Industrial Robots

Wojciech Lisowski

1B
Mechanics of manipulators
Introduction

Assumptions

Class of manipulators under consideration
- nonredundant
- open kinematic chain
- rigid links
- prismatic or revolute joints

Kinematic chain of manipulators posesses one end attached to the base and the other end free

The free end serves as an assembly base for
- a gripper
- an assembly tool
- a technological tool
- an inspection tool

Types of mathematic models of manipulators

GEOMETRICAL MODEL – a set of algebraic equations describing position and orientation of end-effector in robot workspace

KINEMATIC MODEL – a set of algebraic equations expressing velocity and acceleration of robot links and end-effector during motion (causes of motion – forces - are disregarded)

STATIC MODEL – a set of algebraic equations describing static force balance of a manipulator

DYNAMIC MODEL - a set of differential equations that defines relationship between **forces** acting on manipulator, its **mass distribution** in space and **kinematic parameters** of its motion.

Mathematical model is an equation or a set of equations in a general form

$$f(x)=0$$

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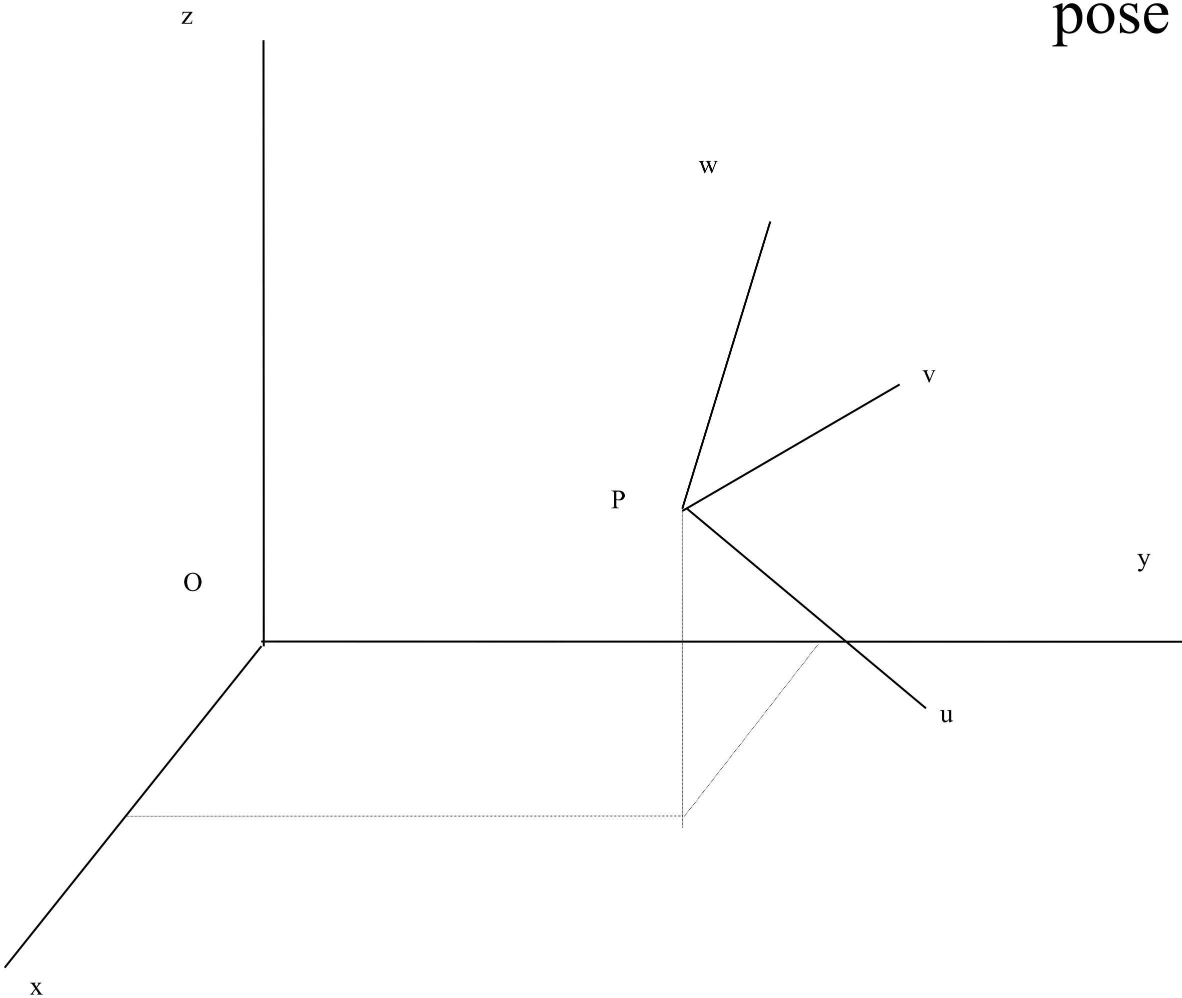
1
Mechanics of manipulators 1
Position and orientation – geometrical description

Problems:

- Homogeneous transformation matrix: interpretation of elements
- Techniques of representing position and orientation
- Convention concerning orienting of a gripper's axes
- Interpretation of assumed orientation basing on directional cosines and RPY angles
- Quaternions

pose = position + orientation



position:

x, y, z

orientation:

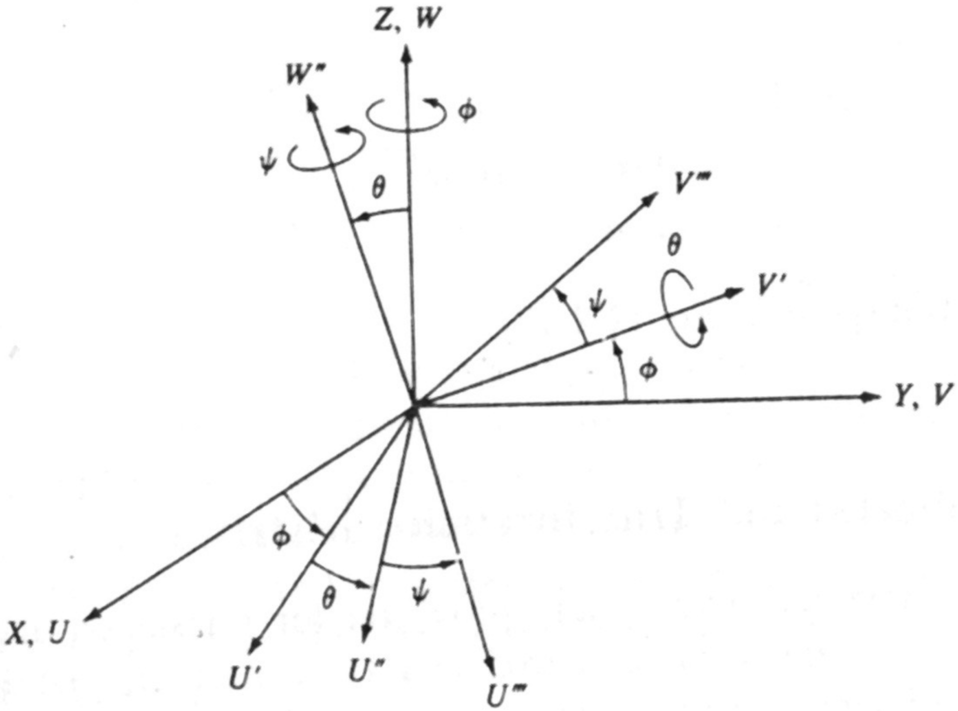
$\varphi,\ \theta,\ \psi$

Geometry model bases on homogeneous transformation matrix

Orientation: $\varphi,\ \theta,\ \psi$

Euler angles (Z-Y-Z)

$$\underline{EU}(\Phi,\Theta,\Psi) = \underline{Rot}(z,\Phi)\underline{Rot}(v,\Theta)\underline{Rot}(w,\Psi)$$

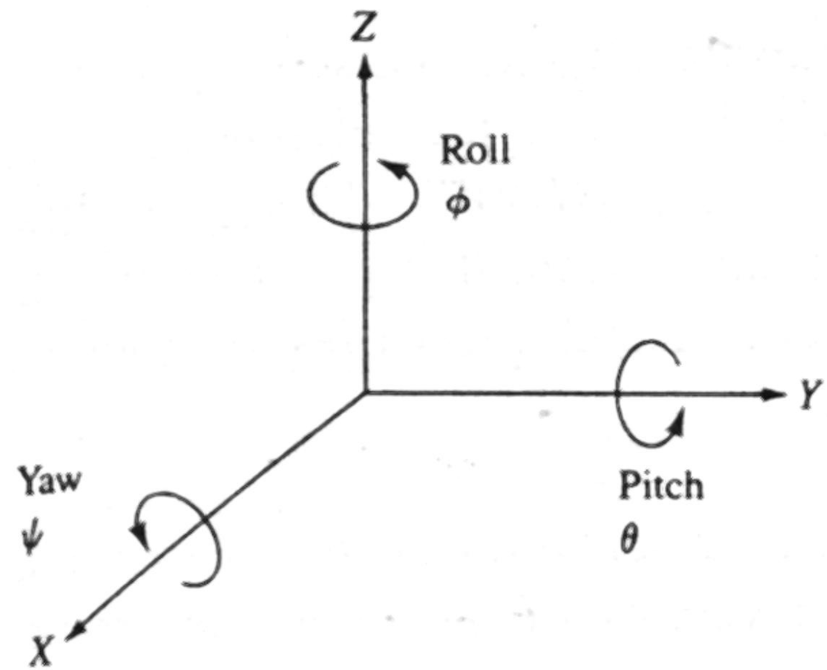


(Fu)

precession - Φ
nutation - Θ
spin- Ψ

RPY angles (Z-Y-X)

$$\underline{RPY}(\phi,\theta,\psi) = \underline{Rot}(z,\phi)\underline{Rot}(y,\theta)\underline{Rot}(x,\psi) =$$



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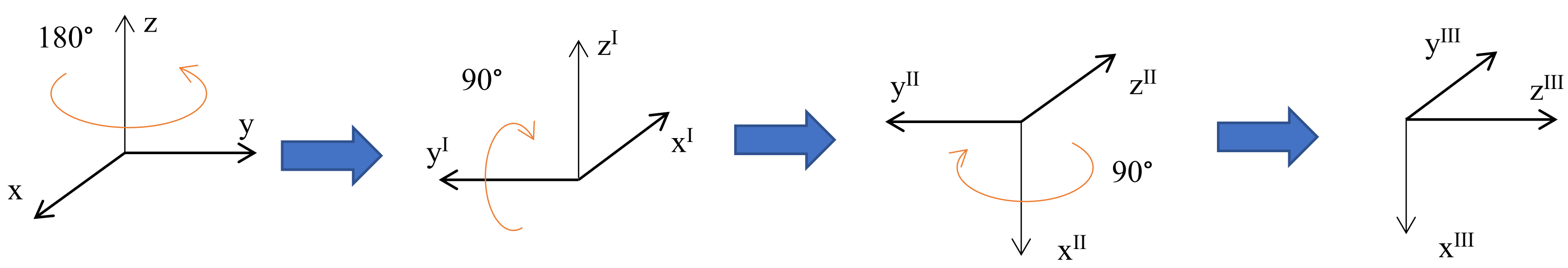
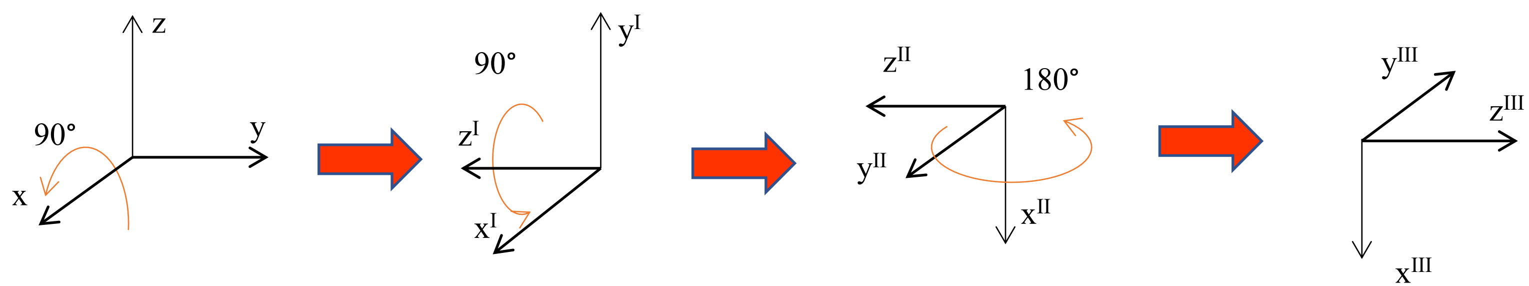
Roll - ϕ
Pitch - θ
Yaw - ψ

Transformation RPY: $Rot(z,180°)Rot(y,90°)Rot(x,90°)$



Reverted sequence: x→y→z
rotations about axes of the first (reference) coordinate frame



The result is **the same !**

Homogeneous co-ordinates:

$$\overline{p} = [p_x, p_y, p_z]^T$$

$$\breve{p} = (sp_x, sp_y, sp_z, s)$$

$$\begin{bmatrix} 3 \\ 4 \\ 5 \\ 1 \end{bmatrix} \equiv \begin{bmatrix} 6 \\ 8 \\ 10 \\ 2 \end{bmatrix} \equiv \begin{bmatrix} -30 \\ -40 \\ -50 \\ -10 \end{bmatrix}$$

zero vector

$$[0,0,0,n], n \neq 0$$

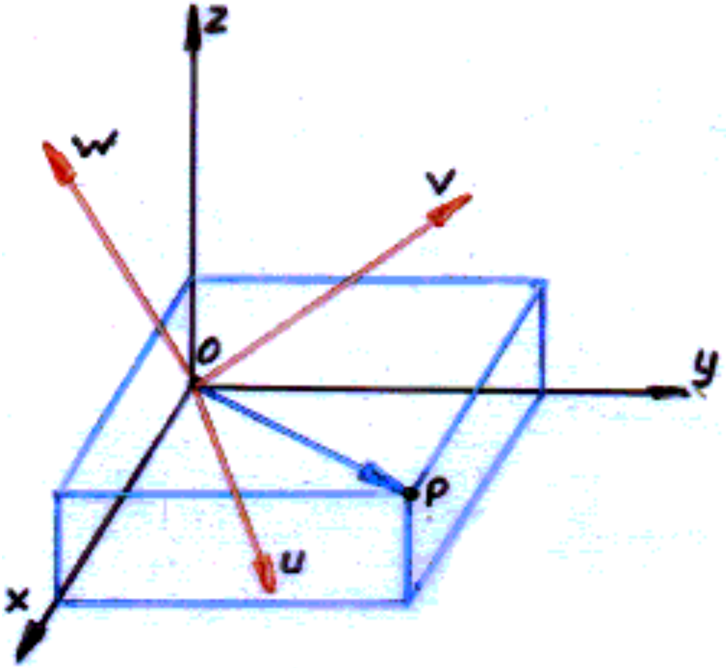
undefined vector

$$[0,0,0,0]$$

directional vector

$$[a, b, c, 0]$$

n -dim. space $\rightarrow$ n+1 –dim. space



$$\bar{p}_{xyz} \equiv \bar{p}_{uvw}$$
$$OXYZ(\hat{i}_x, \hat{j}_y, \hat{k}_z)$$
$$OUVW(\hat{i}_u, \hat{j}_v, \hat{k}_w)$$

$$\bar{p}_{uvw} = p_u \hat{i}_u + p_v \hat{j}_v + p_w \hat{k}_w$$

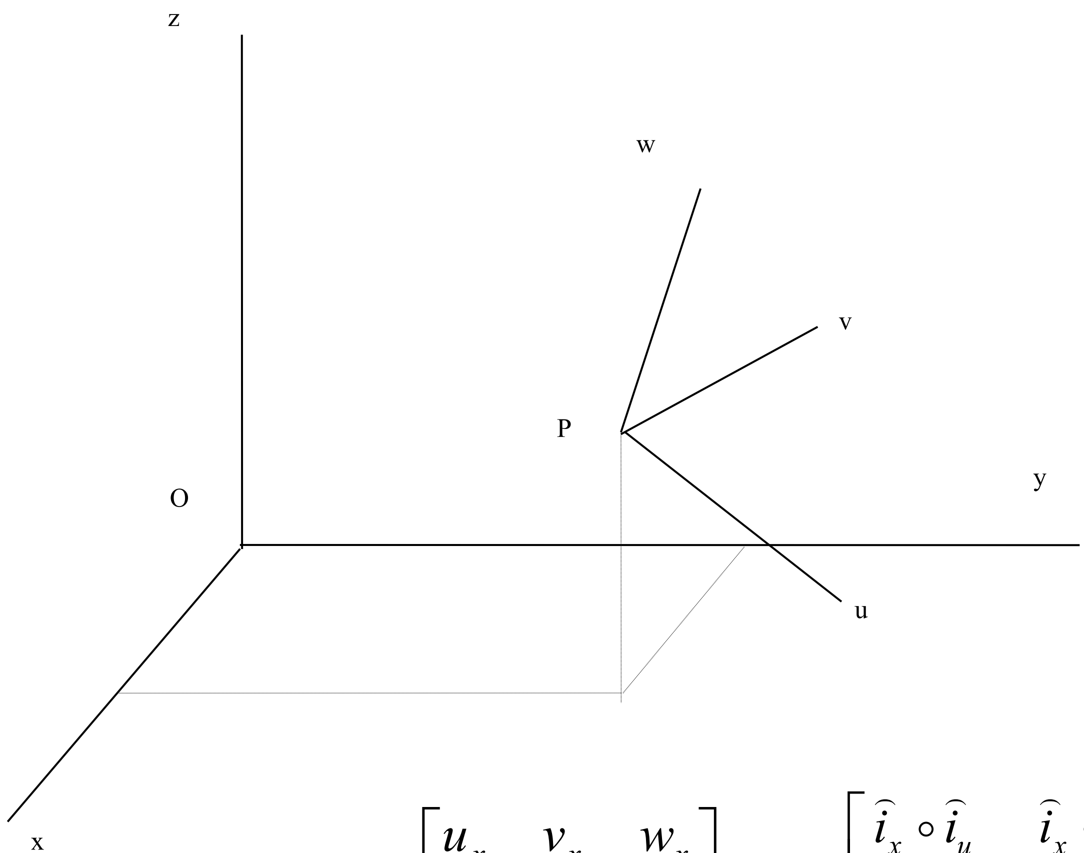
$$p_r = p\cos\angle(\bar{p}, \hat{r}) = p\frac{\bar{p}\circ\hat{r}}{p1} = \bar{p}\circ\hat{r}$$

$$p_x = \hat{i}_x \circ \bar{p}_{uvw} = \hat{i}_x \circ \hat{i}_u p_u + \hat{i}_x \circ \hat{j}_v p_v + \hat{i}_x \circ \hat{k}_w p_w$$
$$p_y = \hat{j}_y \circ \bar{p}_{uvw} = \hat{j}_y \circ \hat{i}_u p_u + \hat{j}_y \circ \hat{j}_v p_v + \hat{j}_y \circ \hat{k}_w p_w$$
$$p_z = \hat{k}_z \circ \bar{p}_{uvw} = \hat{k}_z \circ \hat{i}_u p_u + \hat{k}_z \circ \hat{j}_v p_v + \hat{k}_z \circ \hat{k}_w p_w$$

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} \hat{i}_x \circ \hat{i}_u & \hat{i}_x \circ \hat{j}_v & \hat{i}_x \circ \hat{k}_w \\ \hat{j}_y \circ \hat{i}_u & \hat{j}_y \circ \hat{j}_v & \hat{j}_y \circ \hat{k}_w \\ \hat{k}_z \circ \hat{i}_u & \hat{k}_z \circ \hat{j}_v & \hat{k}_z \circ \hat{k}_w \end{bmatrix} \begin{bmatrix} p_u \\ p_v \\ p_w \end{bmatrix}$$

$$p_u = \hat{i}_u \circ \bar{p}_{xyz}$$
$$p_v = \hat{j}_v \circ \bar{p}_{xyz}$$
$$p_w = \hat{k}_w \circ \bar{p}_{xyz}$$

Homogenous matrix enables to express position and orientation of *Puvw* local coordinate system with respect to the *Oxyz* reference system

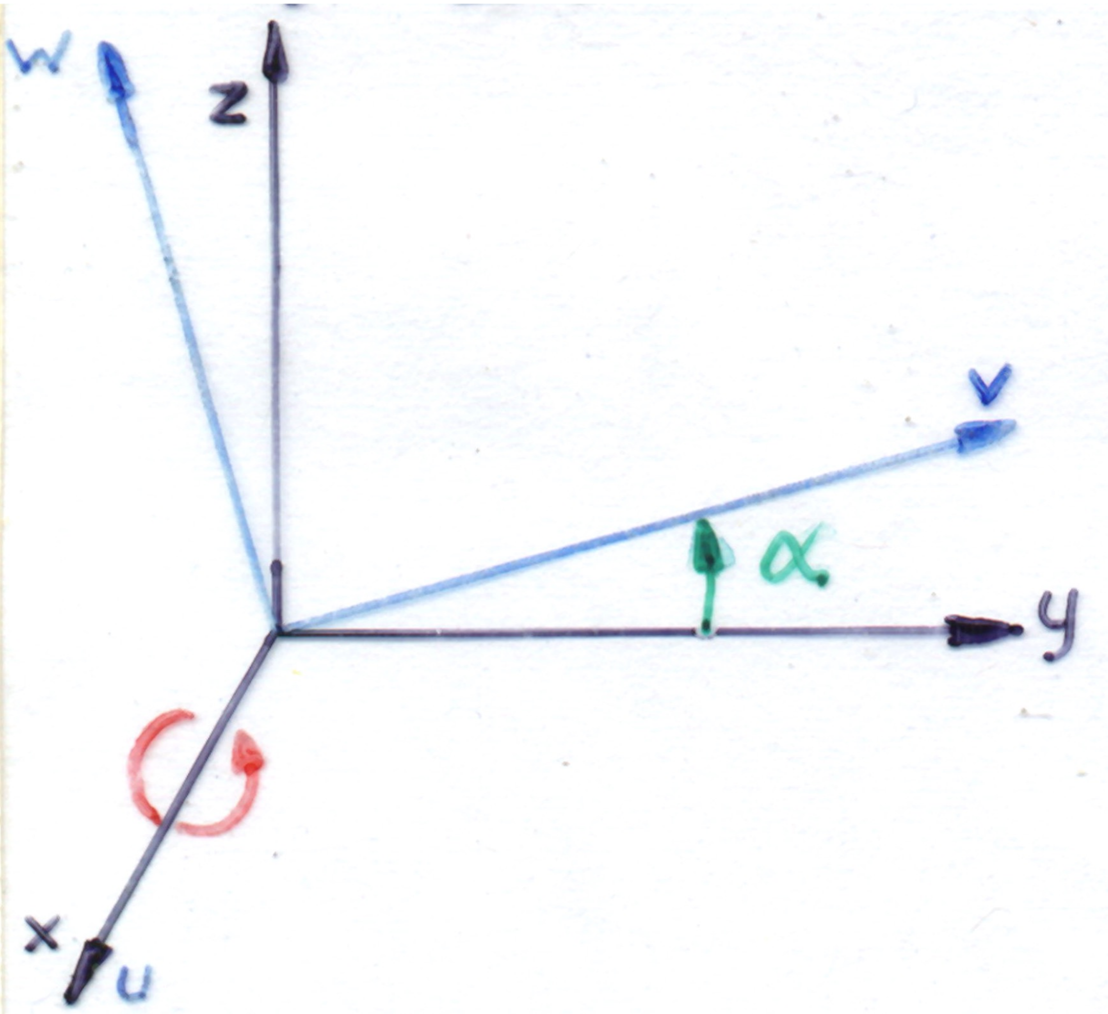


$$\underline{A} = \begin{bmatrix} u_x & v_x & w_x & x_P \\ u_y & v_y & w_y & y_P \\ u_z & v_z & w_z & z_P \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{R} = \begin{bmatrix} u_x & v_x & w_x \\ u_y & v_y & w_y \\ u_z & v_z & w_z \end{bmatrix} = \begin{bmatrix} \widehat{i}_x \circ \widehat{i}_u & \widehat{i}_x \circ \widehat{j}_v & \widehat{i}_x \circ \widehat{k}_w \\ \widehat{j}_y \circ \widehat{i}_u & \widehat{j}_y \circ \widehat{j}_v & \widehat{j}_y \circ \widehat{k}_w \\ \widehat{k}_z \circ \widehat{i}_u & \widehat{k}_z \circ \widehat{j}_v & \widehat{k}_z \circ \widehat{k}_w \end{bmatrix}$$

$$\underline{R}^{-1} = \underline{R}^{T}$$

Determination of the basic rotation matrices



$$\underline{R}(x,\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\alpha & -S\alpha \\ 0 & S\alpha & C\alpha \end{bmatrix}$$

4 basic homogeneous transformation matrices

$$\underline{A} = \underline{Tra}(a,b,c)\underline{Rot}(z,\theta)\underline{Rot}(y,\varphi)\underline{Rot}(x,\alpha)$$

$$\underline{Rot}(x,\alpha) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C\alpha & -S\alpha & 0 \\ 0 & S\alpha & C\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{Rot}(y,\varphi) = \begin{bmatrix} C\varphi & 0 & S\varphi & 0 \\ 0 & 1 & 0 & 0 \\ -S\varphi & 0 & C\varphi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{Rot}(z,\theta) = \begin{bmatrix} C\theta & -S\theta & 0 & 0 \\ S\theta & C\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{Tra}(a,b,c) = \underline{Tra}(x,a) \cdot \underline{Tra}(y,b) \cdot \underline{Tra}(z,c) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Attention!

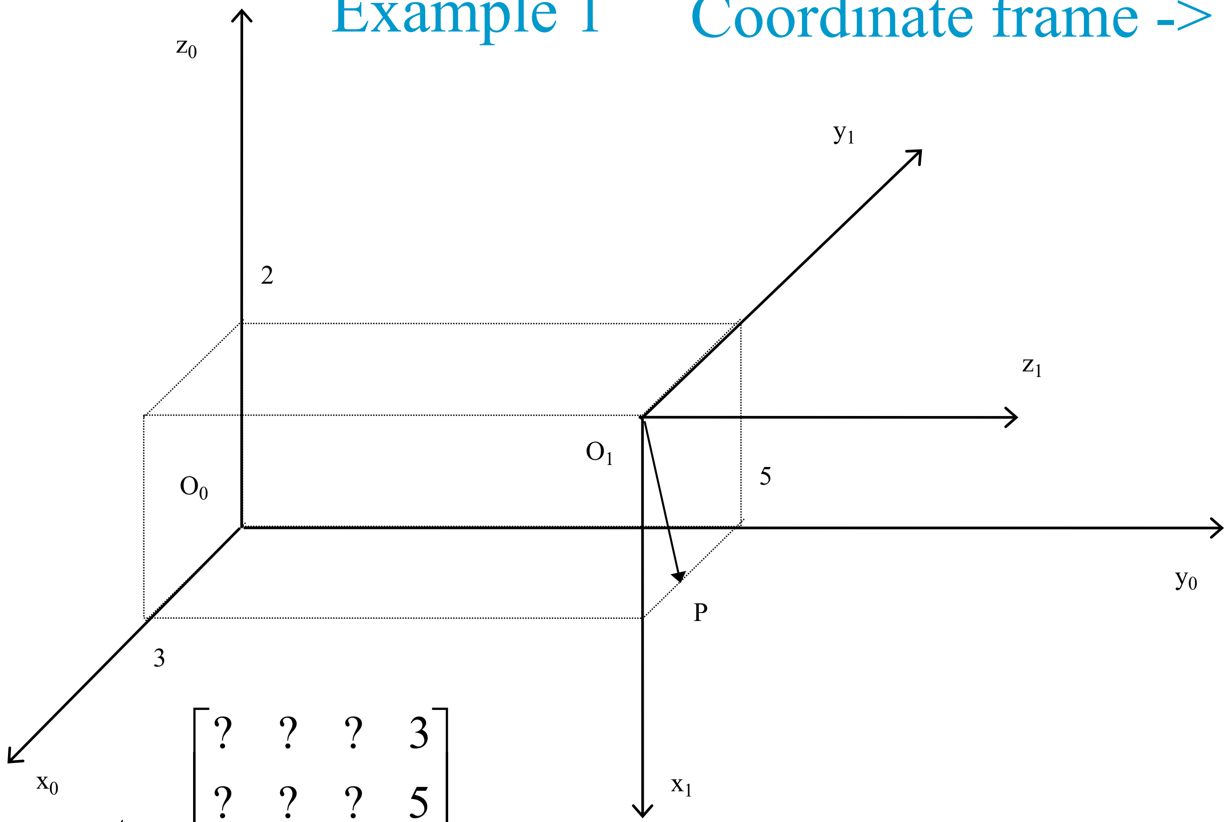
homogeneous transformation is non-commutative

$$\underline{Rot}(x,\alpha)\underline{Rot}(y,\varphi) \neq \underline{Rot}(y,\varphi)\underline{Rot}(x,\alpha)$$

Inverse transformation enables expressig position and orientation of the reference frame with respect to the local coordinate frame assigned to the considered link

$$\underline{T}^{-1} = \begin{bmatrix} N_x & N_y & N_z & -\overline{P} \circ \overline{N} \\ O_x & O_y & O_z & -\overline{P} \circ \overline{O} \\ A_x & A_y & A_z & -\overline{P} \circ \overline{A} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \underline{R}^T & -\underline{R}^T \overline{P} \\ \overline{0}^T & 1 \end{bmatrix}$$

Example 1 Coordinate frame -> Matrix



$$\overline{O_1P} = {}^1\overline{P} = [2,1,0]^T$$

$${}^1\breve{P} = [2,1,0,1]^T$$

$${}^0\breve{P} = \underline{A}_1 \, {}^1\breve{P}$$

$$\underline{A}_1 = \begin{bmatrix} ? & ? & ? & 3 \\ ? & ? & ? & 5 \\ ? & ? & ? & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \Rightarrow \underline{A}_1 = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 \\ 5 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 0 & 0 & 1 & 5 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\bar{x}_1 = [0,0,-1]^T$$

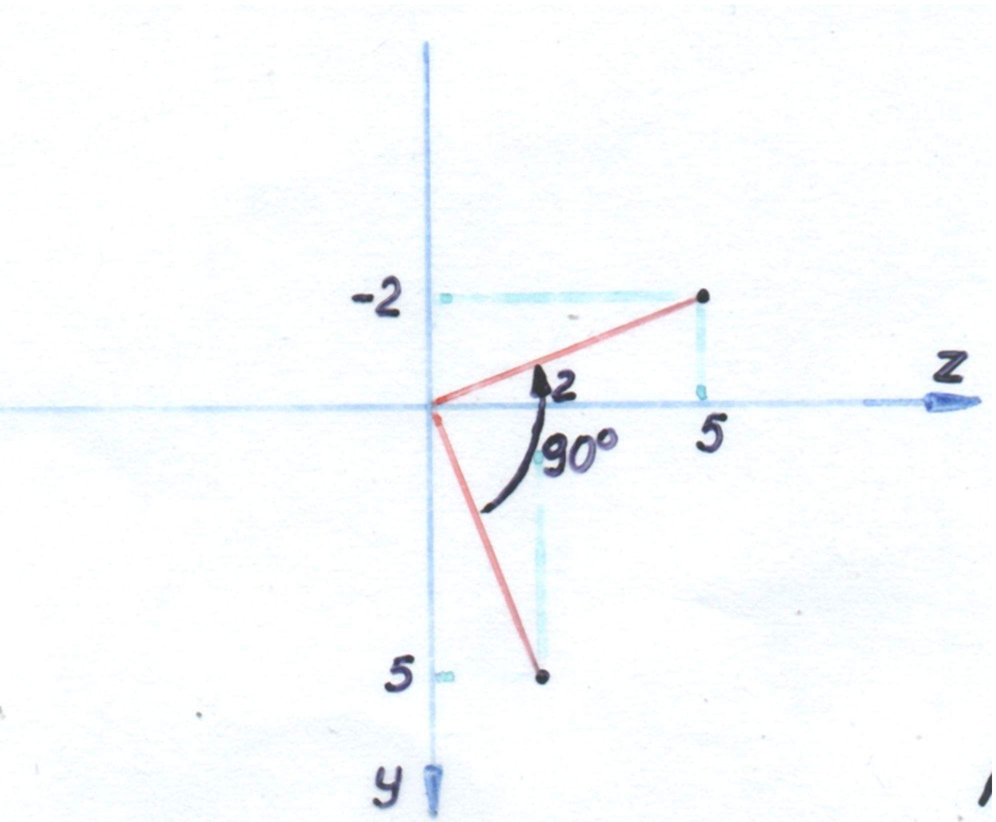
$$\bar{y}_1 = [-1,0,0]^T$$

$$\bar{z}_1 = [0,1,0]^T$$

$$\overline{O_0P} = {}^0\overline{P} = [2,5,0]^T$$

Example 2

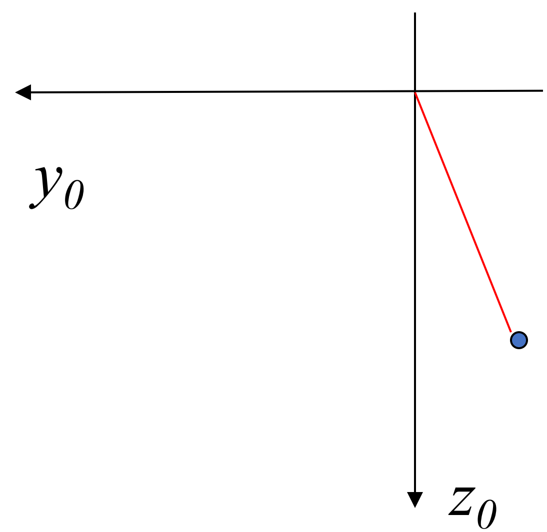
$$Rot(x,90^\circ)\begin{bmatrix}1 & 5 & 2 & 1\end{bmatrix}^T$$



Rotation of a point

$$\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & -1 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}1\\5\\2\\1\end{bmatrix}=\begin{bmatrix}1\\-2\\5\\1\end{bmatrix}$$

Reference frame $Ox_0y_0x_0$

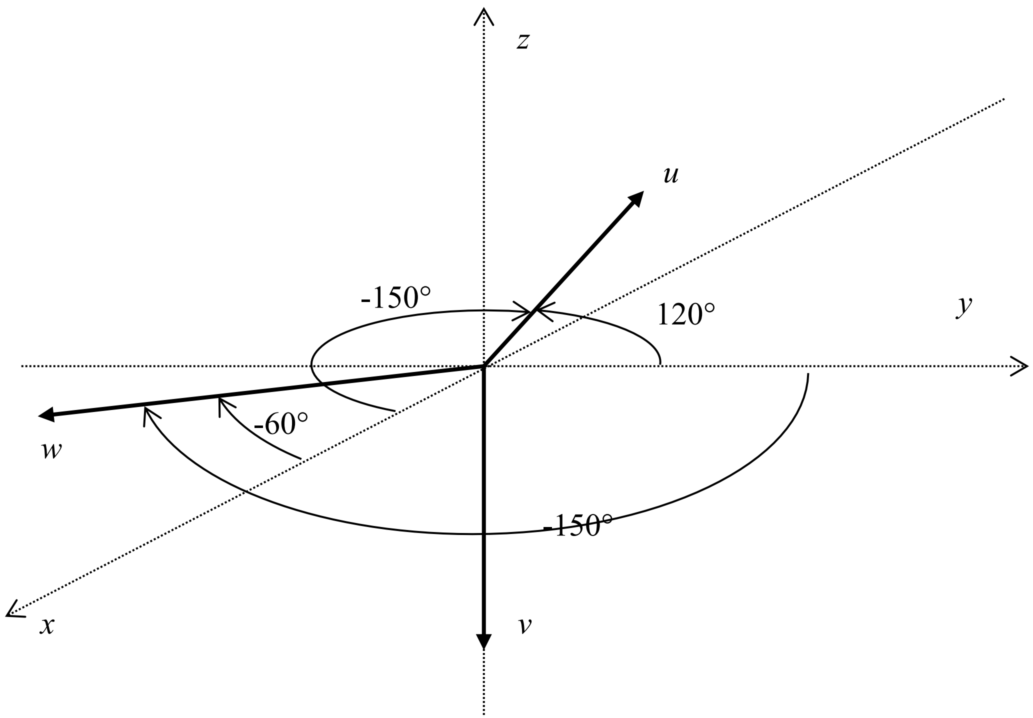


$$\underline{T}_1^{-1}=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & -1 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$$

Example 3

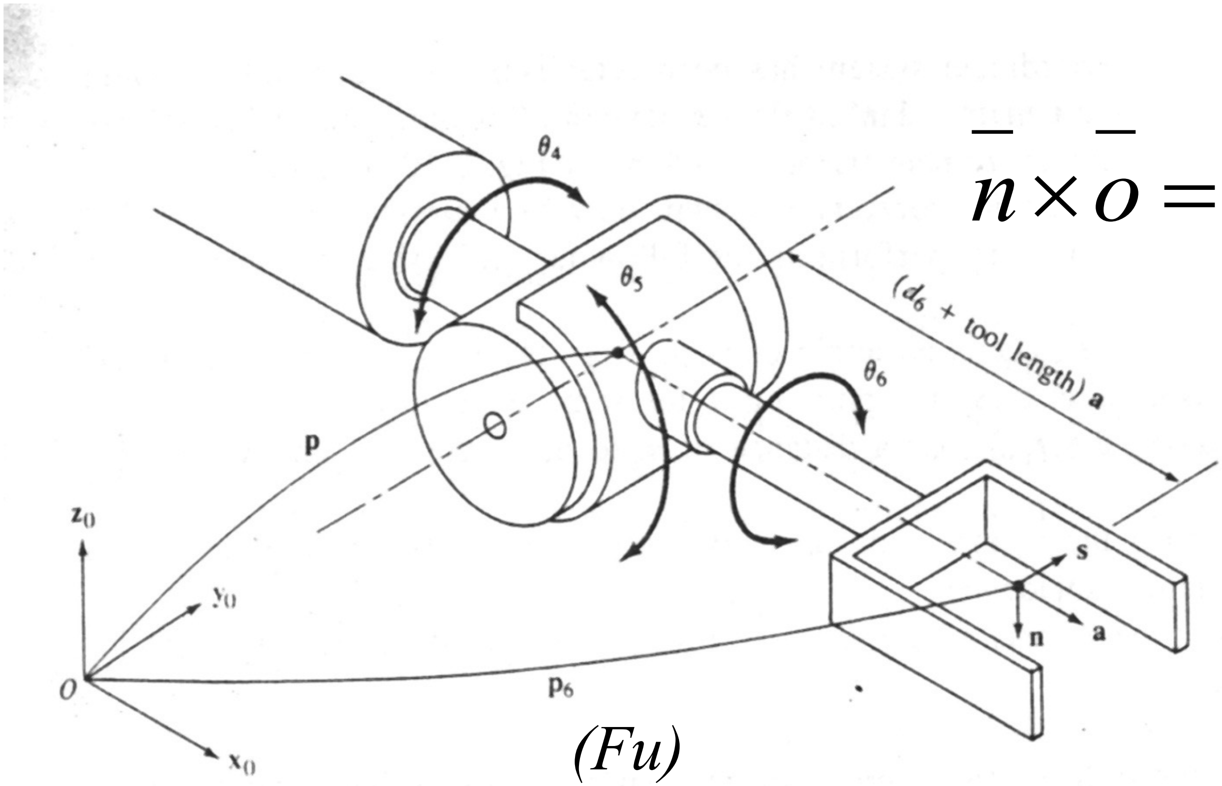
$$\underline{A}_1 = \begin{bmatrix} -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

	u	v	w
x	$\pm 150^\circ$	$\pm 90^\circ$	$\pm 60^\circ$
y	$\pm 120^\circ$	$\pm 90^\circ$	$\pm 150^\circ$
z	$\pm 90^\circ$	$\pm 180^\circ$	$\pm 90^\circ$



$$\begin{bmatrix} \hat{i}_x \circ \hat{i}_u & \hat{i}_x \circ \hat{j}_v & \hat{i}_x \circ \hat{k}_w \\ \hat{j}_y \circ \hat{i}_u & \hat{j}_y \circ \hat{j}_v & \hat{j}_y \circ \hat{k}_w \\ \hat{k}_z \circ \hat{i}_u & \hat{k}_z \circ \hat{j}_v & \hat{k}_z \circ \hat{k}_w \end{bmatrix}$$

A way of orienting of the axes of the local frame assigned to the end-effector



$$\bar{n} \times \bar{o} = \bar{a}$$

(Fu)

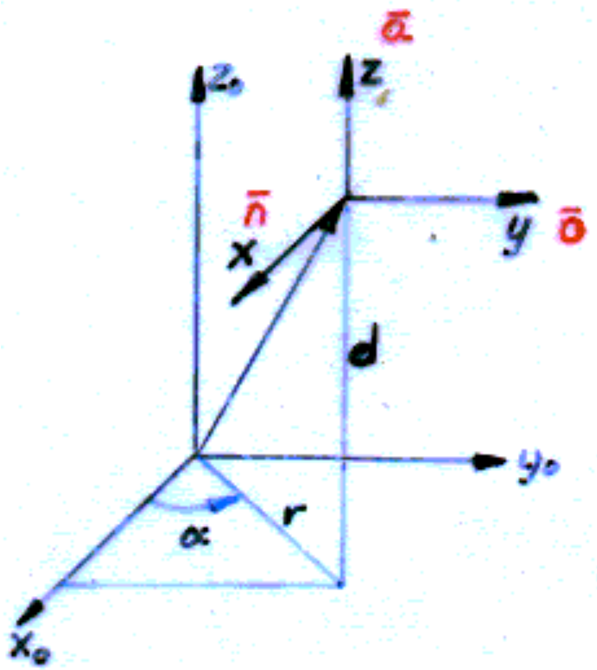
vectors: **n** – normal (x_e)
o (s) – orientation (y_e)
a – approach (z_e)

Uniform position description:

Cartesian co-ordinates

$$\underline{Car}(x,y,z) = \underline{Tra}(x,x)\cdot\underline{Tra}(y,y)\cdot\underline{Tra}(z,z) = \begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

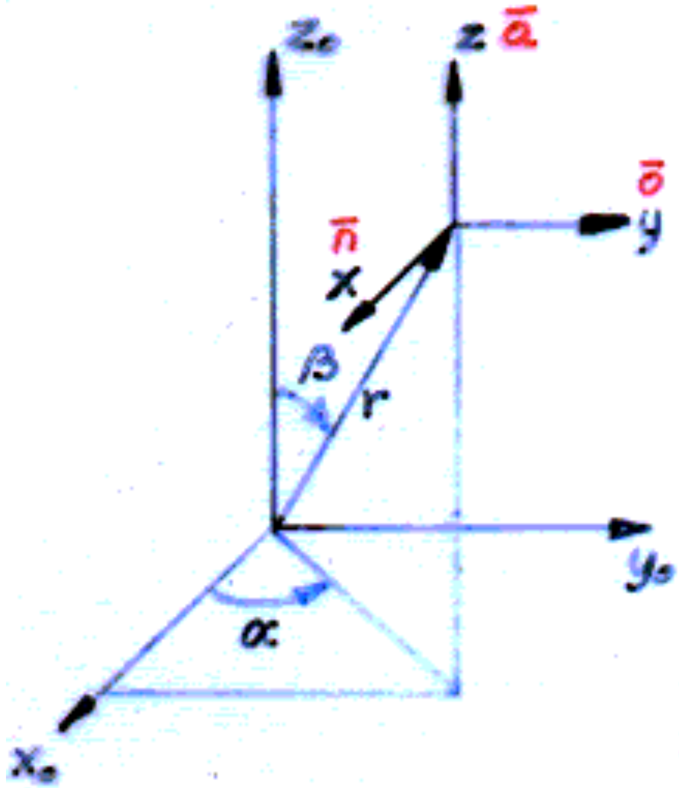
Cylindrical co-ordinates



$$\underline{Cyl}(d,\alpha,r) = \underline{Tran}(z,d)\underline{Rot}(z,\alpha)\underline{Tran}(x,r)\underline{Rot}(z,-\alpha) = \begin{bmatrix} 1 & 0 & 0 & rC\alpha \\ 0 & 1 & 0 & rS\alpha \\ 0 & 0 & 1 & d \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Spherical co-ordinates

$$\underline{Sfe}(\alpha,\beta,r)=\underline{Rot}(z,\alpha)\underline{Rot}(y,\beta)\underline{Tran}(z,r)\underline{Rot}(y,-\beta)\underline{Rot}(z,-\alpha)=\begin{bmatrix}1&0&0&rC\alpha S\beta\\0&1&0&rS\alpha S\beta\\0&0&1&rC\beta\\0&0&0&1\end{bmatrix}$$



Uniform orientation description:

Homogeneous transformation matrix

$$\underline{T} = \begin{bmatrix} N_x & O_x & A_x & P_x \\ N_y & O_y & A_y & P_y \\ N_z & O_z & A_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \underline{R} & \overline{P} \\ \overline{0}^T & 1 \end{bmatrix}$$

$$\underline{EU}(\Phi,\Theta,\Psi) = \underline{Rot}(z,\Phi)\underline{Rot}(y',\Theta)\underline{Rot}(z'',\Psi) =$$

Euler angles

$$= \begin{bmatrix} C\Phi C\Theta C\Psi - S\Phi S\Psi & -C\Phi C\Theta S\Psi - S\Phi C\Psi & C\Phi S\Theta & 0 \\ S\Phi C\Theta C\Psi + C\Phi S\Psi & -S\Phi C\Theta S\Psi + C\Phi C\Psi & S\Phi S\Theta & 0 \\ -S\Theta C\Psi & S\Theta S\Psi & C\Theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\underline{RPY}(\phi,\theta,\psi) = \underline{Rot}(z,\phi)\underline{Rot}(y,\theta)\underline{Rot}(x,\psi) =$$

RPY angles
(Roll, Pitch, Yaw)

$$= \begin{bmatrix} C\phi C\theta & -S\phi C\psi + C\phi S\theta S\psi & S\phi S\psi + C\phi S\theta C\psi & 0 \\ S\phi C\theta & C\phi C\psi + S\phi S\theta S\psi & -C\phi S\psi + S\phi S\theta C\psi & 0 \\ -S\theta & C\theta S\psi & C\theta C\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Evaluation of RPY angles

$$\underline{T} = \begin{bmatrix} N_x & O_x & A_x & P_x \\ N_y & O_y & A_y & P_y \\ N_z & O_z & A_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

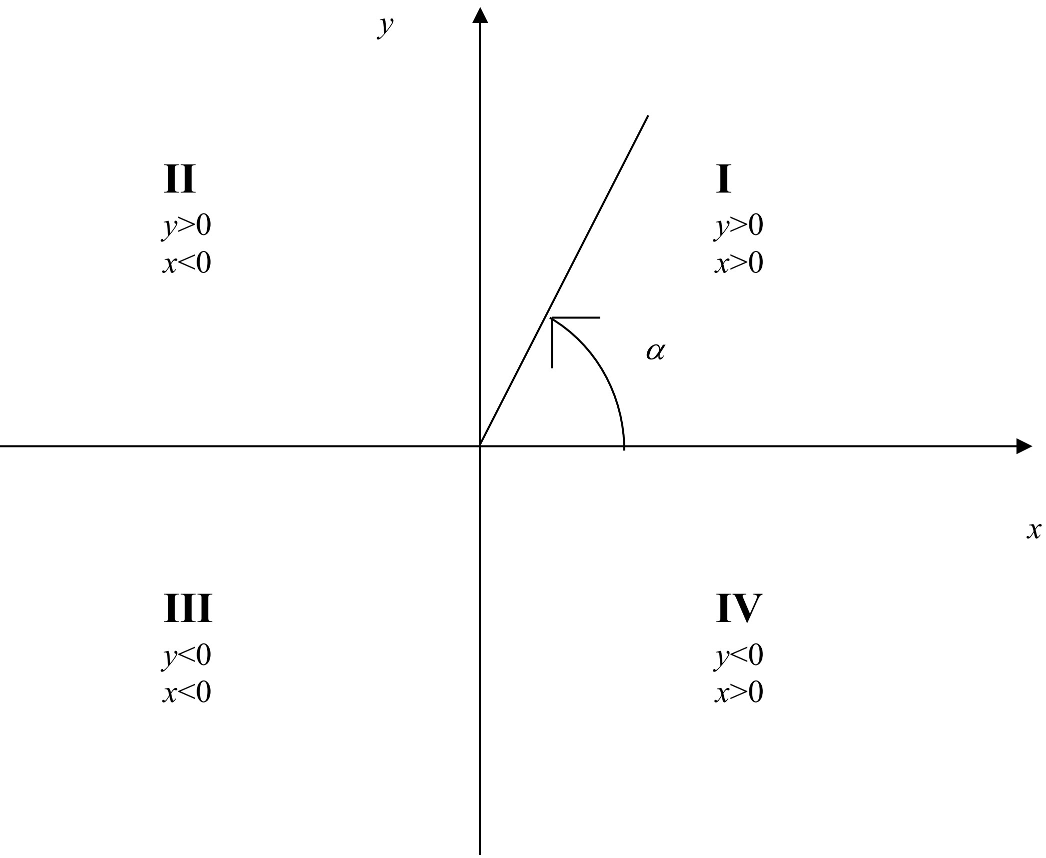
$$\underline{RPY}(\phi,\theta,\psi) = \begin{bmatrix} C\phi C\theta & -S\phi C\psi + C\phi S\theta S\psi & S\phi S\psi + C\phi S\theta C\psi & 0 \\ S\phi C\theta & C\phi C\psi + S\phi S\theta S\psi & -C\phi S\psi + S\phi S\theta C\psi & 0 \\ -S\theta & C\theta S\psi & C\theta C\psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

exceptions:

		$\theta = 90^\circ$	$\theta = -90^\circ$
$\phi = \arctan\dfrac{N_y}{N_x}$	Yaw (z)		
$\theta = \arctan\dfrac{-N_z}{\sqrt{1-N_z^{\,2}}} = \arcsin(-N_z)$	Pitch (y)	$\sin(\psi-\phi) = O_x$ $\cos(\psi-\phi) = O_y$	$-\sin(\psi+\phi) = O_x$ $\cos(\psi+\phi) = O_y$
$\psi = \arctan\dfrac{O_z}{A_z}$	Roll (x)	$\psi - \varphi = \arctan\dfrac{O_x}{O_y}$	$\psi + \varphi = \arctan\dfrac{-O_x}{O_y}$

Ranges: $\varphi \epsilon [-180°, 180°]$ $\theta \epsilon [-90°, 90°]$ $\psi \epsilon [-180°, 180°]$

Atan2 function

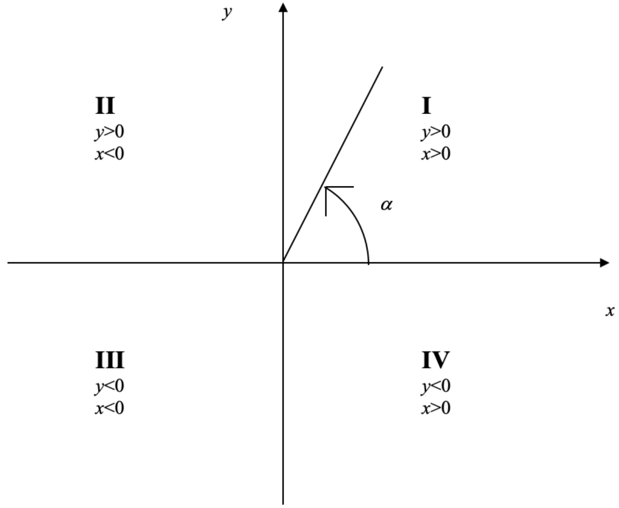


For I and IV quadrant:

$$\operatorname{atan2}\left(\frac{y}{x}\right) = \arctan\left(\frac{y}{x}\right)$$

For II and III quadrant:

$$\operatorname{atan2}\left(\frac{y}{x}\right) = \arctan\left(\frac{y}{x}\right) \pm \pi$$



QUADRANT I $$atan2\left(\frac{1}{\sqrt{2}}\right) = 45°$$

QUADRANT II $$atan2\left(\frac{1}{-\sqrt{2}}\right) = -45° + 180° = 135°$$

QUADRANT III $$atan2\left(\frac{-1}{-\sqrt{2}}\right) = 45° - 180° = -135°$$

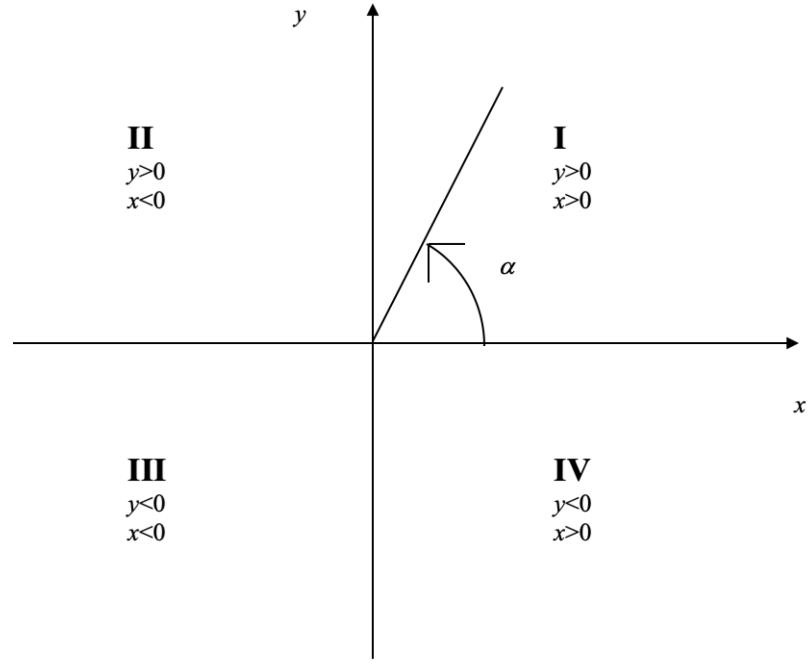
QUADRANT IV $$atan2\left(\frac{-1}{\sqrt{2}}\right) = -45°$$

$$atan2\left(\frac{0}{1}\right) = 0°$$

$$atan2\left(\frac{1}{0}\right) = 90°$$

$$atan2\left(\frac{0}{-1}\right) = 180°$$

$$atan2\left(\frac{-1}{0}\right) = -90°$$

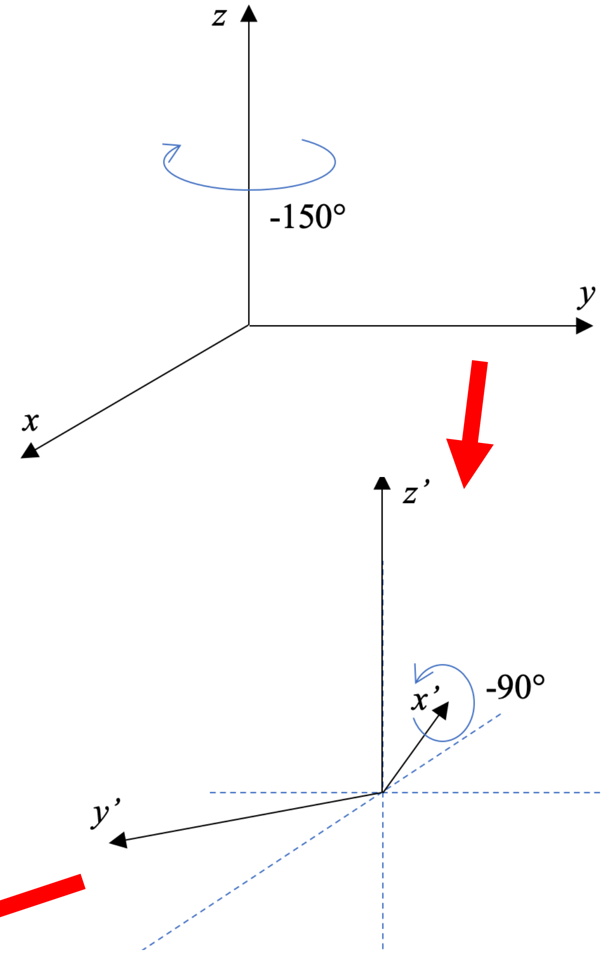


Example Matrix -> Coordinate frame

$$tg(\phi)=\frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}=\frac{-1}{-\sqrt{3}} \Rightarrow \phi=-150^\circ$$

$$tg(\theta)=\frac{0}{\sqrt{1-0}}=0 \Rightarrow \theta=0$$

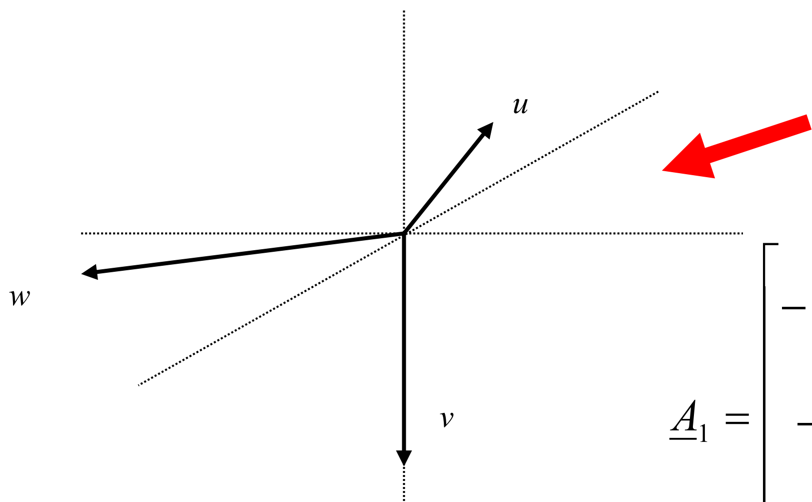
$$tg(\psi)=\frac{-1}{0}=-\infty \Rightarrow \psi=-90^\circ$$



$$\phi=arctg\frac{N_y}{N_x}$$

$$\theta=arctg\frac{-N_z}{\sqrt{1-N_z^{\,2}}}$$

$$\psi=arctg\frac{O_z}{A_z}$$

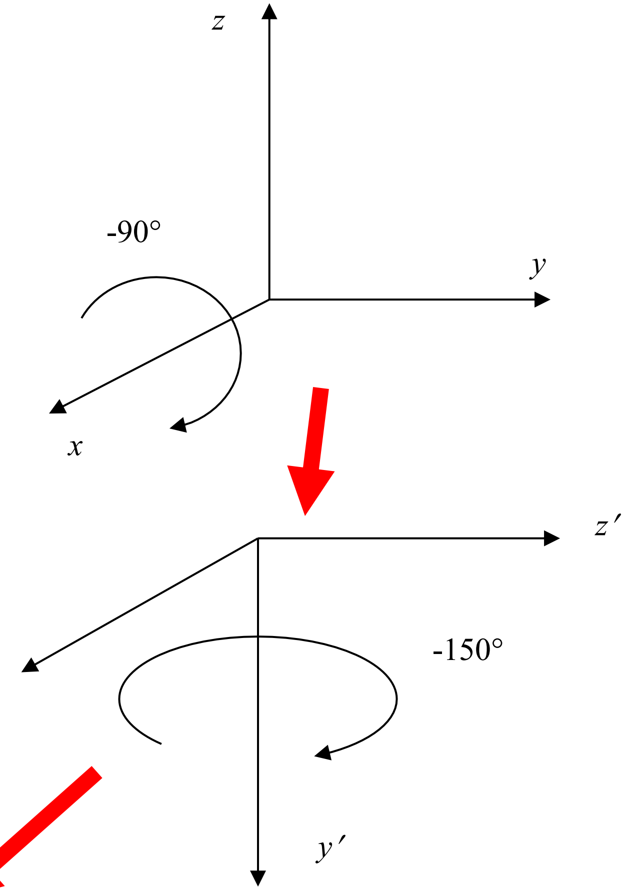


$$\underline{A}_1=\begin{bmatrix} -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\tan(\phi) = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{-1}{-\sqrt{3}} \Rightarrow \varphi = -150^{\circ}$$

$$\tan(\theta) = \frac{0}{\sqrt{1-0}} = 0 \Rightarrow \theta = 0$$

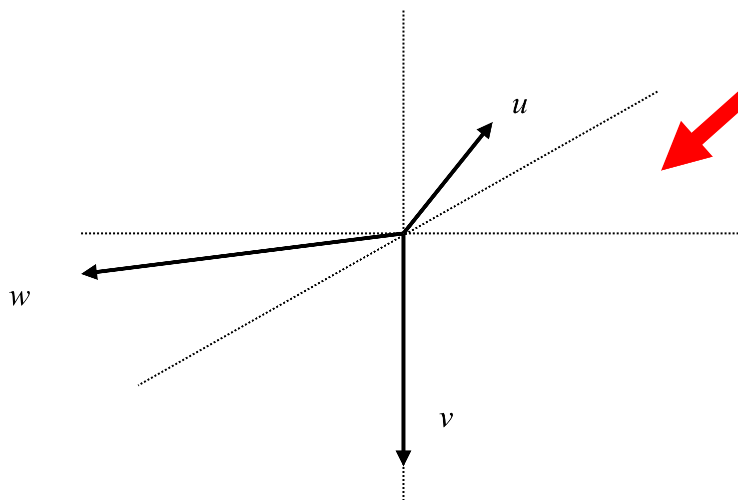
$$\tan(\psi) = \frac{-1}{0} = -\infty \Rightarrow \psi = -90^{\circ}$$



$$\phi = \arctan\frac{N_y}{N_x}$$

$$\theta = \arctan\frac{-N_z}{\sqrt{1-N_z^{\,2}}}$$

$$\psi = \arctan\frac{O_z}{A_z}$$



$$\underline{A}_1 = \begin{bmatrix} -\frac{\sqrt{3}}{2} & 0 & \frac{1}{2} & 0 \\ -\frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Techniques of description of orientation - quaternions

RPY angles
φ, θ, ψ

⟺

Rotation matrix

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

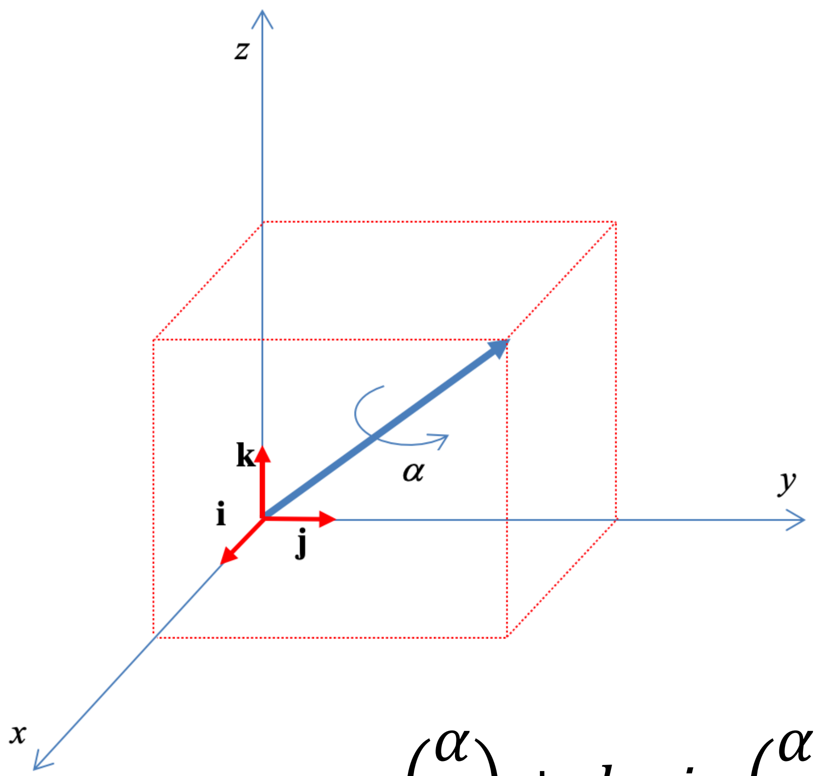
⟺

Quaternions
$\{a, b, c, d\}$

Quaternions

$$q = a + bi + cj + dk$$

$$i^2 = j^2 = k^2 = ijk = -1$$



Unit quaternion

$$a^2 + b^2 + c^2 + d^2 = 1$$

$$q = cos\left(\frac{\alpha}{2}\right) + \ l_x sin\left(\frac{\alpha}{2}\right) i + l_y sin\left(\frac{\alpha}{2}\right) j + l_z sin\left(\frac{\alpha}{2}\right) k$$

operations

$$q = a + bi + cj + dk \qquad i^2 = j^2 = k^2 = ijk = -1$$

$$q_1 = a_1 + b_1 i + c_1 j + d_1 k \qquad q_2 = a_2 + b_2 i + c_2 j + d_2 k$$

$$q_1 \oplus q_2 = (a_1 + a_2) + (b_1 + b_2)i + (c_1 + c_2)j + (d_1 + d_2)k$$

$$Q = \begin{bmatrix} a & -b & -c & -d \\ b & a & -d & c \\ c & d & a & -b \\ d & c & b & a \end{bmatrix}$$

$$r = q \odot p$$

$$\begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} a & -b & -c & -d \\ b & a & -d & c \\ c & d & a & -b \\ d & c & b & a \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$

Rotation matrix→ Quaternions

$$a = \sqrt{\frac{R_{11} + R_{22} + R_{33} + 1}{4}}$$

$$b = sign(R_{32} - R_{23})\sqrt{\frac{R_{11} - R_{22} - R_{33} + 1}{4}}$$

$$c = sign(R_{13} - R_{31})\sqrt{\frac{R_{22} - R_{11} - R_{33} + 1}{4}}$$

$$d = sign(R_{21} - R_{12})\sqrt{\frac{R_{33} - R_{11} - R_{22} + 1}{4}}$$

Quaternions → Rotation matrix

$$\begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} =$$

$$= \begin{bmatrix} 2(a^2+b^2)-1 & 2(bc-ad) & 2(bd+ac) \\ 2(bc+ad) & 2(a^2+c^2)-1 & 2(cd-ab) \\ 2(bd-ac) & 2(cd+ab) & 2(a^2+d^2)-1 \end{bmatrix}$$

Exemplary values

A	B	C
0°	0°	0°

$$\underline{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

q1	q2	q3	q4
1	0	0	0

A	B	C
0°	90°	0°

$$\underline{R} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$$

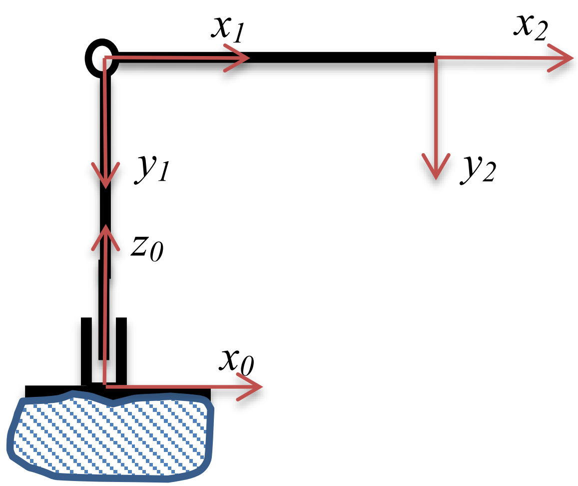
q1	q2	q3	q4
0.7071	0	0.7071	0

A	B	C
45°	45°	45°

$$\underline{R} = \begin{bmatrix} 0.5 & -0.1464 & 0.8536 \\ 0.5 & 0.8536 & -0.1464 \\ -0.7071 & 0.5 & 0.5 \end{bmatrix}$$

q1	q2	q3	q4
0.8446	0.1913	0.4619	0.1913

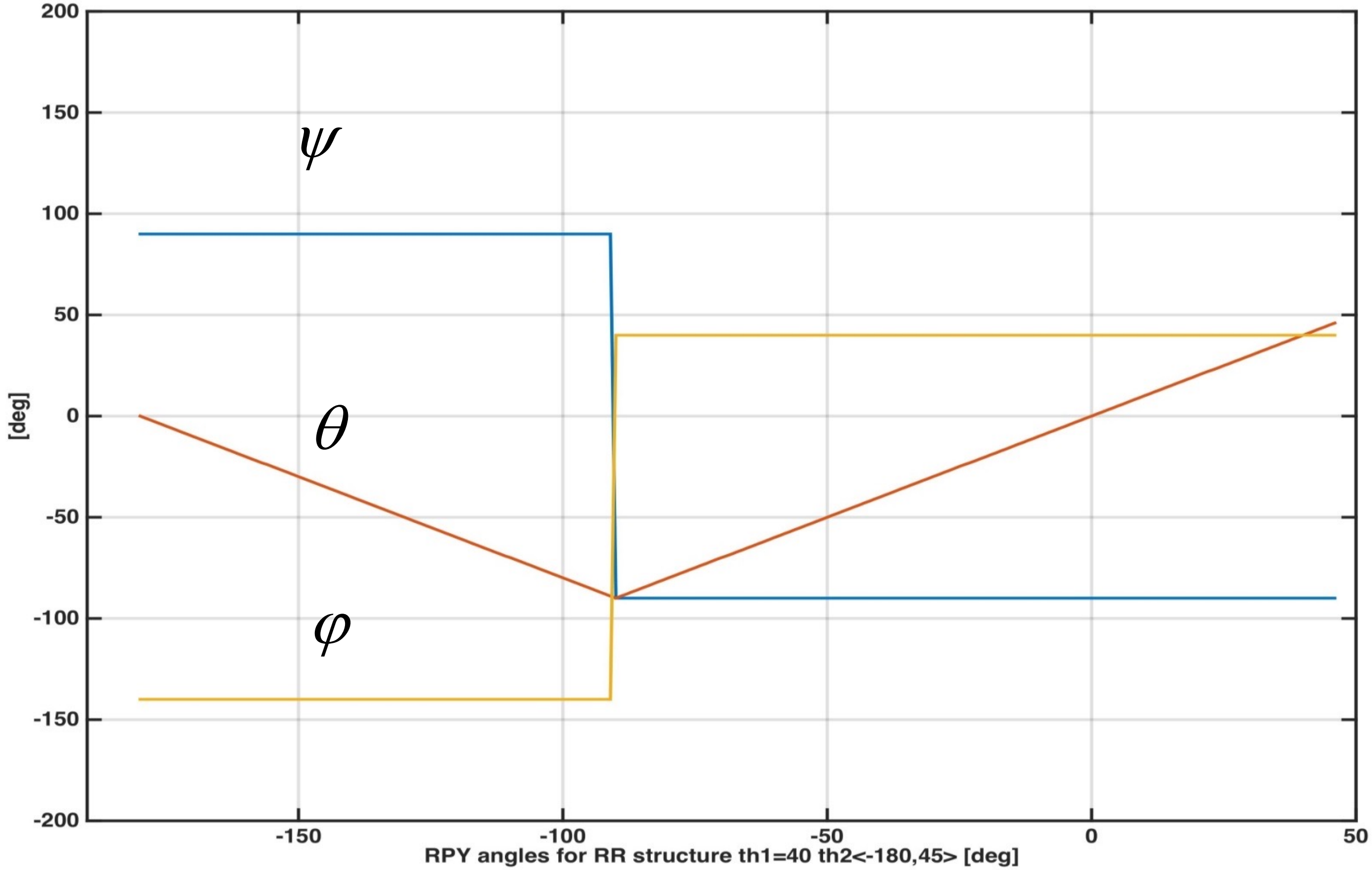
Example of RR manipulator



No.	θ	d	a	α
1	θ_1	d_1	0	-90
2	θ_2	0	a_2	0

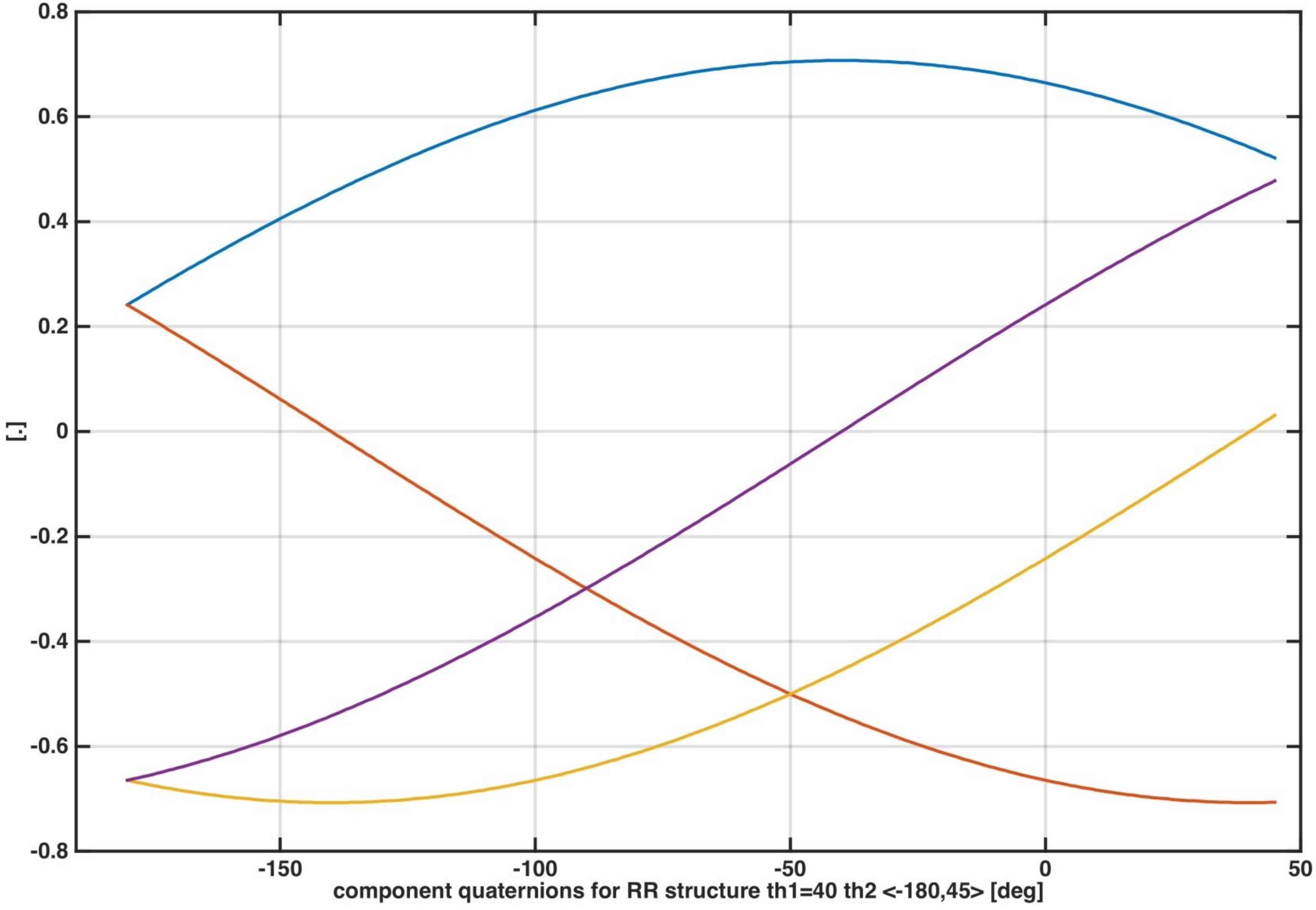
$$
{}^{0}T_2 = \begin{bmatrix} C_1C_2 & -C_1S_2 & -S_1 & a_2C_1C_2 \\ S_1C_2 & -S_1S_2 & C_1 & a_2S_1C_2 \\ -S_2 & -C_2 & 0 & d_1 - a_2S_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

RPY angles



$$\psi = arctan\left(\frac{-C_2}{0}\right) \qquad \theta = arcsin(S_2) \qquad \varphi = arctan\left(\frac{S_1 C_2}{C_1 C_2}\right)$$

Quaternions



1B
Mechanics of manipulators
Introduction. Position and orientation.