



Kinematics and Dynamics of Mechatronic Systems

Exercise B4

Kinematics – algorithms of path and trajectory planning

For the selected kinematic structure of a manipulator the following tasks are to be completed. The list of tasks is as follows.

1. The motion path

Plan 2 motion paths for the considered mechanism's last link.

Depending on the type of the analyzed mechanism the listed below task should be completed.

CASE A of the manipulator composed of 4 links: the last rotation axis is perpendicular to the last link.

Plan 2 paths corresponding to a manipulating operation: grasping of an object, lifting, fast motion, approach with slow down and releasing the object.

One path should be considerably longer than the other.

CASE B of the manipulator composed of 4 links: the last rotation axis is the longitudinal axis of the last link.

Plan 2 paths, one a straight-line segment and one a segment of a circle.

The paths should be **composed of 8 points** in the both cases.

During the planned motion **no joint may be motionless!** (trivial paths are unacceptable)

The joint coordinates as well as the position (x, y, z) and orientation (RPY angles) of the last link should be listed in a table for each of the planned path (2 tables to be prepared).

2. Planning of the joint trajectories

For vectors of joint variables Q_1, Q_2, \dots, Q_N corresponding to the prepared motion paths plan the spline joint trajectory of the type 5-(5)-5.

Presented below, there is an exemplary script corresponding to planning of 10 nodal points trajectory for a 3 motion axis manipulator.

```
% planning of 10 segment polynomial spline joint trajectories
% corresponding to the assumed path
Q1=[0 0.1 0.3 0.6 1.1 1.7 2.2 2.6 2.85 3.0];
Q2=[0 0.05 0.12 0.20 0.22 0.21 0.19 0.13 0.06 0];
Q3=[1.6 1.4 1.1 0.6 0 0.1 0.5 1 1.3 1.5];
% setting the duration time of each trajectory segment (in seconds)
T=[0.2,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.1,0.2];
% setting values of initial and final joint velocity as well as
% the initial and final joint acceleration (usually they are set to 0s)
V=[0 0];A=[0 0];
% planning of the initial trajectory of type 555 (evaluation of the
% coefficients)
y1=fun_path(Q1,T,V,A);
y2=fun_path(Q2,T,V,A);
```

```
y3=fun_path(Q3,T,V,A);
% setting the time axis resolution
dt=0.01
% calculate joint displacements, velocities and accelerations for 3 joints
wb1=waitbar(0,'calculate joint displacement');
[q1,v1,aa1,tt,ti]=fun_graph(y1,T,dt,'r');
i=1;waitbar(i/3,wb1)
[q2,v2,aa2,tt,ti]=fun_graph(y2,T,dt,'b');
i=2;waitbar(i/3,wb1)
[q3,v3,aa3,tt,ti]=fun_graph(y3,T,dt,'g');
i=3;waitbar(i/3,wb1)
close(wb1);
```

3. Present graphically and assess the time histories of the joint variables' displacements, velocities and accelerations (4 x 3 plots for each path)

4. Evaluate, present graphically and assess the resultant Cartesian path and the broken line resulting from composition of Q1, Q2, ... QN vectors for the both paths (in a single picture)

```
syms th1 d2 th3 a2 a3
% calculate displacement in 3D - formulas
XX=cos(th1)*(a2+a3*cos(th3))
YY=sin(th1)*(a2+a3*cos(th3))
ZZ=d2-a3*sin(th3)
% calculate the 3D path
X=double(subs(XX,{th1,d2,th3,a2,a3},{q1,q2,q3,0.1,0.5}));
Y=double(subs(YY,{th1,d2,th3,a2,a3},{q1,q2,q3,0.1,0.5}));
Z=double(subs(ZZ,{th1,d2,th3,a2,a3},{q1,q2,q3,0.1,0.5}));
% calculate the 3D broken line
XQ=double(subs(XX,{th1,d2,th3,a2,a3},{Q1,Q2,Q3,0.1,0.5}));
YQ=double(subs(YY,{th1,d2,th3,a2,a3},{Q1,Q2,Q3,0.1,0.5}));
ZQ=double(subs(ZZ,{th1,d2,th3,a2,a3},{Q1,Q2,Q3,0.1,0.5}));
```

5. Calculate the length of the joint path and of the broken line. Compare the results obtained for the both planned trajectories

The report should contain:

- 2 tables with parameter of the 2 planned paths
- plots of joint displacements, velocities and accelerations
- plots of the planned paths and broken lines joining the nodal points in 3D
- comparison of lengths of the planned 3D paths and corresponding broken lines
- conclusions

The dedicated procedures of trajectory planning, to be used during the lab class, are described in the following tables.

Function	call	Description
<i>fun_path</i>	<code>y=fun_path(X,T,V,A)</code>	<p>Determination of polynomials defining trajectory for each its segment.</p> <p>\mathbf{y} – output vector $\mathbf{y}(t)$ of spline functions corresponding to consequent segments of the planned joint trajectory.</p> <p>\mathbf{X} – vector of displacements corresponding to the beginning of the first segment followed by the ends of the consequent segments. Translational displacement should be set in meters and the rotational one in radians.</p> <p>\mathbf{T} – vector of time corresponding to the beginning of the first segment followed by the ends of the consequent segments.</p> <p>\mathbf{V} – vector containing values of velocity at the beginning and at the end of the trajectory.</p> <p>\mathbf{A} – vector containing values of acceleration at the beginning and at the end of the trajectory.</p>

<i>fun_graph</i>	<code>[xt,vt,at,t,ti]= fun_graph(y,T,dt,col)</code>	<p>Graphical presentation of displacement, velocity and acceleration of the motion described by the planned trajectory. The 3 following plots are generated:</p> <p><i>xt</i> – displacement <i>vt</i> – velocity <i>at</i> – acceleration <i>t</i> – time vector <i>ti</i> – numbers of time samples which join the consequent segments of the trajectory</p> <p><i>y</i> – output vector $y(t)$ of spline functions corresponding to consequent segments of the planned joint trajectory.</p> <p><i>T</i> – vector of time corresponding to the beginning of the first segment followed by the ends of the consequent segments.</p> <p><i>dt</i> – sampling time in seconds (it is advised to use approx. $6\div 10$ time samples for the shortest trajectory segment)</p> <p><i>col</i> – selects the curve color e.g.,</p> <ul style="list-style-type: none"> b blue g green r red c cyan m magenta y yellow k black w white
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