Mathematical description of orientation

Exercise 1

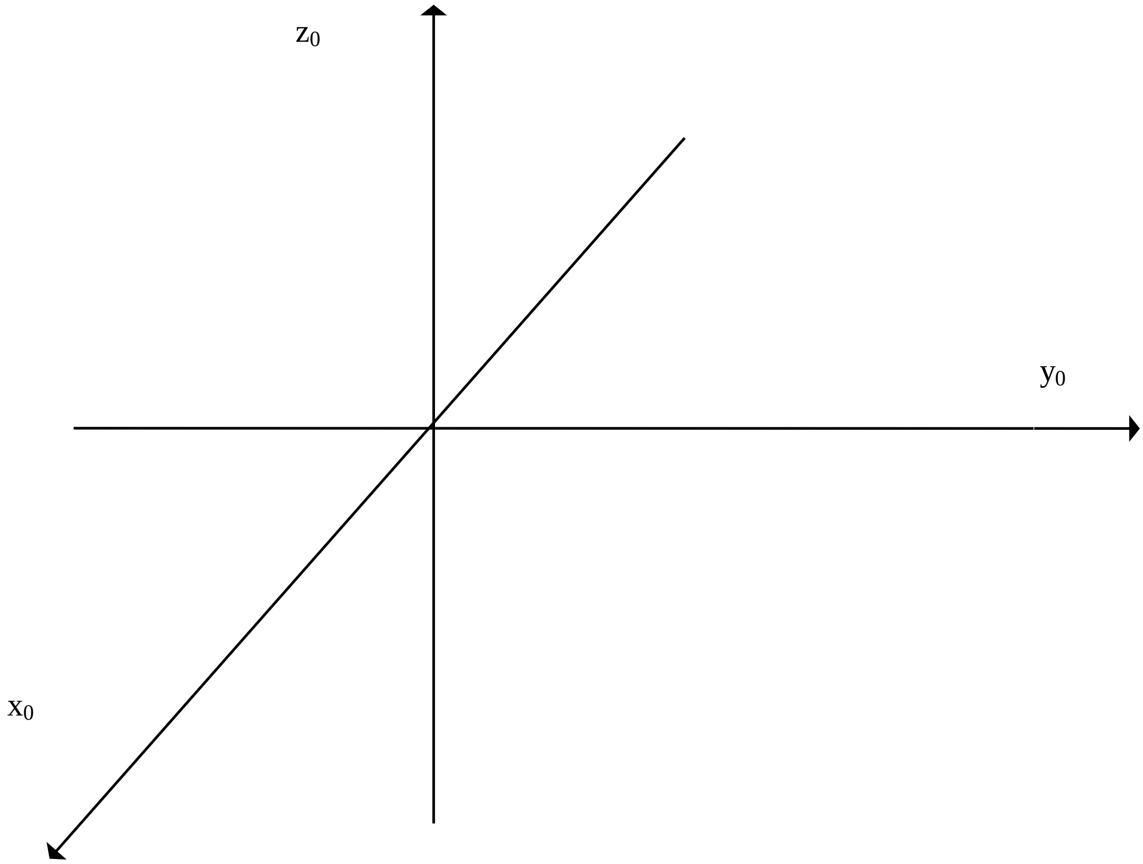
Sketch a local co-ordinate frame $x_1y_1z_1$ in the reference frame $x_0y_0z_0$. Position and orientation of the local co-ordinate system is described by the following $\underline{A}_1$ matrix:

$$\underline{A}_1 = \begin{bmatrix} -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 2 \\ 0 & 0 & 1 & 3 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

First find the angles between pairs of axes of the both systems. Write down the angles values to the following table:

	x_1	y_1	z_1
x_0			
y_0			
z_0			

Now, make a sketch.



Exercise 2

Next calculate values of RPY angles basing on matrix $\underline{A_1}$ and using the following formulas:

$$\phi = arctg\,\frac{N_y}{N_x}$$

$$\theta = arctg\,\frac{-N_z}{\sqrt{1-N_z^{\,2}}}$$

$$\psi = arctg\,\frac{O_z}{A_z}$$

With 2 exceptions for: N_x=0 i N_y=0 (and O_z=0, A_z=0), for which:

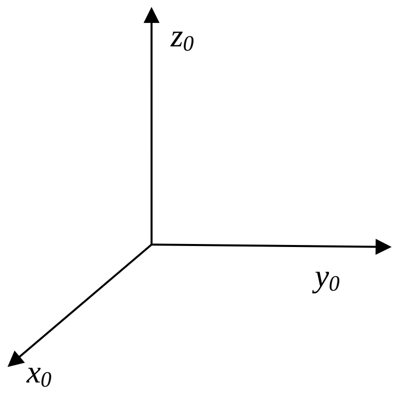
$$\theta = 90^\circ \qquad\qquad \theta = -90^\circ$$

$$\sin(\psi - \phi) = O_x \qquad\qquad -\sin(\psi + \phi) = O_x$$

$$\cos(\psi - \phi) = O_y \qquad\qquad \cos(\psi + \phi) = O_y$$

$$\psi - \phi = arctg\,\frac{O_x}{O_y} \qquad\qquad \psi + \phi = arctg\,\frac{-O_x}{O_y}$$

In calculations use atan2 function (pay attention to a sign of a numerator and a denominator). Present graphically the RPY transformation starting from the reference frame orientation. Perform the 3 subsequent rotations around the original reference frame axes: rotation around x_0 axis by ψ angle, followed by rotation around y_0 axis by angle $\theta \in [-90^\circ, 90^\circ]$., and final rotation around z_0 axis by angle ϕ.



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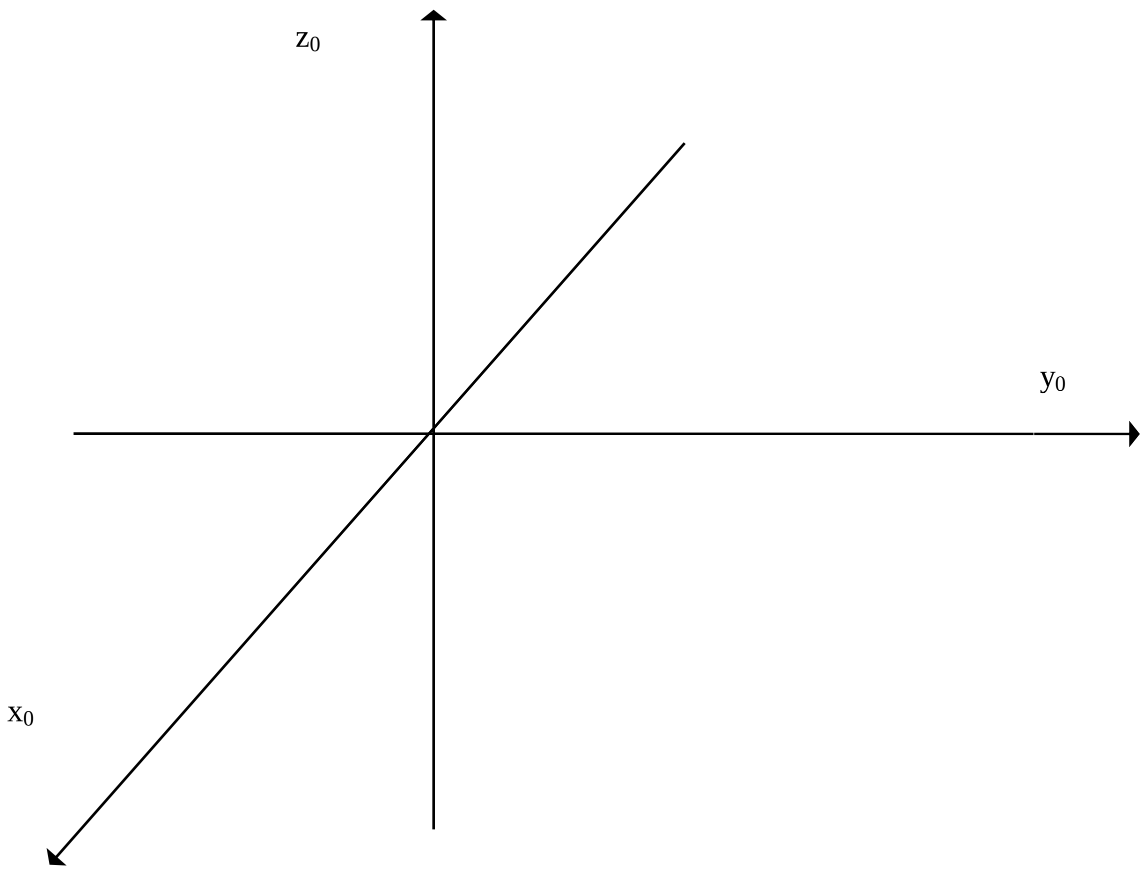
$$\underline{A}_1 = \begin{bmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 3 \\ 0 & 0 & -1 & 4 \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

First find the angles between pairs of axes of the both systems. Write down the angles values to the following table:

	x_1	y_1	z_1
x_0			
y_0			
z_0			

Now, make a sketch.



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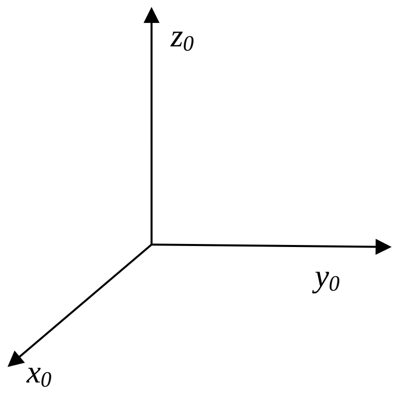

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$$\underline{A}_1 = \begin{bmatrix} 0 & 0 & 1 & 2 \\ \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & -4 \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

First find the angles between pairs of axes of the both systems. Write down the angles values to the following table:

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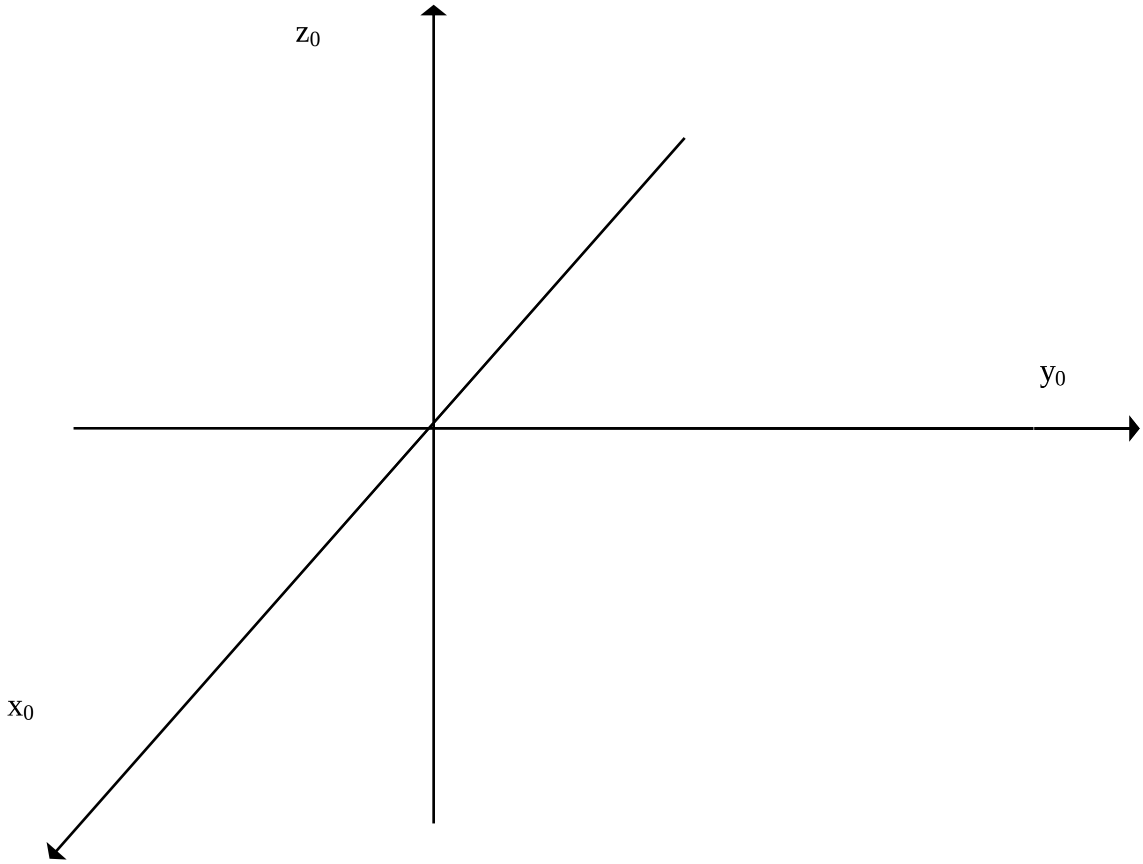
y_0			
z_0			

Now, make a sketch.



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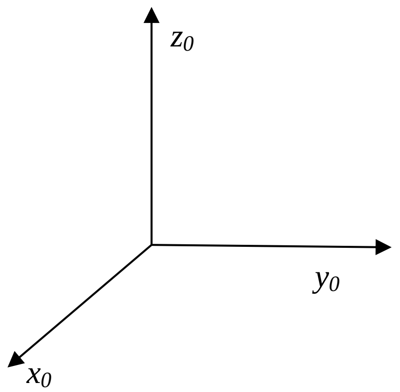

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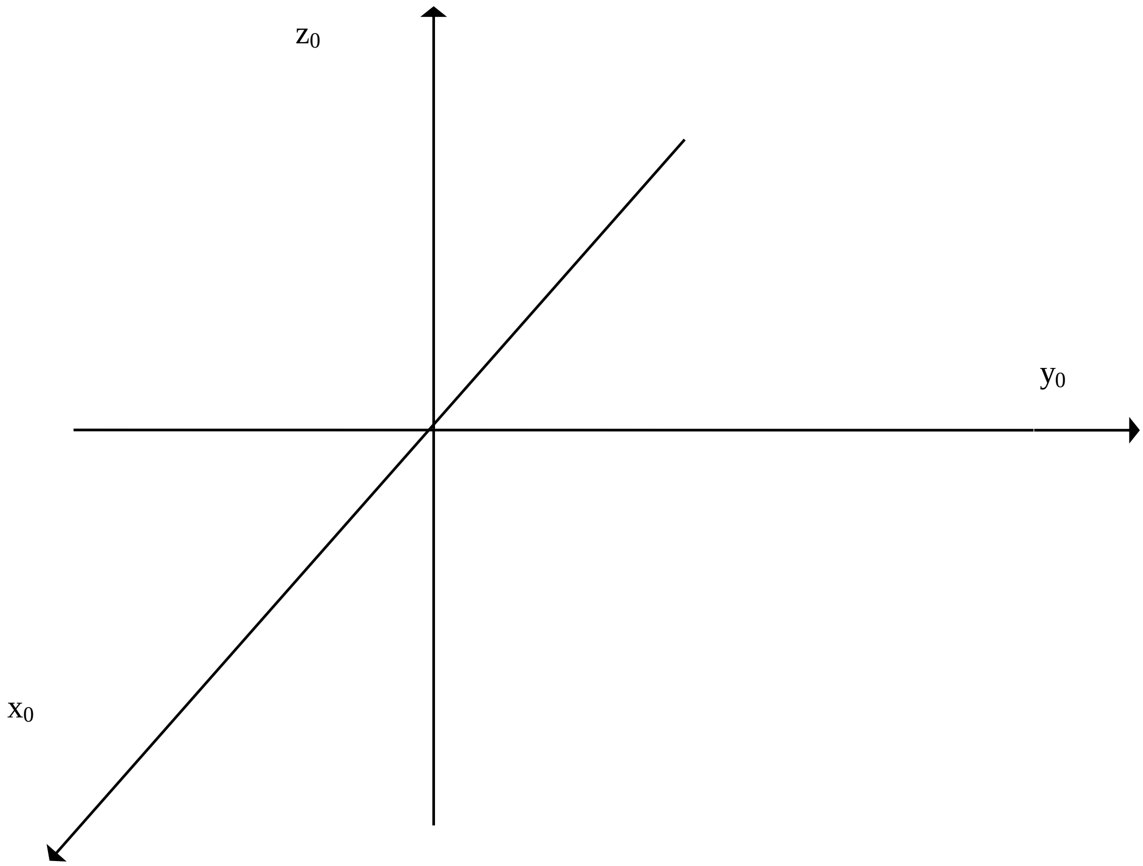

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$\sin(\psi-\phi) = O_x$ $\cos(\psi-\phi) = O_y$	$-\sin(\psi+\phi) = O_x$ $\cos(\psi+\phi) = O_y$
$\psi-\phi = arctg\frac{O_x}{O_y}$	$\psi+\phi = arctg\frac{-O_x}{O_y}$

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