Dynamics modeling of the two-wheeled mobile robot

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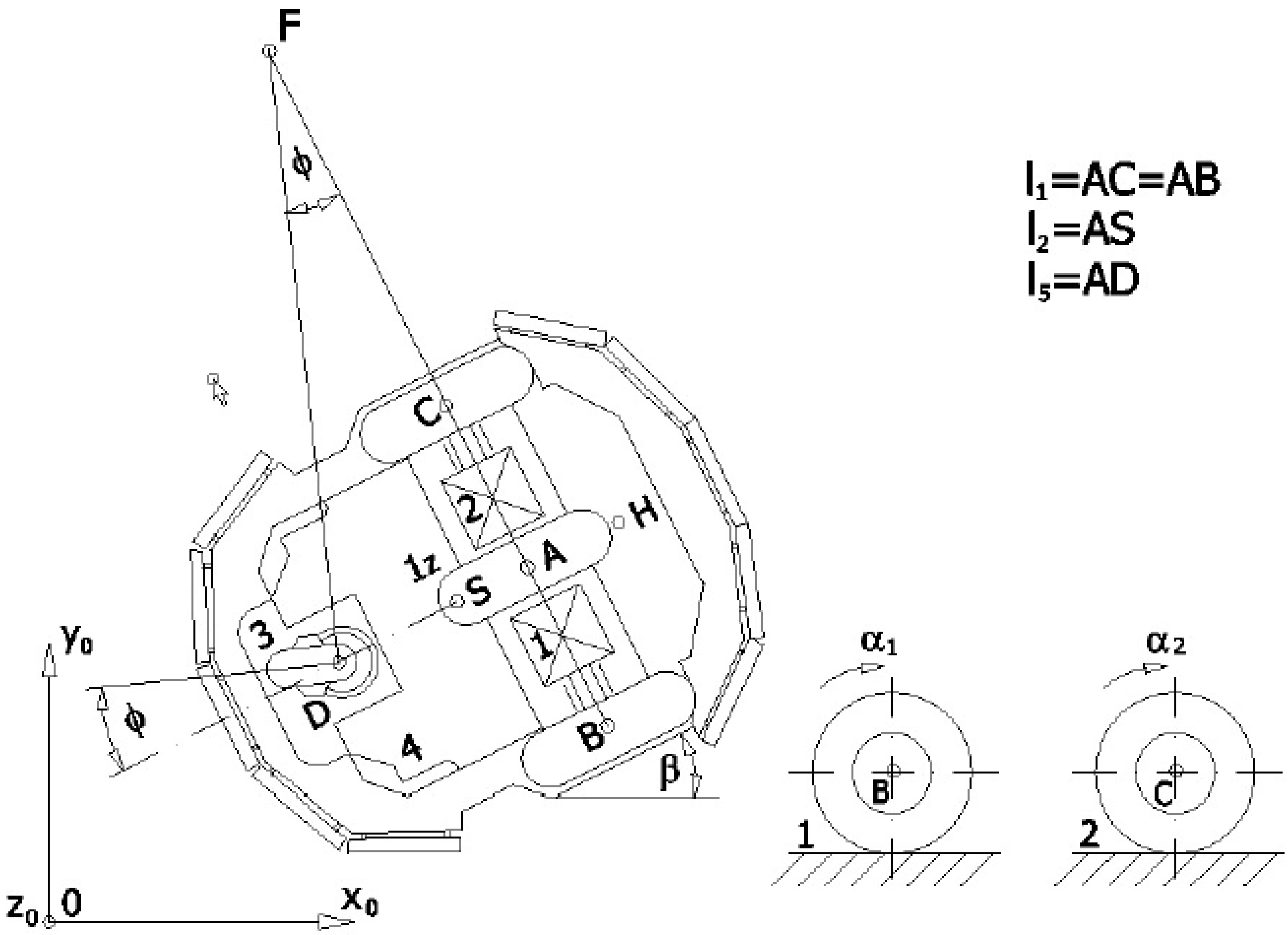
Assumptions:

We asume a two wheeled mobile robot along a path consisting of 5 different segments: Startup, straight line movement, arc movement, straignt line movement,deceleration and stop. Values presented below are also depicted in figure 1.

- Acceleration time: 2s
- Max speed: 1m/s
- Straight line movement: 1s
- Arc movement: 2s R=4m
- Straight line movement: 1s
- Decceleration time: 2s



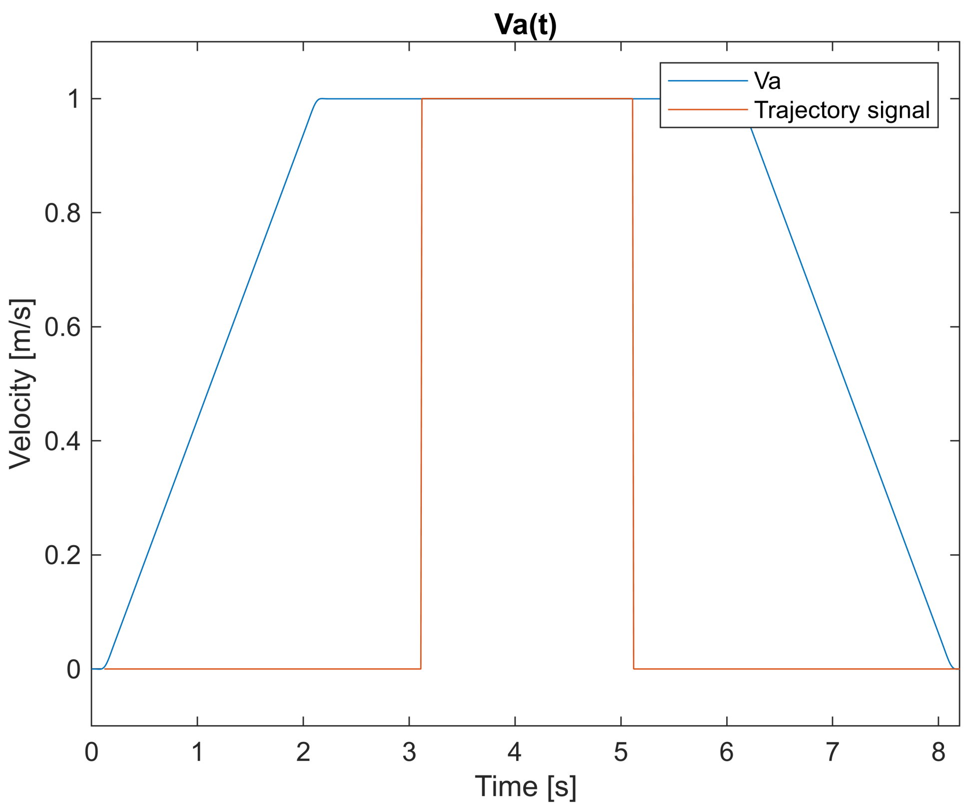
Sketch 1: Robot geometry. Source: prof. Tomasz Buratowski.

l_2 [m]	m_1 [kg]	m_2 [kg]	m_3 [kg]	m_4 [kg]	Ix_1[kgm^2]	Ix_2[kgm^2]	Ix_3[kgm^2]	Iz_1[kgm^2]
0.07	1.5	1.5	0.5	5.67	0.02	0.02	0.005	0.051
Iz_2[kgm^2]	Iz_3[kgm^2]	Iz_4[kgm^2]	N_1 [N]	N_2 [N]	N_3 [N]	f_1	f_2	f_3
0.051	0.002	0.154	31.25	31.25	29.20	0.01	0.01	0.0014

Table 1: Values assumed. Source: prof. Tomasz Buratowski.

```
clear
fs=100;
dt=1/fs;
t=0:dt:8-dt;
Va=0.5*t(1:(2*fs));
Va=[Va,ones(1,4*fs)];
Va=[Va,-0.5*t(6*fs+1:8*fs)+4];
trajSig=[zeros(1,3*fs),ones(1,2*fs),zeros(1,3*fs)];
N=length(t);
df=fs/N;
fv = (0:N-1)*df; % frequency vector
[Vas,group_delay] = FilterData([Va,zeros(1,0.3*fs)]);
```

```
t2=0:dt:8-dt+0.3;
figure(1)
plot(t2,Vas);hold on
title('Va(t)')
xlabel('Time [s]')
ylabel('Velocity [m/s]')
plot(t2+group_delay*dt,[trajSig,zeros(1,30)]);hold off
legend('Va','Trajectory signal')
xlim([0,8.2])
ylim([-0.1,1.1])
```



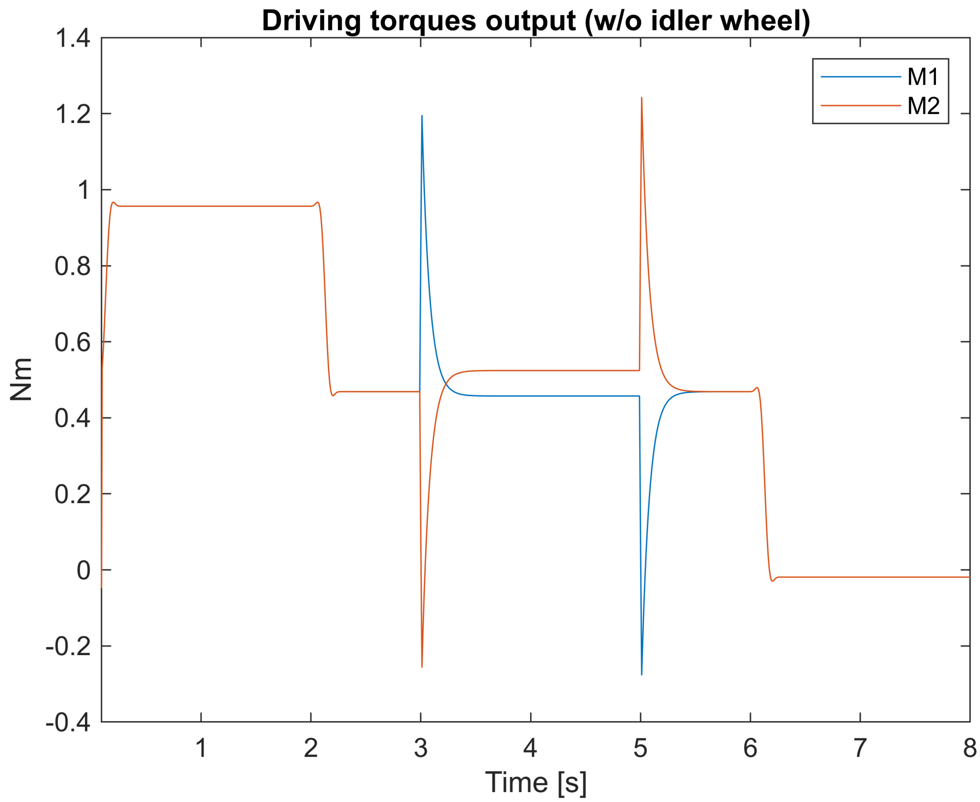

```
Va=Vas;
Ts=[trajSig,zeros(1,30)];
Va = timeseries(Vas ,t2);
Ts = timeseries([trajSig,zeros(1,30)],t2);
```

```
tsim = 8;
R = 3;
r1 = 0.0825 ;
r2 = 0.0825;
r3 = 0.035;
l1 = 0.163;
l3 = 0.07;
l5 = 0.07;
```

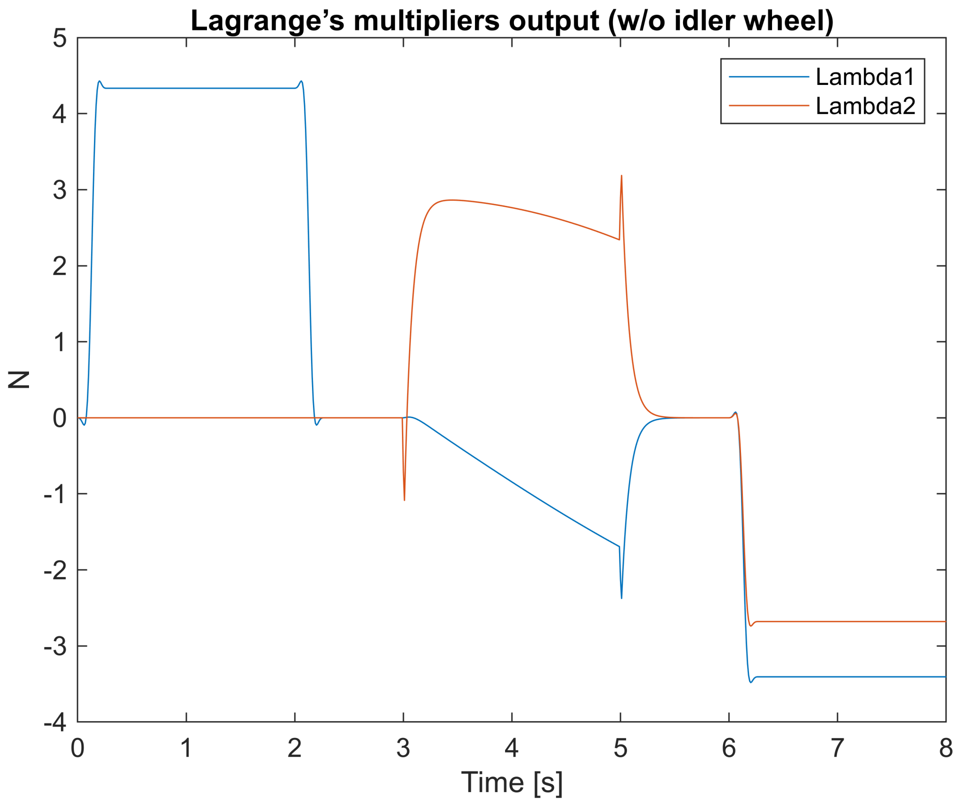
```
r = r1;
l2 = 0.07;
m1 = 1.5;
m2 = 1.5;
m3 = 0.5;
m4 = 5.67 ;
Ix1 = 0.02;
Ix2 = 0.02;
Ix3 = 0.005;
Iz1 = 0.051;
Iz2 = 0.051;
Iz3 = 0.002;
Iz4 = 0.154;
N1 = 31.25;
N2 = 31.25;
N3 = 29.2;
f1 = 0.015;
f2 = 0.015;
f3 = 0.0015;
h= l1/r1;
```

```
out=sim("inverse_dynamics2.slx");
```

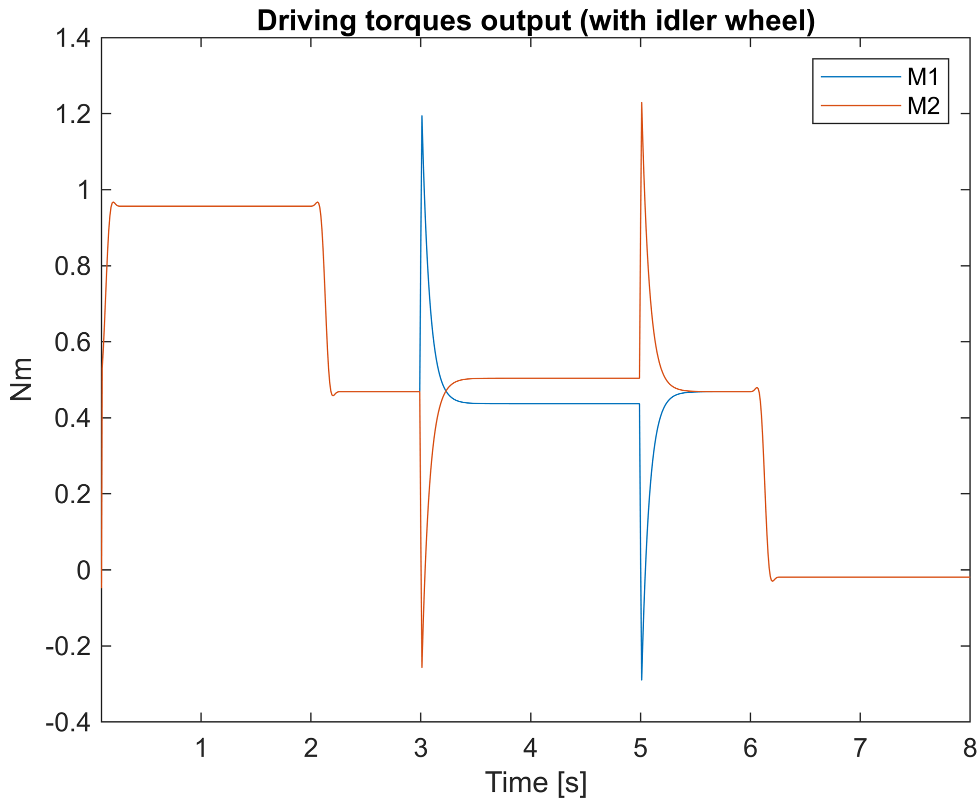
```
Time=out.M1M2.Time;
M1=out.M1M2.Data(:,1);
M2=out.M1M2.Data(:,2);
figure(2)
plot(Time,M1,Time,M2)
title('Driving torques output (w/o idler wheel)')
xlabel('Time [s]')
ylabel('Nm')
legend('M1','M2')
xlim([0.095 8])
```



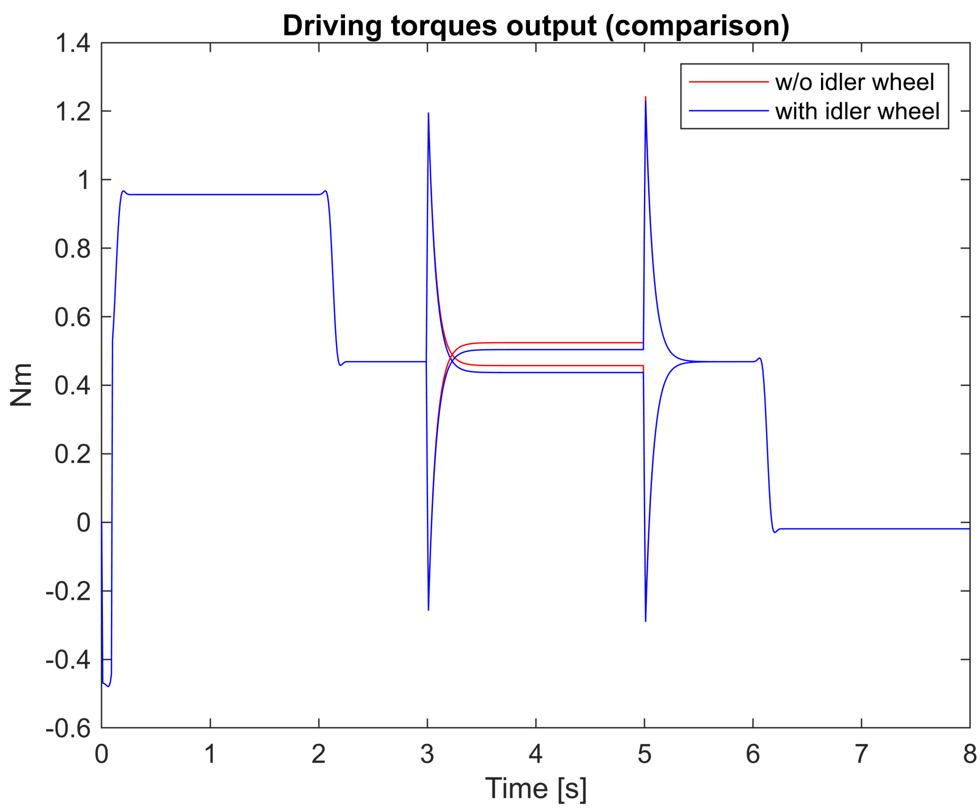

```
Lambda1=out.Lambda1Lambda2.Data(:,1);
Lambda2=out.Lambda1Lambda2.Data(:,2);
figure(3)
plot(Time,Lambda1,Time,Lambda2)
title('Lagrange’s multipliers output (w/o idler wheel)')
xlabel('Time [s]')
ylabel('N')
legend('Lambda1','Lambda2')
```




```
Time=out.M1M2.Time;
M12=out.M1M2withidlerwheel.Data(:,1);
M22=out.M1M2withidlerwheel.Data(:,2);
figure(4)
plot(Time,M12,Time,M22)
title('Driving torques output (with idler wheel)')
xlabel('Time [s]')
ylabel('Nm')
legend('M1','M2')
xlim([0.095 8])
```

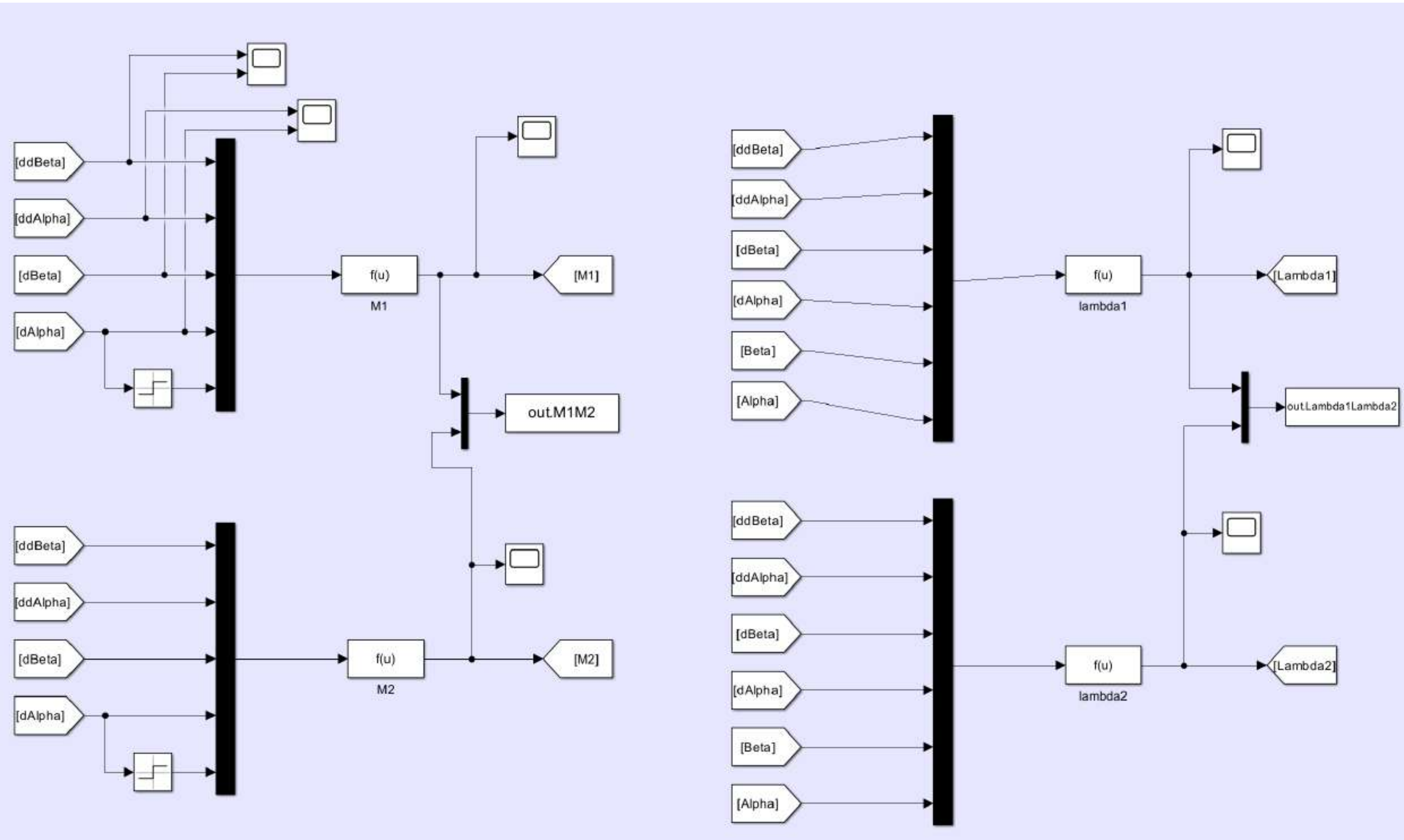



```
figure(5)
plot(Time,M1,'r',Time,M12,'b');hold on
plot(Time,M2,'r',Time,M22,'b')
xlabel('Time [s]')
ylabel('Nm')
title('Driving torques output (comparison)')
legend('w/o idler wheel','with idler wheel')
```

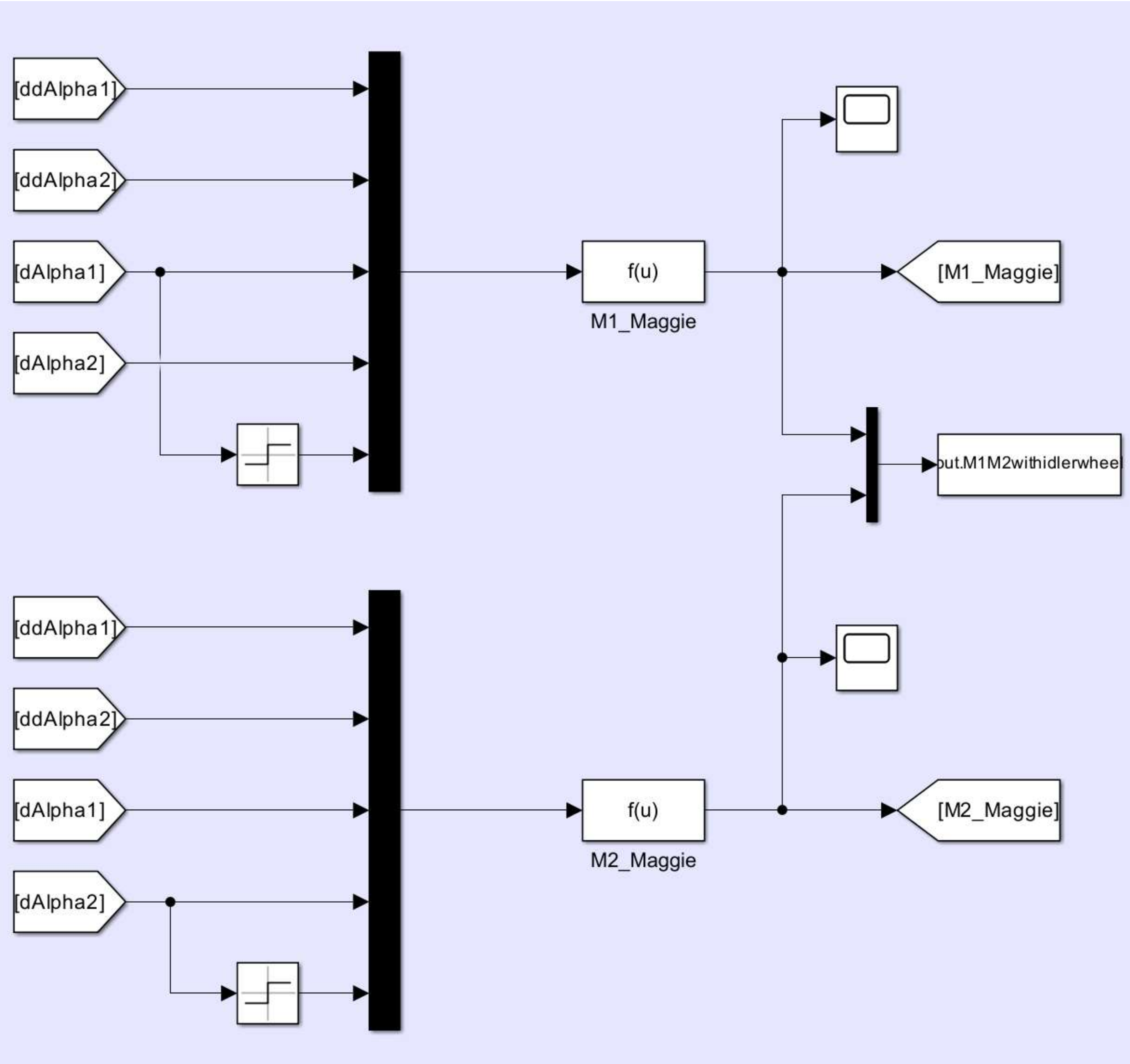



```
function [dataFiltered,group_delay] = FilterData(data)

[n,m]=size(data);

%Forcing row vectros
if n>m
    data=data';
    n=temp;
    n=m;
    m=temp;
end

%Init Filter
Hd=filterF;

%Filtering and scaling all signals one by one
for i=1:n

%Applying filter
x=Hd.filter(data(i,:));

%Scaling the filtered signal
group_delay = floor((length(Hd.Numerator) - 1)/2);
% x = x(group_delay+1:end);
```

```
scale_factor = max(data(i,:)) / max(x);
x = x * scale_factor;

%Appending Filtered and scaled signal
dataFiltered(i,:)=x;
end
end


function Hd = filterF
%FILTERF Returns a discrete-time filter object.

% MATLAB Code
% Generated by MATLAB(R) 9.14 and Signal Processing Toolbox 9.2.
% Generated on: 25-Mar-2024 10:36:58

% Equiripple Lowpass filter designed using the FIRPM function.

% All frequency values are in Hz.
Fs = 100;  % Sampling Frequency

Fpass = 5;            % Passband Frequency
Fstop = 15;           % Stopband Frequency
Dpass = 0.057501127785;  % Passband Ripple
Dstop = 0.0001;          % Stopband Attenuation
dens  = 20;              % Density Factor

% Calculate the order from the parameters using FIRPMORD.
[N, Fo, Ao, W] = firpmord([Fpass, Fstop]/(Fs/2), [1 0], [Dpass, Dstop]);


% Calculate the coefficients using the FIRPM function.
b  = firpm(N, Fo, Ao, W, {dens});
Hd = dfilt.dffir(b);

end
```



[ddBeta]
[ddAlpha]
[dBeta]
[dAlpha]
f(u)
M1
[M1]
out.M1M2
[ddBeta]
[ddAlpha]
[dBeta]
[dAlpha]
f(u)
M2
[M2]
[ddBeta]
[ddAlpha]
[dBeta]
[dAlpha]
[Beta]
[Alpha]
f(u)
lambda1
[Lambda1]
out.Lambda1Lambda2
[ddBeta]
[ddAlpha]
[dBeta]
[dAlpha]
[Beta]
[Alpha]
f(u)
lambda2
[Lambda2]


[ddAlpha1]
[ddAlpha2]
[dAlpha1]
[dAlpha2]
f(u)
M1_Maggie
[M1_Maggie]
out.M1M2withidlerwheel
[ddAlpha1]
[ddAlpha2]
[dAlpha1]
[dAlpha2]
f(u)
M2_Maggie
[M2_Maggie]