Kinematics and Dynamics of Mechatronic Systems

Exercise B.1. Geometrical model of a positioning mechanism

Determination of position and orientation of an end-effector in a manipulator workspace with use of the homogeneous transformation – preparation of data for further analysis

Geometrical model of a manipulator defines relationships between joint variables ($q_1, q_2, \ldots, q_n$) and Cartesian variables ($x, y, z, \phi, \theta, \psi$), that determine position and orientation of an end-effector in a manipulator workspace.
During classes a geometrical model of a selected kinematic structure should be formulated

List of tasks:

T1. For a provided kinematic diagram of a manipulator prepare a table of the geometrical model parameters in the following form.

Link No.	θ_i	d_i	a_i	α_i	ranges
…	…	…	…	…	…
2	θ_2	0	a_2	-90°	-120°÷120°
…	..	..	..	..	..

List values of the model constants and ranges of motion of each link (ranges of the joint variables' values).
It should be noted that for the reference links' setting the values of the rotating joint variables are set to zeros.

T2. Next, basing on the prepared table of parameters, evaluate symbolic form of:

- ${}^{0}\underline{T}_{3}$ – a matrix describing position and orientation of a local coordinate frame assigned to the 3rd link (the last link of the manipulator arm)
- ${}^{3}\underline{T}_{e}$ – a matrix describing position and orientation of the end-effector with respect to the frame assigned to the link No. 3
- $\boldsymbol{p}_a$ – a position vector of the manipulator arm tip with respect to the reference frame is evaluated as a product of ${}^{0}\underline{T}_{3}$ matrix and ${}^{3}\boldsymbol{p}_a$ vector (the position vector of the manipulator arm tip with respect of the local frame assigned to the 3rd manipulator link; usually ${}^{3}\boldsymbol{p}_a=[0,0,0,1]^T$ vector, when the local frame No. 3 is located at the tip of the link No. 3)

The above indicated calculations should be carried out with use of functions of the „model" library prepared for use in Matlab and using functions of Symbolic Toolbox.
If the functions of the "model" library are not available, please download the files from the course's webpage, save it on a disk, and make the library available - an appropriate path should be added to the list of Matlab paths.

Below there is presented an example of evaluation of ${}^{0}\underline{T}_{3}$ matrix for a RRP kinematic structure.

```
% this script determines a Homogeneous Transformation matrix
clear all
% declaration of symbols
syms th1 a1 th2 a2 a3 q1 q2
% determination of a symbolic form of HT matrices –
% application of mA function
A1=mA(th1,0,a1,0)
A2=mA(th2,0,a2,0)
A3=mA(0,0,a3,sym(pi/2))
% multiplication of matrices
T03=A1*A2*A3
% substitution of rotational joint variables
% for the simplification purpose
T03v=subs(T03,{th1,th2},{q1,q2});
% indication of joint coordinates
% variables: th1,th2 and a3 indicated by '1's
zmie=[[1,0,0,0];[1,0,0,0];[0,0,1,0]]
% a simplified form of the evaluated HT matrices
% for interpretation purpose for a user
T03u=zam(zmie,T03v,'q')
```

T3. Assume a set of values of all the joint variables in their operational ranges (except zero or a multiple of the right angle). Next, calculate the exemplary values of the ${}^{0}\underline{T}_{e}$ matrix elements by substitution of numerical values for the symbols.

```
% example of substitution of the join variables values
% and constant values into the T0e matrix for the RRP manipulator example
% please use meters and radians
T03n=subs(T03,{th1,a1,th2,a2,a3},{pi/6,0.6,pi/4,0.4,0.4})
```

T4. Present the considered mechanism graphically for the selected set of values of the joint variables. Indicate all joint variables, the reference frame and position of the end-effector (*X*, *Y* and *Z* coordinates) on the drawing.

All the obtained results should be presented in the report. The report concerning tasks: 1, 2, 3 should be delivered to the tutor at the end of the class. The report of task 4 should be sent to the tutor's e-mail address.

Names of the sent files (formats *.pdf, *.doc, *.m, *.rar, *.zip itp.) should be created in the following way:

Surname1_Surname2_*GR*_*n*…..

where: *GR* – is an identifier of the group, *n* – is a number of an exercise

Remarks.

A list of procedures to be used in the geometrical model formulation:

Function	Call	Description
mA	mA(θ,a,d,α)	Determination of the homogeneous transformation matrix A ***Θ-*** angle of rotation around ***z*** axis ***a-*** displacement along *z* axis ***d-*** displacement along *x* axis ***α-*** angle of rotation around *x* axis
zam	zam(zmie,M,’q’)	This procedure simplifies the form of a homogeneous transformation matrix M by substitution of S1 instead of sin(q1), S2 instead of sin(q2), C1 instead of cos(q1), S12 instead sin(q1+q2), C12 instead of cos(q1+q2) and S1_2 instead of sin(q1-q2) etc. ***zmie* –** is a matrix defining which parameter of the geometrical model is a variable (value 1) and which is constant (value of 0). Each row of the *zmie* matrix corresponds to one row of the homogeneous matrix M ***M* –** a matrix that is to be simplified
subs	subs(Y,OLD,NEW)	The procedure is intended for substitution of a symbol or a set of symbols (NEW) instead of another symbol or another set of symbols (OLD). A set of symbols is listed in brackets e.g. {q1,q2} **Y** – an expression in which the symbols are to be substituted