Kinematics and Dynamics of Mechatronic Systems

Exercise B2

Geometrical model – inverse kinematics

1. Analytical formulation of an algorithm

For the selected kinematic structure of a manipulator the following tasks are to be completed.

T1. Determine analytically relationships expressing the joint coordinates q_i i=1..4 in terms of position of the last link's tip .
In case of rotary joints the '**arctan'** function or a pair of '**arcsin**' and '**arccos**' (unless an assumed joint motion range enables using only one of 'arcsin' or 'arccos') functions should be used.

The report should contain **complete derivation** (stages 1- 3) of the relationships starting from the position and orientation of the manipulator last link described by a proper homogeneous transformation matrix ${}^{0}\underline{T}_e$ in the following general symbolic form:

$$ {}^{o}\underline{T}_e = \begin{bmatrix} N_x & O_x & A_x & P_x \\ N_y & O_y & A_y & P_y \\ N_z & O_z & A_z & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

All the elements of the above matrix are numbers denoted by symbols to make the analysis general. The matrix and the constant parameters of the geometrical model constitute the input data for the algorithm of inverse kinematics.

STAGE 1
Report the algorithm of determination of the $\mathbf{p_a}$ vector as follows:
- determination of $\mathbf{p_w}$ vector

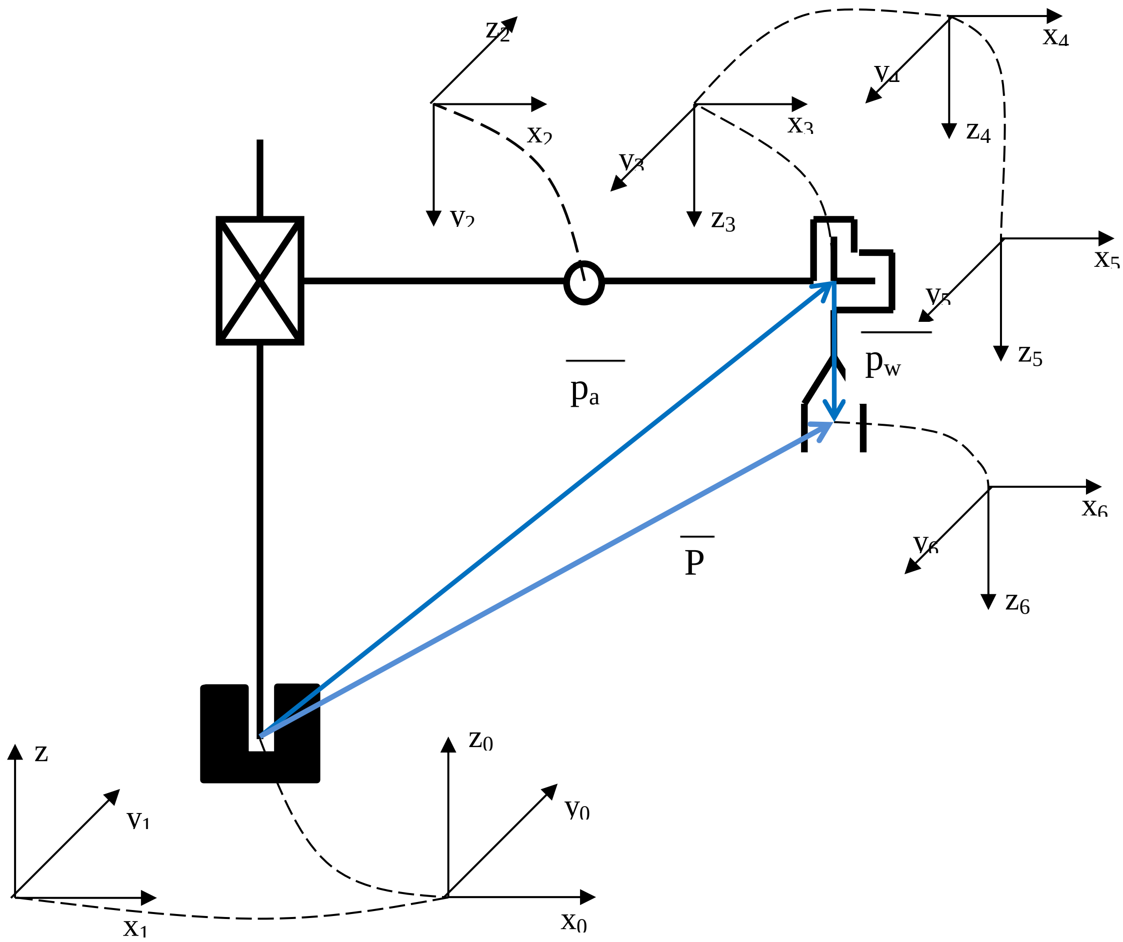
$$ p_w = \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} \cdot \left|\overline{p_w}\right| $$

- determination of $\boldsymbol{p_a}$ vector

$$ p_a = P - p_w $$

$$ p = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} $$

Present the $\boldsymbol{p_a}$ and $\boldsymbol{p_w}$ vectors on a kinematic diagram of the considered manipulator like in the following example:



Cartesian coordinates – p_{ax}, p_{ay}, p_{az} are 3 numbers that define the location of the tip of the 3rd link with respect to the reference frame.

STAGE 2

Next, the manipulator geometrical model defines the form of dependence of the above mentioned Cartesian coordinates on joint coordinates – algebraic, usually non-linear functions $f_1(q_1,q_2,q_3)$, $f_2(q_1,q_2,q_3)$ and $f_3(q_1,q_2,q_3)$ that lead to the following set of equations:

$$p_{ax} = f_1(q_1, q_2, q_3)$$
$$p_{ay} = f_2(q_1, q_2, q_3)$$
$$p_{az} = f_3(q_1, q_2, q_3)$$

like in the following example:

$$p_{ax} = d_3 C_1 S_2$$
$$p_{ay} = d_3 S_1 S_2$$
$$p_{az} = d_1 + d_3 C_2$$

Note that in case when the 3rd coordinate frame is located at the beginning of the 3rd link the location of the tip of the 3rd link with respect to the reference frame might be determined by the following product:

$$p_a = {}^0\underline{T}_3 \cdot \begin{bmatrix} {}^3p_{ax} \\ {}^3p_{ay} \\ {}^3p_{az} \\ 1 \end{bmatrix}$$

Solution of the above equations leads to formulas expressing joint coordinates q_1, q_2, q_3 in terms of the already known values of p_{ax}, p_{ay} and p_{az}.

like in the following example:

$$\frac{d_3 S_1 S_2}{d_3 C_1 S_2} = \frac{p_{ay}}{p_{ax}} \Rightarrow \mathrm{tg}(\theta_1) = \frac{p_{ay}}{p_{ax}}$$

STAGE 3

The fourth coordinate q_4 should be determined via substitution of the determined q_1, q_2, q_3 values to ${}^0\underline{R}_3$ matrix, resulting in ${}^0\underline{r}_3$ matrix. Next, the following product of matrices is calculated:

$${}^0\underline{r}_3{}^T \cdot {}^0\underline{r}_e = \begin{bmatrix} n_{wx} & o_{wx} & a_{wx} \\ n_{wy} & o_{wy} & a_{wy} \\ n_{wz} & o_{wz} & a_{wz} \end{bmatrix}$$

Comparison of elements (algebraic expressions) of ${}^3\underline{R}_e$ matrix with corresponding elements (numbers) of the above product leads to determination of a formula that expresses q_4 coordinate.
In the above derivation only the rotation matrices are used instead of the full homogeneous transformation matrices as only the orientation angles are to be determined.

T2. Find a <u>number of solutions</u> of the inverse problem for the considered example of a manipulator. In case of the multiple solution present all the solutions graphically on the drawing of the kinematic chain of the manipulator. Indicate the solution within and outside of the selected joint variables' motion ranges.

T3. Analyze conditions of existence of the achieved solutions to the inverse kinematics. Compare the conditions with the assumed joint's ranges of motion.
The typical components of the derived relationships that should be analyzed are:

$$arctan\left(\frac{n}{d}\right) \qquad n \neq 0 \; or \; d \neq 0$$

$$\sqrt{a} \qquad a \geq 0$$

The report should contain:
- graphical presentation of $\boldsymbol{p_a}$ and $\boldsymbol{p_w}$ vectors
- full derivation of the analytical solution to the inverse kinematics of the mechanism (scanned clear handwritten relationships are accepted) – task T1
- discussion of number and existence of the solutions – T2 and T3
- conclusions