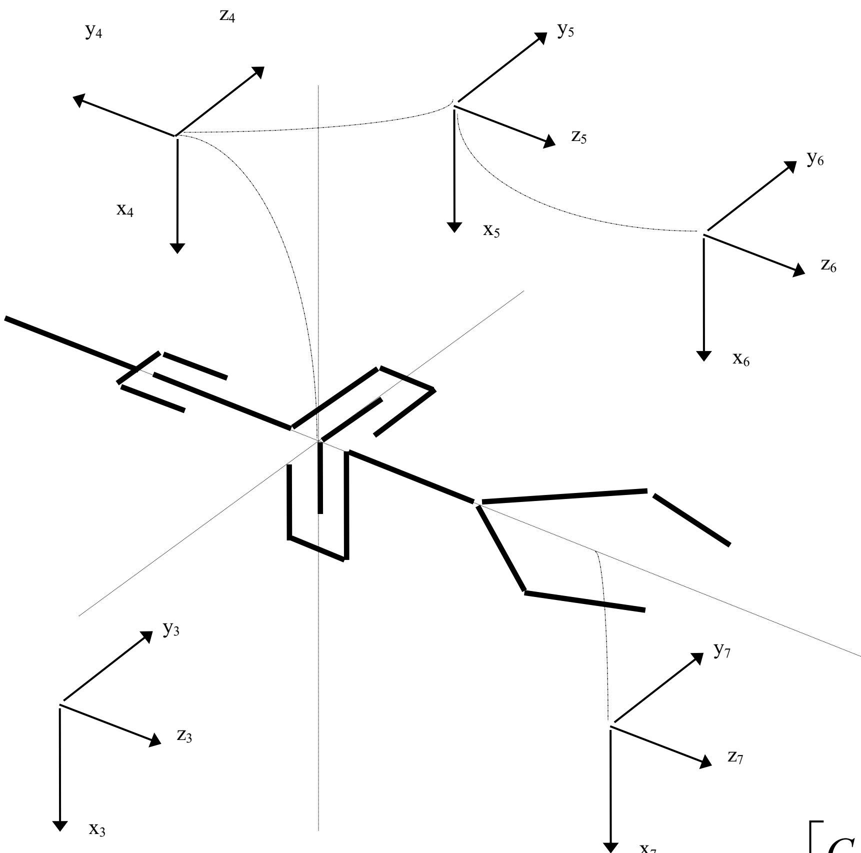
No. of link	θ	d	a	α
4	θ_4 v	d_4	0	-90°
5	θ_5 v	0	0	90°
6	θ_6 v	d_6	0	0

$$
{}^{3}\underline{T}_{6} = \begin{bmatrix} C_4C_5C_6 - S_4S_6 & -C_4C_5S_6 - S_4C_6 & C_4S_5 & d_6C_4S_5 \\ S_4C_5C_6 + C_4S_6 & -S_4C_5S_6 + C_4C_6 & S_4S_5 & d_6S_4S_5 \\ -S_5C_6 & S_5S_6 & C_5 & d_4 + d_6C_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Spherical wrist (Euler - type RBR)



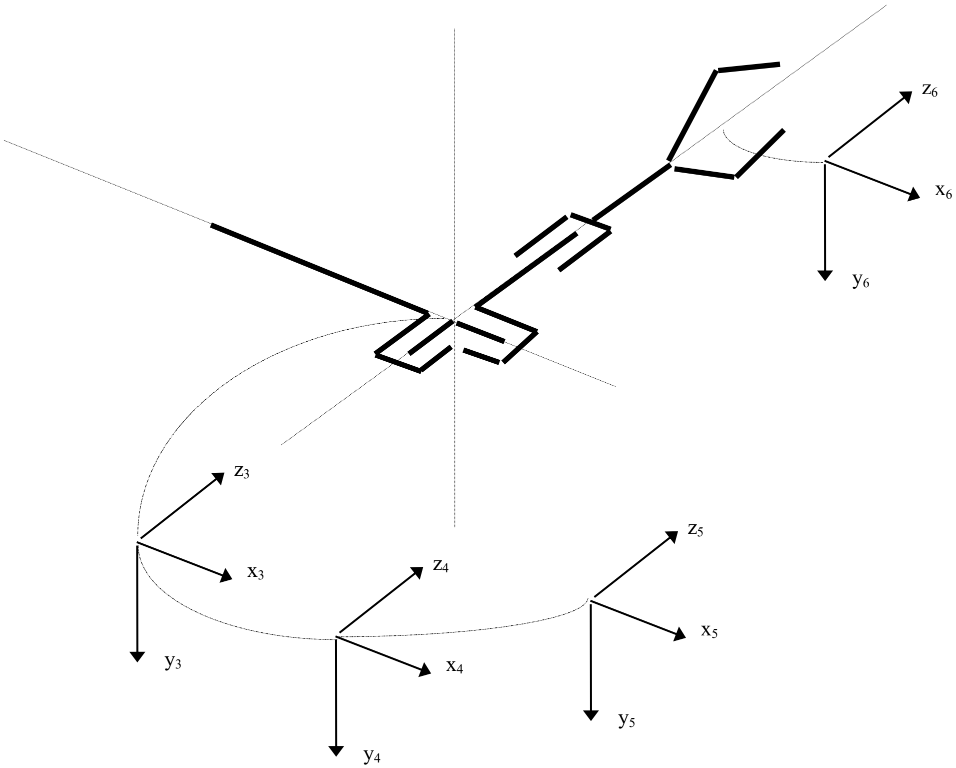
Spherical wrist (RPY - type RBB)



No. of link	θ	d	a	α
4	θ_4 v	d_4	0	-90°
5	θ_5 v	0	0	90°
6	0	0	0	α_6 v
6	0	d_7	0	0

$$
{}^{3}\underline{T}_{6} = \begin{bmatrix} C_4C_5 & C_4S_5S_6 - S_4C_6 & C_4S_5C_6 + S_4S_6 & d_7\left(C_4S_5C_6 + S_4S_6\right) \\ S_4C_5 & S_4S_5S_6 + C_4C_6 & S_4S_5C_6 - C_4S_6 & d_7\left(S_4S_5C_6 - C_4S_6\right) \\ -S_5 & C_5S_6 & C_5C_6 & d_4 + d_7C_5C_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

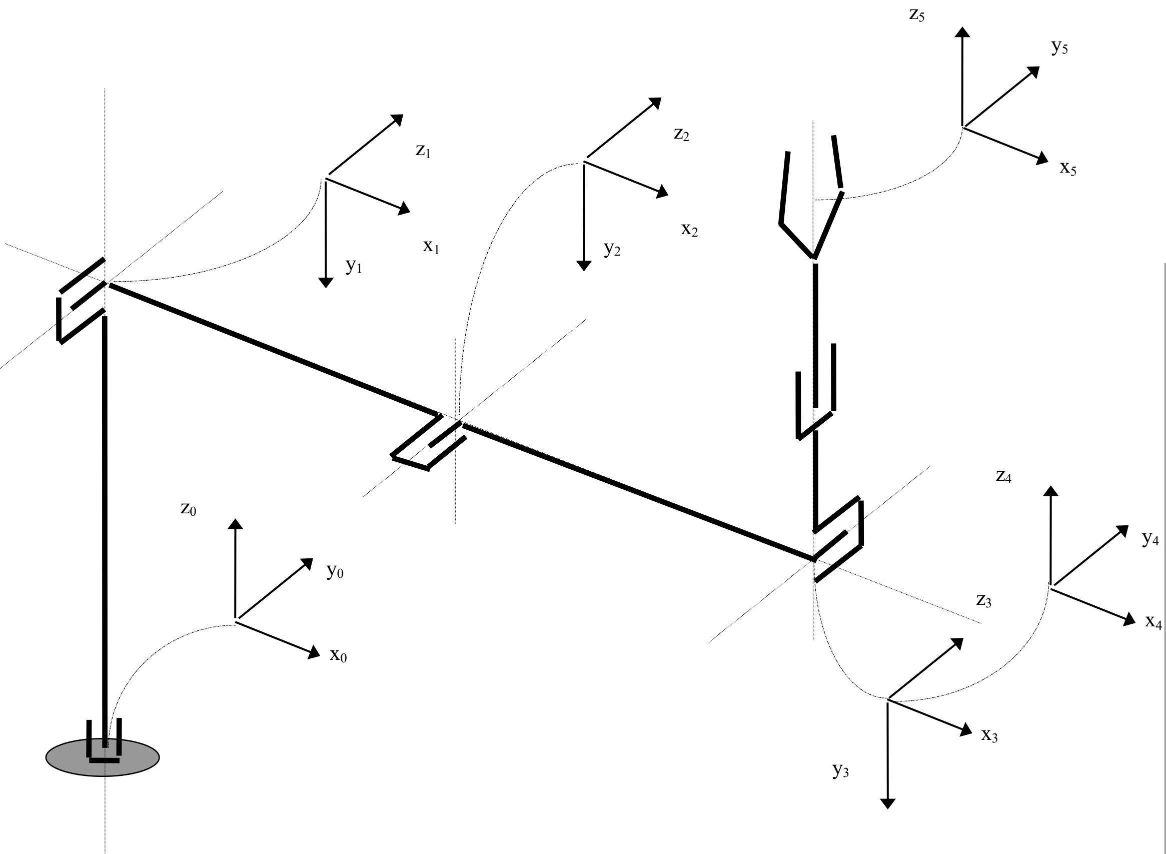
Spherical wrist (RPY - type RBB)



No. of link	θ	d	a	α
4	θ_4 v	0	0	0
5	0	0	0	α_5 v
6	θ_6 v	d_6	0	0

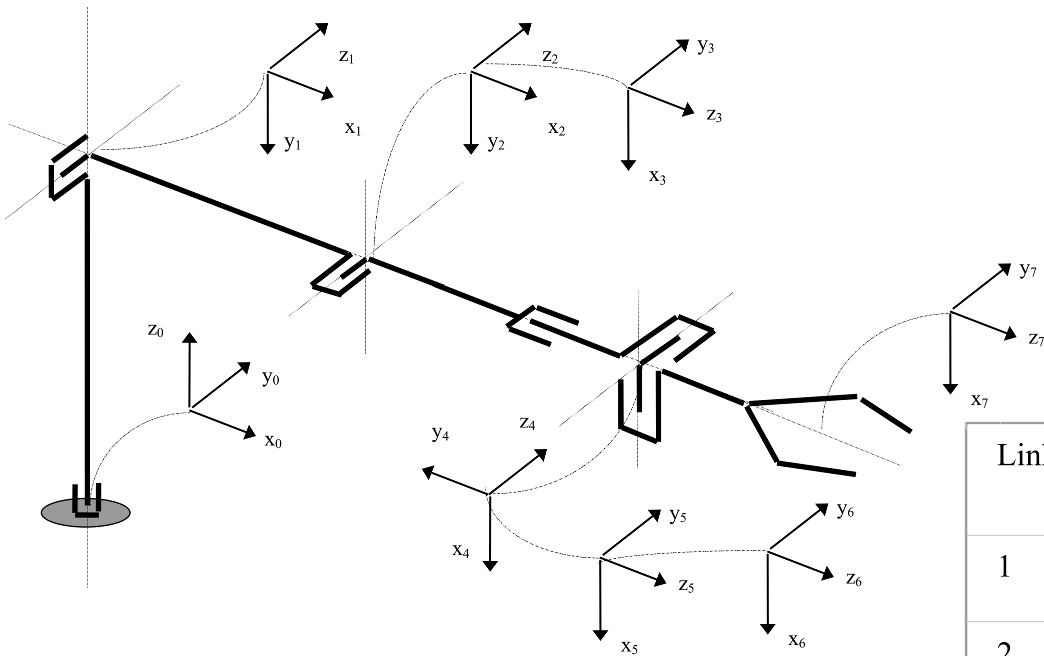
$$
{}^{3}\underline{T}_{6} = \begin{bmatrix} -S_4C_5S_6 + C_4C_6 & -S_4C_5C_6 - C_4S_6 & S_4S_5 & d_6S_4S_5 \\ C_4C_5S_6 + S_4C_6 & C_4C_5C_6 - S_4S_6 & -C_4S_5 & -d_6C_4S_5 \\ S_5S_6 & S_5C_6 & C_5 & d_6C_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Spherical wrist (RPY - type RBB)



Link No.	θ	d	a	α
1	θ_1 v	d_1	0	-90º
2	θ_2 v	0	a_2	0
3	θ_3 v	0	a_3	0
4	θ_4 v	0	0	90º
5	θ_5 v	d_5	0	0

RRR RR Manipulator



Link No.	θ	d	a	α
1	θ_1 v	d_1	0	-90°
2	θ_2 v	0	a_2	0
3	θ_3+90° v	0	0	90°
4	θ_4 v	d_4	0	-90°
5	θ_5 v	0	0	90°
6	0	0	0	α_6 v
6	0	d_7	0	0

RRRRRR (RRR+RPY) manipulator

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