Kinematics and Dynamics of Mechatronic Systems

Exercise B3

Geometrical model – inverse kinematics – implementation of the elaborated algorithm, the manipulator workspace

For the selected kinematic structure of a manipulator the following tasks are to be completed.

1. Implementation and testing of the inverse kinematics algorithm

Basing on the description of the consequent stages of the inverse kinematic problem solution elaborated during the previous class, prepare the graphical presentation of the algorithm (of a block diagram type). <u>Present this algorithm to the tutor, and after acceptance deliver it at the end of the class</u>.
Using the formulated algorithm prepare a Matlab procedure for calculation of values of joint coordinates basing on the position and orientation of the last link defined by a corresponding homogeneous transformation matrix.
To test the procedure, please, <u>randomly</u> choose a set of values of the joint coordinates: q_1, q_2, q_3 and q_4 from their ranges of motion.
Then calculate elements of the corresponding homogeneous transformation matrix ${}^{0}\underline{T}_{e}$ that expresses position and orientation of the last link. Next, use this matrix as the input data for the procedure that solves the inverse kinematic problem. One of the achieved solutions should be identical with the values of the joint coordinates q_1, q_2, q_3 and q_4 that were assumed at the start of the testing procedure. Repeat the testing for other input values. Report the results.

The structure of the procedure:
<u>Forward kinematics</u> (preparation of the input data)
1. Random selection of the values of joint coordinates and setting values of the geometrical model constants
2. Calculation of position and orientation of the end effector – a matrix ${}^{0}\underline{T}_{e}$

<u>Inverse kinematics</u> (actual inverse kinematics problem solution)
1. Calculation of $\mathbf{p_a}$
2. Calculation of values of q_1, q_2 and q_3 coordinates with use of the derived relationships
3. Checking whether the values of the joint variables lie within their assumed ranges of motion
4. Checking whether the values of p_{ax}, p_{ay} and p_{az} achieved by substitution of the calculated values of q_1, q_2 and q_3 to symbolic formulas on p_{ax}, p_{ay} and p_{az} are consistent with the values obtained during the step 1.
5. Calculation of q_4 joint coordinate for each set of q_1, q_2 and q_3 values calculated in the step 2

Prepared by W. Lisowski

Some examples of commands useful in the prepared script are listed below.

The elements of the homogeneous transformation matrix are referred to as follows:

```
pwx=T0E(1,3)*pwd;
……………………..
pax=T0E(1,4)-pwx;
……………………..
```

The function atan2 is called like in the following lines:

```
th21=atan2((d1-paz),sqrt(paz^2+pay^2))
th22=atan2((d1-paz),-sqrt(paz^2+pay^2))
```

An example of a definition of the motion ranges:

```
z=[-pi/2,-pi/2,0.5;pi/2,pi/3,1;0.1,0.5]
```

Creation of vectors *t1*, *t2* and *t3* containing various sets of solutions:

```
t1(1)=th11
t1(2)=th12
t2(1)=th21
t2(2)=th22
t3(1)=d31
t3(2)=d32
```

Calculation of p_a values for the achieved values of the joint variables:

```
pas=subs(pa,{th1,th2,d3,a1………},{t1,t2,t3,a1s………});
```

dpax, *dpay*, *dpaz* should be equal 0

```
dpax=pax-pas(1,1);
…………………..
```

Checking of the motion ranges (determined by the matrix *z*):

```
k=1
for i=1:2
        if (t1(i)>z(1,1)) && (t1(i)<z(2,1)) && (t2(i)>z(1,2)) && (t2(i)<z(2,2)) &&
        (t3(i)<z(1,3)) && (t3(i)<z(2,3)) && abs(dpax)<0.00001 && …
                qr(k,1)=t1(i)
                qr(k,2)=t2(i)
                qr(k,3)=t2(i)
        end
end
```

Please note that all the angles used in the trigonometric functions should be expressed in radians as well as all the distances/positions in meters.

2. The workspace

Taking into account the assumed joint motion ranges and dimensions of links present graphically the workspace of the modelled mechanism. The position should correspond to the tip of the third link. The horizontal and the vertical cross-sections of the workspace should be presented. Show manipulator links on the cross-sections.
For structures for which $\mathbf{p}_w=\mathbf{0}$, the presented workspace will actually correspond to the positions of the origin of the frame assigned to the last link of the manipulator.

The report should contain:
- graphical presentation of the algorithm of determination of the inverse kinematic problem (a block diagram)
- report of testing of the implemented algorithm for at least 10 different sets of data
- graphical presentation of the workspace
- conclusions