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WHEEL MOBILE ROBOTS

LECTURE- 04 – DYNAMICS Maggie

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Dynamics modeling of the two-wheeled mobile robot

In dynamics modeling of the mobile robots we often use Lagrange equation of type II with multipliers or Maggi's equation which are based on Lagrange's ones.

Maggi's equations allow to omit decoupling transformation, due to the fact that number of generalized coordinates is equal to number of DOFs.

Let's look closer to the Maggi's equations.

Dynamics modeling of the two-wheeled mobile robot

General form of Maggi's equations:

$$\sum_{j=1}^n C_{ij} \left[\frac{d}{dt} \left(\frac{\partial E}{\partial \dot{q}_j} \right) - \left(\frac{\partial E}{\partial q_j} \right) \right] = \Theta_i \quad (4.1)$$

Where s denotes number of independent parameters of the system in generalized coordinates in the number of DOFs

Generalized velocities have form of:

$$\dot{q}_j = \sum_{i=1}^s C_{ij} \dot{e}_i + G_j \quad (4.2)$$

In (4.2) equation \dot{e}_i is called characteristics or kinematic parameters of the system in generalized coordinates.

Dynamics modeling of the two-wheeled mobile robot

Right hand sides of the (4.1) equation are called coefficients at variations δe_i in expression for prepared work of external forces and defined as:

$$\sum_{i=1}^s \Theta_i \delta e_i = \sum_{i=1}^s \delta e_i \sum_{j=1}^n C_{ij} Q_j \quad (4.3)$$

Equation (4.1) in a matrix form:

$$\sum_{j=1}^n C_{ij} L_j = \Theta_i \quad (4.4)$$

where: $L = M(q)\ddot{q} + C(q, \dot{q})\dot{q}$

Dynamics modeling - Maggie

For symbolic calculation generalized coordinate vector was selected in form of:

$$q = [x_A, y_A, \beta, \alpha_1, \alpha_2]^T \quad (4.5)$$

Kinetic energy for a system without idler wheel has form of:

$$\begin{aligned}
 E_k = & [(m_1 + m_2 + m_4)\dot{x}_A + ((m_1 - m_2)l_1 \cos(\beta) + m_4 l_2 \sin(\beta))\dot{\beta}] \dot{x}_A + \\
 & [(m_1 + m_2 + m_4)\dot{y}_A + ((m_1 - m_2)l_1 \sin(\beta) - m_4 l_2 \cos(\beta))\dot{\beta}] \dot{y}_A + \\
 & [((m_1 - m_2)l_1 \cos(\beta) + m_4 l_2 \sin(\beta))\dot{x}_A + ((m_1 - m_2)l_1 \sin(\beta) - m_4 l_2 \cos(\beta))\dot{y}_A + \\
 & + ((m_1 + m_2)l_1^2 + Ix_1 + Ix_2 + Iz_4 + m_4 l_2^2)\dot{\beta}] \dot{\beta} + [Iz_1 \dot{\alpha}_1] \dot{\alpha}_1 + [Iz_2 \dot{\alpha}_2] \dot{\alpha}_2
 \end{aligned} \quad (4.6)$$

Dynamics modeling - Maggie

Taking into account idler wheel kinetic energy was expanded to form of:

$$\begin{aligned}
 E_k = & \frac{1}{2}(m_1 + m_2 + m_3 + m_4)\dot{x}_A^2 + \frac{1}{2}(m_1 + m_2 + m_3 + m_4)\dot{y}_A^2 + (((m_1 - m_2)l_1 \cos(\beta) + (m_3l_5 + m_4l_2) \sin(\beta))\dot{x}_A + \\
 & + ((m_1 - m_2)l_1 \sin(\beta) - (m_3l_5 + m_4l_2) \cos(\beta))\dot{y}_A + \frac{1}{2}(Ix_1 + m_1l_1^2 + Ix_2 + m_2l_1^2 + Ix_3 + m_3l_5^2 + Iz_4 + m_4l_2^2)\dot{\beta})\dot{\beta} + \\
 & + \frac{1}{2}Iz_1\dot{\alpha}_1^2 + \frac{1}{2}Iz_2\dot{\alpha}_2^2 + \frac{Ix_3l_1\dot{\beta}\ddot{\beta}v_A}{v_A^2 + l_1^2\dot{\beta}^2} + \frac{1}{2}\frac{Ix_3l_1^2\ddot{\beta}^2v_A^2}{(v_A^2 + l_1^2\dot{\beta}^2)^2} + \frac{1}{2}\frac{Iz_3(v_A^2 + l_1^2\dot{\beta}^2)}{r_3^2}
 \end{aligned} \tag{4.7}$$

Dynamics modeling - Maggie

In an analogical way to Lagrange equations, matrix of inertia was derived (case w/o idler wheel):

$$M = \begin{bmatrix} 2m_1 + m_4 & 0 & m_4 l_2 \sin(\beta) & 0 & 0 \\ 0 & 2m_1 + m_4 & -m_4 l_2 \cos(\beta) & 0 & 0 \\ m_4 l_2 \sin(\beta) & -m_4 l_2 \cos(\beta) & 2m_1 l_1^2 + 2Ix_1 + Iz_4 + m_4 l_2^2 & 0 & 0 \\ 0 & 0 & 0 & Iz_1 & 0 \\ 0 & 0 & 0 & 0 & Iz_1 \end{bmatrix} \quad (4.8)$$

Dynamics modeling - Maggie

(case w/ idler wheel):

$$M = \begin{bmatrix} 2m_1 + m_3 + m_4 & 0 & (m_3l_5 + m_4l_2)\sin(\beta) & 0 & 0 \\ 0 & 2m_1 + m_3 + m_4 & -(m_3l_5 + m_4l_2)\cos(\beta) & 0 & 0 \\ (m_3l_5 + m_4l_2)\sin(\beta) & -(m_3l_5 + m_4l_2)\cos(\beta) & 2m_1l_1^2 + 2Ix_1 + 2Iz_1h^2 + Iz_4 + m_4l_2^2 + Ix_3 + m_3l_5^2 + \%1 & 0 & 0 \\ 0 & 0 & 0 & Iz_1 & 0 \\ 0 & 0 & 0 & 0 & Iz_1 \end{bmatrix}$$

$$\%1 = \frac{Ix_3l_1v_A}{v_A^2 + l_1^2\dot{\beta}^2} - \frac{8Ix_3l_1^3\dot{\beta}^2v_A}{(v_A^2 + l_1^2\dot{\beta}^2)^2} + \frac{8Ix_3l_1^5\dot{\beta}^4v_A - 6Ix_3l_1^4\dot{\beta}^2v_A^2}{(v_A^2 + l_1^2\dot{\beta}^2)^3} + \frac{12Ix_3l_1^6\dot{\beta}^4v_A^2}{(v_A^2 + l_1^2\dot{\beta}^2)^4} + \frac{Iz_3l_1^2}{r_3^2} \quad (4.9)$$

Dynamics modeling - Maggie

Coriolis and centrifugal force matrix (case w/o idler wheel):

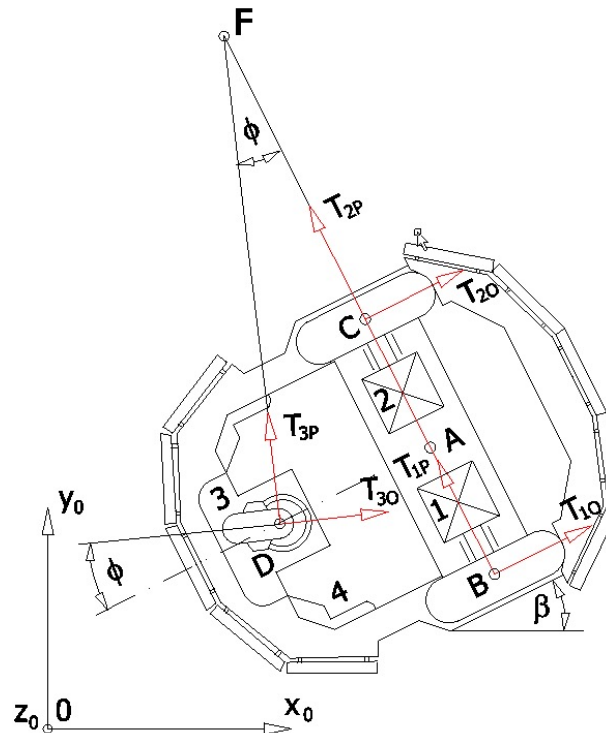
$$C = \begin{bmatrix} 0 & 0 & m_4 l_2 \dot{\beta} \cos(\beta) & 0 & 0 \\ 0 & 0 & m_4 l_2 \dot{\beta} \sin(\beta) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.10)$$

(case w/ idler wheel):

$$C = \begin{bmatrix} 0 & 0 & (m_3 l_5 + m_4 l_2) \dot{\beta} \cos(\beta) & 0 & 0 \\ 0 & 0 & (m_3 l_5 + m_4 l_2) \dot{\beta} \sin(\beta) & 0 & 0 \\ 0 & 0 & \frac{-(l_1^6 \dot{\beta}^6 + (12 l_1^5 v_A - 13 v_A^2 l_1^4) \dot{\beta}^4 - (30 l_1^3 v_A^3 + 5 l_1^2 v_A^4) \dot{\beta}^2 + 6 l_1 v_A^5 + 9 v_A^6) \ddot{\beta} \dot{\beta} l x_3 l_1^3 v_A^3}{(v_A^2 + l_1^2 \dot{\beta}^2)^5} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (4.11)$$

Dynamics modeling - Maggie

After deriving external forces acting on a system we must derive forces of friction. Force distribution is presented on the figure below:



Dynamics modeling - Maggie

Generalized force for the system w/o idler wheel:

$$\begin{aligned}
 Q_1 &= T_{1,0} \cos(\beta) - T_{1,P} \sin(\beta) + T_{2,0} \cos(\beta) - T_{2,P} \sin(\beta) \\
 Q_2 &= T_{1,0} \sin(\beta) + T_{1,P} \cos(\beta) + T_{2,0} \sin(\beta) + T_{2,P} \cos(\beta) \\
 Q_3 &= T_{1,0} l_1 - T_{2,0} l_1 \\
 Q_4 &= M_1 - N_1 f_1 \operatorname{sgn}(\dot{\alpha}_1) - T_{1,0} r \\
 Q_5 &= M_2 - N_2 f_2 \operatorname{sgn}(\dot{\alpha}_2) - T_{2,0} r
 \end{aligned} \tag{4.12}$$

For the system w/ idler wheel:

$$\begin{aligned}
 Q_1 &= T_{1,0} \cos(\beta) - T_{1,P} \sin(\beta) + T_{2,0} \cos(\beta) - T_{2,P} \sin(\beta) - \frac{N_3 f_3 \operatorname{sgn}(\dot{\alpha}_3)}{r_3} \\
 Q_2 &= T_{1,0} \sin(\beta) + T_{1,P} \cos(\beta) + T_{2,0} \sin(\beta) + T_{2,P} \cos(\beta) - \frac{N_3 f_3 \operatorname{sgn}(\dot{\alpha}_3)}{r_3} \\
 Q_3 &= T_{1,0} l_1 - T_{2,0} l_1 \\
 Q_4 &= M_1 - N_1 f_1 \operatorname{sgn}(\dot{\alpha}_1) - T_{1,0} r \\
 Q_5 &= M_2 - N_2 f_2 \operatorname{sgn}(\dot{\alpha}_2) - T_{2,0} r
 \end{aligned} \tag{4.13}$$

Dynamics modeling - Maggie

Generalized velocities from (4.2) have form of:

$$\begin{aligned} \dot{q}_1 = \dot{x}_A &= \frac{1}{2} r \dot{e}_1 \cos(\beta) + \frac{1}{2} r \dot{e}_2 \cos(\beta) \\ \dot{q}_2 = \dot{y}_A &= \frac{1}{2} r \dot{e}_1 \sin(\beta) + \frac{1}{2} r \dot{e}_2 \sin(\beta) \\ \dot{q}_3 = \dot{\beta} &= \frac{1}{2} \frac{r}{l_1} \dot{e}_1 - \frac{1}{2} \frac{r}{l_1} \dot{e}_2 \\ \dot{q}_4 = \dot{\alpha}_1 &= \dot{e}_1 \\ \dot{q}_5 = \dot{\alpha}_2 &= \dot{e}_2 \end{aligned} \tag{4.14}$$

Dynamics modeling - Maggie

Coefficients C_{ij} i G_j from (4.2):

$$C_{11} = \frac{1}{2} r \cos(\beta), C_{21} = \frac{1}{2} r \cos(\beta), G_1 = 0$$

$$C_{12} = \frac{1}{2} r \sin(\beta), C_{22} = \frac{1}{2} r \sin(\beta), G_2 = 0$$

$$C_{13} = \frac{1}{2} \frac{r}{l_1}, C_{22} = -\frac{1}{2} \frac{r}{l_1}, G_3 = 0$$

$$C_{14} = 1, C_{24} = 0, G_4 = 0$$

$$C_{15} = 0, C_{25} = 1, G_5 = 0$$

(4.15)

Dynamics modeling - Maggie

Finally with the use of previous dependencies and (4.4) form we can write Maggie's equation for our system (w/o idler wheel)

$$\begin{aligned}
 & \frac{(r^2 m_4 l_1^2 + 4r^2 m_1 l_1^2 + 2r^2 I x_1 + 4I z_1 l_1^2 + r^2 I z_4 + r^2 m_4 l_2^2)}{4l_1^2} \ddot{\alpha}_1 + \frac{(r^2 m_4 l_1^2 - r^2 m_4 l_2^2 - 2r^2 I x_1 - r^2 I z_4)}{4l_1^2} \ddot{\alpha}_2 - \frac{r^3 m_4 l_2}{4l_1^2} \dot{\alpha}_1 \dot{\alpha}_2 + \frac{r^3 m_4 l_2}{4l_1^2} \dot{\alpha}_2^2 = \\
 & = M_1 - N_1 f_1 \operatorname{sgn}(\dot{\alpha}_1) \\
 & \frac{(r^2 m_4 l_1^2 + 4r^2 m_1 l_1^2 + 2r^2 I x_1 + 4I z_1 l_1^2 + r^2 I z_4 + r^2 m_4 l_2^2)}{4l_1^2} \ddot{\alpha}_2 + \frac{(r^2 m_4 l_1^2 - r^2 m_4 l_2^2 - 2r^2 I x_1 - r^2 I z_4)}{4l_1^2} \ddot{\alpha}_1 - \frac{r^3 m_4 l_2}{4l_1^2} \dot{\alpha}_1 \dot{\alpha}_2 + \frac{r^3 m_4 l_2}{4l_1^2} \dot{\alpha}_1^2 = \\
 & = M_2 - N_2 f_2 \operatorname{sgn}(\dot{\alpha}_2)
 \end{aligned}
 \tag{4.16}$$

Dynamics modeling - Maggie

(w/ idler wheel) - part 1

(4.17)

$$\begin{aligned}
 & \frac{r^2(2m_3l_1^2\ddot{\alpha}_1 + m_4l_1^2r\dot{\alpha}_2 + m_3l_5r\dot{\alpha}_2^2 - (2m_3l_5r + 2m_4l_2r)\dot{\alpha}_1\dot{\alpha}_2 + m_3l_5r\dot{\alpha}_1^2 + 2m_4l_1^2\ddot{\alpha}_2 + 2m_4l_1^2\ddot{\alpha}_1 + 2m_3l_1^2\ddot{\alpha}_2 + 4m_2l_1^2\ddot{\alpha}_2 + m_4l_2r\dot{\alpha}_1^2)}{8l_1^2} + \\
 & + \frac{r}{2} \left(((m_1 - m_2) \cos(\frac{r(\alpha_1 - \alpha_2)}{2l_1})l_1 + (m_4l_2 + m_3l_5) \sin(\frac{r(\alpha_1 - \alpha_2)}{2l_1})) \left(\frac{1}{2} (\ddot{\alpha}_1 + \ddot{\alpha}_2) \cos(\frac{r(\alpha_1 - \alpha_2)}{2l_1})r + \frac{(\alpha_2^2 - \alpha_1^2) \sin(\frac{r(\alpha_1 - \alpha_2)}{2l_1})r^2}{4l_1} \right) + \right. \\
 & + \left. ((m_1 - m_2) \sin(\frac{r(\alpha_1 - \alpha_2)}{2l_1})l_1 - (m_4l_2 + m_3l_5) \cos(\frac{r(\alpha_1 - \alpha_2)}{2l_1})) \left(\frac{1}{2} (\ddot{\alpha}_1 + \ddot{\alpha}_2) \sin(\frac{r(\alpha_1 - \alpha_2)}{2l_1})r + \frac{(\alpha_1^2 - \alpha_2^2) \cos(\frac{r(\alpha_1 - \alpha_2)}{2l_1})r^2}{4l_1} \right) + \right. \\
 & + \frac{1}{2l_1} (Ix_1 + m_2l_1^2 + Iz_4 + Ix_3 + m_4l_2^2 + Ix_2 + m_1l_1^2 + m_3l_5^2 + \frac{Iz_3l_1^2}{r_3^2} - \frac{6Ix_3l_1^2r^2(\dot{\alpha}_1 - \dot{\alpha}_2)^2v_A^2}{4(v_A^2 + \frac{r^2}{4}(\dot{\alpha}_1 - \dot{\alpha}_2)^2)^3} + \frac{12Ix_3l_1^2r^4(\dot{\alpha}_1 - \dot{\alpha}_2)^4v_A^2}{16(v_A^2 + \frac{r^2}{4}(\dot{\alpha}_1 - \dot{\alpha}_2)^2)^4} + \frac{Ix_3l_1v_A}{v_A^2 + \frac{r^2}{4}(\dot{\alpha}_1 - \dot{\alpha}_2)^2} + \\
 & - \frac{8Ix_3l_1r^2(\dot{\alpha}_1 - \dot{\alpha}_2)^2v_A}{4(v_A^2 + \frac{r^2}{4}(\dot{\alpha}_1 - \dot{\alpha}_2)^2)^2} + \frac{8Ix_3l_1r^4(\dot{\alpha}_1 - \dot{\alpha}_2)^4v_A}{4(v_A^2 + \frac{r^2}{4}(\dot{\alpha}_1 - \dot{\alpha}_2)^2)^3})r - \frac{1}{8} Ix_3v_Ar^3(\dot{\alpha}_1 - \dot{\alpha}_2)^2(\ddot{\alpha}_1 - \ddot{\alpha}_2) \left(\frac{r^6(\dot{\alpha}_1 - \dot{\alpha}_2) + (12l_1v_A - 13v_A^2)\frac{r^4}{16}(\dot{\alpha}_1 - \dot{\alpha}_2)^4}{(v_A^2 + \frac{r^2}{4}(\dot{\alpha}_1 - \dot{\alpha}_2)^2)^5} + \right. \\
 & \left. - \frac{(30l_1v_A^3 + 5v_A^2)\frac{r^2}{2}(\dot{\alpha}_1 - \dot{\alpha}_2)^2 + 6l_1v_A^5 + 9v_A^6}{(v_A^2 + \frac{r^2}{4}(\dot{\alpha}_1 - \dot{\alpha}_2)^2)^5} \right) + Iz_1\ddot{\alpha}_1 = M_1 - N_1f_1 \operatorname{sgn}(\dot{\alpha}_1) - \frac{(\cos(\frac{r}{2l_1}(\alpha_1 - \alpha_2)) + \sin(\frac{r}{2l_1}(\alpha_1 - \alpha_2)))rN_3f_3 \operatorname{sgn}(\dot{\alpha}_3)}{2r_3}
 \end{aligned}$$

Dynamics modeling - Maggie

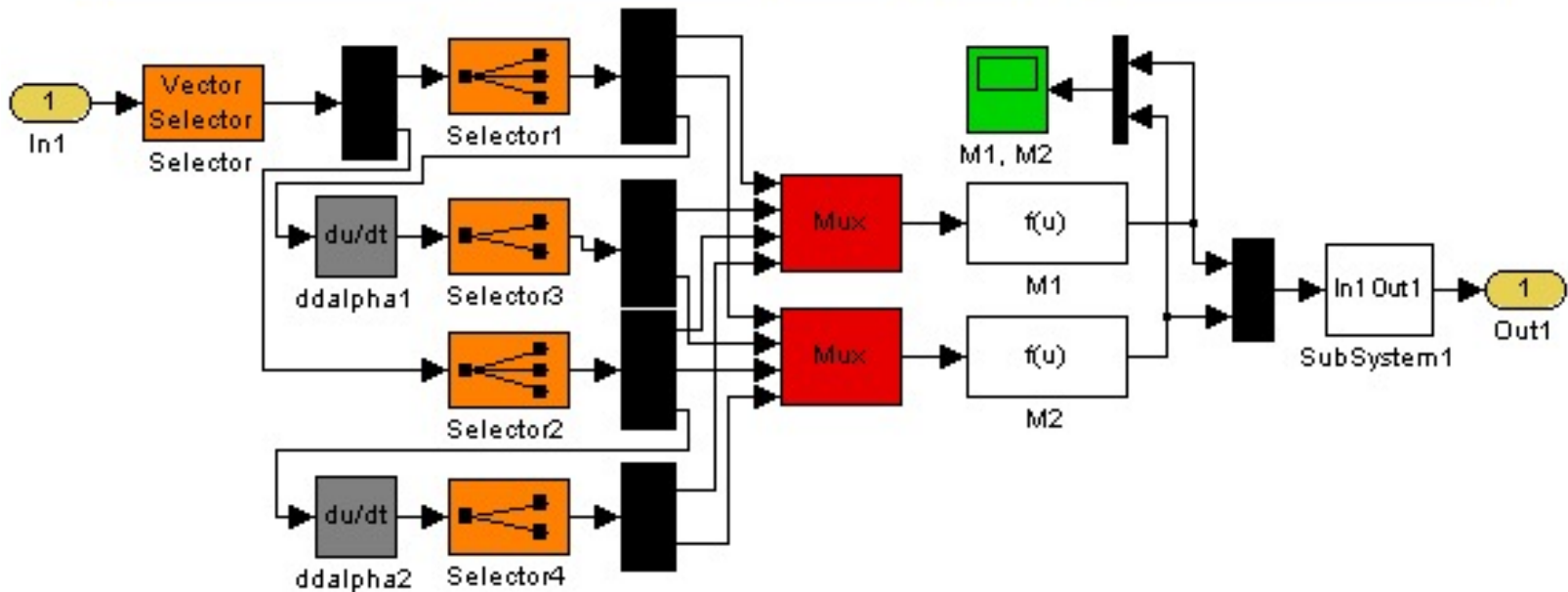
(w/ idler wheel) - part 2

(4.18)

$$\begin{aligned}
 & \frac{r^2(2m_3l_1^2\ddot{\alpha}_1 + m_4l_1^2r\dot{\alpha}_2 + m_3l_5r\dot{\alpha}_2^2 - (2m_3l_5r + 2m_4l_2r)\dot{\alpha}_1\dot{\alpha}_2 + m_3l_5r\dot{\alpha}_1^2 + 2m_4l_1^2\ddot{\alpha}_2 + 2m_4l_1^2\ddot{\alpha}_1 + 2m_3l_1^2\ddot{\alpha}_2 + 4m_2l_1^2\ddot{\alpha}_2 + m_4l_2r\dot{\alpha}_1^2)}{8l_1^2} + \\
 & -\frac{r}{2} \left(((m_1 - m_2) \cos(\frac{r(\alpha_1 - \alpha_2)}{2l_1})l_1 + (m_4l_2 + m_3l_5) \sin(\frac{r(\alpha_1 - \alpha_2)}{2l_1})) \left(\frac{1}{2} (\ddot{\alpha}_1 + \ddot{\alpha}_2) \cos(\frac{r(\alpha_1 - \alpha_2)}{2l_1})r + \frac{(\alpha_2^2 - \alpha_1^2) \sin(\frac{r(\alpha_1 - \alpha_2)}{2l_1})r^2}{4l_1} \right) + \right. \\
 & + ((m_1 - m_2) \sin(\frac{r(\alpha_1 - \alpha_2)}{2l_1})l_1 - (m_4l_2 + m_3l_5) \cos(\frac{r(\alpha_1 - \alpha_2)}{2l_1})) \left(\frac{1}{2} (\ddot{\alpha}_1 + \ddot{\alpha}_2) \sin(\frac{r(\alpha_1 - \alpha_2)}{2l_1})r + \frac{(\alpha_1^2 - \alpha_2^2) \cos(\frac{r(\alpha_1 - \alpha_2)}{2l_1})r^2}{4l_1} \right) + \\
 & + \frac{1}{2l_1} (Ix_1 + m_2l_1^2 + Iz_4 + Ix_3 + m_4l_2^2 + Ix_2 + m_1l_1^2 + m_3l_5^2 + \frac{Iz_3l_1^2}{r_3^2} - \frac{6Ix_3l_1^2r^2(\dot{\alpha}_1 - \dot{\alpha}_2)^2v_A^2}{4(v_A^2 + \frac{r^2}{4}(\dot{\alpha}_1 - \dot{\alpha}_2)^2)^3} + \frac{12Ix_3l_1^2r^4(\dot{\alpha}_1 - \dot{\alpha}_2)^4v_A^2}{16(v_A^2 + \frac{r^2}{4}(\dot{\alpha}_1 - \dot{\alpha}_2)^2)^4} + \frac{Ix_3l_1v_A}{v_A^2 + \frac{r^2}{4}(\dot{\alpha}_1 - \dot{\alpha}_2)^2} + \\
 & - \frac{8Ix_3l_1r^2(\dot{\alpha}_1 - \dot{\alpha}_2)^2v_A}{4(v_A^2 + \frac{r^2}{4}(\dot{\alpha}_1 - \dot{\alpha}_2)^2)^2} + \frac{8Ix_3l_1r^4(\dot{\alpha}_1 - \dot{\alpha}_2)^4v_A}{4(v_A^2 + \frac{r^2}{4}(\dot{\alpha}_1 - \dot{\alpha}_2)^2)^3})r - \frac{1}{8} Ix_3v_Ar^3(\dot{\alpha}_1 - \dot{\alpha}_2)^2(\ddot{\alpha}_1 - \ddot{\alpha}_2) \left(\frac{\frac{r^6}{64}(\dot{\alpha}_1 - \dot{\alpha}_2) + (12l_1v_A - 13v_A^2)\frac{r^4}{16}(\dot{\alpha}_1 - \dot{\alpha}_2)^4}{(v_A^2 + \frac{r^2}{4}(\dot{\alpha}_1 - \dot{\alpha}_2)^2)^5} + \right. \\
 & \left. - \frac{(30l_1v_A^3 + 5v_A^2)\frac{r^2}{2}(\dot{\alpha}_1 - \dot{\alpha}_2)^2 + 6l_1v_A^5 + 9v_A^6}{(v_A^2 + \frac{r^2}{4}(\dot{\alpha}_1 - \dot{\alpha}_2)^2)^5} \right) + Iz_2\ddot{\alpha}_2 = M_2 - N_2f_2 \operatorname{sgn}(\dot{\alpha}_2) - \frac{(\cos(\frac{r}{2l_1}(\alpha_1 - \alpha_2))) + \sin(\frac{r}{2l_1}(\alpha_1 - \alpha_2))}{2r_3} rN_3f_3 \operatorname{sgn}(\dot{\alpha}_3)
 \end{aligned}$$

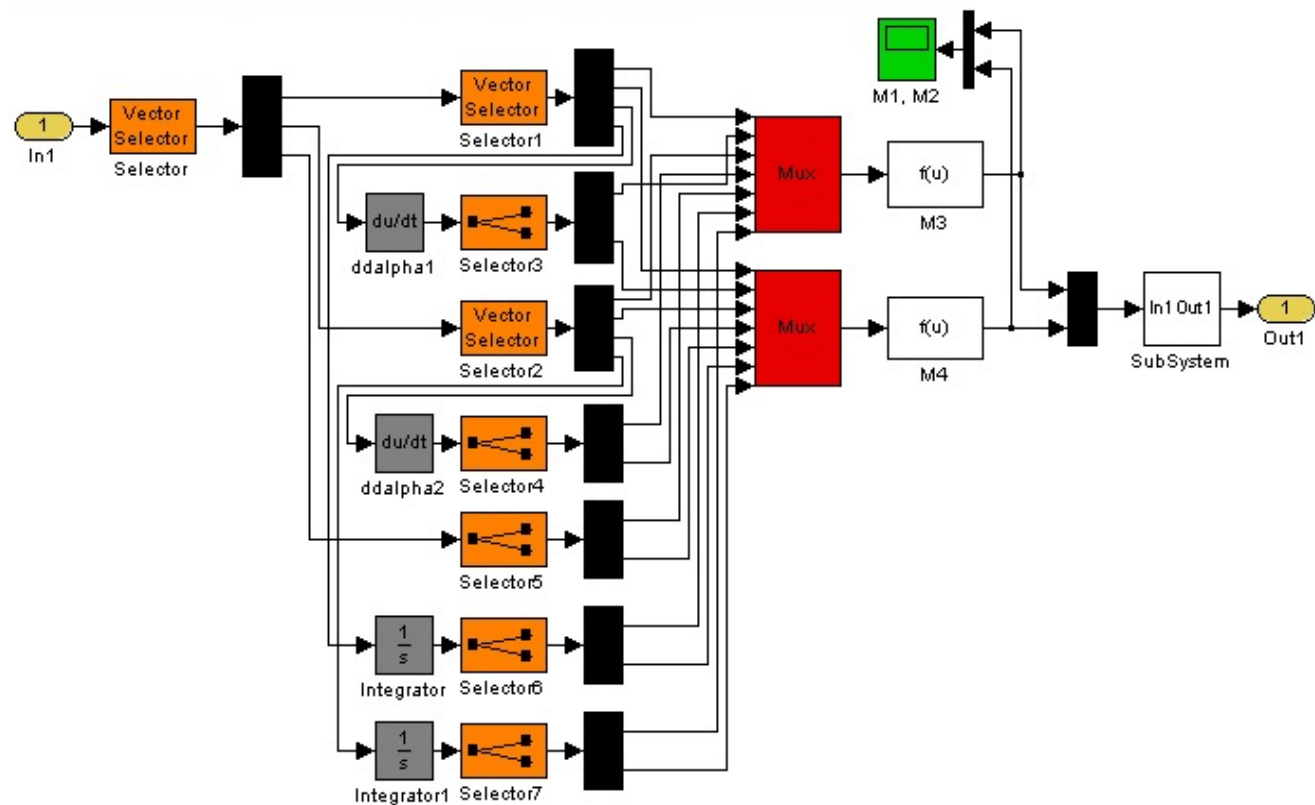
Dynamics simulation - Maggie

In order to conduct a simulation of an inverse dynamics problem for Maggie's equation proper model was prepared (case w/o idler wheel):



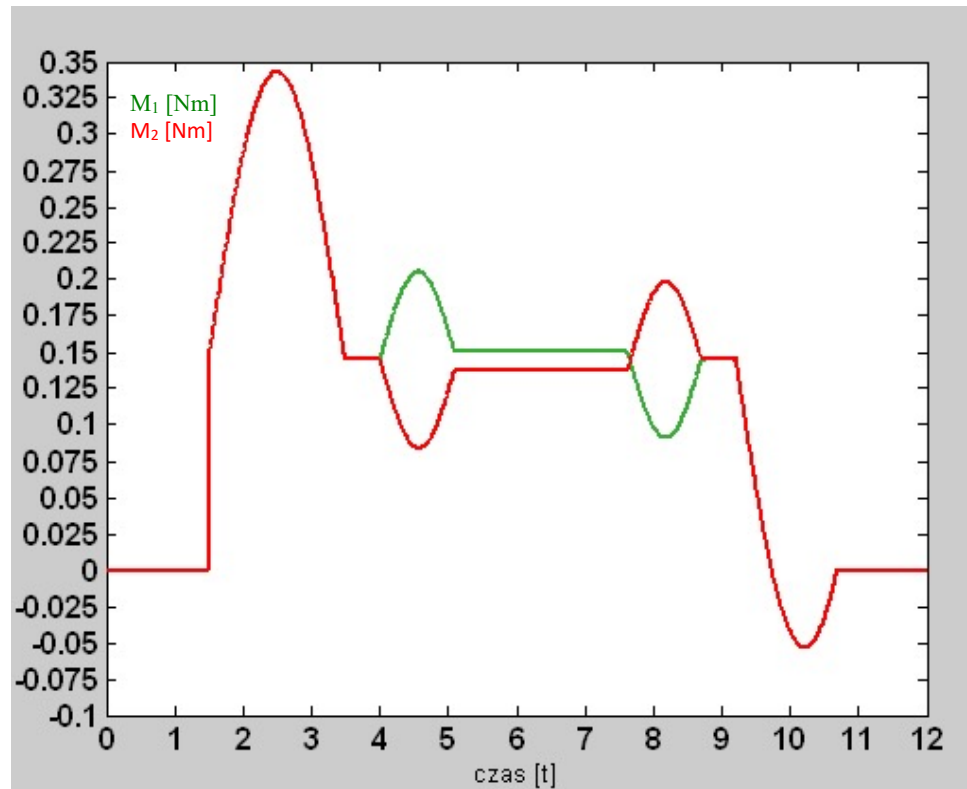
Dynamics simulation - Maggie

(case w/ idler wheel) in addition on input is present
speed of idler wheel:



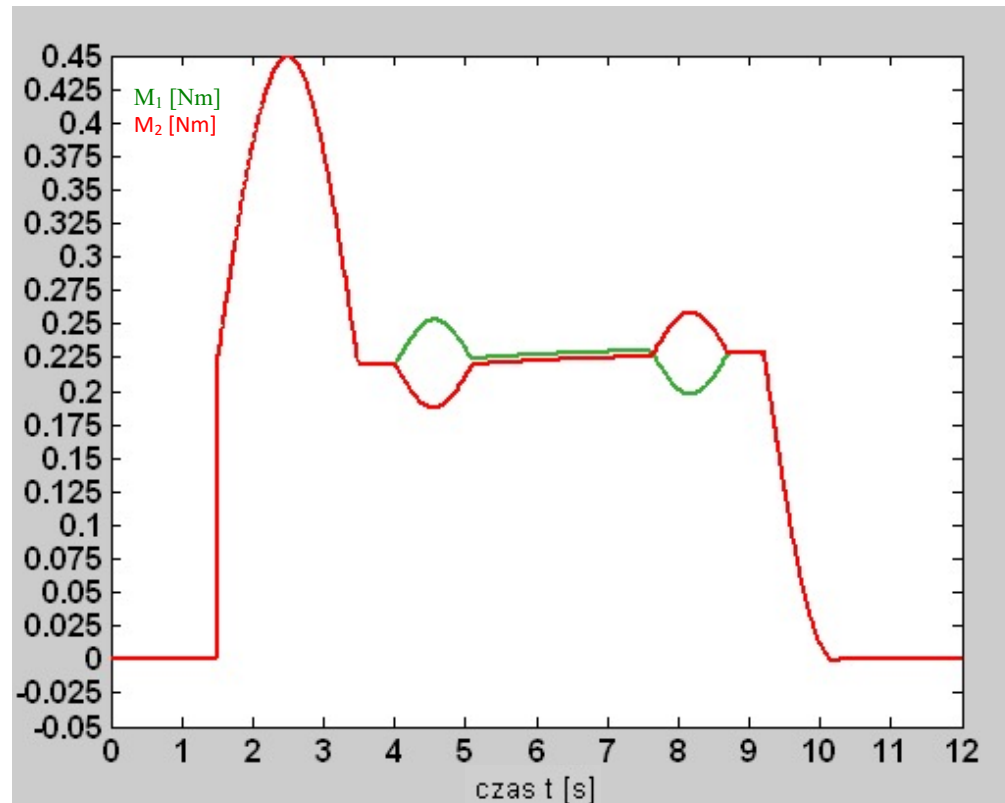
Dynamics simulation - Maggie

Driving moments derived from model for Maggie's equations (case w/o idler wheel):



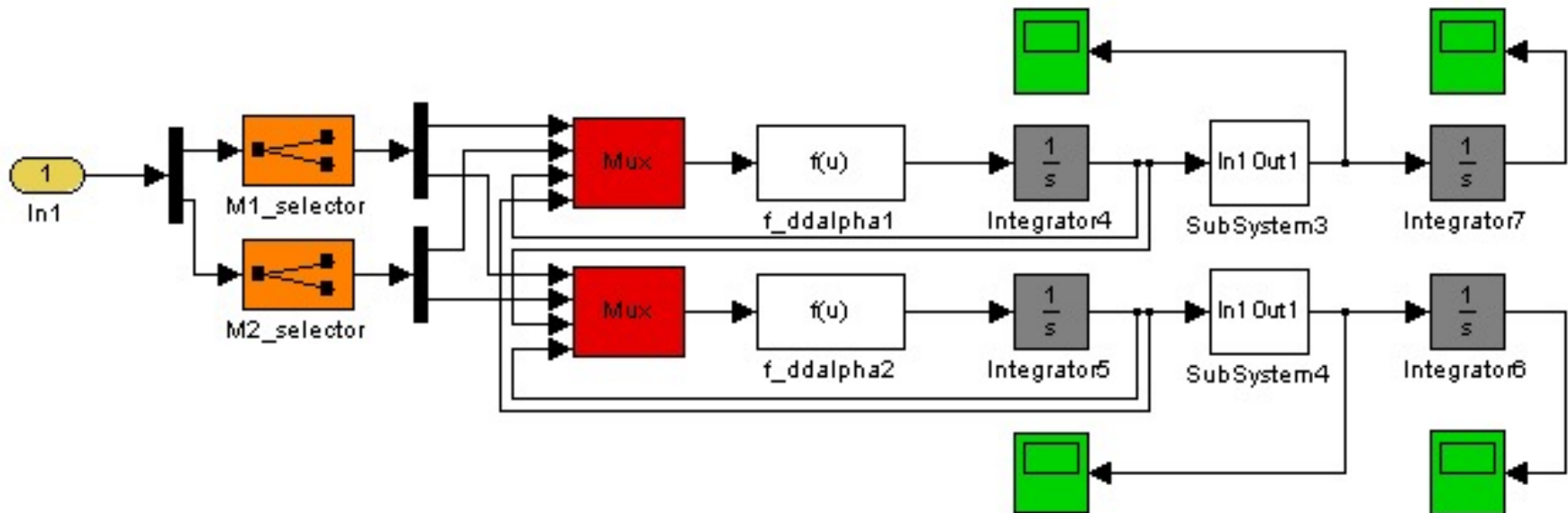
Dynamics simulation - Maggie

Driving moments derived from model for Maggie's equations (case w/ idler wheel):



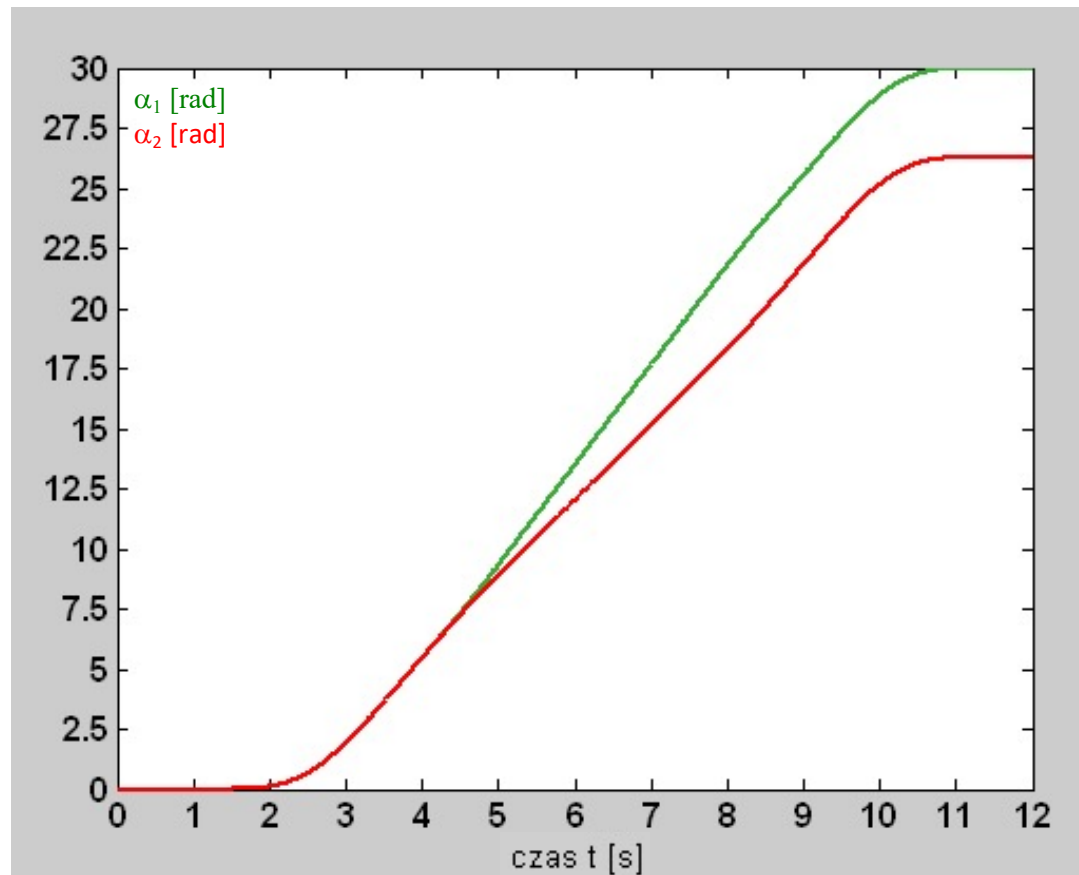
Dynamics simulation - Maggie

Simple dynamics problem with use of Maggie's equations:



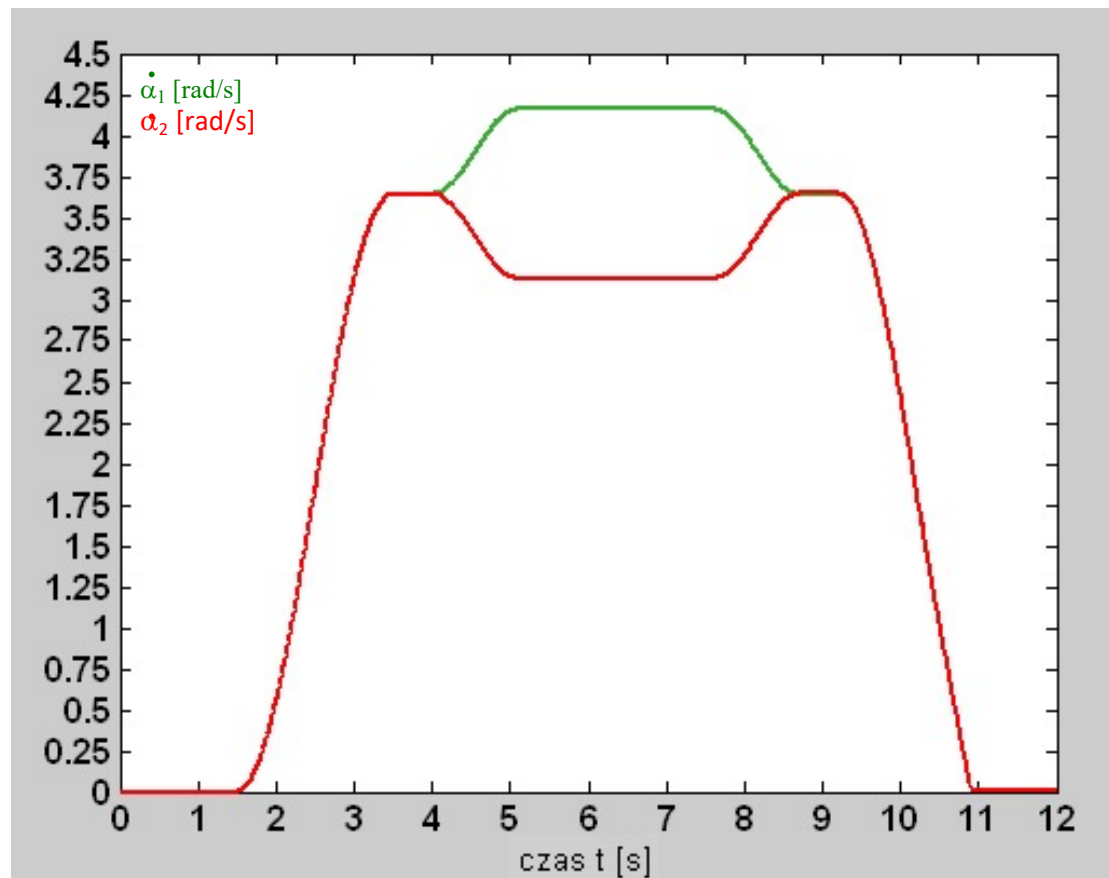
Dynamics simulation - Maggie

Angle of rotation for wheel 1 and 2



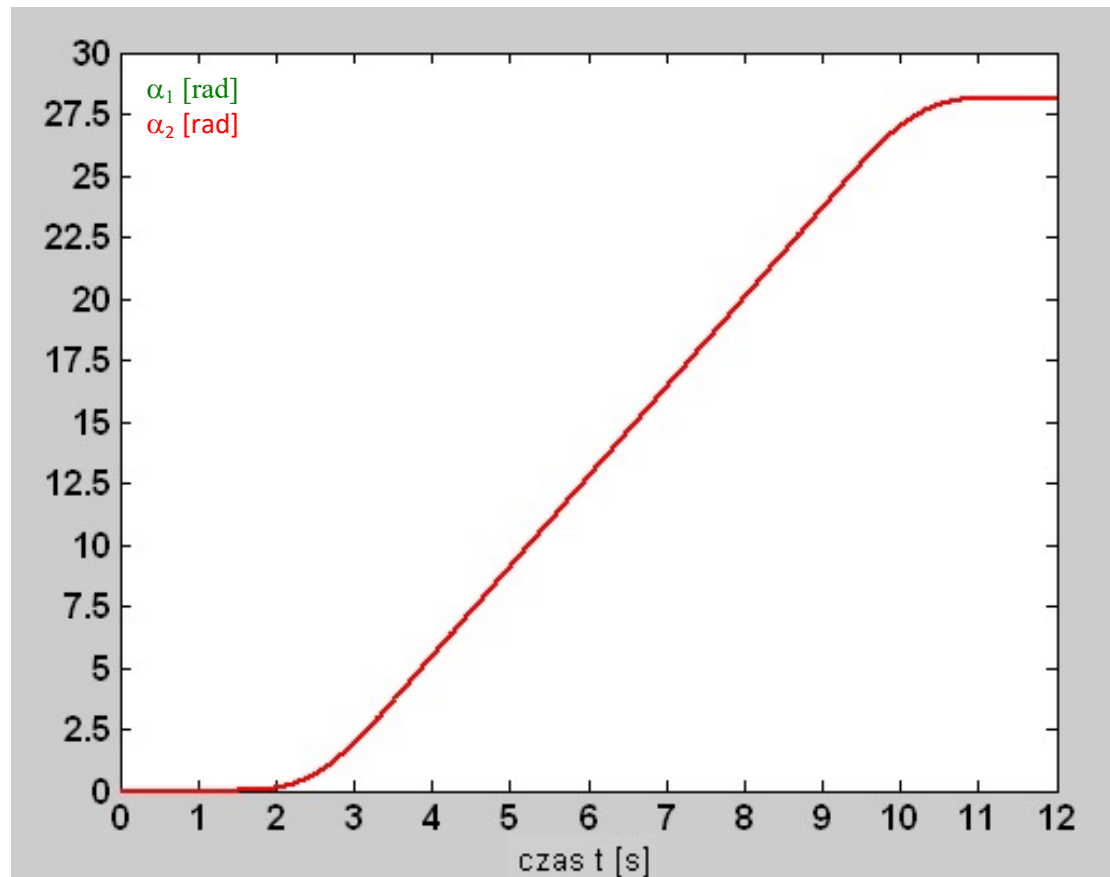
Dynamics simulation - Maggie

Angular velocities of wheel 1 and 2



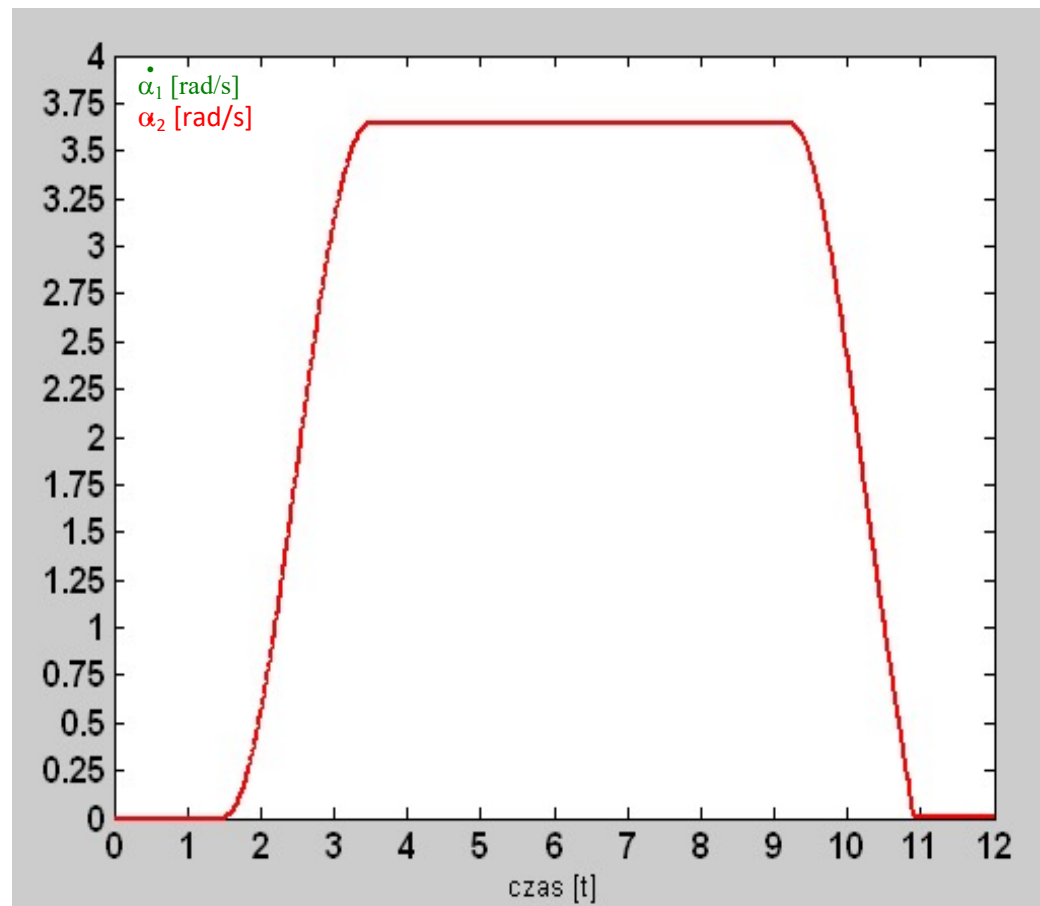
Dynamics simulation - Maggie

Angle of rotation for wheel 1 and 2 (for a straight path)



Dynamics simulation - Maggie

Angular velocities of wheel 1 and 2 (for a straight path)

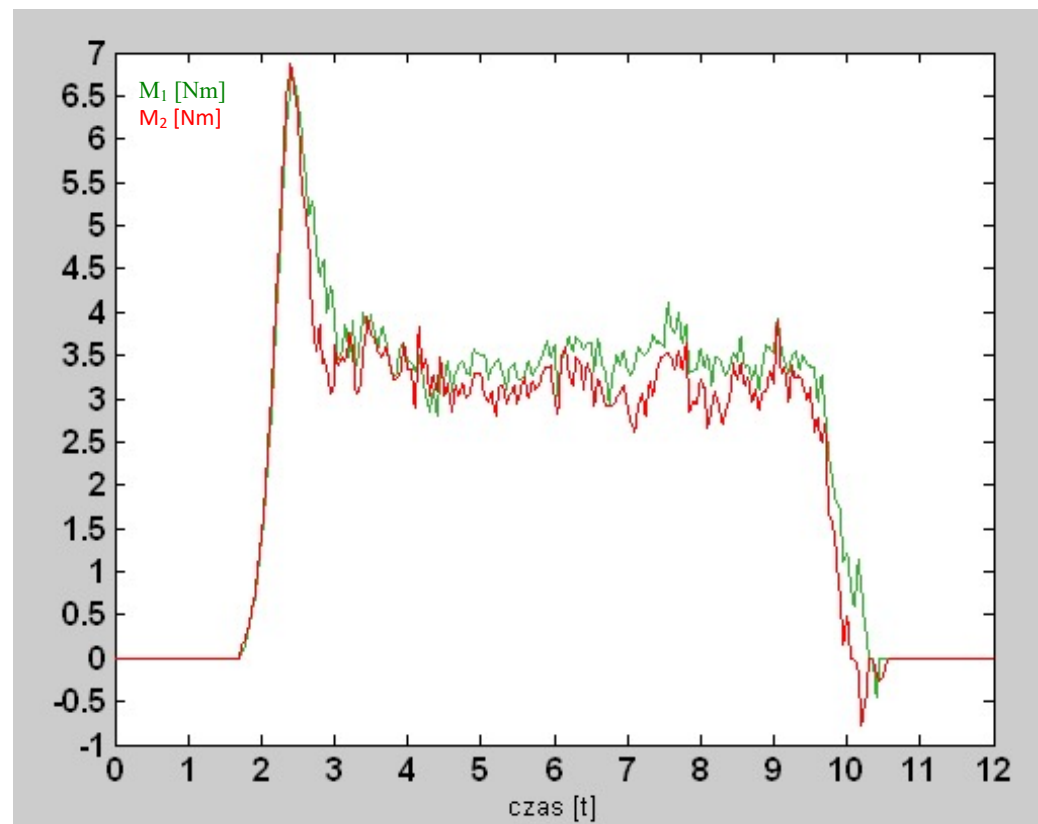


Experimental measurements

In order to check correctness of mathematical model experimental measurements with the use of Pioneer2DX robot and PC equipped with Dispace measuring card and Matlab/Simulink software were conducted. Results are presented in the next few slides.

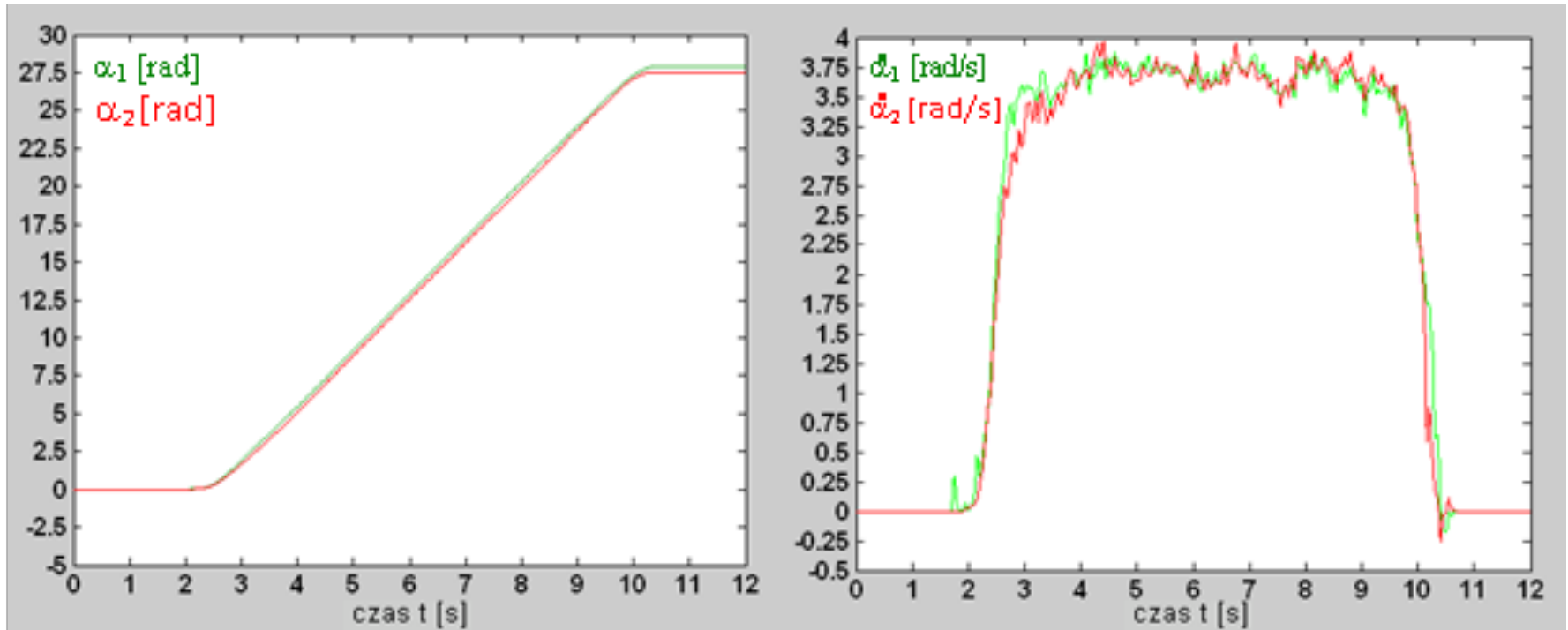
Experimental measurements

Driving torques for straight path motion



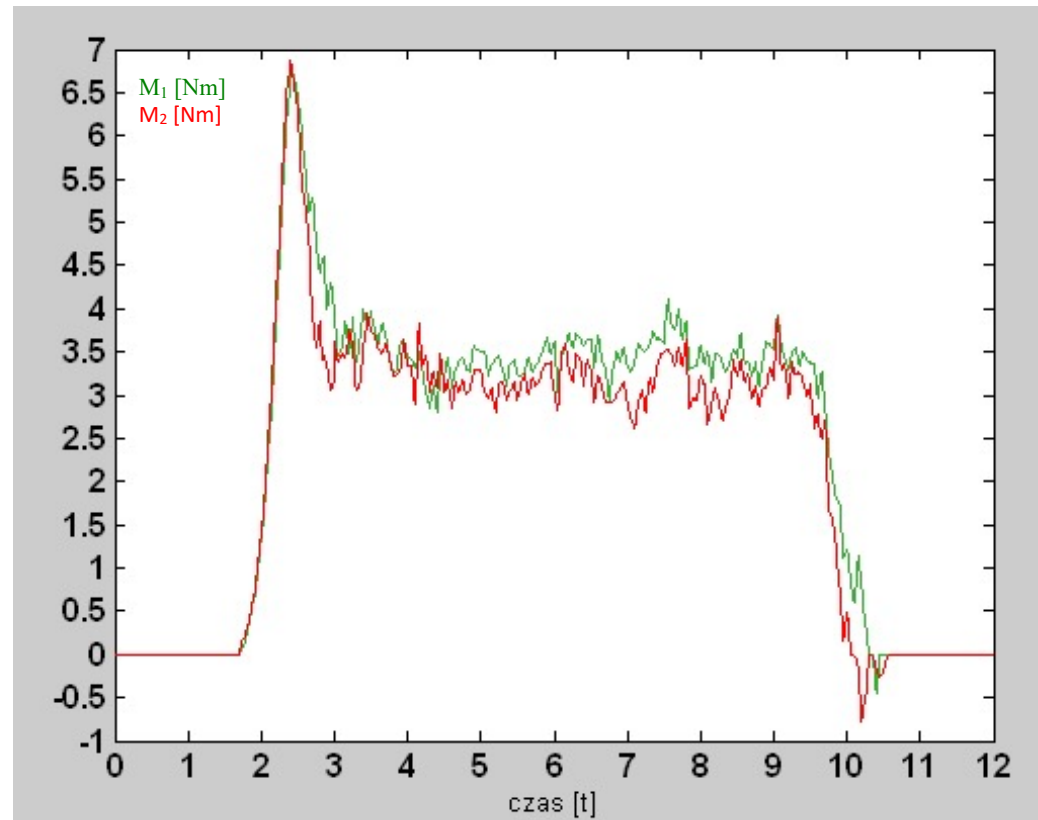
Experimental measurements

Angle of rotation and angular velocity for wheel 1 and 2
(straight path)



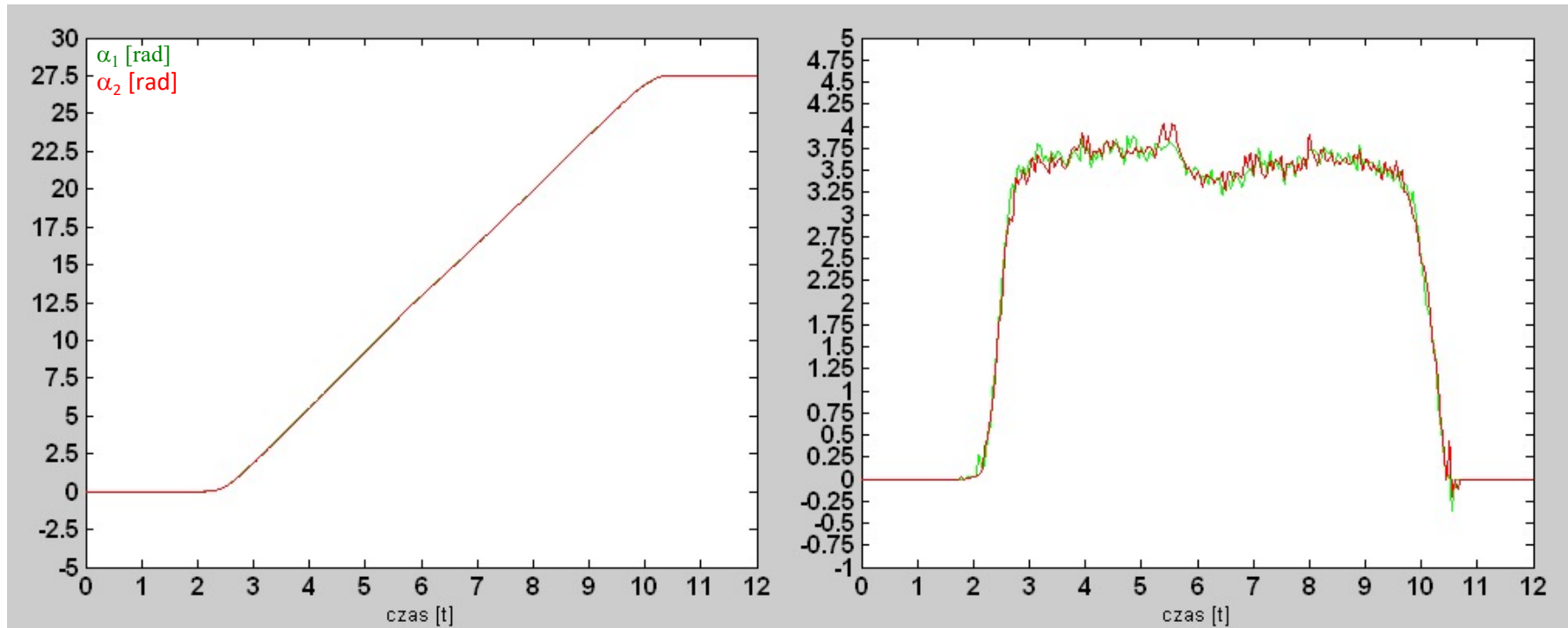
Experimental measurements

Driving torques for straight path motion with parametric disturbance



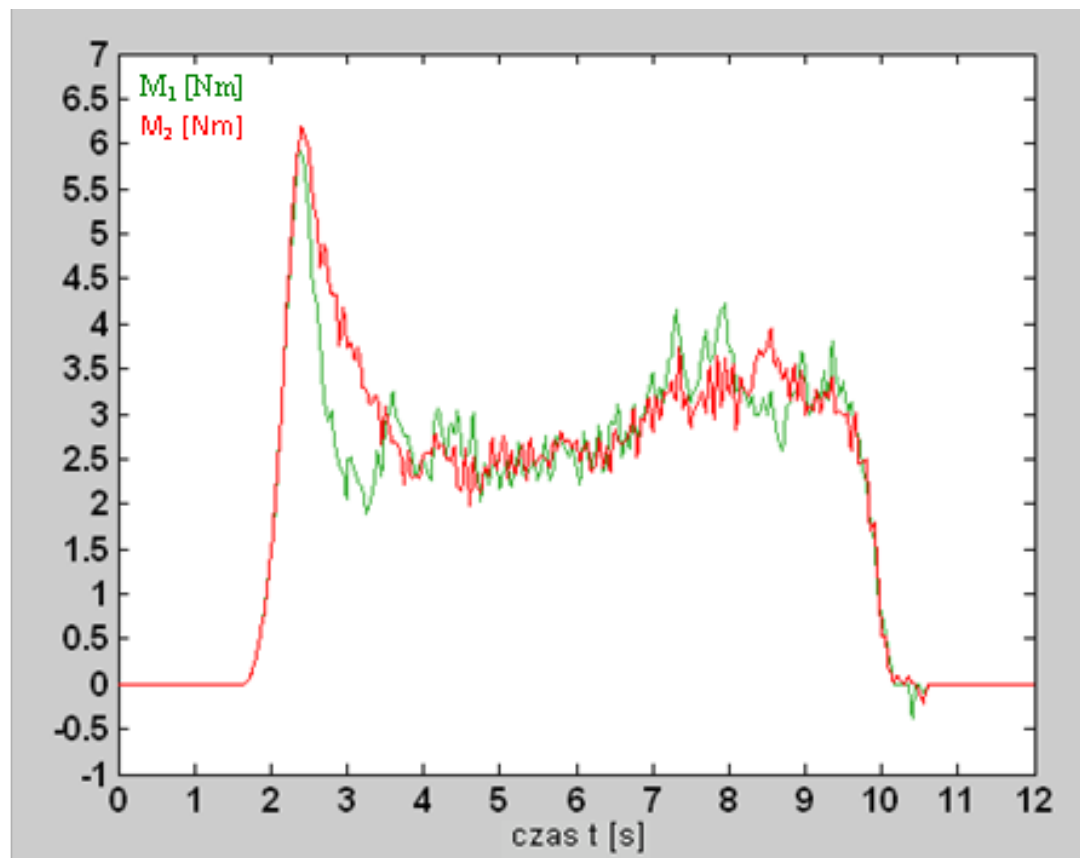
Experimental measurements

Angle of rotation and angular velocity for wheel 1 and 2
(straight path) with parametric disturbance



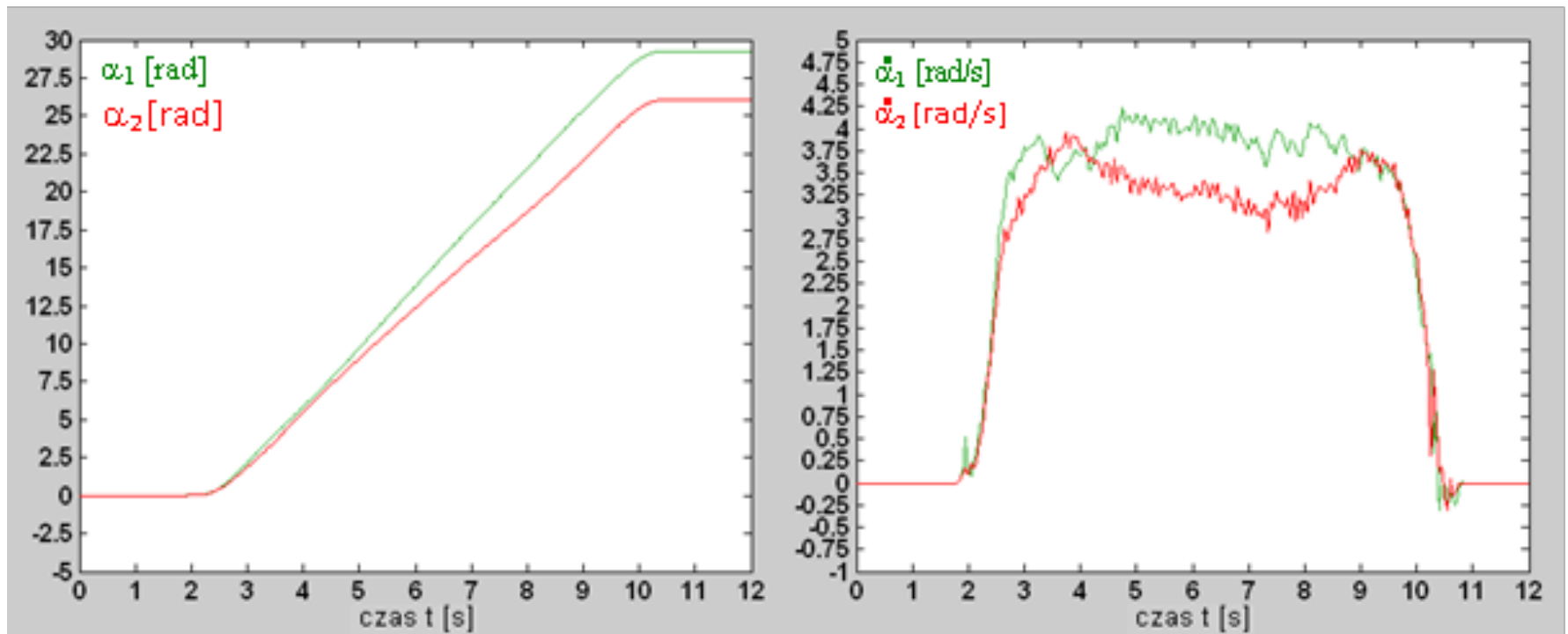
Experimental measurements

Driving torques for curve path motion



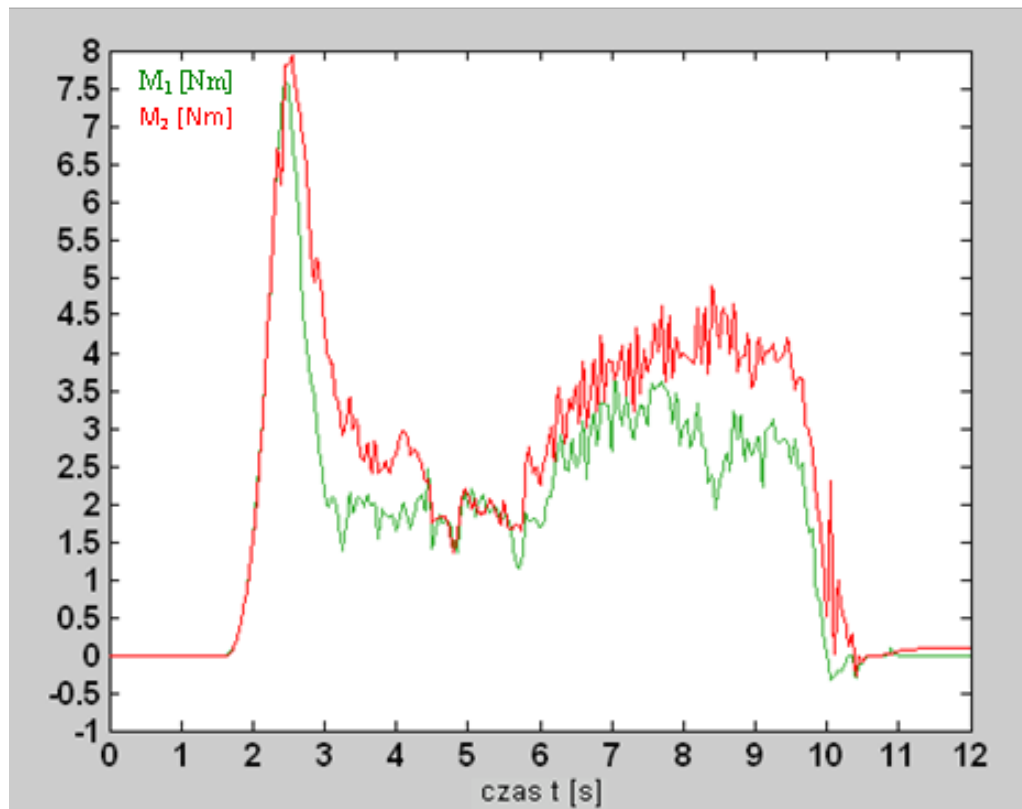
Experimental measurements

Angle of rotation and angular velocity for wheel 1 and 2
(curve path)



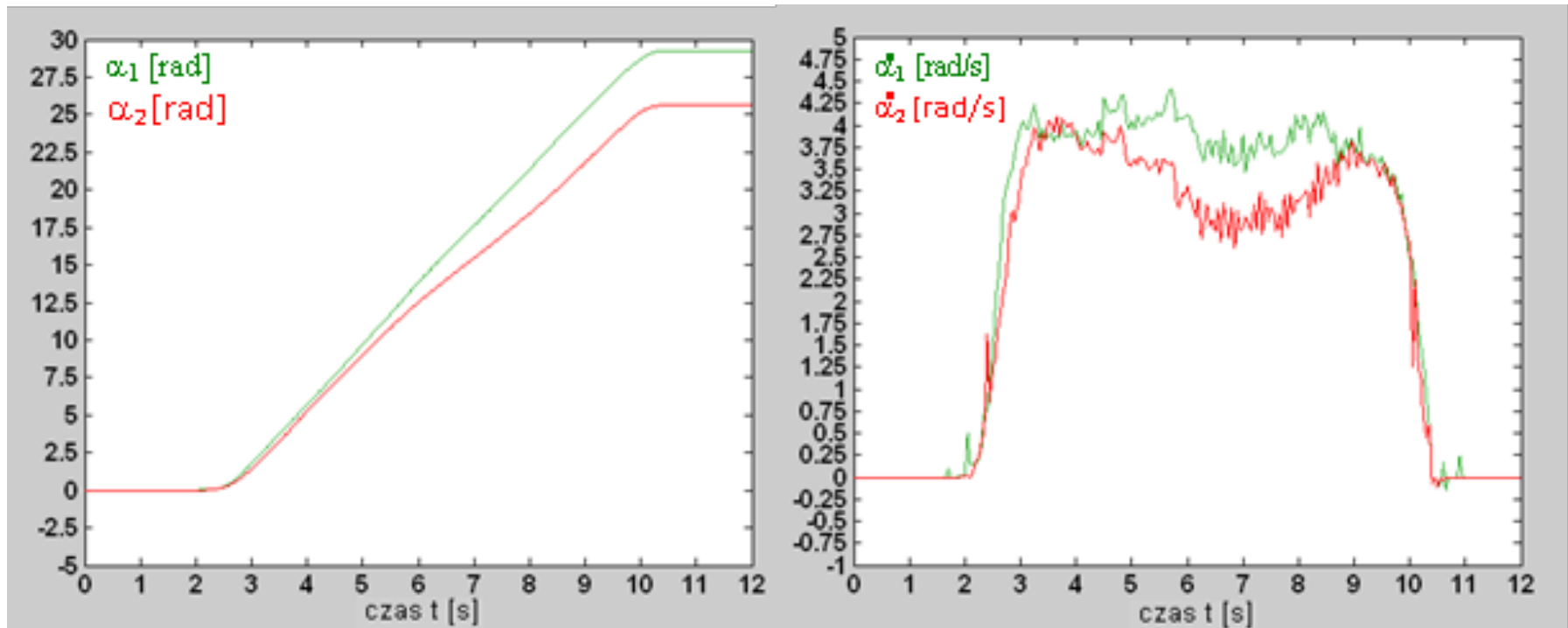
Experimental measurements

Driving torques for curve path motion with parametric disturbance



Experimental measurements

Angle of rotation and angular velocity for wheel 1 and 2
(curve path) with parametric disturbance





THANK YOU