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WHEEL MOBILE ROBOTS
LECTURE– 02 - KINEMATICS

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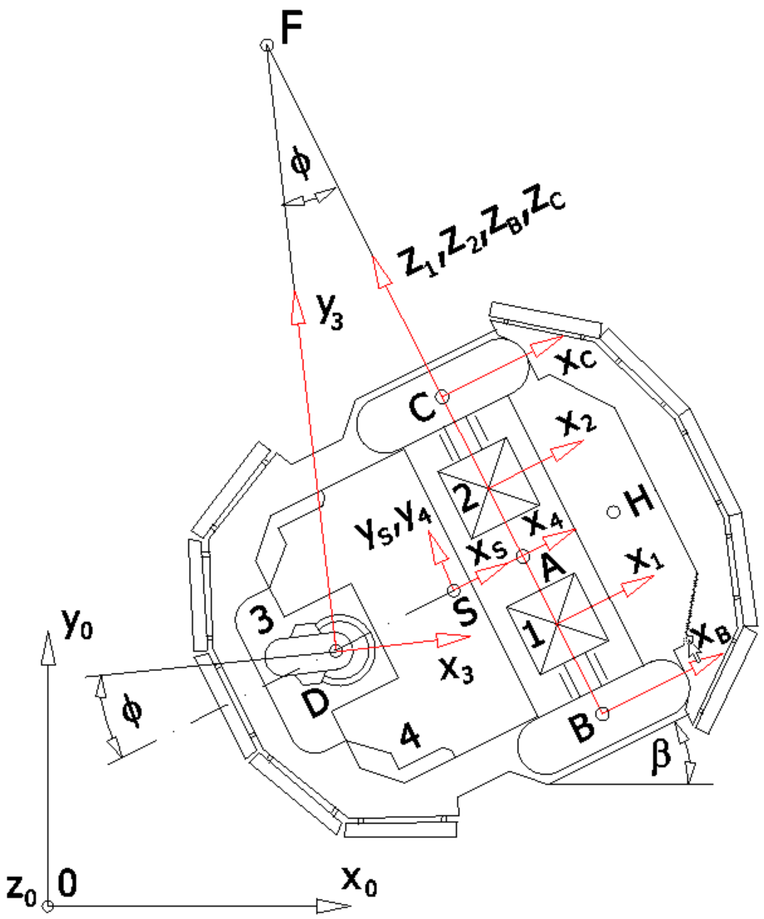
Mobile robots - kinematics

- Kinematics of arbitrarily selected point belonging to a particular section of the system can be analyzed with the use of so-called kinematic equations
- Each movable part must be connected with

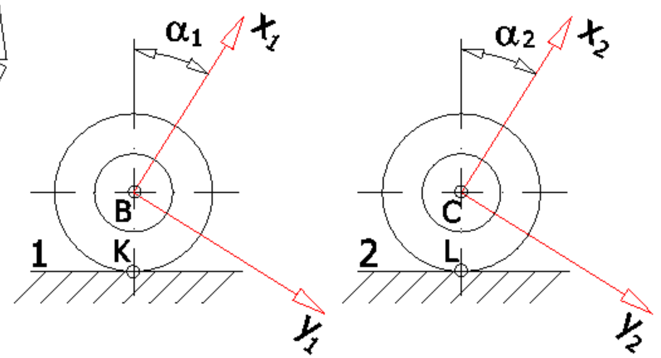
Kinematics modeling of the two-wheels mobile robot



$l_1 = AC_{z1} = AB_{z1}$
$l_2 = AS_{xS}$
$l_3 = AH_{xS}$
$l_4 = AH_{zS}$
$l_5 = AD_{xS}$

Robot consists of:

- chassis *(4)*
- drive system *(1&2)*
- idler wheel *(3)*



Kinematics modeling of the two-wheels mobile robot

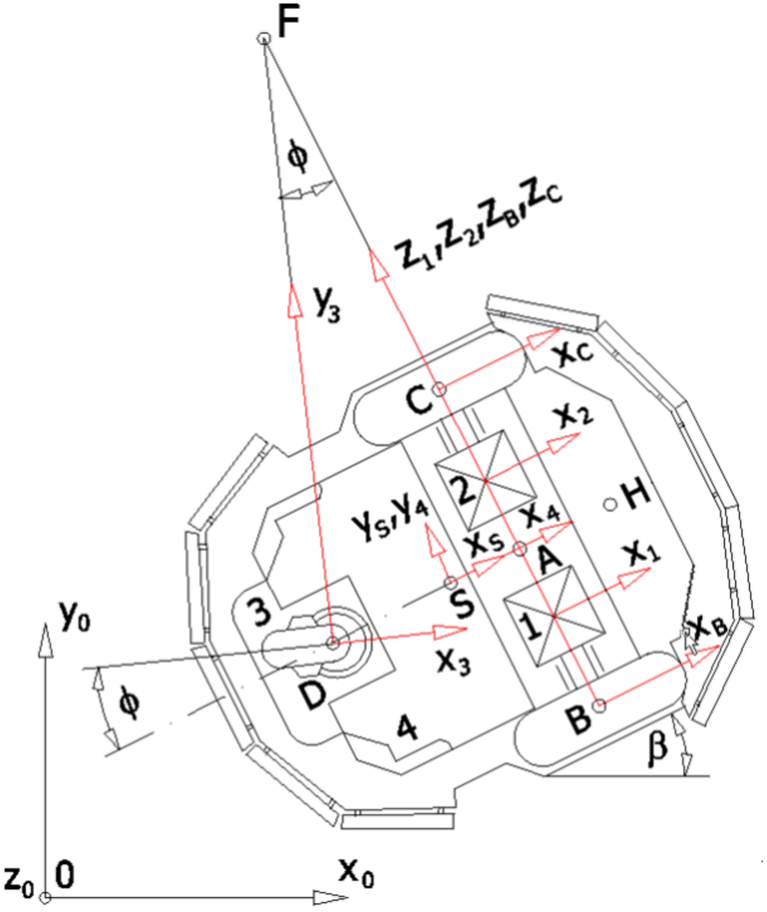
Proper coordinate system was connected with particular parts of the model:

- chassis (4) and $\mathbf{x_4y_4z_4}$ coordinate system (CS) with the origin in the mass center of the part
- $\mathbf{x_1y_1z_1}$ and $\mathbf{x_2y_2z_2}$ CSs are connected with driving systems with origins in points **B** and **C**
- $\mathbf{x_0y_0z_0}$ CS is stationary coordinate system and is base frame of reference

Kinematic description of the model is based on equations for characteristic points of the robot and assumes that it travels with constant velocity of the point A V_A

Kinematics modeling of the two-wheels mobile robot

Kinematic equation for point H has form of:



$$\bar{r}_H = T_{4,0,H}\,\bar{\rho}_H$$

Kinematics modeling of the two-wheels mobile robot

Transformation matrix of the frame of reference $\mathbf{x_4 y_4 z_4}$ to $\mathbf{x_0 y_0 z_0}$ for point H has form of:

$$T_{4,0,H} = \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 & x_A + l_3\cos(\beta) \\ \sin(\beta) & \cos(\beta) & 0 & y_A + l_3\sin(\beta) \\ 0 & 0 & 1 & r_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Positioning vector of the point H with reference to the $\mathbf{x_4 y_4 z_4}$ has form of:

$$\rho_H = \begin{bmatrix} 0 \\ 0 \\ -l_4 \\ 1 \end{bmatrix}$$

Kinematics modeling of the two-wheels mobile robot

After evaluation of the formula $\bar{r}_H = T_{4,0,H}\bar{\rho}_H$ we obtained equation of motion of the point H written in two equivalent forms:

$$r_H = \begin{bmatrix} x_A + l_3\cos(\beta) \\ y_A + l_3\sin(\beta) \\ r_1 - l_4 \\ 1 \end{bmatrix} \qquad \begin{bmatrix} x_H \\ y_H \\ z_H \\ 1 \end{bmatrix} = \begin{bmatrix} x_A + l_3\cos(\beta) \\ y_A + l_3\sin(\beta) \\ r_1 - l_4 \\ 1 \end{bmatrix}$$

Kinematics modeling of the two-wheels mobile robot

In the next step the equation for the circular path are presented:

$$x_H = R\sin(\phi)$$

$$y_H = R(1-\cos(\phi))$$

$$z_H = r_1 - l_4$$

After comparison of the equation of motion with path formulae we get:

$$R\sin(\phi) = x_A + l_3\cos(\beta)$$

$$R(1-\cos(\phi) = y_A + l_3\sin(\beta)$$

Kinematics modeling of the two-wheels mobile robot

Differentiating with respect to time equations for point H

$$R\sin(\phi) = x_A + l_3\cos(\beta) \,\&\, R(1-\cos(\phi) = y_A + l_3\sin(\beta)$$

we obtain equation for speed of the point H along x and y axis

$$\dot{x}_A - l_3\dot{\beta}\sin(\beta) - R\dot{\phi}\cos(\phi) = 0$$

$$\dot{y}_A + l_3\dot{\beta}\cos(\beta) - R\dot{\phi}\sin(\phi) = 0$$

Kinematics modeling of the two-wheels mobile robot

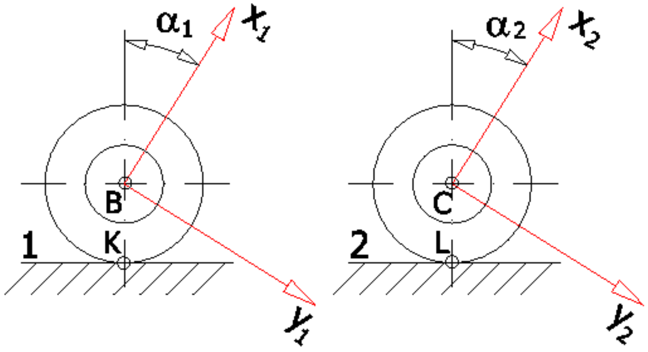
Next characteristic point is point A for which constant speed was assumed and projection of speed vectors on to X and Y axis of the base reference frame was made.

$$\dot{x}_A - v_A \cos(\beta) = 0$$
$$\dot{y}_A - v_A \sin(\beta) = 0$$

Kinematics modeling of the two-wheels mobile robot

To fully describe parameters of motion of our structure we need angular velocities and rotation angles of the wheels (*1&2*).

Those velocities are derived from equations for velocity of the tangent points between wheel and ground (assuming no slip)



for wheel 1:

$$\bar{r}_K = T_{4,0,K} T_{1,4} \bar{\rho}_K$$

Kinematics modeling of the two-wheels mobile robot

Transformation matrix of the frame of reference $\mathbf{x_4 y_4 z_4}$ to $\mathbf{x_0 y_0 z_0}$ for point K has form of:

$$T_{4,0,K} = \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 & x_A + l_1 \sin(\beta) \\ \sin(\beta) & \cos(\beta) & 0 & y_A - l_1 \cos(\beta) \\ 0 & 0 & 1 & r_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Kinematics modeling of the two-wheels mobile robot

Transformation matrix of the frame of reference $\mathbf{x_1y_1z_1}$ to $\mathbf{x_4y_4z_4}$ for point K has form of:

$$T_{1,4} = \begin{bmatrix} \sin(\alpha_1) & \cos(\alpha_1) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \cos(\alpha_1) & -\sin(\alpha_1) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Kinematics modeling of the two-wheels mobile robot

Positioning vector for the point K with respect to $\mathbf{x_1 y_1 z_1}$ reference frame has form of:

$$\rho_K = \begin{bmatrix} -r_1 \cos(\alpha_1) \\ r_1 \sin(\alpha_1) \\ 0 \\ 1 \end{bmatrix}$$

After differentiating of equation of motion for the point K we get:

$$\bar{v}_K = \dot{T}_{4,0,K} \dot{T}_{1,4} \bar{\rho}_K$$

Kinematics modeling of the two-wheels mobile robot

After substitution of all part of the equation of motion for point K we get equations for velocities for point K

$$\begin{bmatrix} \dot{x}_K \\ \dot{y}_K \\ \dot{z}_K \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{x}_A - r_1\dot{\alpha}_1 \cos(\beta) + l_1\dot{\beta}\cos(\beta) \\ \dot{y}_A - r_1\dot{\alpha}_1 \sin(\beta) + l_1\dot{\beta}\sin(\beta) \\ 0 \\ 0 \end{bmatrix}$$

Kinematics modeling of the two-wheels mobile robot

Having in mind the fact that $V_K = 0$ (no-slip condition) in scalar form we get:

$$\dot{x}_A - r_1\dot{\alpha}_1 \cos(\beta) + l_1\dot{\beta}\cos(\beta)$$

$$\dot{y}_A - r_1\dot{\alpha}_1 \sin(\beta) + l_1\dot{\beta}\sin(\beta)$$

Kinematics modeling of the two-wheels mobile robot

For the second wheel we proceed in similar manner.

The equation for motion of the point L (tangent point between 2 wheel and the ground) was derived:

$$\bar{r}_L = T_{4,0,L} T_{2,4} \overline{\rho}_L$$

Kinematics modeling of the two-wheels mobile robot

Transformation matrix of the frame of reference $\mathbf{x_4y_4z_4}$ to $\mathbf{x_0y_0z_0}$ for point L has form of:

$$T_{4,0,L} = \begin{bmatrix} \cos(\beta) & -\sin(\beta) & 0 & x_A - l_1\sin(\beta) \\ \sin(\beta) & \cos(\beta) & 0 & y_A + l_1\cos(\beta) \\ 0 & 0 & 1 & r_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Kinematics modeling of the two-wheels mobile robot

Transformation matrix of the frame of reference $\mathbf{x_2y_2z_2}$ to $\mathbf{x_4y_4z_4}$ for point L has form of:

$$T_{2,4} = \begin{bmatrix} \sin(\alpha_2) & \cos(\alpha_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \cos(\alpha_2) & -\sin(\alpha_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Kinematics modeling of the two-wheels mobile robot

Positioning vector for the point L with respect to $\mathbf{x_2 y_2 z_2}$ reference frame has form of:

$$\rho_L = \begin{bmatrix} -r_2\cos(\alpha_2) \\ r_2\sin(\alpha_2) \\ 0 \\ 1 \end{bmatrix}$$

After differentiating of equation of motion for the point L we get:

$$\bar{v}_L = \dot{T}_{4,0,L}\dot{T}_{2,4}\bar{\rho}_L$$

Kinematics modeling of the two-wheels mobile robot

After substitution of all part of the equation of motion for point L we get equations for velocities for point L

$$\begin{bmatrix} \dot{x}_L \\ \dot{y}_L \\ \dot{z}_L \\ 0 \end{bmatrix} = \begin{bmatrix} \dot{x}_A - r_2\dot{\alpha}_2\cos(\beta) - l_1\dot{\beta}\cos(\beta) \\ \dot{y}_A - r_2\dot{\alpha}_2\sin(\beta) - l_1\dot{\beta}\sin(\beta) \\ 0 \\ 0 \end{bmatrix}$$

Kinematics modeling of the two-wheels mobile robot

Having in mind the fact that $V_L = 0$ (no-slip condition) in scalar form we get:

$$\dot{x}_A - r_2\dot{\alpha}_2\cos(\beta) - l_1\dot{\beta}\cos(\beta)$$
$$\dot{y}_A - r_2\dot{\alpha}_2\sin(\beta) - l_1\dot{\beta}\sin(\beta)$$

Kinematics modeling of the two-wheels mobile robot

Summing up, we obtained system of equations describing motion of the two-wheeled robot:

$$\dot{x}_A - l_3\dot{\beta}\sin(\beta) - R\dot{\phi}\cos(\phi) = 0$$
$$\dot{y}_A + l_3\dot{\beta}\cos(\beta) - R\dot{\phi}\sin(\phi) = 0$$
$$\dot{x}_A - v_A\cos(\beta) = 0$$
$$\dot{y}_A - v_A\sin(\beta) = 0$$
$$\dot{x}_A - r_1\dot{\alpha}_1\cos(\beta) + l_1\dot{\beta}\cos(\beta) = 0$$

calculation of:

- displacement
- velocity
- acceleration
- angle of rotation
- angular velocities
- solving simple and inverse kinematic problems

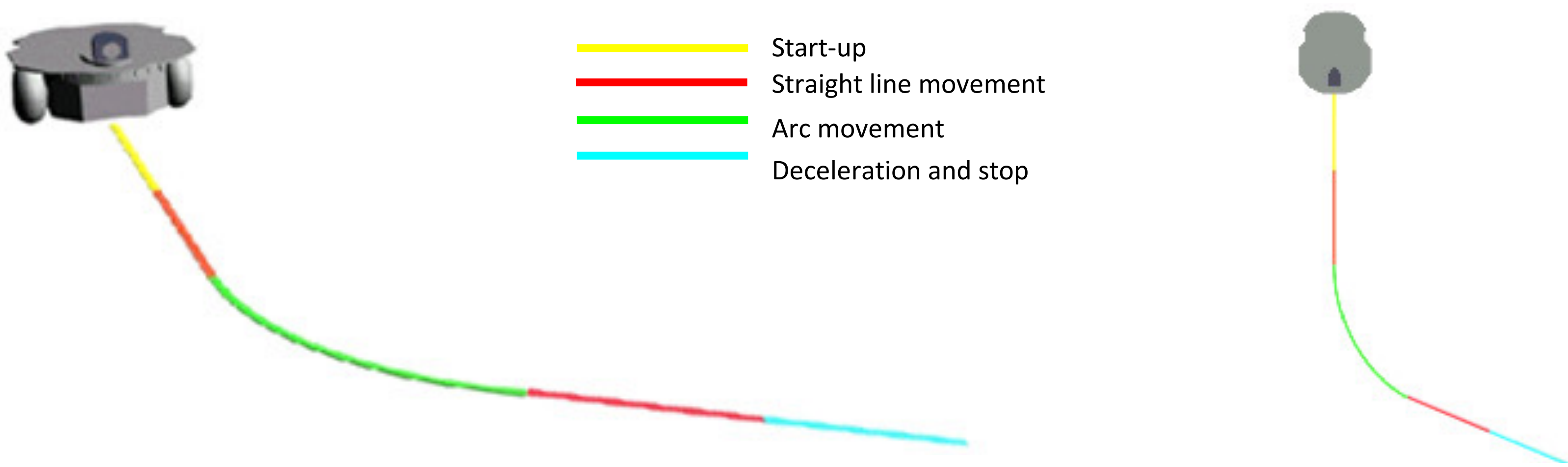
$$\sqrt{l_5^2\dot{\beta}^2 + v_A^2} - r_3\dot{\alpha}_3 = 0$$

calculation of:

- kinematic parameters of idler wheel

Simulation of the inverse kinematic problem with the use of Matlab Simulink software

Based on previosly derived equations of motion for our robot we conduct simulation of inverse kinematic problem for proposed path:



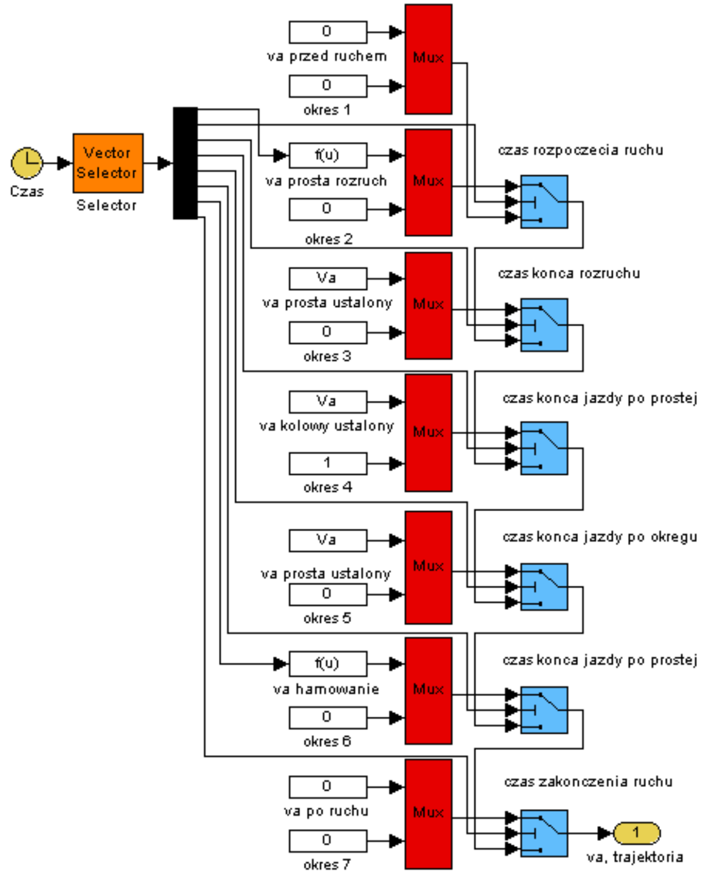
Simulation of the inverse kinematic problem with the use of Matlab Simulink software

We assume working conditions and physical features of the robot:

- Point A velocity **v_A=0.3 [m/s]**,
- Start-up **2 [s],**
- Straight line movement **0.5 [s],**
- Acr movement **4.7 [s]** with **R=1.5 [m],**
- Angle of chassis rotation **0°-54°**,
- Straight line movement **0.5 [s]**,
- Deceleration and stop **2 [s]**.

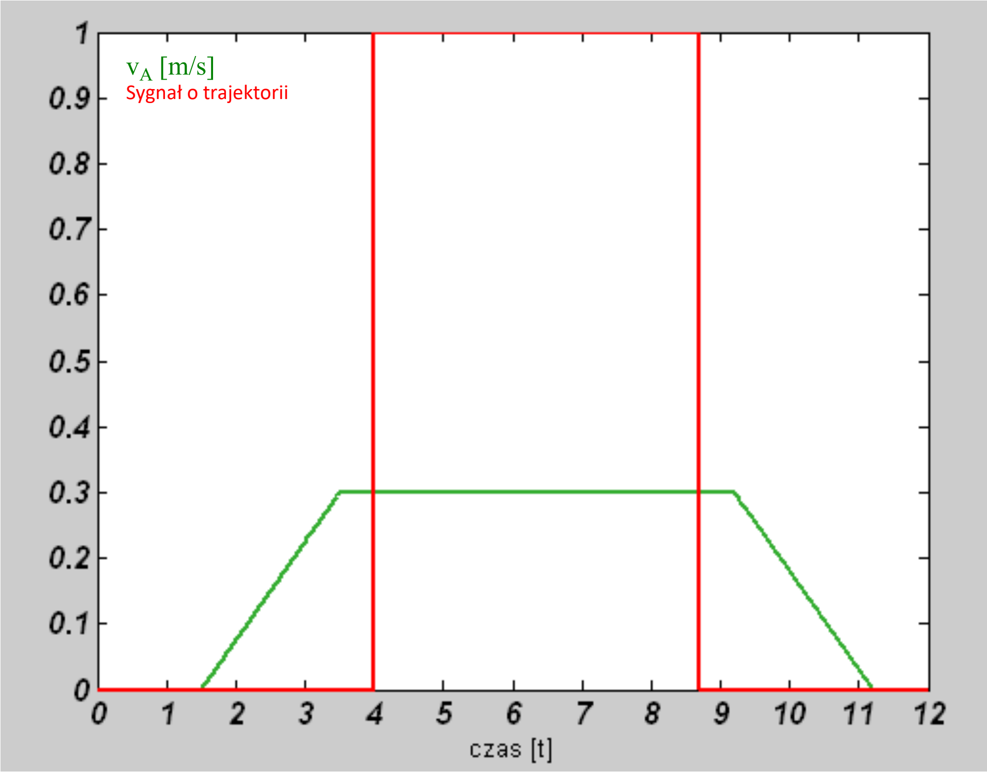
l_1 [m]	l_3 [m]	l_4 [m]	l_5 [m]	r_1 [m]	r_2 [m]	r_3 [m]
0.163	0.07	0.2	0.07	0.0825	0.0825	0.035

Simulation of the inverse kinematic problem with the use of Matlab Simulink software



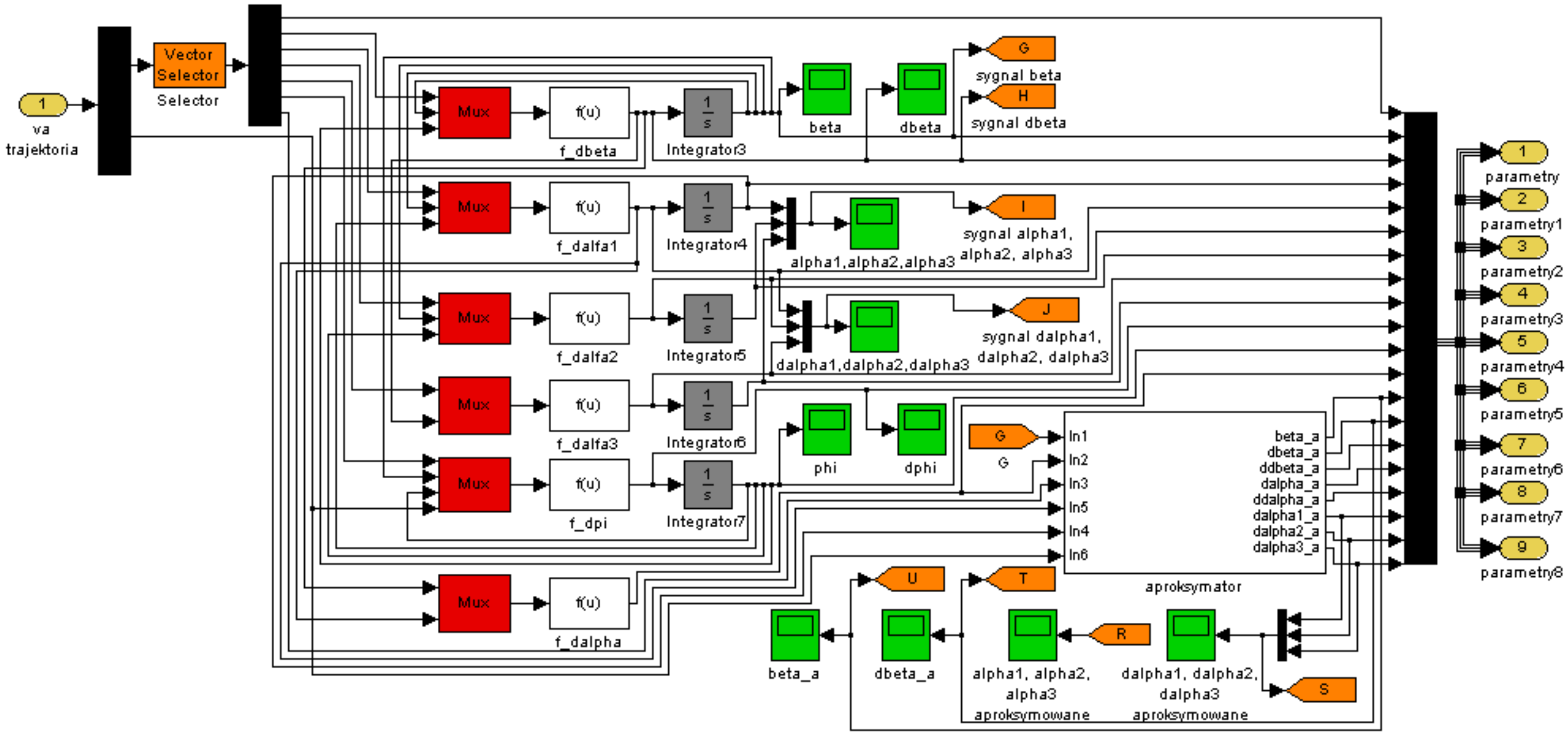
Speed and trajectory generator model

Simulation of the inverse kinematic problem with the use of Matlab Simulink software



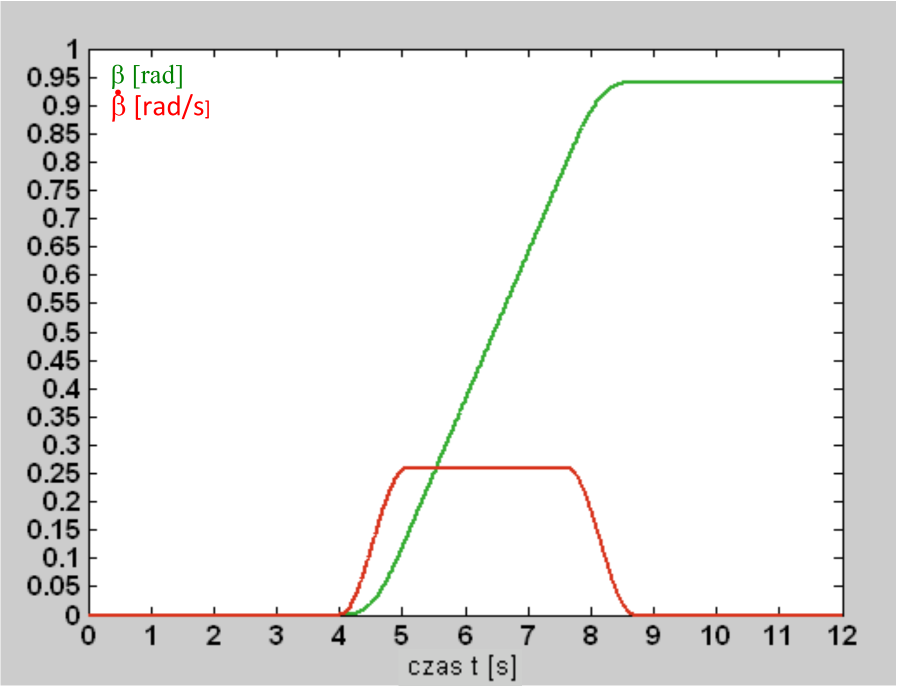
Output from speed and trajectory generator model

Simulation of the inverse kinematic problem with the use of Matlab Simulink software



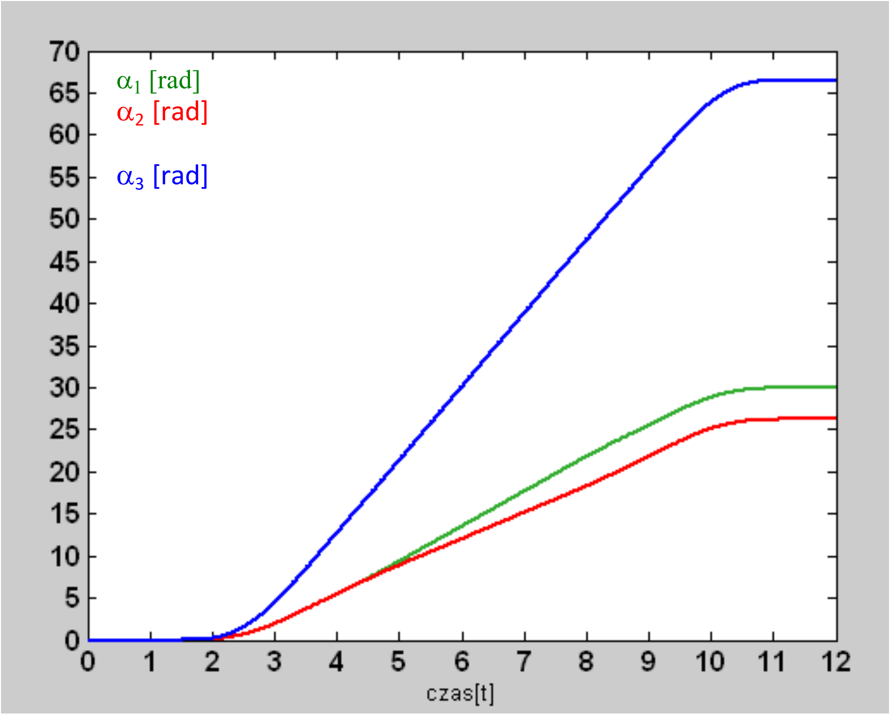
Model of inverse kinematic solver (as input there is trajectory from previously discused model)

Simulation of the inverse kinematic problem with the use of Matlab Simulink software



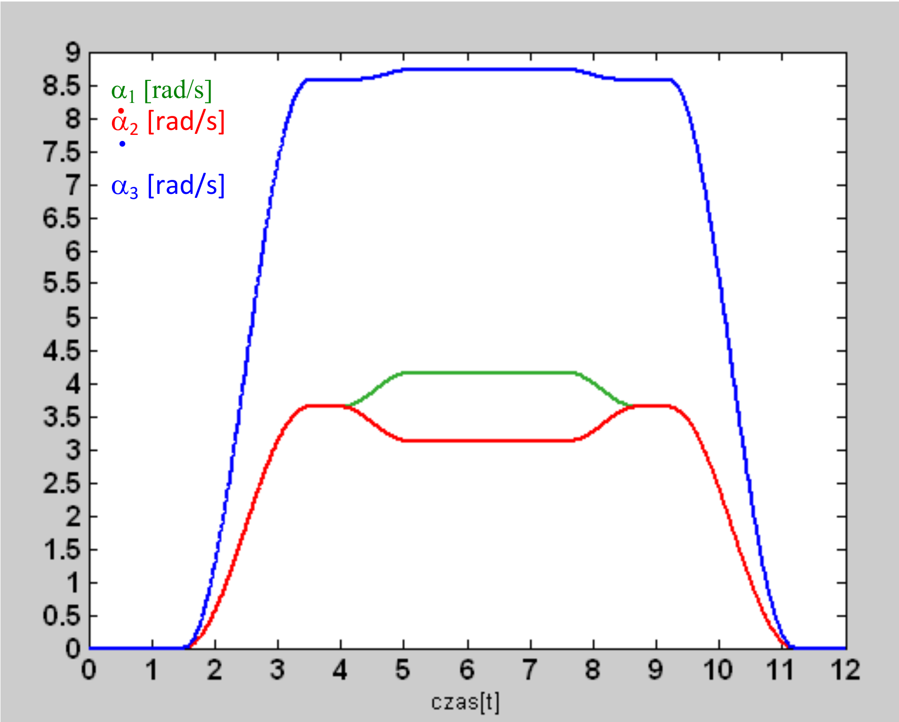
Output for the angle of rotation of the chassis and its angular velocity

Simulation of the inverse kinematic problem with the use of Matlab Simulink software



Output for the angle of rotation of wheels 1,2 and 3

Simulation of the inverse kinematic problem with the use of Matlab Simulink software



Output for the angular speed of wheels 1,2 and 3

Simulation of the simple kinematic problem with the use of Matlab Simulink software

Based on the results from inverse kinematic problem we can check correctness of our results by plotting the path as a result of simple kinematic problem.

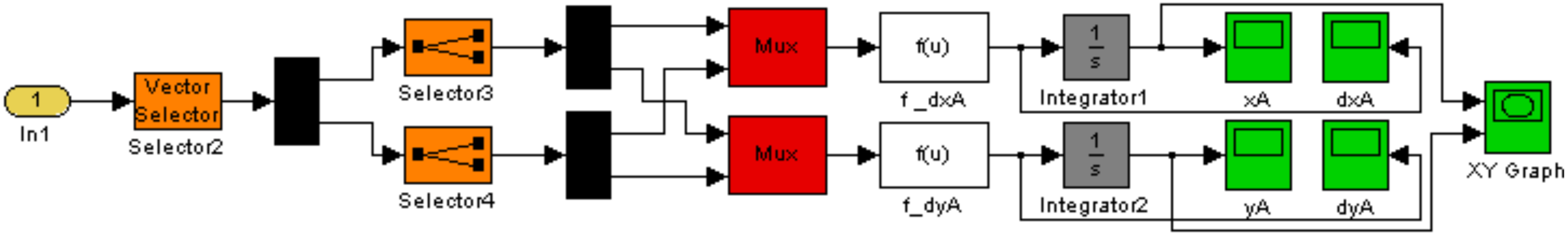


fig. Model solving simple kinematics problem

Simulation of the simple kinematic problem with the use of Matlab Simulink software

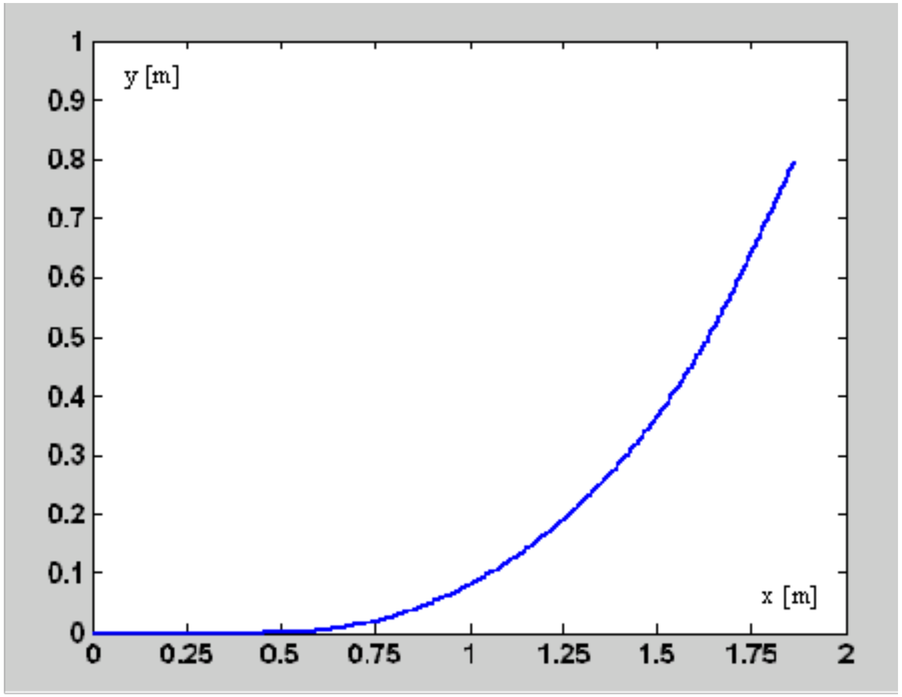
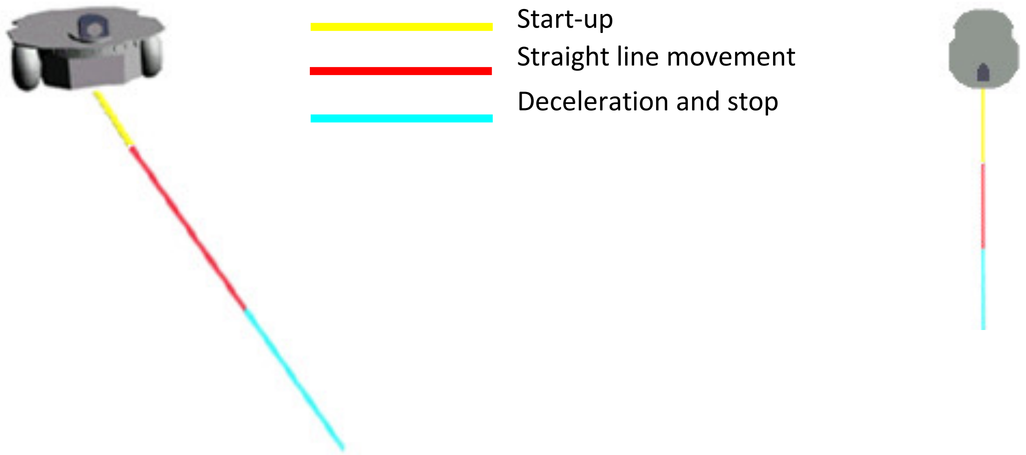


Fig. Output from simple kinematics solver - path

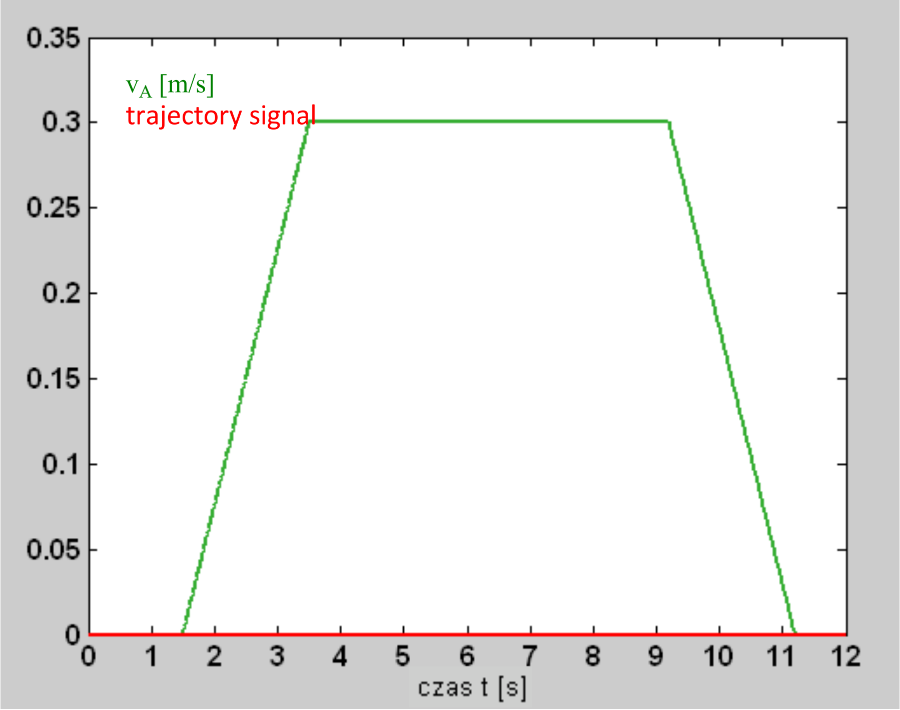
As a result we obtain previosly defined path for inverse kinematics problem. That means our model is made in a proper way.

Simulation of the inverse kinematic problem with the use of Matlab Simulink software (straight line path)

Lets assume different path – straight line

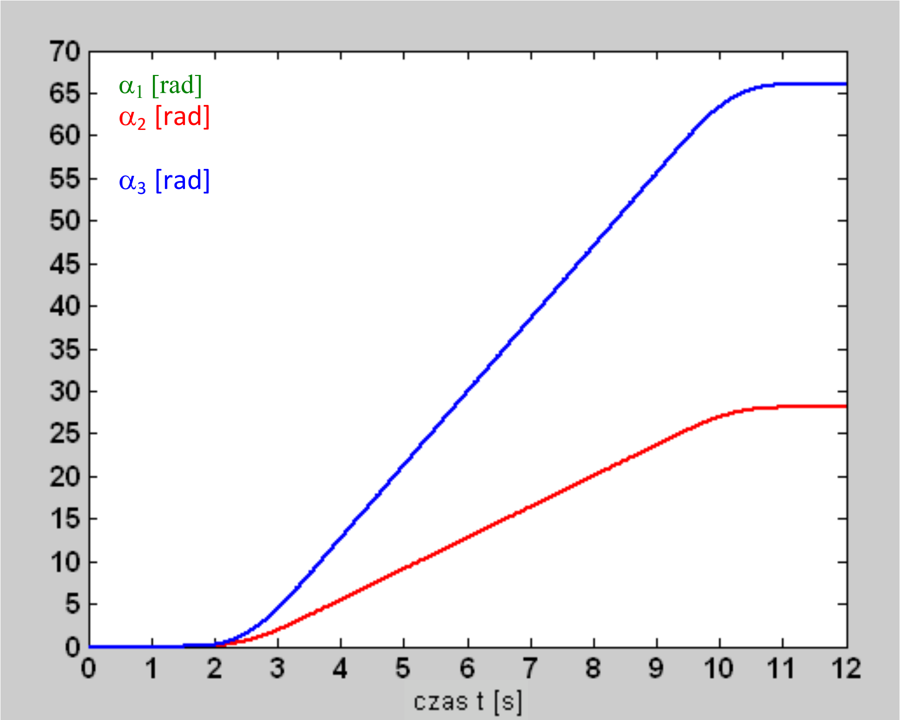


Simulation of the inverse kinematic problem with the use of Matlab Simulink software



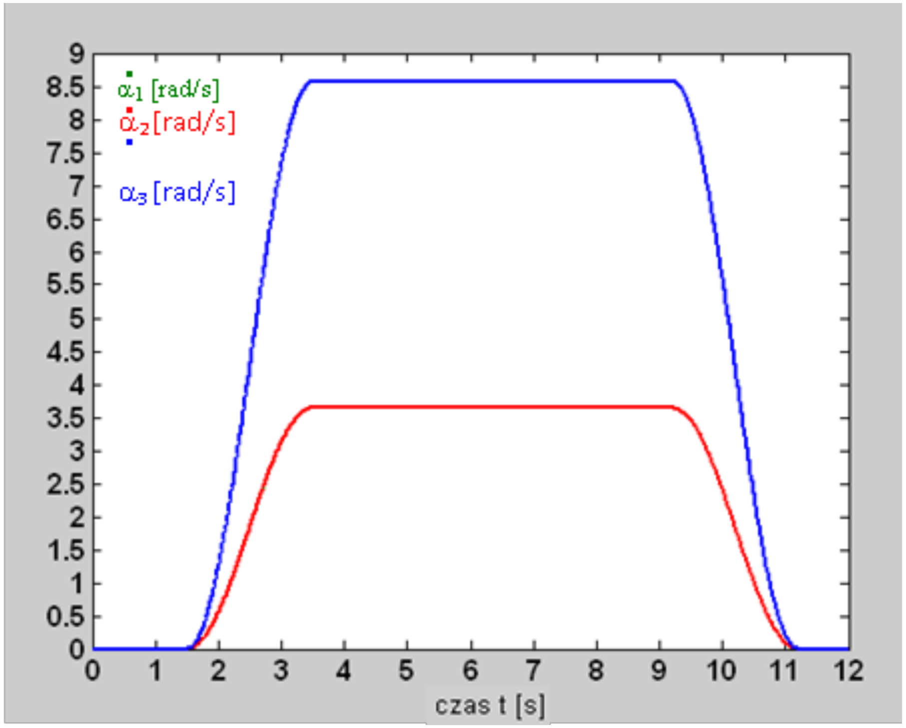
Output from speed and trajectory generator model

Simulation of the inverse kinematic problem with the use of Matlab Simulink software



Output for the angle of rotation of wheels 1,2 and 3

Simulation of the inverse kinematic problem with the use of Matlab Simulink software



Output for the angular speed of wheels 1,2 and 3

Simulation of the simple kinematic problem with the use of Matlab Simulink software

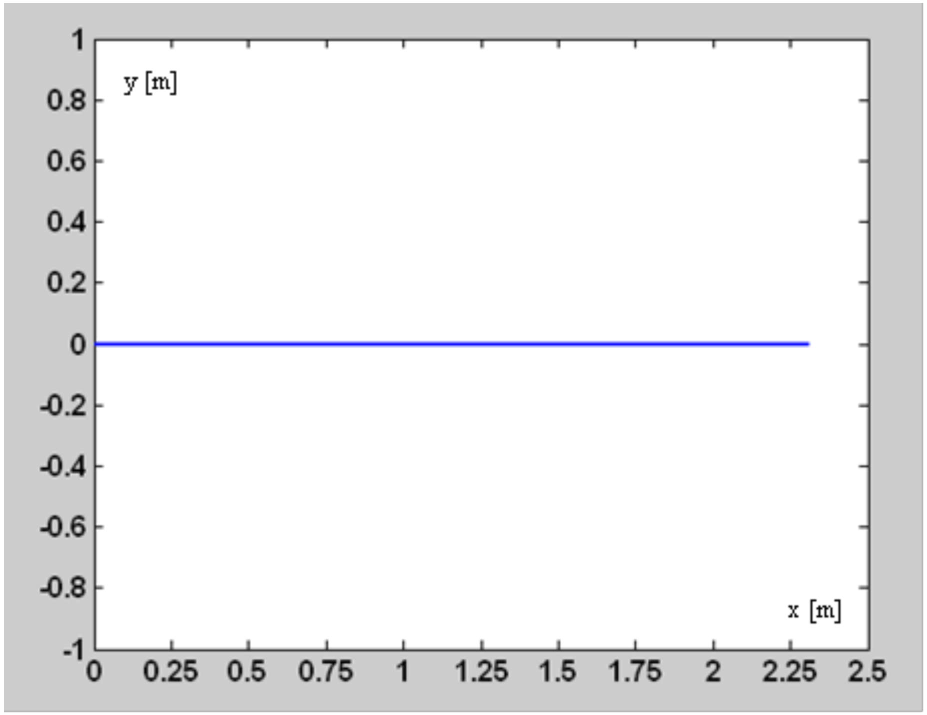


Fig. Output from simple kinematics solver - path

As a result we obtain previosly defined path for inverse kinematics problem. That means our model is made in a proper way.

THANK YOU