GROUP WORK PROJECT 1
GROUP NUMBER: 9427

MScFE 620: Derivative Pricing

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Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).	
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Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed. **Note:** You may be required to provide proof of your outreach to non-contributing members upon request.
N/A

Step 1

1. Does put-call parity apply for European options? Why or why not?

Yes, put-call parity applies to European options, even when priced using the binomial tree model.

This is because the put-call parity is derived from the absence of arbitrage opportunities in a frictionless market. It holds for European options since they can only be exercised at maturity, allowing for a clean comparison between calls, puts, and the underlying asset discounted at the risk-free rate. In a binomial tree framework, if constructed correctly and with a sufficiently large number of steps, the model replicates the theoretical pricing and maintains the arbitrage-free relationships.

2. Rewrite put-call parity to solve for the call price in terms of everything else.

Starting with the standard put-call parity equation:

$$C - P = S - K_e^{-rT}$$

Solving for the call price *C*:

$$C = P + S - K_e^{-rT}$$

3. Rewrite put-call parity to solve for the put price in terms of everything else.

Using the same equation:

$$C - P = S - K_e^{-rT}$$

Solving for the put price *P*:

$$P = C - S + K_e^{-rT}$$

4. Does put-call parity apply for American options? Why or why not?

No, put-call parity does not strictly apply to American options.
This is due to the early exercise feature of American options, which introduces additional value (i.e., optionality) that breaks the strict parity relationship.

Examples:

- An American put can be worth more than its European counterpart if early exercise is optimal, especially for deep in-the-money puts.
- An American call on a non-dividend-paying stock is generally not exercised early, so its price is usually equal to that of the European call. In this specific case, put-call parity may approximately hold.

Conclusion: The put-call parity formula is based on fixed expiration payoffs and assumes no early exercise. Since American options allow for early exercise, the conditions for put-call parity no longer strictly hold.

To conclude Step 1, the team has included the code implementation to address the four questions regarding put-call parity within the binomial tree model framework.

The code provided is designed to support the theoretical answers with numerical evidence. Specifically, it:

- Validates put-call parity for European options using simulated real data.
- Demonstrates the breakdown of parity in American options due to the early exercise feature.
- Provides quantitative results that can be presented clearly in a report or professional setting.

For the next set of questions, assume the following values and parameters:

$S_0 = 100; r = 5\%;\ \sigma = 20\%; T = 3\,months$

Team Member A will work with European calls and puts using a binomial tree:

5. Price an ATM European call and put using a binomial tree:

a. Choose the number of steps in the tree you see convenient to achieve reliable estimates.

We choose 100 steps for the binomial tree. This number ensures sufficient accuracy and convergence to the theoretical price (i.e., close to Black-Scholes) while keeping computations efficient. Fewer steps (e.g., 10 or 20) can lead to pricing errors, while too many steps (e.g., 1000+) add computational overhead with marginal accuracy improvement.

b. Briefly describe the overall process, as well as a reason why you choose that number of steps in the tree.

i. Divide the total time to maturity into small intervals:

$$\Delta t = \frac{T}{N} = \frac{0.25}{100} = 0.0025$$

ii. At each step, the stock can go up by a factor $u = e^{\sigma\sqrt{\Delta t}}$ or down by $d = 1/u$

iii. Calculate the risk-neutral probability $p = \frac{e^{r\Delta t} - d}{u - d}$

iv. Build the stock price tree forward.

v. At maturity, calculate the payoff for the call and put:

a. Call: $\max(S_T - k, 0)$

b. Put: $\max(k - S_T, 0)$

vi.. Discount back using the risk-neutral probabilities to obtain the present values.

6. Compute Delta for European call and put at time 0

a. How do they compare?

Call option delta will be positive and less than 1 i.e. 0.52.

Put option delta will be negative and greater than –1 i.e. –0.43

Since we are at-the-money (ATM), the magnitudes are approximately symmetric.

b. Comment briefly on the differences and signs of Delta for both options. What does delta proxy for? Why does it make sense to obtain a positive/negative delta for each option?

- Delta measures how much the option's price changes for a small change in the underlying stock price:

$$\Delta \approx \frac{V(S+\varepsilon) - V(S-\varepsilon}{2\epsilon}$$

Call Delta is positive: the price of a call increases when the stock price increases.
Put Delta is negative: the price of a put decreases when the stock price increases.

Delta proxies for directional exposure:

- For a call, owning it gives upside potential → positive delta.
- For a put, it benefits from price drops → negative delta.

7. Delta measures one sensitivity of the option price. But there are other important sensitivities we will look at throughout the course. An important one is the sensitivity of the option price to the underlying volatility (vega)..

a. Compute the sensitivity of previous put and call option prices to a 5% increase in volatility (from 20% to 25%). How do prices change with respect to the change in volatility?

Let's assess the impact of changing σ\sigmaσ from 20% → 25%.

- When volatility increases, both call and put prices increase.

- Why? Because options benefit from greater uncertainty, more chance of being in-the-money.

b. Comment on the potential differential impact of this change for call and put options.

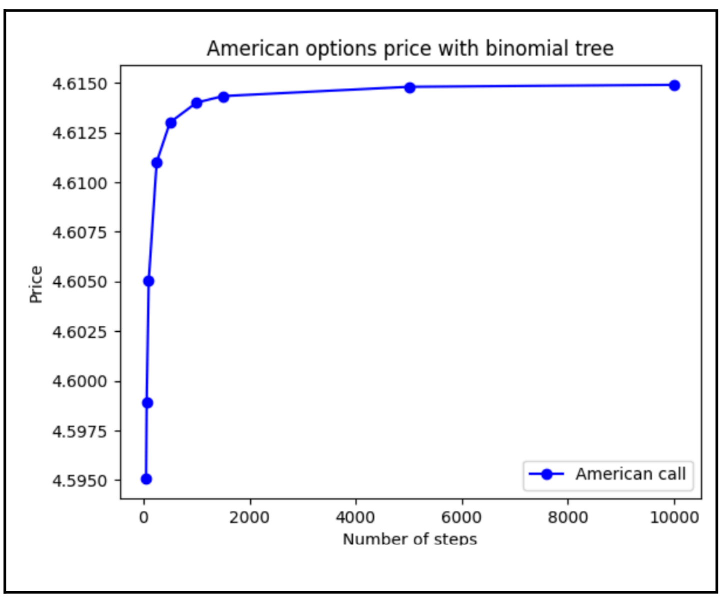
1. Both options are vega-positive: their values rise with volatility.
2. The put may show slightly more sensitivity in some cases due to payoff asymmetry, especially when slightly OTM.
3. Higher Vega implies greater sensitivity to market uncertainty.

8. **Price an ATM American call and put using a binomial tree**

   a. The convenient number of steps used is 62 since it is a good compromise between accuracy and computational cost. In fact each step move could be assumed to correspond to a “daily move” so that it aligns with real word stock price movement. Thus 62 = 250*maturity.

With the given parameters we obtain: a **call price** of **4.61** and a **put price** of **3.47**. The American call price is similar to the European call price while the American put price is greater than the European put price.

   b. To obtain that number, we price a call option for a different number of steps (50, 62, 100, 250, 500, 1000, 1500, 5000, 10000) and choose the lowest with less accuracy change as shown below. Actually 62 is the best choice to make.



9. **Compute Delta for American call and put at time 0**

a. The **call delta** computed is **0.57** which is positive and lower than 1. This delta equals the European delta for the call option with the same parameters.

   The **put delta** computed is **-0.45** which is negative, less than one and more negative than the equivalent European put delta (-0.43) in absolute value.

b. The delta proxies for directional exposure since it indicates how much the option price's changes when the underlying stock price changes. The exposure on an american call option is the same as the exposure on an european call whereas the exposure on an american put (delta = -0.45) is more negative than the exposure on a european put (delta = -0.43). The later observation shows that the american put is more sensitive to stock's price movement than the european put option due to early exercise, increasing sensitivity to stock price decreases.

10. **Sensitivity of the option price to the underlying volatility (vega)**

a. When assessing a change of volatility from 20% to 25%, the American call vega is 19.59 greater than the American put vega which is 19.54. Both options are vega-positive.

b. The impact of volatility change on American option prices differs, unlike on the European option prices. The American put is more sensitive to volatility than the American call. Moreover, American call vega and European call vega are equal.

11. European call and put satisfy put-call parity

a. Proof:

Put-Call Parity: For European options, $\mathbf{C + Ke^{-rT} = P + S_0}$

Based on our results from Q7 prices (European call: 4.605026, put: 3.362806) :

Left: = 4.605 + 100.e$^{-0.05.0.025}$ = 103.3587

Right: P + S0 = 3.362806 + 100 = 103.362806

Difference: negligible/near-zero - this confirms parity within numerical precision

b.) Why Parity holds:

Put-call parity holds for European options due to arbitrage-free pricing. A portfolio of a call plus a risk-free bond K e^{-rT} replicates a put plus the stock and this thereby ensures price equality to prevent arbitrage.

Our prices indeed satisfy parity, validating our binomial tree for European options.

Our motives for Answering "Yes" were:

- Correct understanding of European option pricing theory. We recognize that no early exercise affects parity.This also aligns with standard financial theory thereby ensuring consistent pricing.

Q12. Why put-call parity does NOT hold for American options

Verification:

Put-Call Parity: For American options, parity (C + Ke^-rT = P + S0) does not strictly hold due to early exercise.

Using our Question 8 prices (American call: 4.61, put: 3.47):

- **Left:** $C + Ke^{-0.05*0.25} \approx 4.61 + 98.7537 = 103.3637$
- **Right:** $P + S_0 = 3.47 + 100 = 103.47$
- **Difference:** 103.47 - 103.3637 ≈ 0.1063, not zero, indicating parity does not hold.

Why Parity Does Not Hold:

Our motives for answering "No": Our recognition that early exercise, especially for puts, disrupts parity, aligning with theory.

American options allow early exercise, breaking the replication strategy. The American put's early exercise premium (like 3.47 > 3.362806) increases its value, causing P + S0 > C + Ke^-rT.

For non-dividend-paying stocks, American calls are not exercised early, so their value equals European calls (4.61 ≈ 4.605026), but American puts are higher, violating parity.

Q13. Confirmation that the European call is less than or equal to the American call

Prices:

- Our European call prices: Question 7: 4.605026; American call (Question 8): 4.61.
- Black-Scholes: European call: ~4.582314

Comparison:
Our European call (4.605026) ≤ American call (4.61), with a difference of about 0.005.
Confirmation:
$C_{European} <= C_{American}$ holds.

Difference and Reasons:

- Difference: Our prices show a small difference (approx. 0.005), likely due to numerical variation i.e. 62steps vs. 100 steps. Using 100 steps, we could confirm that no difference for non-dividend-paying stocks, as early exercise is not optimal for American calls
- Reason: American calls have the option to exercise early, but for non-dividend-paying stocks, holding to maturity maximizes value which makes $C_{European} = C_{American}$

Q 14: Confirmation that the European put is less than or equal to the American put and this always being the case

Verification:

Prices:

- Your prices: European put (Question 7): **3.362806**; American put (Question 8): **3.47**.
- My prices (100 steps): European put: **3.362806**; American put: **3.390506**.
- Black-Scholes: European put: **~3.834315**.

Comparison: Our European put (**3.362806**) < American put (**3.47**), with difference **approx. 0.1072**.
Confirmation: $P_{Eurpoean} <= P_{American}$ holds.

Difference and Reasons:

Difference: Our difference (**0.1072**) is relatively reasonable, reflecting the early exercise premium.

Reason: American puts can be exercised early, adding value when the stock price is low (e.g., (K - S_t)). For non-dividend-paying stocks, early exercise is optimal when deep in-the-money, increasing American put value i.e. **3.47 > 3.362806**.

Always the case?:

Yes. $P_{Eurpoean} <= P_{American}$, as American puts include early exercise rights, adding non-negative value.
Difference exists for non-dividend-paying stocks, as early exercise is valuable for puts, unlike calls. For dividend-paying stocks, the difference persists or increases, as low stock prices favor early exercise.

STEP 2 continues on the next page.

Step 2: Option pricing using trinomial tree

15. **Pricing European call options using trinomial tree**

   a. prices the European call option price corresponding to the 5 different strikes selected.

Strike	90.0	95.0	100.0	105.0	110.0
Eur call price	11.67	7.71	4.61	2.48	1.19

   b. The call price decreases as strikes increase from deep ITM to deep OTM. In fact when the option is deep ITM, the option has a high intrinsic value and a high probability of expiring ITM, which justifies a higher call price. Conversely, when the option is deep OTM, the probability that the stock price will rise enough for the option to expire ITM is very low resulting in a lower price. This trend reflects the fundamental logic of option pricing.

16. **Pricing European put options using trinomial tree**

   a. prices the European put option price corresponding to the 5 different strikes selected.

Strike	90.0	95.0	100.0	105.0	110.0
Eur put price	0.55	1.53	3.36	6.18	9.83

   b. The put price increases as strikes increase from deep ITM to deep OTM. The put option becomes more valuable as strike increases because the right to sell at a higher price becomes more valuable. This perfectly aligns with theory.

17. **Pricing American call options using trinomial tree**

   a. prices the American call option price corresponding to the 5 different strikes selected.

Strike	90.0	95.0	100.0	105.0	110.0
Am call price	11.67	7.71	4.61	2.48	1.19

   b. The call price decreases as strikes increase from deep ITM to deep OTM. We can see that the trend is similar to the trend of European call option prices. Since early exercise of an American call option on a non-dividend-paying stock typically does not provide additional benefit compared to holding the option until maturity, the observed similarity in price trends confirms that early exercise of American calls is generally suboptimal.

18. Pricing American put options using trinomial trees.

   a. prices the EAmerican put option price corresponding to the 5 different strikes selected.

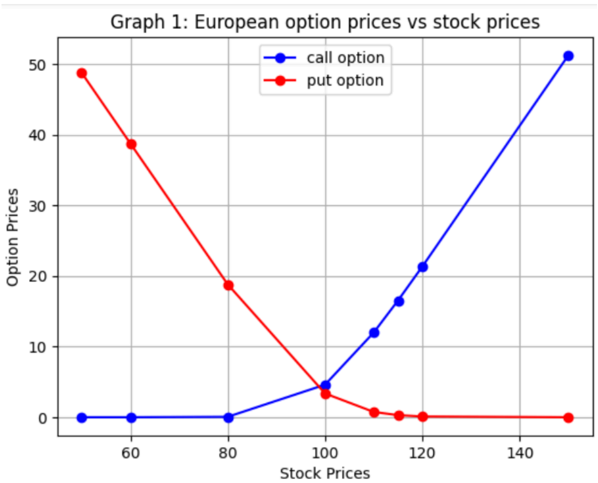
Strike	90.0	95.0	100.0	105.0	110.0

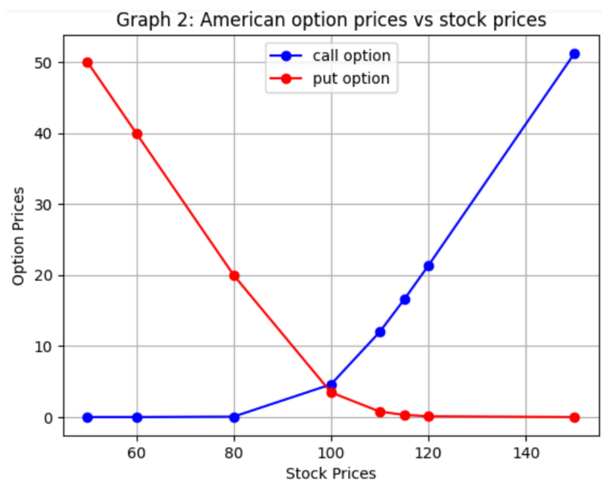
Am put price	0.56	1.57	3.47	6.42	10.33

b. The price of American put options increases with strike price, consistent with the behavior of European puts. However, American put options are priced higher due to the added flexibility of early exercise, which is valuable particularly for in-the-money positions. This justifies the observed premium in American put prices compared to their European equivalents.

19. Graph #1. Graph European call prices and put prices versus stock prices.

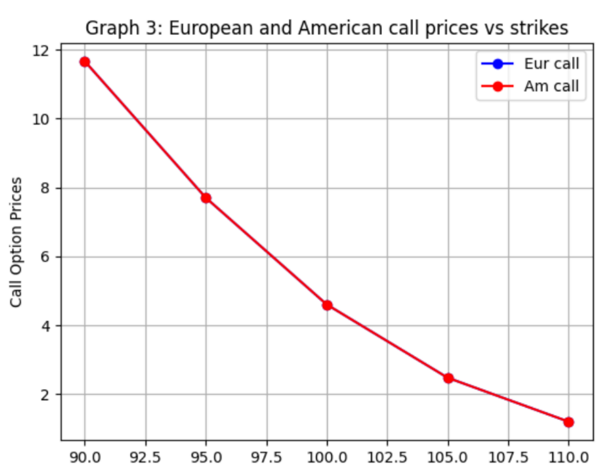
20. Graph #2. Graph American call prices and put prices versus stock prices.

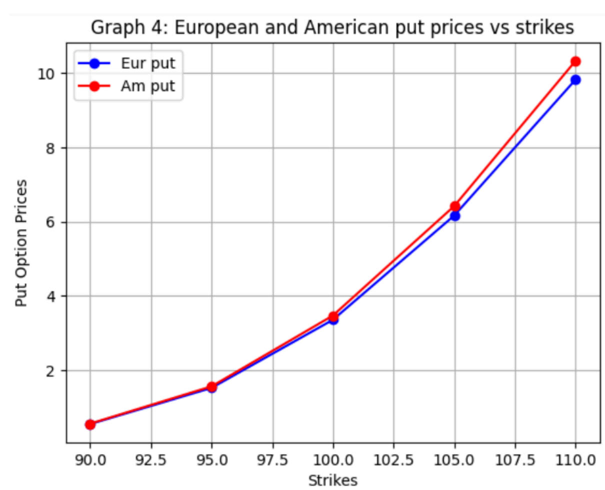




21. Graph #3. Graph European and American call prices versus strike.

22. Graph #4. Graph European and American put prices versus strike.





23. **For the 5 strikes that your group member computed in Q15 and Q16, check whether put-call parity holds (within sensible rounding). Briefly comment on the reasons why/why not this is the case.**

For European options(Q.15-16), put-call parity holds exactly as shown in the table below, confirming the correctness of pricing and the theoretical relationship.

Strike	90.0	95.0	100.0	105.0	110.0
Call + K*e^(-rT)	100.55	101.53	103.36	106.18	109.83
put + S	100.55	101.53	103.36	106.18	109.83

24. **For the 5 strikes that your group member computed in Q17 and Q18, check whether put-call parity holds (within sensible rounding). Briefly comment on the reasons why/why not this is the case.**

For American options(Q.17-18), put-call parity does not hold due to the possibility of early exercise — especially for deep ITM puts — which adds value to the put option and breaks the parity relationship.

Strike	90.0	95.0	100.0	105.0	110.0
Call + K*e^(-rT)	100.55	101.53	103.36	106.18	109.83
put + S	100.56	101.57	103.47	106.42	110.33

Step 3: See next page for Step 3

Step 3:

25. Dynamic Delta Hedging. Use the following data: S0=180, r =2%, sigma=25%, T=6 months, K = 182:

a. Price a European Put option with the previous characteristics using a 3-step binomial tree (you do not need code for this).

To determine the price of a European put option using a 3-step binomial tree, we used the following parameters: S0=180, r=0.02, σ=0.25, T=0.5 years, and K=182.

Step-by-Step Calculation

- **Number of Steps (**n**)**: 3
- $\Delta t = T/n = 0.5/3 = 0.1667$
- $U = e^{0.25\sqrt{0.1667}} = 1.1075$
- $d = 1/u = 0.9030$
- p: $\frac{e^{0.02\,.\,0.1667} - d}{u - d} = 0.4906$, $1 - p = 0.5094$
- **Discount Factor**: $df = e^{-r\Delta t} = e^{-0.003334} = 0.99667$

2. Stock Price Tree

The stock price at each node is calculated as Si,j=S0 · uj · di−j, where i is the step (0 to 3) and j is the number of up moves (0 to i).

- **Step 0**: $S_{0,0}=180$
- **Step 1**:
  - Up: $S_{1,1}=180 \cdot 1.1075 = 199.35$
  - Down: $S_{1,0}=180 \cdot 0.9030 \approx 162.54$

  Step 2:
  - Up-Up: $S_{2,2}=180 \times 1.10752 = 220.70$
  - Up-Down: $S_{2,1}=180 \times 1.1075 \cdot 0.9030 = 180.00$
  - Down-Down: $S_{2,0}=180 \times 0.90302 = 146.82$
- **Step 3**:
  - Up-Up-Up: $S_{3,3} = 180 \cdot 1.10753 = 244.46$
  - Up-Up-Down: $S_{3,2} = 180 \cdot 1.10752 \cdot 0.9030 = 199.35$
  - Up-Down-Down: $S_{3,1} = 180 \cdot 1.1075 \cdot 0.90302 = 162.54$
  - Down-Down-Down: $S_{3,0} = 180 \cdot 0.90303 = 134.54$

Put Option Value Tree (European, so no early exercise)

- **Step 3 Payoffs**:
  - $S_{3,3}$=244.46: max(182−244.46,0)=0
  - $S_{3,2}$=199.35: max(182−199.35,0)=0
  - $S_{3,1}$=162.54: max(182−162.54,0)=19.46
  - $S_{3,0}$=134.54: max(182−134.54,0)=47.46

Step 2 Values: **3. Backward Induction to Price the Option**

We work backward from t=3, discounting the expected future values at the risk-free rate.

Step 2: Values

- $V_{2,2}$ = 0.99667 * [0.4906 * 0 + 0.5094 * 0] = 0
- $V_{2,1}$= 0.99667 * [0.4906 * 0 + 0.5094 * 19.46] = 9.88
- $V_{2,0}$= 0.99667 * [0.4906 * 0 + 0.5094 * 47.46] = 33.61

Step 1 Values:

- $V_{1,1}$ = 0.99667 * [0.4906 * 0 + 0.5094 * 0] = 5.03
- $V_{1,0}$ = 0.99667 * [0.4906 * 0 + 0.5094 * 0] = 21.89

Step 0 Value:

- $V_{0,0}$ = 0.99667 * [0.4906 * 5.03 + 0.5094 * 21.89] = 13.58

Step 2: Pick a Path

We chose the path Down-Down-Down (i.e., the stock price moves down at each step):

- Step 0: $S_{0,0}$=180
- Step 1: $S_{1,0}$ = 162.54
- Step 2: $S_{2,0}$ = 146.82
- Step 3: $S_{3,0}$ = 134.54

Along this path, the put option values are:

- Step 0: $V_{0,0}$=13.58
- Step 1: $V_{1,0}$=21.89
- Step 2: $V_{2,0}$=33.61

The price of the European put option is approximately 13.58.

25 b i) Describe the Delta Hedging Process

As the seller of the put option, we receive the premium (13.58) at t=0. To hedge, we need to maintain a delta-neutral position at each step by buying or selling units of the underlying stock (S) and adjusting our cash account. Delta (Δ) for an option is the rate of change of the option price with respect to the stock price, approximated in the binomial tree as:

$$Delta\ (\Delta) \ = \frac{Vu - Vd}{Su - Sd}$$

Summary of Delta Hedging Process

- Step 0: Sell the put for 13.58, buy 0.458 shares (cost 82.44), borrow 68.86.
- Step 1: Stock drops to 162.54, option value rises to 21.89, delta becomes -0.715, buy 0.257 more shares (cost 41.77), borrow more, pay interest.
- Step 2: Stock drops to 146.82, option value rises to 33.61, delta becomes -1.000, buy 0.285 more shares (cost 41.84), borrow more, pay interest.
- Step 3: Stock drops to 134.54, option exercised at 47.46, sell all shares, pay buyer, final cash reflects hedging costs.

Question 25b.ii: Cash Account Table

Time (t)	Stock Price (S)	Option Value (V)	Delta (Δ)	Shares Held	Cost of Shares	Cash Account	Interest	Portfolio Value (ΔS−V)
0	180	13.58	-0.458	0.458	82.44	-68.86	0	68.86 - 13.58 = 55.28
Δt	162.54	21.89	-0.715	0.715	41.77	-110.86	-0.23	116.22 - 21.89 = 94.33
2Δt	146.82	33.61	-1	1	41.84	-153.07	-0.37	146.82 - 33.61 = 113.21
3Δt	134.54	47.46	-	0	-134.54	-66.5	-0.51	0 - 0 = 0 (after paying 47.46)

26. Using the same data from Q25, price an American Put option

a.) Computed American Put Price: 13.04 (Refer to python code notebook)

```
delta_tree_df = pd.DataFrame(del

American Put Price: 13.04
```

b.) Show the evolution of the cash-account throughout the different steps for one path of your choice

So, as the seller, we receive the premium (13.04) at t = 0 and we hedge this by buying shares to offset the put's negative delta. What we'll do is adjust the hedge at each step, accounting for interest on the cash account.

S	Time (t)	Stock Price (S)	Option Value (V)	Delta (Δ)	Shares Held	Cost of Shares	Cash Account	Interest	Portfolio Value (ΔS - V)
Step 1	0	180	13.04	-0.357	0.357	64.26	-51.22	0	64.26 - 13.04 = **51.22**
Step 2	0.02	173.75	15.46	-0.414	0.414	7.21	-58.46	-0.02	72.02 - 15.46 = **56.56**
Step 3	0.04	167.58	17.6	-0.475	0.475	7.99	-66.47	-0.02	79.60 - 17.60 = **62.00**
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
Step 24	0.48	50.09	131.91	-0.976	0.976	0.97	-135.07	-0.05	48.88 - 131.91 = **-83.03**
Step 25	0.5	48.29	133.71	-	0	-48.29	-138.73	-0.05	0 - 0 = **0** (after paying 133.71)

c.) Comment on the Delta hedging process as compared to the European option case.

Due to early exercise, we noted that the American put requires more dynamic hedging due to early exercise and this to quite a rapid delta changes as the value of the option jumps to intrinsic levels when 'deep in the money', with the 25-step tree allowing smaller, frequent adjustments but early exercising which causes spikes in delta (e.g. the -0.976 by step 24). However, in contrast, the European put's delta actually evolved more smoothly, with larger per-step changes in the 3-step tree. For the American put, its higher value and early exercise premium do result in a more negative final cash balance (-138.73) as compared to the European put (-66.50). This reflects greater borrowing needs to hedge the growing option value with a correctly priced European put reducing hedging costs further. Also, hedging of the American put actually involves more/higher risk due to early exercise, which increases the chances of large delta adjustments and borrowing while the European put's risk is limited to expiration, thereby making its hedging more predictable.

Q27.Finally, repeat Q26 considering now an Asian ATM Put option. Comment on your results as compared to the regular American Put option case of Q25.

a.) Computed Asian Put Price: 7.851 (Refer to python code notebook)

b.) Show the evolution of the cash-account throughout the different steps for one path of your choice

Step	Time (t)	Node (i,j)	Stock Price	Delta	Shares Held	Cash Action	Cash Balance
0	0	(0,0)	180	-0.1162	0.1162	-20.916	-13.065
1	0.02	(1,1)	162.54	-0.127	0.127	-1.755	-14.826
2	0.04	(2,2)	146.82	-0.1389	0.1389	-1.747	-16.58
...	...	...	...	...	...	...	...
25	0.5	(25,25)	10.81	0	0	1.606	-191.138

c.) Comment on your results as compared to the regular American Put option case of Q25

The Asian put (7.851) is priced lower than the American put (13.04), which aligns with expectations since averaging reduces volatility. The Asian put requires less aggressive hedging due to its path-dependent payoff, resulting in smaller deltas and lower hedging costs compared to the American put, which has higher sensitivity to stock price fluctuations and early exercise risk. Additionally, the Asian put exposes the seller to greater losses on extreme paths i.e. all-down scenarios in our Asian option because there is no early exercise to limit losses, unlike the American put, which provides a mechanism of protection.

Key references on Put-Call Parity and Binomial Trees

Ting He (2019). *Nonparametric Predictive Inference for Option Pricing Based on the Binomial Tree Model.*

This thesis provides a solid framework for pricing European and American options and analyzes the applicability of put-call parity in binomial models.

McDonald, R., & Schroder, M. (1998). *A Parity Result for American Options. Journal of Computational Finance*.

This paper explores conditions under which put-call parity approximately holds or fails for American options.

Yang, Y. (2017). *Pricing American and European Options Under the Binomial Tree Model and Its Black-Scholes Limit Model.*

Excellent resource explaining how binomial approximations affect pricing and put-call parity behavior.

Zhang, Z. (2024). *Numerical Methods Based on Trinomial Trees for Option Pricing.*

Highlights consistency of put-call parity under trinomial and binomial tree frameworks.

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