Akademia Górniczo-Hutnicza
Faculty of Mechanical Engineering and Robotics
Mechatronic Engineering

Kinematics and Dynamics of Mechatronic Systems

Lab Report

Dynamics

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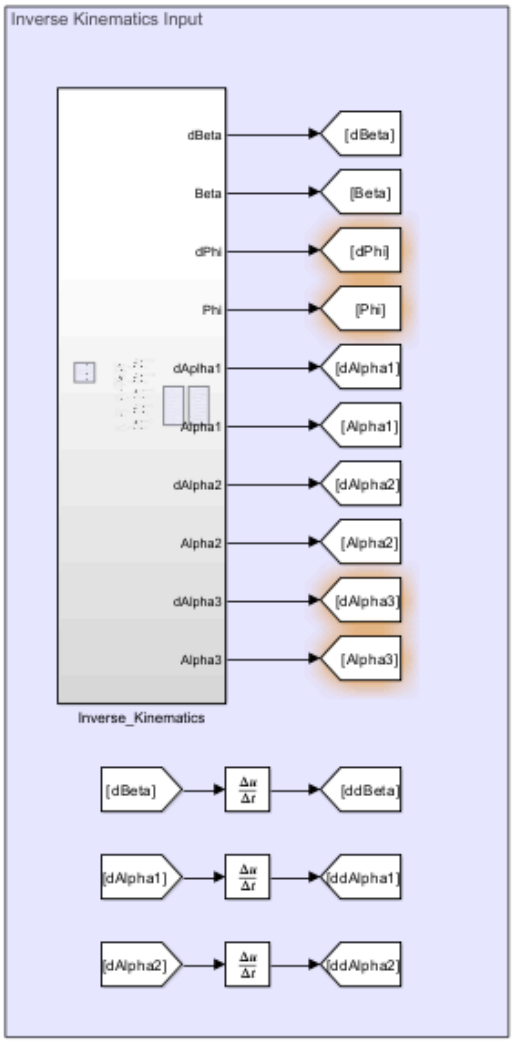
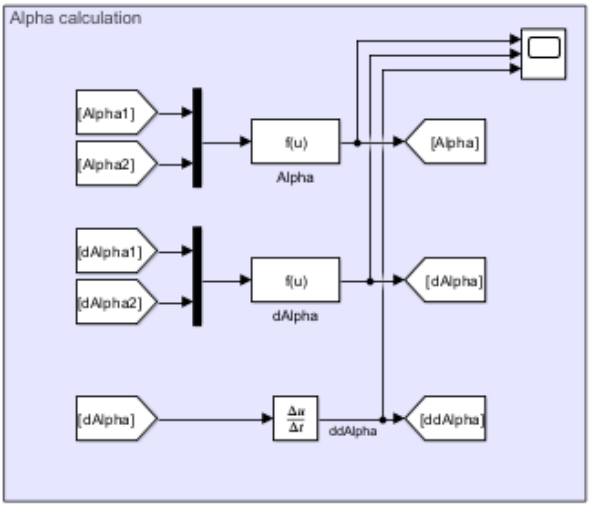
Prerequisites

Values used

```
%% Values
r = r1;
l2 = 0.07;
m1 = 1.5;
m2 = 1.5;
m3 = 0.5;
m4 = 5.67 ;
Ix1 = 0.02;
Ix2 = 0.02;
Ix3 = 0.005;
Iz1 = 0.051;
Iz2 = 0.051;
Iz3 = 0.002;
Iz4 = 0.154;
N1 = 31.25;
N2 = 31.25;
N3 = 29.2;
f1 = 0.015;
f2 = 0.015;
f3 = 0.0015;
h= l1/r1;
```

Simulink input and additional calculations





Lagrange

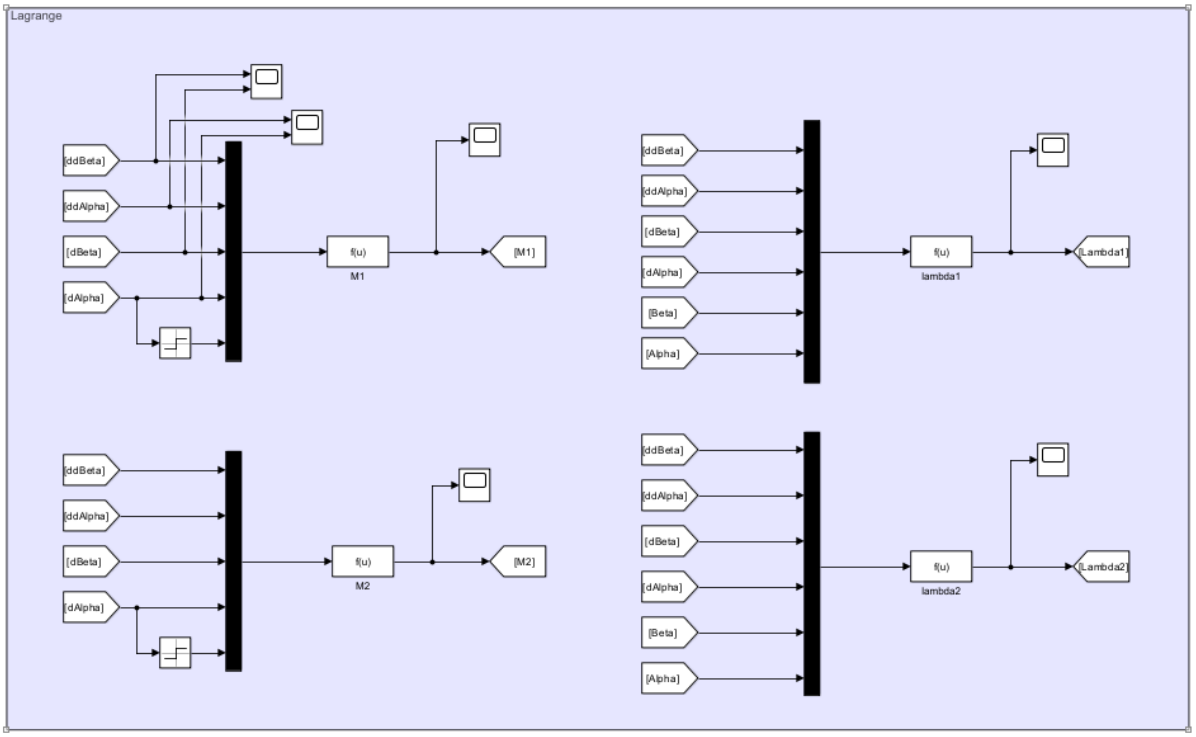
Equations

$$A = 2Iz1h^2 + 2m1l1^2 + 2Ix1 + Iz4 + m4l2^2$$
$$B = r^2m4 + 2r^2m1 + 2Iz1$$

$$M1 = 0.5\,(B\ddot{\alpha} + m4l2\dot{\beta}^2 + \frac{1}{h}(A\ddot{\beta} - m4l2r\dot{\beta}\dot{\alpha}) + N1f1sgn(\alpha)$$
$$M2 = 0.5\,(B\ddot{\alpha} + m4l2\dot{\beta}^2 - \frac{1}{h}(A\ddot{\beta} - m4l2r\dot{\beta}\dot{\alpha}) + N2f2sgn(\alpha)$$

$$\lambda1 = m4l2sin(\beta)\,\ddot{\beta} + (m4 + 2m1)rcos(\beta)\,\ddot{\alpha} + m4l2cos(\beta)\dot{\beta}^2 - (m4 + 2m1)rsin(\beta)\dot{\beta}\,\dot{\alpha}$$
$$\lambda2 = -m4l2cos(\beta)\,\ddot{\beta} + (m4 + 2m1)rsin(\beta)\,\ddot{\alpha} + m4l2sin(\beta)\dot{\beta}^2 + (m4 + 2m1)rcos(\beta)\dot{\beta}\,\dot{\alpha}$$

Simulink model



Maggie

Equations

$$A = (r^2 m4 l1^2 + 4r^2 m1 l1^2 + 2r^2 Ix1 + 4Iz1 l1^2 + r^2 Iz4 + r^2 m4 l2^2) / (4l1\text{^}2)$$

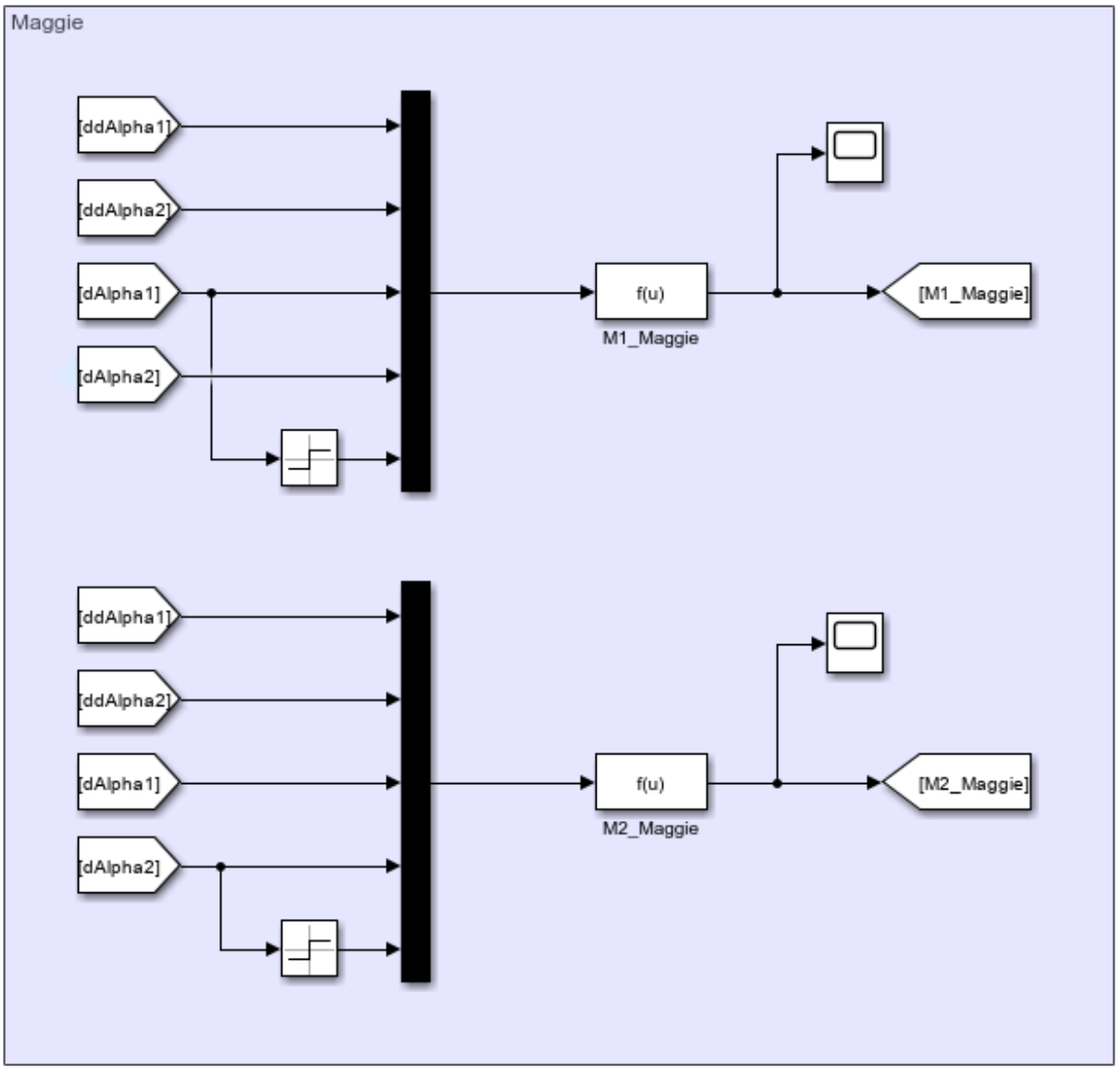
$$B = (r^2 m4 l1^2 - r^2 m4 l2^2 - 2r^2 Ix1 - r^2 Iz4) / (4l1^2)$$

$$C = (r^3 m4 l2) / (4l1^2)$$

$$M1 = A\ddot{\alpha}_1 + B\ddot{\alpha}_2 - C\dot{\alpha}_1\dot{\alpha}_2 + C\dot{\alpha}_2^2 + N1 f1 sgn(\dot{\alpha}_1)$$

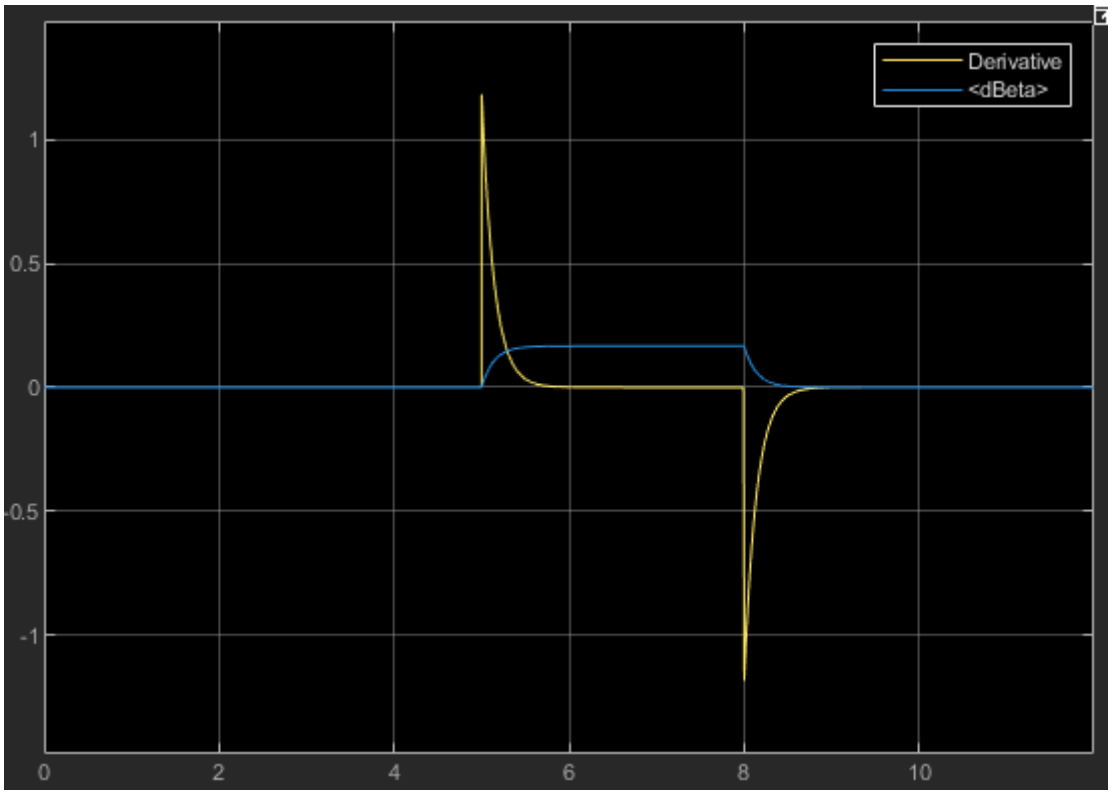
$$M2 = A\ddot{\alpha}_2 + B\ddot{\alpha}_1 - C\dot{\alpha}_1\dot{\alpha}_2 + C\dot{\alpha}_1^2 + N2 f2 sgn(\dot{\alpha}_2)$$

Simulink model

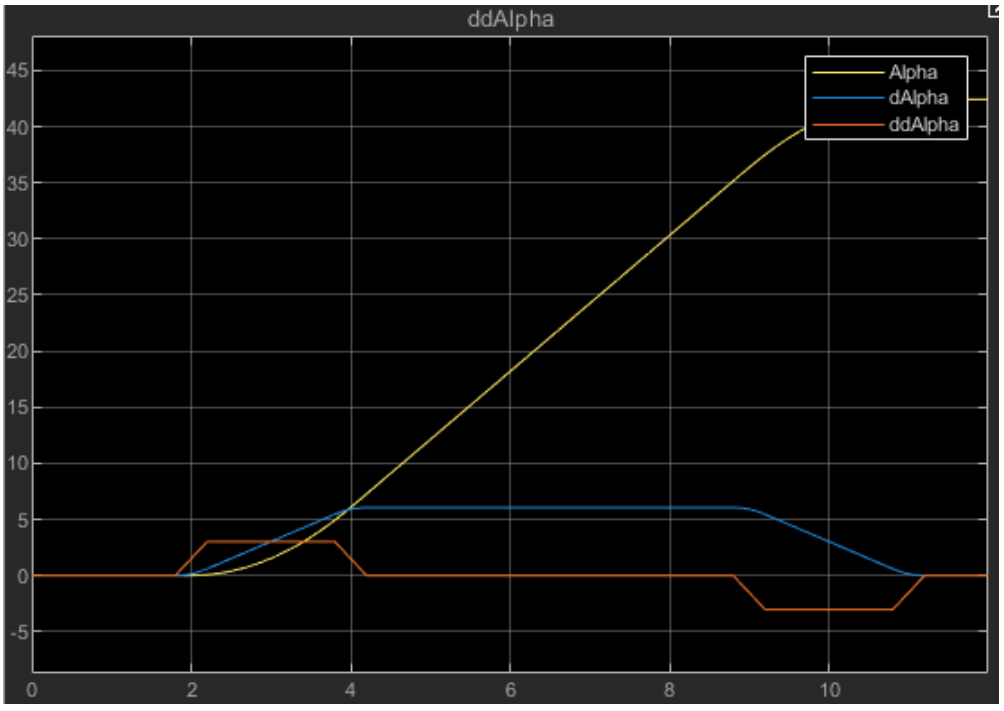


Results

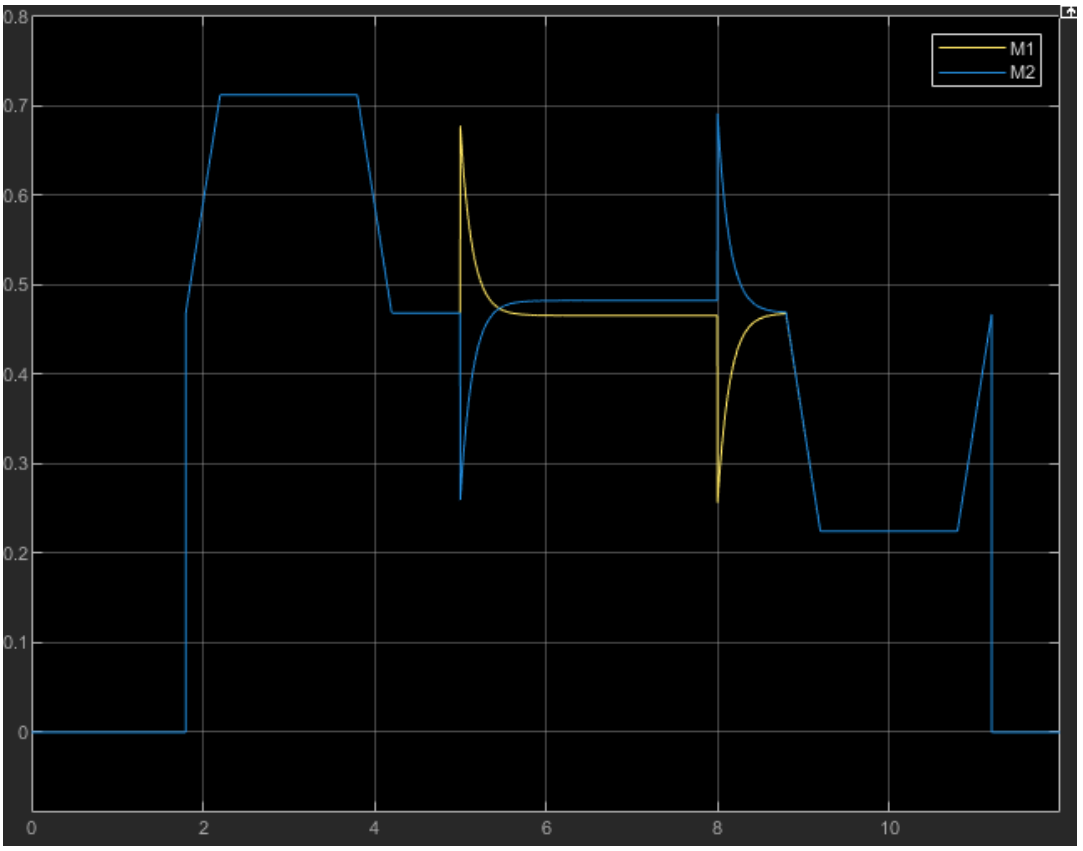
dBeta and ddBeta

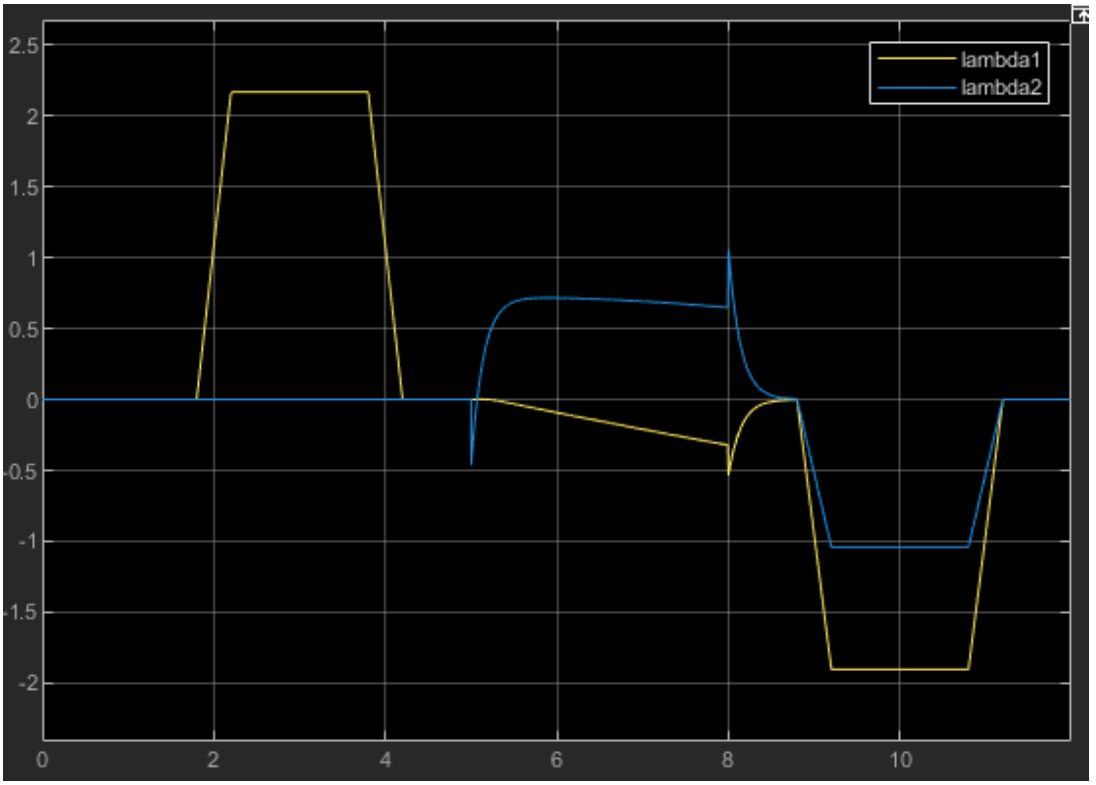


Alphas

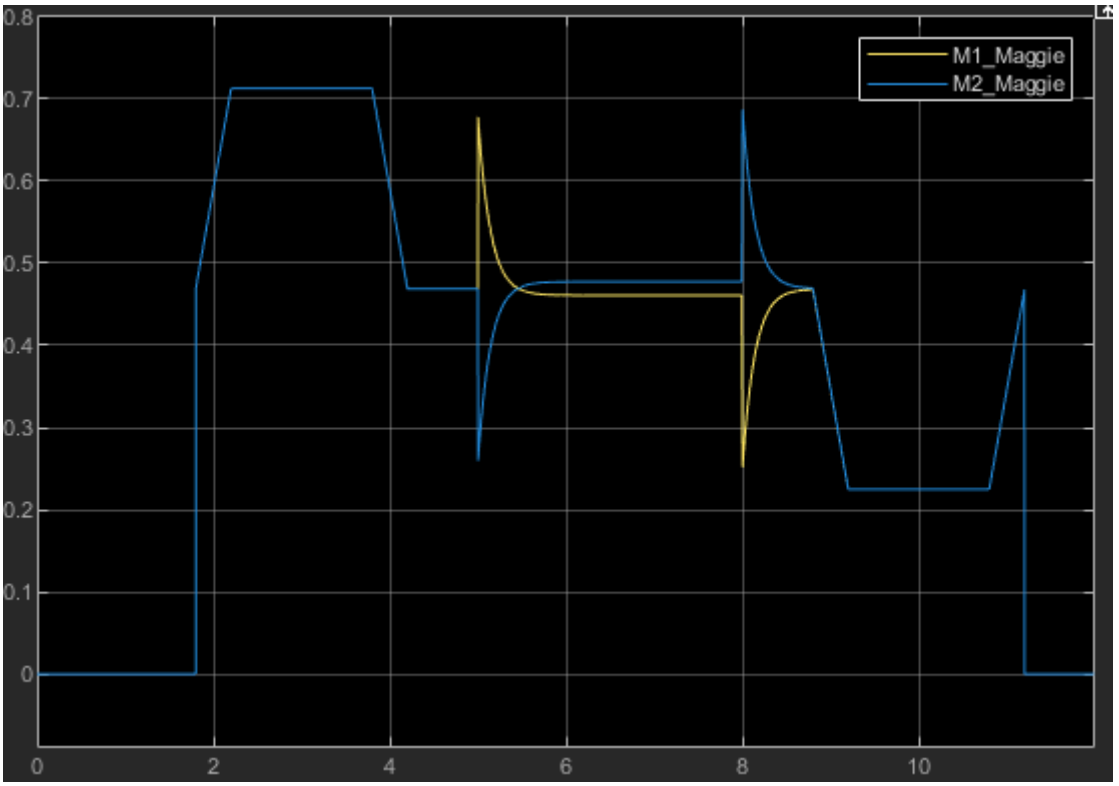


Lagrange

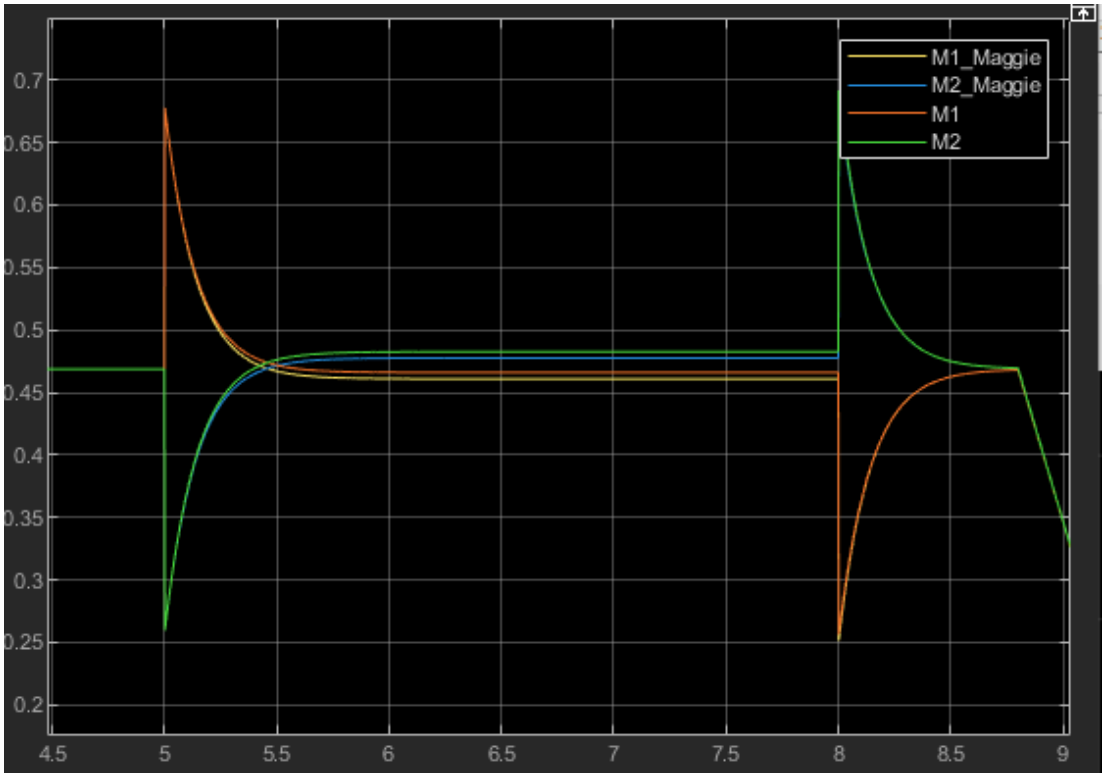

M1
M2
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10

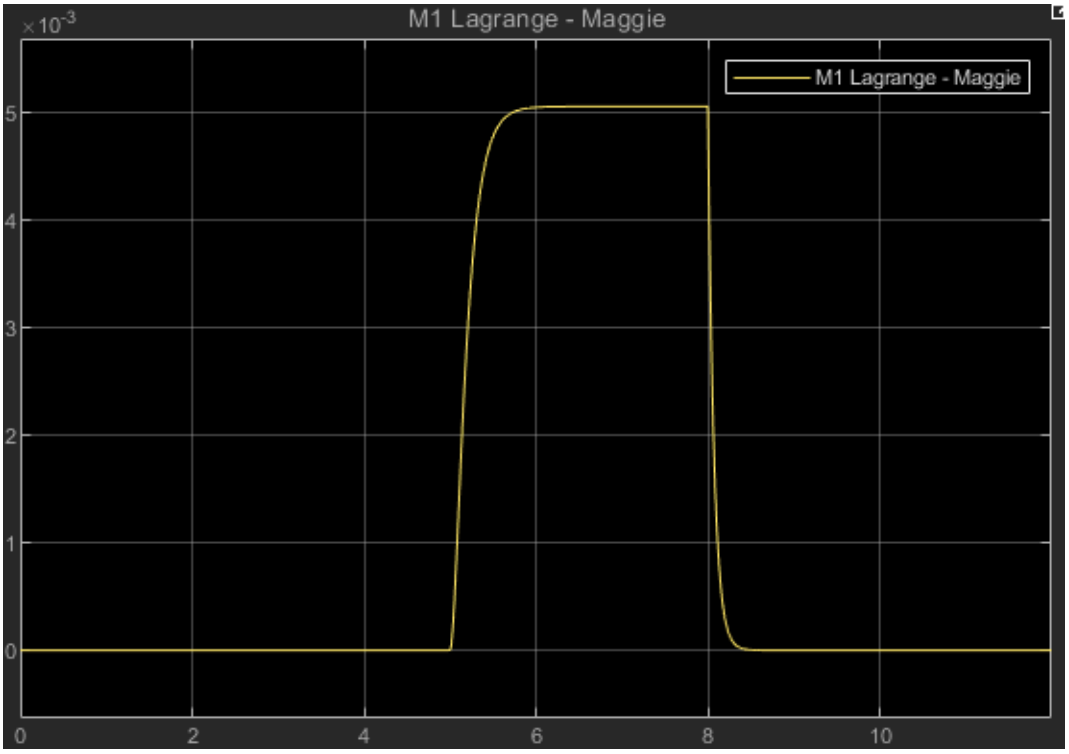

lambda1
lambda2
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
2
4
6
8
10

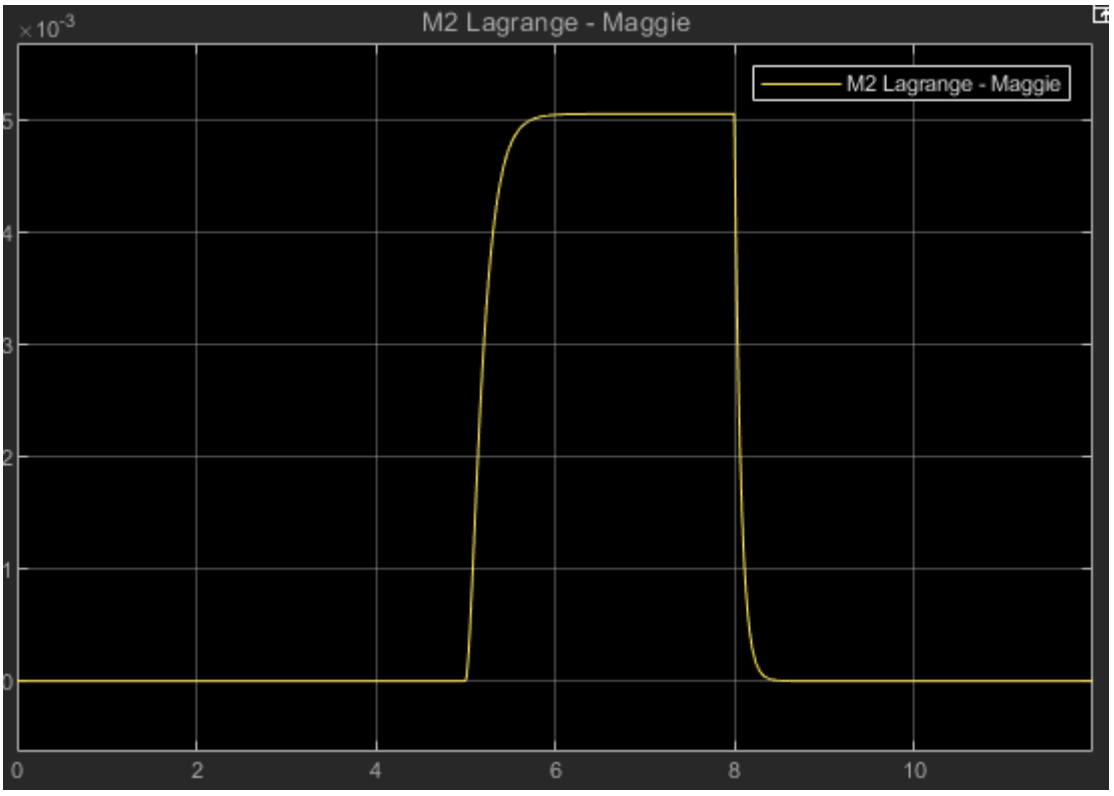
Maggie



Comparison




M1 Lagrange - Maggie
×10^-3
M1 Lagrange - Maggie
5
4
3
2
1
0
0
2
4
6
8
10


M2 Lagrange - Maggie
×10^-3
M2 Lagrange - Maggie
5
4
3
2
1
0
0
2
4
6
8
10

Conclusions

Abrupt peaks that can be seen in M1 and M2 for both Lagrange and Maggie as well as Lambdas for Lagrange are expected and are caused by the shape of the ddBeta. Difference between Lagrange and Maggie methods is in the 10^{-3} range.