Foreward Kinematics

0T3

```
% this script determines a Homogeneous Transformation matrix
clear
% declaration of symbols
syms th1 d2 a3 q1
% determination of a symbolic form of HT matrices -
% application of mA function
A1=mA(th1,0,0,0)
```

A1 =

$$\begin{pmatrix} \cos(\mathrm{th}_1) & -\sin(\mathrm{th}_1) & 0 & 0 \\ \sin(\mathrm{th}_1) & \cos(\mathrm{th}_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A2=mA(0,d2,0,0)
```

A2 =

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A3=mA(0,0,a3,0)
```

A3 =

$$\begin{pmatrix} 1 & 0 & 0 & a_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
% multiplication of matrices
T03=A1*A2*A3
```

T03 =

$$\begin{pmatrix} \cos(\mathrm{th}_1) & -\sin(\mathrm{th}_1) & 0 & a_3\cos(\mathrm{th}_1) \\ \sin(\mathrm{th}_1) & \cos(\mathrm{th}_1) & 0 & a_3\sin(\mathrm{th}_1) \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
% substitution of rotational joint variables
% for the simplification purpose
T03v=subs(T03,{th1},{q1});
% indication of joint coordinates
```

```
% variables: th1,th2 and a3 indicated by '1's
zmie=[[1 0 0 0];[0 1 0 0];[0 0 1 0]]
```

```
zmie = 3×4
     1     0     0     0
     0     1     0     0
     0     0     1     0
```

```
% a simplified form of the evaluated HT matrices
% for interpretation purpose for a user
T03u=zam(zmie,T03v,"q")
```

T03u =

$$\begin{pmatrix} C_1 & -S_1 & 0 & C_1\,a_3 \\ S_1 & C_1 & 0 & S_1\,a_3 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
% example of substitution of the join variables values
% and constant values into the T0e matrix for the RRP manipulator example
% please use meters and radians
%T04n=subs(T04,{d1,th2,th3,th4},{0.2,pi/2,-pi/2,0})
```

0Te

```
% declaration of symbols
syms th1 d2 a3 th4 d4 q1 q4
% determination of a symbolic form of HT matrices -
% application of mA function
A1=mA(th1,0,0,0)
```

A1 =

$$\begin{pmatrix} \cos(\mathrm{th}_1) & -\sin(\mathrm{th}_1) & 0 & 0 \\ \sin(\mathrm{th}_1) & \cos(\mathrm{th}_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A2=mA(0,d2,0,0)
```

A2 =

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A3=mA(0,0,a3,0)
```

A3 =

$$\begin{pmatrix} 1 & 0 & 0 & a_3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
A4=mA(th4,-d4,0,sym(pi))
```

A4 =

$$\begin{pmatrix} \cos(\mathrm{th}_4) & \sin(\mathrm{th}_4) & 0 & 0 \\ \sin(\mathrm{th}_4) & -\cos(\mathrm{th}_4) & 0 & 0 \\ 0 & 0 & -1 & -d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
% multiplication of matrices
T04=A1*A2*A3*A4
```

T04 =

$$\begin{pmatrix} \cos(\mathrm{th}_1)\cos(\mathrm{th}_4) - \sin(\mathrm{th}_1)\sin(\mathrm{th}_4) & \sigma_1 & 0 & a_3\cos(\mathrm{th}_1) \\ \sigma_1 & \sin(\mathrm{th}_1)\sin(\mathrm{th}_4) - \cos(\mathrm{th}_1)\cos(\mathrm{th}_4) & 0 & a_3\sin(\mathrm{th}_1) \\ 0 & 0 & -1 & d_2 - d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

where

$$\sigma_1 = \cos(\mathrm{th}_1)\sin(\mathrm{th}_4) + \cos(\mathrm{th}_4)\sin(\mathrm{th}_1)$$

```
% substitution of rotational joint variables
% for the simplification purpose
T04v=subs(T04,{th1,th4},{q1,q4});
% indication of joint coordinates
% variables: th1,th2 and a3 indicated by ‘1’s
zmie=[[1 0 0 0];[0 1 0 0];[0 0 1 0];[1 0 0 0]]
```

```
zmie = 4×4
     1     0     0     0
     0     1     0     0
     0     0     1     0
     1     0     0     0
```

```
% a simplified form of the evaluated HT matrices
% for interpretation purpose for a user
T04u=zam(zmie,T04v,"q")
```

T04u =

$$\begin{pmatrix} C_{14} & S_{14} & 0 & C_1\,a_3 \\ S_{14} & -C_{14} & 0 & S_1\,a_3 \\ 0 & 0 & -1 & d_2 - d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
% example of substitution of the join variables values
% and constant values into the T0e matrix for the RRP manipulator example
% please use meters and radians
%T04n=subs(T04,{d1,th2,th3,th4},{0.2,pi/2,-pi/2,0})
```

Variables substitution:

```
th1_val=pi/2;
d2_val=270;
a3_val=130;
th4_val=-pi/2;
d4_val=89;
T04n=subs(T04,{th1,d2,a3,th4,d4},{th1_val,d2_val,a3_val,th4_val,d4_val})
```

```
T04n =
```

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 130 \\ 0 & 0 & -1 & 181 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T03n=subs(T03,{th1,d2,a3},{th1_val,d2_val,a3_val})
```

```
T03n =
```

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 130 \\ 0 & 0 & 1 & 270 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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```
T03u*[0;0;-d4;1]
```

```
ans =
```

$$\begin{pmatrix} C_1\,a_3 \\ S_1\,a_3 \\ d_2 - d_4 \\ 1 \end{pmatrix}$$

```
T04u
```

```
T04u =
```

$$\begin{pmatrix} C_{14} & S_{14} & 0 & C_1\,a_3 \\ S_{14} & -C_{14} & 0 & S_1\,a_3 \\ 0 & 0 & -1 & d_2-d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T3e=subs(mA(th4,-d4,0,sym(pi)),{th4},{q4})
```

T3e =

$$\begin{pmatrix} \cos(q_4) & \sin(q_4) & 0 & 0 \\ \sin(q_4) & -\cos(q_4) & 0 & 0 \\ 0 & 0 & -1 & -d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
zmie=[1,0,0,0]
```

zmie = 1×4

```
     1     0     0     0
```

```
T3u=zam(zmie,T3e,'q')
```

T3u =

$$\begin{pmatrix} \cos(q_4) & \sin(q_4) & 0 & 0 \\ \sin(q_4) & -\cos(q_4) & 0 & 0 \\ 0 & 0 & -1 & -d_4 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

```
T3un=subs(T3e,{q4,d4},{th4_val,d4_val})
```

T3un =

$$\begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -89 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

P, pw and pa Vectors

```
P=T04u(1:3,4)
```

P =

$$\begin{pmatrix} C_1\,a_3 \\ S_1\,a_3 \\ d_2-d_4 \end{pmatrix}$$

```
pw=T04u(1:3,3)*d4
```

pw =

$$\begin{pmatrix} 0 \\ 0 \\ -d_4 \end{pmatrix}$$

```
pa=P-pw %same as T03u
```

pa =

$$\begin{pmatrix} C_1\, a_3 \\ S_1\, a_3 \\ d_2 \end{pmatrix}$$

```
T03u
```

T03u =

$$\begin{pmatrix} C_1 & -S_1 & 0 & C_1\, a_3 \\ S_1 & C_1 & 0 & S_1\, a_3 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Inverse kinematics

```
%Inverse kinematiks joint variables:
syms pax pay paz q2 q3
eq=[pax;pay]==[cos(q1)*q3;sin(q1)*q3];
result=solve(eq,q1,q3);
result.q1
```

ans =

$$\begin{pmatrix} -2\,\mathrm{atan}\left(\frac{\mathrm{pax}-\sqrt{\mathrm{pax}^2+\mathrm{pay}^2}}{\mathrm{pay}}\right) \\ -2\,\mathrm{atan}\left(\frac{\mathrm{pax}+\sqrt{\mathrm{pax}^2+\mathrm{pay}^2}}{\mathrm{pay}}\right) \end{pmatrix}$$

```
result.q3(1)
```

ans = $\sqrt{\mathrm{pax}^2+\mathrm{pay}^2}$

```
paz = q2;
```

```
%Inverse kinematics with values:
T03n(1:3,4)
```

ans =

$$\begin{pmatrix} 0 \\ 130 \\ 270 \end{pmatrix}$$

```
q2 = T03n(3,4) %d2
```

q2 = 270

```
q3=subs(result.q3(1),{pax,pay},{0,130})
```

q3 = 130

```
q1=subs(result.q1(1),{pax,pay},{0,130})
```

q1 =

$$\frac{\pi}{2}$$

```
T03u(1:3,1:3)'*T04u(1:3,1:3)
```

ans =

$$\begin{pmatrix} C_{14}\,\overline{C_1} + S_{14}\,\overline{S_1} & S_{14}\,\overline{C_1} - C_{14}\,\overline{S_1} & 0 \\ S_{14}\,\overline{C_1} - C_{14}\,\overline{S_1} & -C_{14}\,\overline{C_1} - S_{14}\,\overline{S_1} & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

```
disp("#### T3e ####")
```

T3e

```
T3e = transpose(T03u)*T04u;
T3e = T3e(1:3,1:3)
```

T3e =

$$\begin{pmatrix} S_1\,S_{14} + C_1\,C_{14} & C_1\,S_{14} - C_{14}\,S_1 & 0 \\ C_1\,S_{14} - C_{14}\,S_1 & -S_1\,S_{14} - C_1\,C_{14} & 0 \\ 0 & 0 & -1 \end{pmatrix}$$