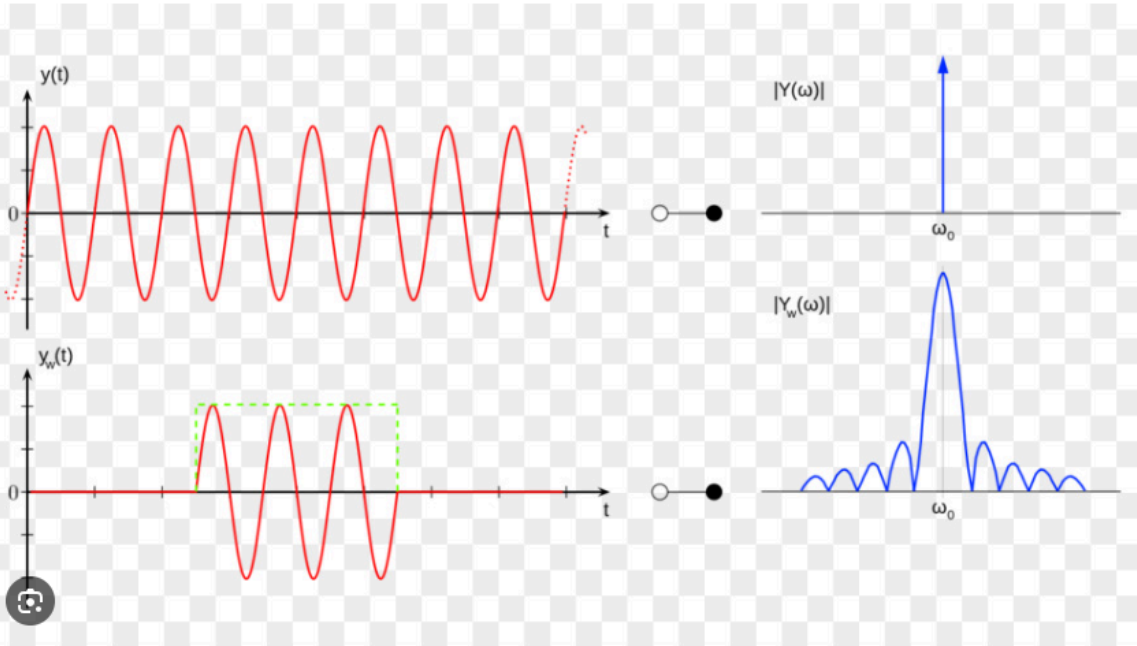
Topics

1. Frequency Resolution

Frequency Resolution: The ability of a system to differentiate between two closely spaced frequency components in a signal. It is usually quantified as the smallest frequency difference that can be distinguished by the system.
Factors that affect it are observation time, sampling rate, and sampling frequency.

2. Spectral Leakage

Spectral leakage is a phenomenon in signal processing that occurs when a signal is transformed from the time domain to the frequency domain using techniques like the Fast Fourier Transform (FFT). It happens because the signal is not perfectly periodic within the observation window, leading to the spread of energy from one frequency bin to adjacent bins in the frequency spectrum.
Windowing Functions: Applying a windowing function (such as Hamming, Hanning, Blackman, or Kaiser windows) to the signal before performing the FFT can reduce the discontinuities at the edges of the observation window. These windows taper the signal smoothly to zero at the edges, thereby reducing spectral leakage.
Longer Observation Windows: Increasing the length of the observation window can help in reducing spectral leakage, as it minimizes the truncation effect.



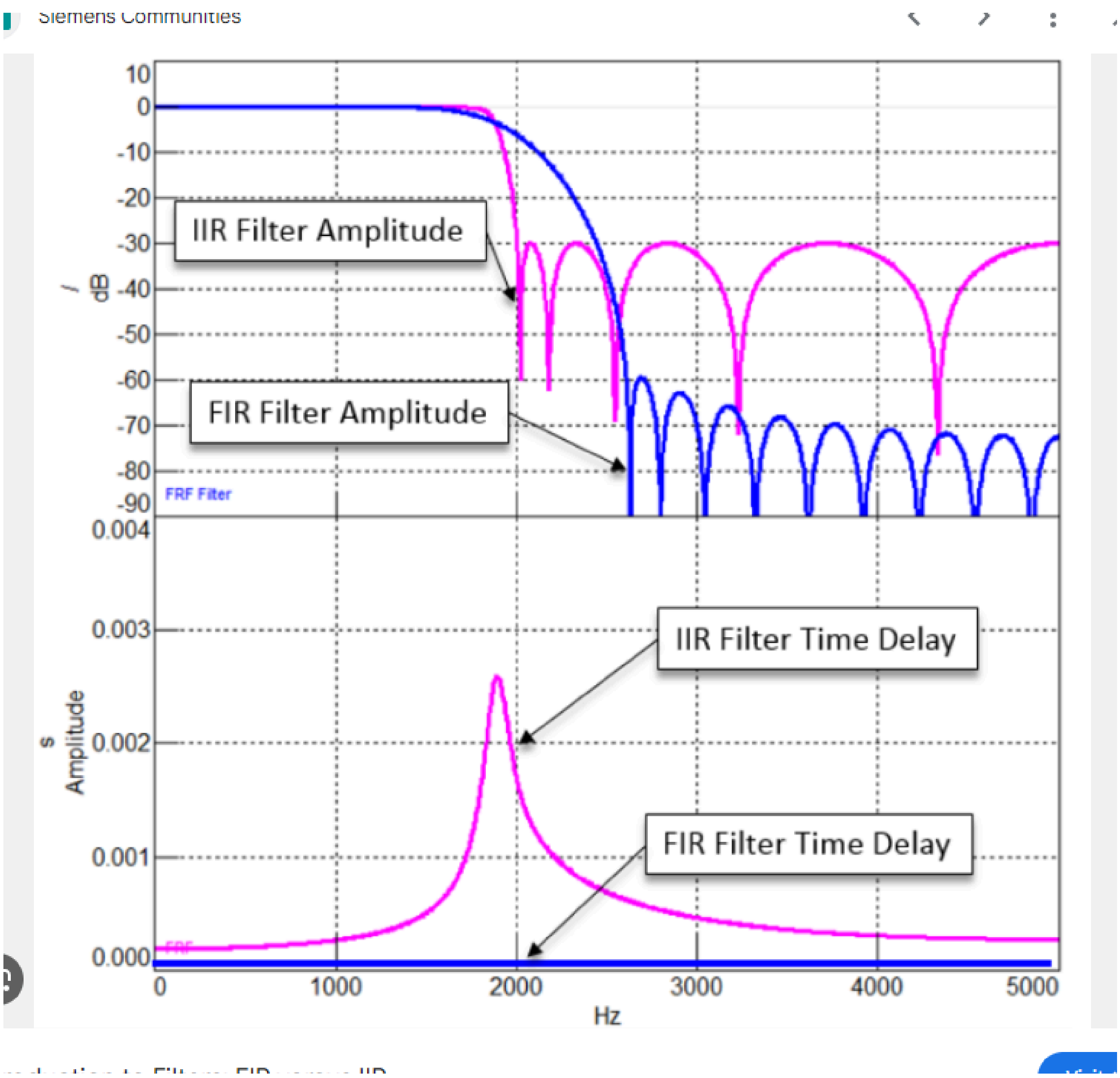
3. Filtering methods and types of filters

Based on Frequency Response: Low Pass, High Pass, Bandpass, Bandstop

Based on Design Characteristics:
FIR Filters (Finite Impulse Response)
Characteristics:
Impulse Response: The impulse response of an FIR filter is finite because it settles to zero in a finite amount of time.

Stability: FIR filters are always stable because they do not have feedback loops.
Phase Response: They can be designed to have a linear phase response, meaning that all frequency components of the input signal are delayed by the same amount of time, which preserves the waveform shape of signals.
Implementation: Implemented using a convolution of the input signal with a finite number of filter coefficients.

IIR Filters (Infinite Impulse Response)
Characteristics:
Impulse Response: The impulse response of an IIR filter is infinite because it continues indefinitely due to the presence of feedback.
Stability: IIR filters can be unstable if the poles of the filter are outside the unit circle in the z-domain.
Phase Response: Generally have a non-linear phase response, which can distort the phase of the input signal.
Implementation: Implemented using a combination of feedforward and feedback components.

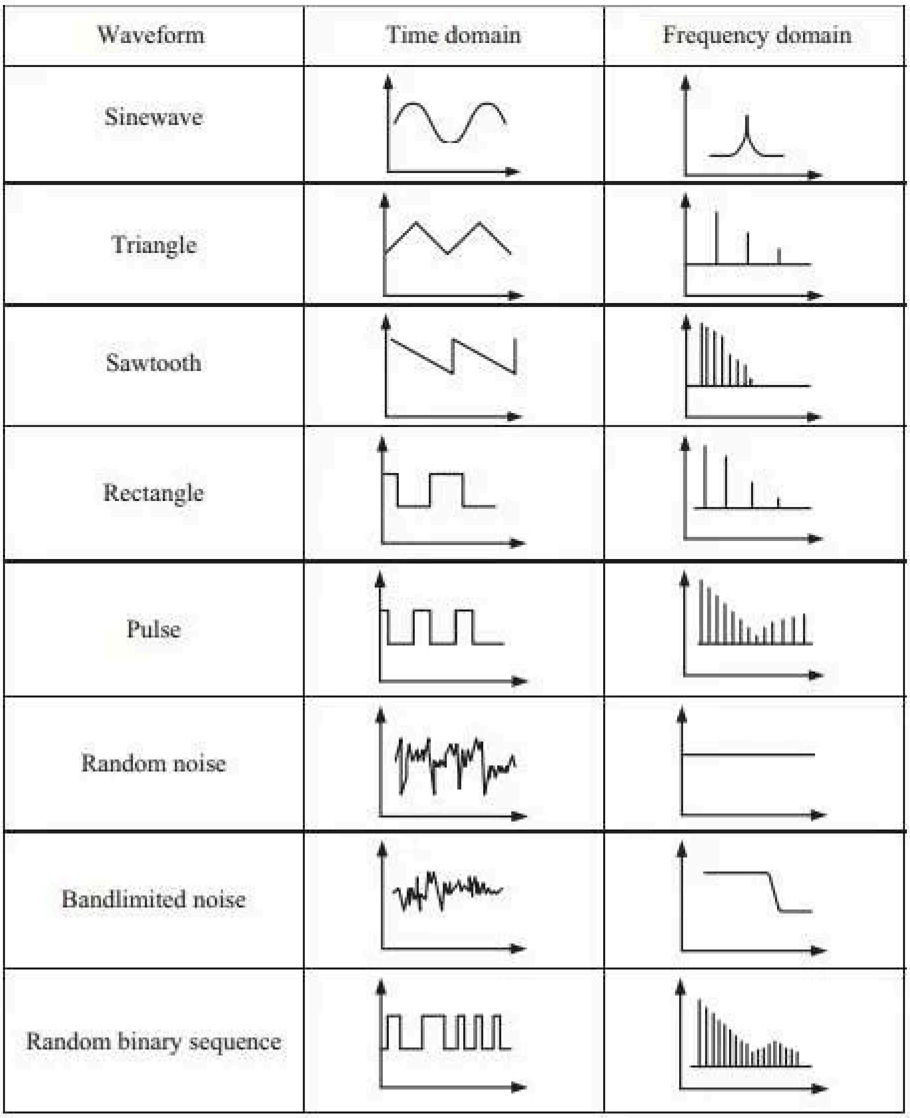


4. Matched Filtering

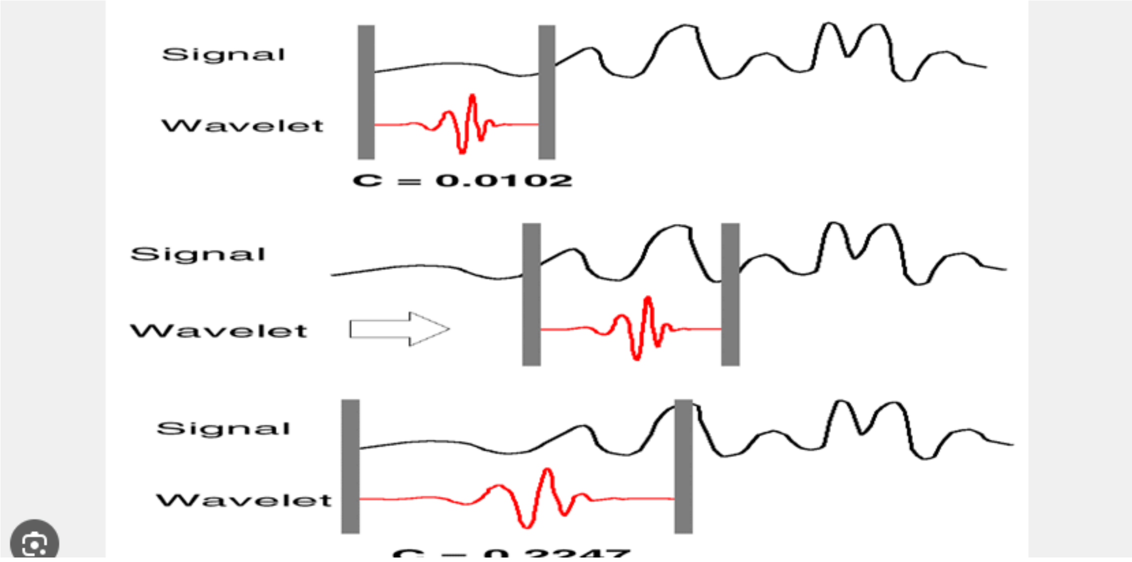
Optimal Detection: Designed to maximize the signal-to-noise ratio (SNR) in the presence of noise, making them optimal for detecting known signals.
Impulse Response: The impulse response of a matched filter is matched to the time-reversed and conjugated version of the expected signal.
Implemented using a convolution operation in the time domain or multiplication in the frequency domain.
Widely used in radar, sonar, communication systems, and other applications where detecting a known signal in the presence of noise is critical.

5. FFT and CTW

FFT is the function that transforms the signal from the time domain to the frequency domain.



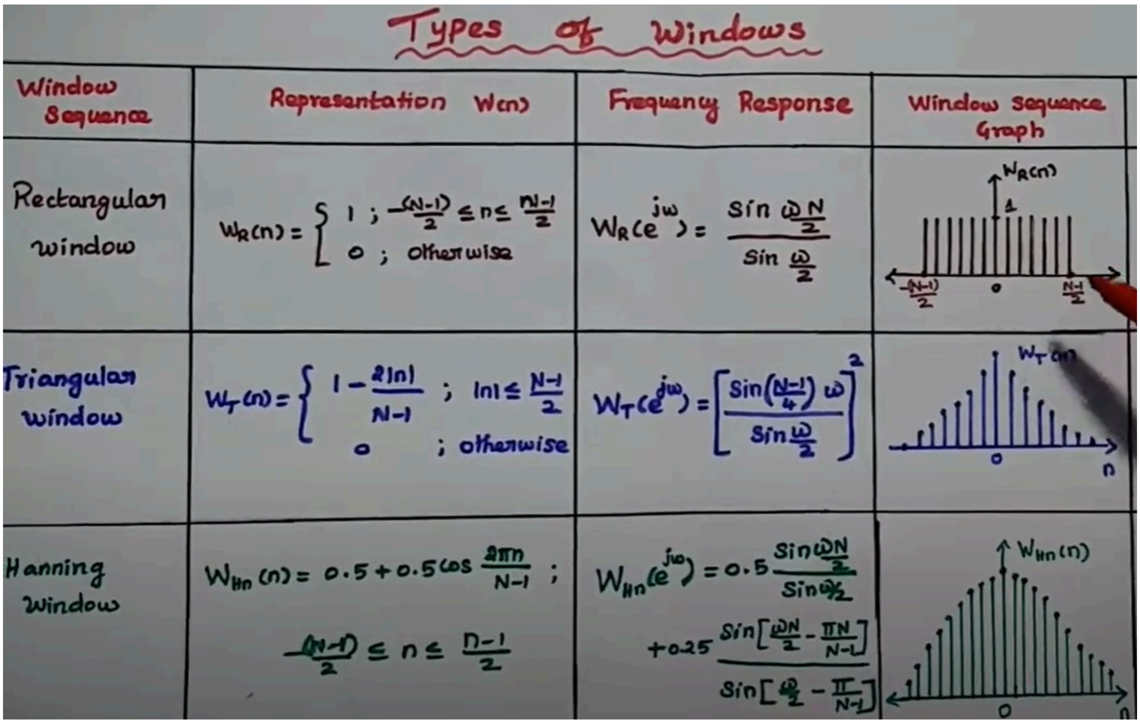
The CWT provides a time-frequency representation of the signal, allowing for the analysis of how the frequency content changes over time.



6. Windowing

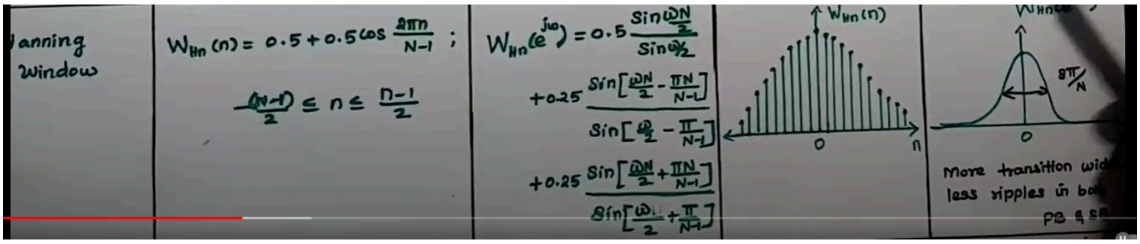
Windowing is a technique in digital signal processing used primarily to reduce spectral leakage during the Fourier Transform of a signal. Spectral leakage occurs when the finite segment of a signal being analyzed doesn't constitute an exact number of periods of a periodic signal, causing the energy of specific frequencies to spread across other frequencies. Windowing helps mitigate this by modifying the signal at the edges so that it smoothly tapers off to zero, effectively minimizing discontinuities at the boundaries.

Types:



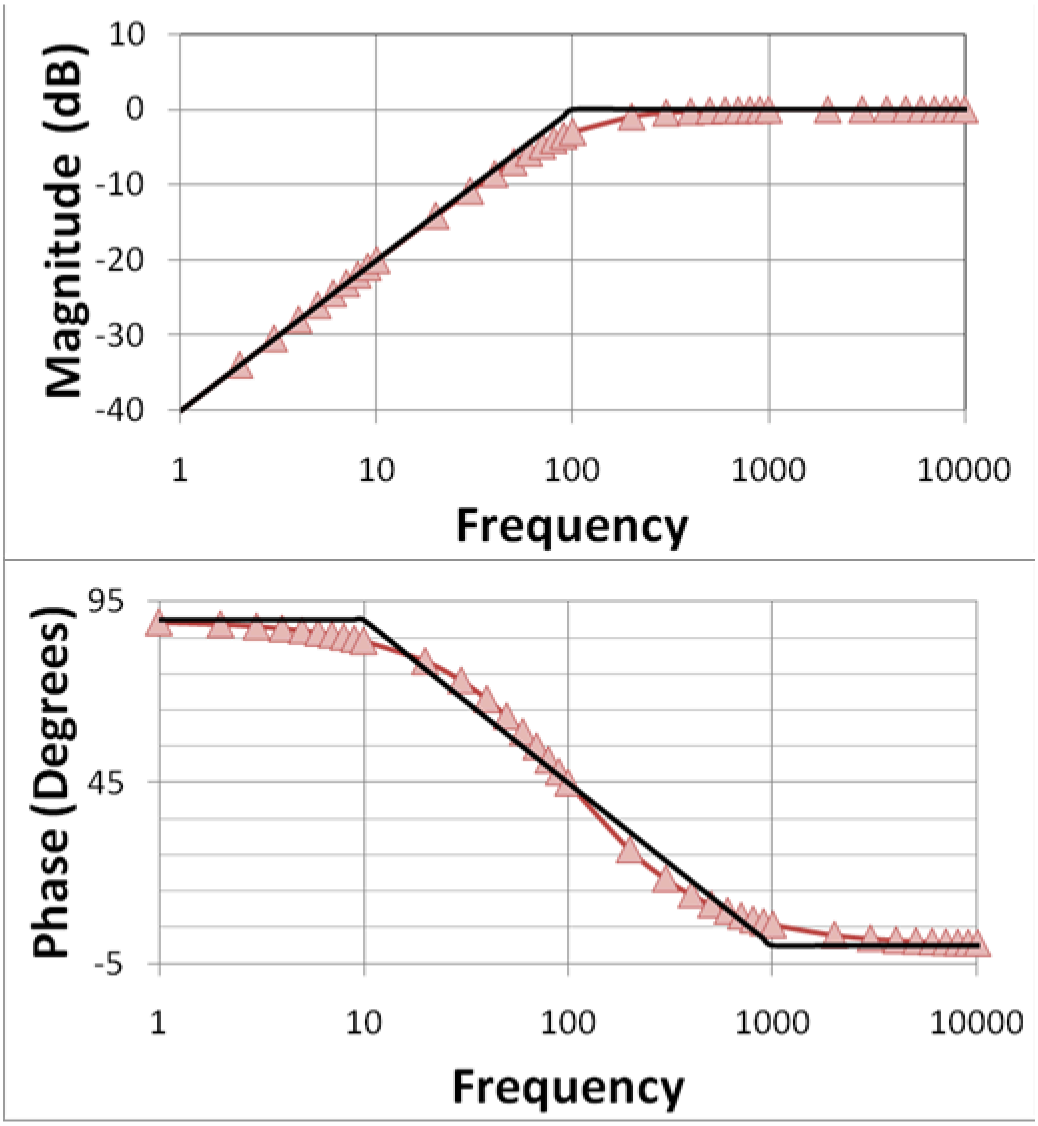
Types of Windows

Window Sequence	Representation W(n)	Frequency Response	Window sequence Graph
Rectangular window	$W_R(n) = \begin{cases} 1; & -\frac{(N-1)}{2} \le n \le \frac{N-1}{2} \\ 0; & \text{otherwise} \end{cases}$	$W_R(e^{j\omega}) = \dfrac{\sin \frac{\omega N}{2}}{\sin \frac{\omega}{2}}$	$W_R(n)$; 1; $-\frac{(N-1)}{2}$, 0, $\frac{N-1}{2}$
Triangular window	$W_T(n) = \begin{cases} 1 - \frac{2\lvert n\rvert}{N-1}; & \lvert n\rvert \le \frac{N-1}{2} \\ 0; & \text{otherwise} \end{cases}$	$W_T(e^{j\omega}) = \left[\dfrac{\sin\left(\frac{N-1}{4}\right)\omega}{\sin\frac{\omega}{2}}\right]^2$	$W_T(n)$; 0, n
Hanning window	$W_{Hn}(n) = 0.5 + 0.5\cos\frac{2\pi n}{N-1}$; $-\frac{(N-1)}{2} \le n \le \frac{N-1}{2}$	$W_{Hn}(e^{j\omega}) = 0.5\dfrac{\sin\frac{\omega N}{2}}{\sin\frac{\omega}{2}} + 0.25\dfrac{\sin\left[\frac{\omega N}{2} - \frac{\pi N}{N-1}\right]}{\sin\left[\frac{\omega}{2} - \frac{\pi}{N-1}\right]}$	$W_{Hn}(n)$; 0, n



7. Bode plots and their functions

Bode plots are graphical representations of a linear, time-invariant system's frequency response, typically used in control systems and signal processing. They consist of two separate plots: the magnitude plot and the phase plot, both plotted against frequency on a logarithmic scale.
Frequency Response: Bode plots provide a visual representation of how a system responds to different frequencies, helping to understand the system's behavior in the frequency domain.
Gain and Phase Margins: They are used to assess the stability and robustness of control systems. Gain margin and phase margin indicate how much gain or phase can be increased before the system becomes unstable.
Bandwidth: The bandwidth of the system can be easily identified, which is the range of frequencies where the system operates effectively.



8. Correlation and cross-correlation

Correlation: is the degree of similarity between two-time series or signals in the same time or sequence while no lag is considered in the magnitude of (-1 to 1).

Cross-correlation: is the degree of similarity between two time series in different times or space while lag can be considered when time is under investigation. The difference between these two time series in different situations like distance, angle, direction etc. can be considered while the space is under investigation respectively. (Two signals correlated at different time ranges)

Auto-correlation: is the cross-correlation of a time series while investigating the persistence between lagged times of the same time series or signal. Correlating the signal by itself

9. Convolution

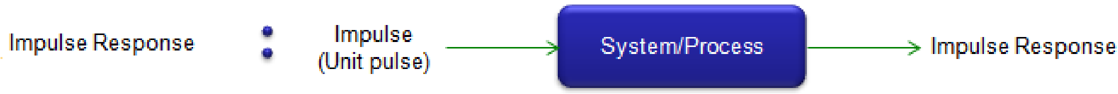
Convolution is a mathematical operation used to express the relation between the input and output of an LTI (Linear Time-Invariant) system. It integrates the product of an input signal and a time-reversed copy of a signal (often an impulse response).

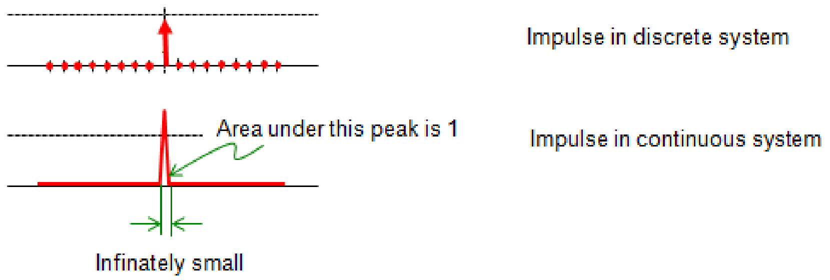
10. Impulse response

An impulse response is the reaction of any dynamic system in response to some external change. In both cases, the impulse response describes the reaction of the system as a function of time (or possibly as a function of some other independent variable that parametrizes the dynamic behavior of the system).

In all these cases, the dynamic system and its impulse response may be actual physical objects, or may be mathematical systems of equations describing such objects.

Since the impulse function contains all frequencies (see the Fourier transform of the Dirac delta function, showing infinite frequency bandwidth that the Dirac delta function has), the impulse response defines the response of a linear time-invariant system for all frequencies.

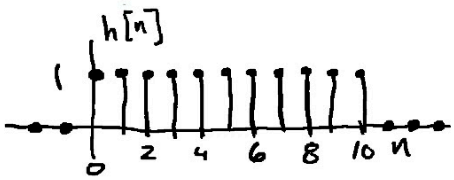




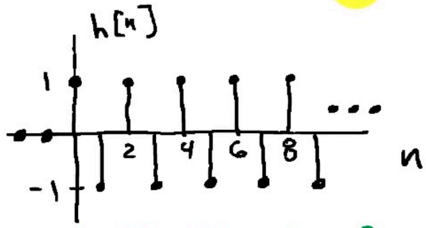
Finite impulse response (FIR): 5

duration of impulse response is finite

$h[n] = \begin{cases} 1 & 0 \le n \le 10 \\ 0 & \text{otherwise} \end{cases}$ $h[n] = \begin{cases} (-1)^n, & n \ge 0 \\ 0 & n < 0 \end{cases}$



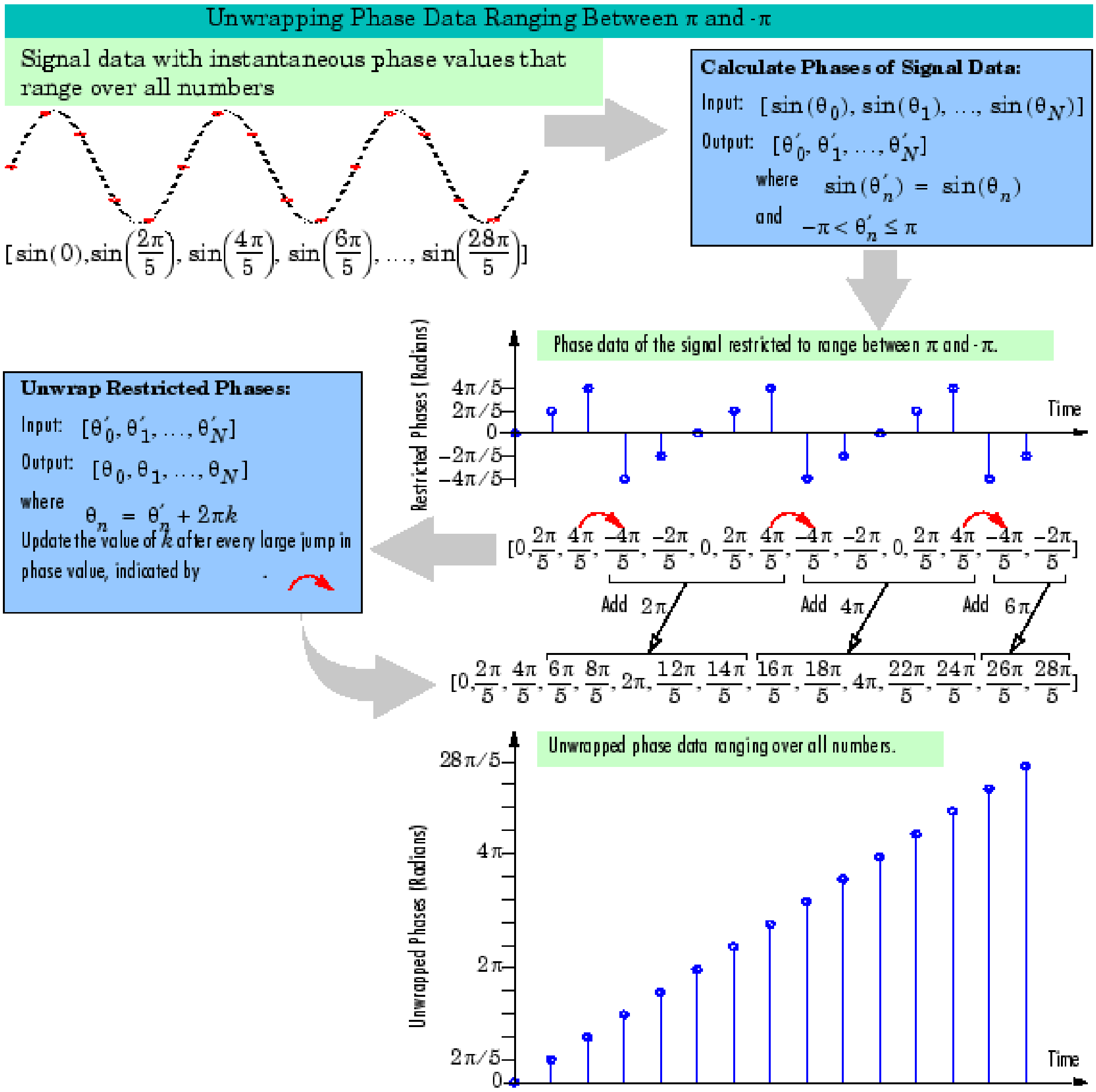
FIR



Infinite Impulse Response (IIR)

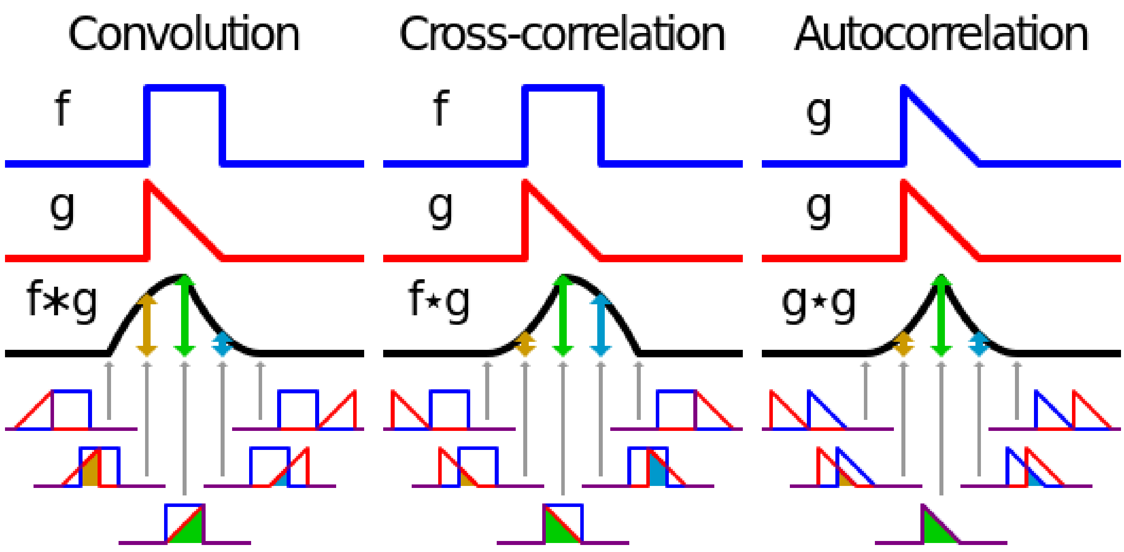
11. Phase unwrapping

Phase unwrapping is the process of modifying the phase of a signal to eliminate discontinuities induced by the transition from +π to -π (or vice versa), providing a continuous phase representation.



12. Difference between correlation and convolution

correlation is a measure of similarity between two signals, convolution is a new signal created by combining two signals.



$$f(x) * g(x) = \int_{-\infty}^{\infty} f(\tau)g(x-\tau)\,d\tau$$

and for the cross correlation

$$(f \star g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f^*(\tau)g(t+\tau)\,d\tau,$$

we come to know that equation-wise the only difference is that, in convolution, before doing sliding dot product we flip the signal across y-axis i.e we change (t) to $(-t)$, while the cross correlation is just the sliding dot product of two signals.

We use the convolution to get output/result of a system which have two blocks/signals and they are directly next to each other (in series) in the time domain.

13. Shannon's theorem & Nyquist frequency

Nyquist-Shannon sampling theorem, which states that a signal must be sampled at least twice the maximum frequency present in the signal (the Nyquist rate) to be accurately reconstructed. Sampling below this rate means that signals with frequencies higher than half the sampling rate (Nyquist frequency) will interfere with lower frequencies, leading to aliasing.

14. Aliasing

Aliasing is a phenomenon in digital signal processing that occurs when a signal is sampled at a rate that is not sufficient to capture its frequency content accurately. This insufficiency in the sampling rate causes higher frequency components of the signal to be indistinguishable from lower frequency components, effectively "folding" back into the lower frequency spectrum. This distortion is not only undesirable but also irreversible, meaning once aliasing has occurred in a sampled signal, the original signal cannot be accurately reconstructed from the sampled data.

Damping Ratio

The damping ratio of the system was calculated using the logarithmic damping decrement, and half power method, the results are as follows:

logarithmic damping decrement :

$$\xi = \frac{\ln(\frac{x_0}{x_1})}{2\pi}$$

Half power method:

$$\xi = \frac{\omega_2 - \omega_1}{2\omega_r}$$

fig 7: formulas for calculating damping using the two methods

- fft Fast Fourier Transform, transforms a signal into its frequency components.
- conv Convolution of two signals
- xcorr Computes cross-correlation of two signals.
- filter digital filter
- filtfilt Zero-phase digital filtering which applies a filter forwards and backwards.
- freqz Computes the frequency response of a digital filter.
- chirp Generates a signal whose frequency increases or decreases with time.
- bode: Generates Bode plots for a system transfer function.