
	<p>Faculty of Mechanical Engineering and Robotics</p> <p>Department of Robotics and Mechatronics</p>	
<p>Mechatronic Systems Identification</p>		
<p>Lab 1: Basics of signal processing</p>		

1. Frequency resolution

Create a 500-sample long signal y with a sampling rate of 500 Hz. The signal is the sum of two sine waves with frequencies of 30 and 30.5 Hz, and amplitudes of 1 and 2, respectively. Calculate the Fourier transform of the signal using the FFT function. Display the amplitude-frequency plot for the frequency range from 0 to the Nyquist frequency.

Use the following code to scale the frequency range

```
Y = fft(y);
N = length(Y);
df = fs/N;           % frequency resolution
fv = (0:N-1)*df;    % frequency vector
figure(1)
plot( fv , abs(Y))
xlabel('frequency (Hz)')
```

Scale the Fourier transform amplitude so that the frequency peaks correspond to the sine amplitudes ($\text{abs}(\text{fft}(y))/\text{length}(y)*2$). Then select such a length of the signal (at the same sampling frequency) that both component frequencies are clearly distinguishable in the spectrum and have the appropriate amplitudes. What is the minimal signal length for which these conditions are met?

Graphs showing the spectra of a signal with a length of 500 samples and a signal with sufficient frequency resolution should be included in the report. Post the Matlab code you used to get the graphs. In the comments, formulate the relationship between the length of the signal and the frequency resolution.

2. Resampling

Create a signal y of a length of 1000 samples, sampling frequency of 1000 Hz, which is the sum of two cosine waves of frequencies of 125 and 375 Hz and an amplitude of 1 each. Calculate the Fourier transform of the signal using the `fft` function. Display the amplitude-frequency plot for the frequency range from 0 to the Nyquist frequency by scaling the Fourier transform amplitude accordingly. Then, subsample the signal, keeping every second sample from the original signal (`y(1:2:end)`). Calculate and display the `fft` result for the new signal by scaling the spectrum amplitude. Compare the spectra and waveforms of both signals. Describe how the spectrum of the subsampled signal differs from that of the original signal, and why this difference arises.

Redo the procedure for the following cases:

- a) Signal $Y1$ consisting of a sum of two sine waves of frequencies of 125 and 375 Hz.
- b) Signal $Y2$ consisting of a difference of cosine waves of frequencies of 125 and 375 Hz.

Comment similarities and differences between all cases. Take into account amplitude, real and imaginary part of the signals.

In the report, include the spectrum plots of the original and subsampled signal, both for the first case and cases a) and b). Post the Matlab code you used to get the graphs. In the comments, describe the reason for the difference between the original signal and the subsampled signal in the tested cases. In the explanation, you can use other graphs of the described signals.

3. Spectral leakage and windowing

Create a signal $y1$ in the time range of 0 to 1 second with a sampling rate of 100 Hz (which gives a signal length of 101 samples). The signal is a sum of a 7 Hz sine wave with an amplitude of 10 and a 20 Hz cosine wave with an amplitude of 0.02. Calculate the Fourier transform of the signal (Matlab function `fft`) and display the spectrum on a logarithmic plot on the $y1$ axis (take $20 \log_{10}$ of the resulting spectrum). Scale the frequency range from 0 to the Nyquist frequency.

In the next step apply a Hann window by multiplying it in the time domain with the $y1$ signal to get the $y2$ signal. Use the Matlab `hann` function. Calculate the Fourier transform of the windowed $y2$ signal and display the spectrum on the graph. Compare the spectrum of the original $y1$ signal with the spectrum of the windowed $y2$ signal. For easier comparison, display both spectra on the same graph, e.g. with the `hold on` function. What are the differences between the spectra of $y1$ and $y2$ signals?

Remember to create the appropriate frequency vector for the created graphs.

Why is windowing necessary for the $y1$ signal?

What are other windows that can be used here? Create additional signal $y3$, which is signal $y1$ apodized with a selected window (other than Hann). Plot spectrum of $y3$ signal and discuss the properties of the window.

The report should include a graph showing the spectra obtained for the $Y1$ signal and for the $Y2$ and $Y3$ windowed signals. Post the Matlab code you used to get the graph. In the comments, describe what causes the differences between the spectra of all signals.