AGH	Faculty of Mechanical Engineering and Robotics Department of Robotics and Mechatronics	R&M
Identification of Mechatronic Systems		
Lab 10: Signal modulations applied to vibration measurement		

1. Introduction

Modulation is a periodic change of signal characteristics, i.e. its amplitude, frequency or phase. The signal that is modulated is called a carrier signal or a carrier wave and is usually a constant frequency signal. In telecommunications, modulations are used to transmit low-frequency useful signals over long distances by modulating them with a high-frequency carrier wave. In laser techniques, modulations are used to measure vibrations. In vibrating mechanisms, modulations are a frequent and usually undesirable phenomenon, as they hinder the proper interpretation of the system's characteristic frequencies.

Let's assume a carrier frequency signal $f_c$in the following form:

$$y(t) = Acos(2\pi f_c t + \phi)$$

The amplitude modulation of this signal with a sine wave $\sin(2\pi f_m t)$takes the following form:

$$y_{am}(t) = A\,sin(2\pi f_m t)\,cos(2\pi f_c t + \phi)$$

The phase modulation of the signal $y$can have the following form:

$$y_{pm}(t) = A\cos(2\pi f_c t + A_m\,sin(2\pi f_m t) + \phi)$$

The frequency modulation of the signal $y$can take the following form:

$$y_{fm}(t) = A\cos(2\pi f_c t + 2\pi A_m \int_0^t sin\,(2\pi f_m t)dt + \phi)$$

There are signal processing techniques that allow you to recover the modulating signal $sin\,(2\pi f_m t)$(performing the so-called demodulation) or remove the modulation if we consider it undesirable. The following activities provide examples of how these signal processing techniques can be used.

2. Amplitude modulations

Create a 1 kHz amplitude modulated (AM) carrier signal with a linear chirp signal from 100 Hz to 200 Hz . The signals should be 1 second long and 10 kHz sampling rate:

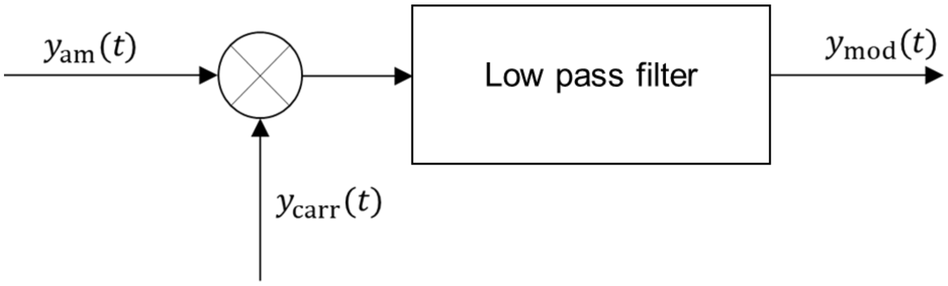
```
fs = 1e4;
t = 0:1/fs:1;
y_mod = 0.5*chirp(t,50,t(end),200);
y_carr = sin(2*pi*t*1e3);
y = y_mod .* y_carr+y_carr ;
```

Task 2.1 Removing modulation

The amplitude modulations of the signal can be removed by separating the signal by its envelope, obtained using the Hilbert transform. Calculate the envelope of the **y signal** and then remove modulations. Display frequency-amplitude plots showing the spectra of the **y- signal** and the signal after modulation has been removed. Comment on the effect of modulation removal on the signal spectrum.

Task 2.2 Demodulation of a signal

There are different methods to perform AM demodulation. One of the simpler methods is to multiply the modulated signal $y_{am}(t)$by the carrier frequency signal $y_{carr}(t)$, and then low-pass filter the modulation signal bandwidth.



According to the diagram above, demodulate the **y signal** . Put on a low-pass filter with a cut-off frequency of 400 Hz . You can use the filter designed in **filterDesigner or the lowpass** function **:**

```
y_filt = lowpass( y_demod,fpass,fs );
```

where **y_demod** is the **y signal** multiplied with the carrier signal, **fpass** is the cut-off frequency and **fs** is the sampling rate. Then remove the constant component from the signal by subtracting the average:

```
y_filt = y_filt - mean( y_filt );
```

To make the end result better match the **y_mod signal** multiply **y_filt** by two, because as a result of filtration we lose the left-hand modulation band. Compare the received signal with the modulation signal in the time and frequency domain.

From task 2.1, the amplitude-time and amplitude-frequency diagrams showing the comparison between the modulated signal and the signal after modulation removal should be included. Discuss how amplitude modulations affect the signal spectrum.

From problem 2.2, an amplitude-frequency plot showing the ideal **y_mod modulating signal should be included** and a signal obtained by demodulation. Discuss the effectiveness of the method.

The report should include the Matlab code used to perform the above tasks.

3. Phase modulations

Create a 10 kHz carrier signal frequency modulated with a 500 Hz signal . The signals should be 1 second long and 200 kHz sampling rate:

```
fs = 2e5;
fc = 1e4;
t = 0:1/fs:1;
y_mod = sin(2*pi*t*500);
y_carr = sin(2*pi*t*fc);
y = sin(2*pi*t* fc+y_mod );
```

Exercise 3

As with amplitude modulation, there are many ways to perform frequency demodulation. One method uses a signal derivative and an envelope detector. Let's assume a carrier frequency signal $\omega_c$with phase modulation by a sine signal with frequency ω_m:

$$y_{pm}(t) = A\sin(\omega_c t + \sin(\omega_m t))$$

If we take the time derivative of this signal, we get the following equation:

$$\frac{dy_{pm}(t)}{dt} = A\cos(\omega_c t + \sin(\omega_m t)) * (\omega_c + \omega_m \cos(\omega_m t))$$

You will notice that the modulation signal appears on the envelope of the new signal, denoted below as $A_{pm}(t)$:

$$\frac{dy_{pm}(t)}{dt} = A * \left(\omega_c + \cos\left(\omega_{pm} t\right)\right) * \cos(\omega_c t + \sin(\omega_m t)) = A_{pm}(t)\cos(\omega_c t + \sin(\omega_m t))$$

Next, an envelope detector can be used to frequency demodulate the signal.

In Matlab, frequency demodulation can be done by first calculating the approximate derivative of the signal using the **diff function** (where **y** is the modulated signal and **dt** is a time step) **:**

```
y_diff = diff(y)/dt;
```

Then, calculate the envelope of the **y_diff signal (** variable **y_env)** and remove the constant component **:**

```
y_env = y_env - mean( y_env );
```

Note that the received signal will be shifted by 90 degrees and rescaled as a result of calculating the derivative. You can compensate for this delay by integrating the signal. For this purpose, integration in the frequency domain can be performed using the included **inteFD function** :

```
y_dem = inteFD (y_env,1/fs);
```

where **fs** is the sampling rate.

Perform frequency demodulation of the given signal. Compare the waveform and spectrum of the final signal with the ideal modulating signal and describe the differences between them. Be sure to compensate for the phase shift of the signal. In addition, discuss the effect of frequency modulation on the signal spectrum and compare it with the effect of amplitude modulation.

As part of task 3, you should include amplitude-frequency plots showing the comparison between the ideal modulating signal and the signal obtained using demodulation. Comment on the effectiveness of the method used. Discuss how frequency modulations differ from amplitude modulations and how they affect the signal spectrum.

The report should include the Matlab code used to complete the task.

4. Measurement of surface vibrations using demodulation

The experimental setup described in the instructions for Laboratory 11 was used to measure mechanical vibrations (Figure 1). The plate previously serving as a wave deflector was replaced with a small glass plate attached to the modal driver (Figure 2). The exciter is powered by a sinusoidal signal from the generator, thanks to which it performs vibrations of a given amplitude and frequency. The ultrasonic signal with a frequency of 100 kHz emitted by the air-coupled transducer was directed to the plate, and after being reflected from it was registered by the second transducer. Due to surface vibrations, the phase of the ultrasonic wave reflected from the vibrating reflector is modulated according to the vibration frequency. During the experiment, plate vibrations were recorded for frequencies of 40 Hz , 100 Hz and 150 Hz .

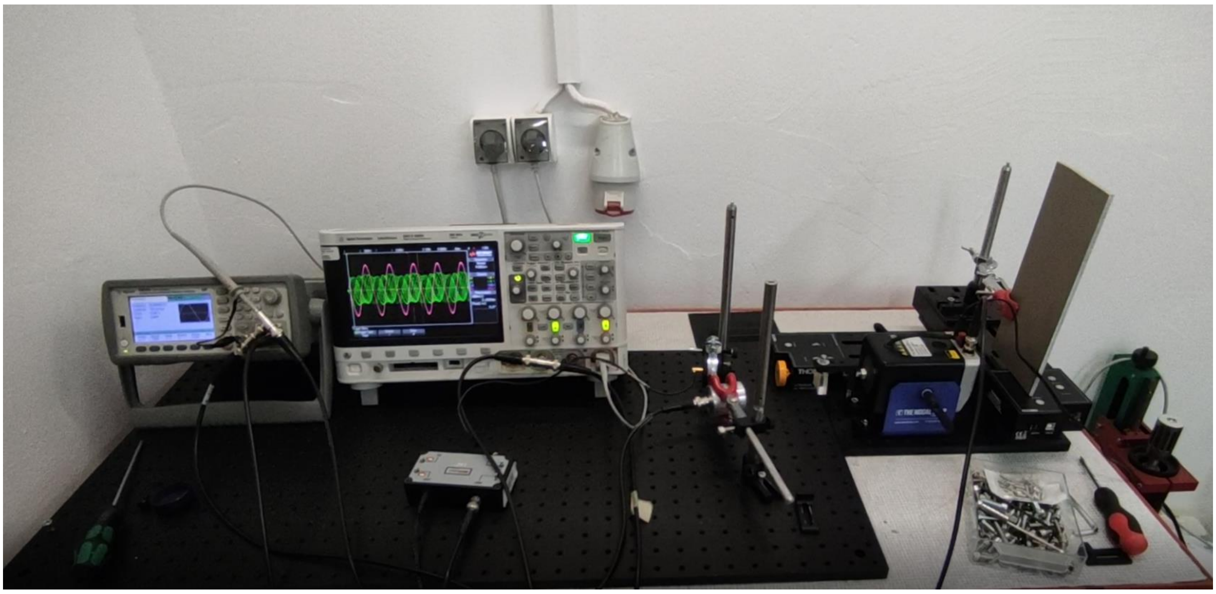


Figure 1: Photograph of the experimental setup for measuring vibrations

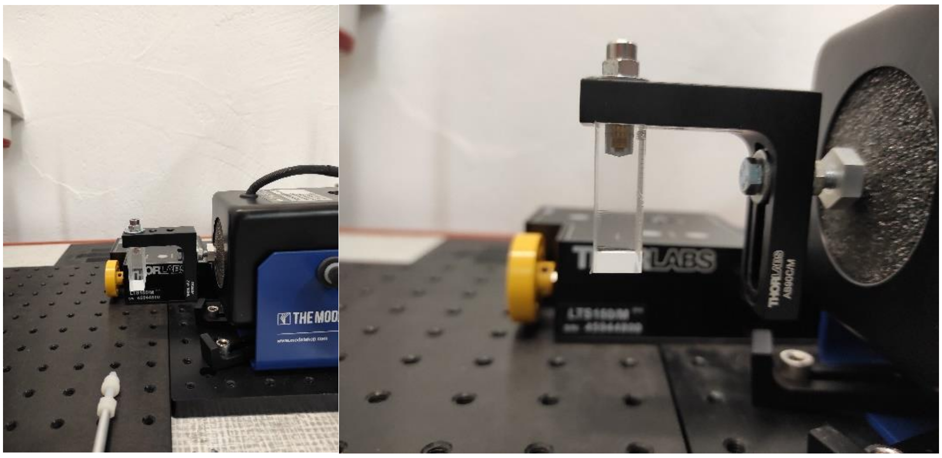


Figure 2: Pictures of a reflector attached to the end of a modal driver

Task 4

Based on the signal measured on channel A, the vibration frequency of the plate surface should be estimated. For this purpose, the following signal processing scheme should be used:

1. Perform a narrowband filter around the 100 kHz carrier frequency to remove noise from the signal while leaving modulation bands. You can use the **bandpass function** with the following parameters:

```
y = bandpass(y,[0.8e5,1.2e5],fs);
```

2. Remove the amplitude modulations of the **y signal** (as in problem 2.1)
3. Perform frequency demodulation using derivative and envelope estimation as in Task 3.
4. Remove the constant component and perform low pass filtering with a cutoff frequency of around 800 Hz .
5. Remove signal processing artifacts at the start and end of the signal by clipping the data vector (sampling).
6. Apply integration using the **inteFD function** as in Exercise 3 **.** If a low-frequency trend is still visible on the received time waveform, it can be removed using the **detrend (y,N) function. y** is the signal from which the trend is removed, and **N** is the order of the trend polynomial (**N = 1 by default**). If the default value **of N** does not remove the trend from the signal, try using higher values.

Display waveforms and spectra of vibration graphs obtained by demodulation for 40Hz.mat, 100Hz.mat and 150Hz.mat files. In order to verify the correctness of the measurement, compare the obtained result with the time course for the frequency of 40 Hz from the file 40Hz_ref.mat. The file contains the measured signal vector **y_meas and** the corresponding time vector **t_meas .** For a better comparison of the spectrum of the signals for the 40 Hz excitation , apply windowing to remove spectrum leakage. When assessing the correctness, take into account not only the fundamental frequency of the excitation, but also its harmonics (multiples of the fundamental frequency).

As part of task 4, time and amplitude-frequency diagrams showing vibration signals obtained as a result of demodulation should be included. Post a comparison with a reference measurement for a 40 Hz excitation . Discuss the differences between a reference measurement and a processing result.

The report should include the Matlab code used to complete the task.