GROUP NUMBER: 9427

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Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).	
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Use the box below to explain any attempts to reach out to a non-contributing member. Type (N/A) if all members contributed. **Note:** You may be required to provide proof of your outreach to non-contributing members upon request.
N/A

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Step 1: **For the next set of questions, assume the following values and parameters:**

$S_0 = 80;\ r = 5.5\%;\ \sigma = 35\%;\ T = 3\,months$

Team Member A: Stochastic Volatility Modeler

$v_0 = 3.2\%;\ \kappa_v = 1.85;\ \theta_v = 0.045$

For all computations involving Monte-Carlo simulation we use a $Number\ of\ simulations = 10\,000$ and consider 500 steps in a year. Reproducibility is made possible under python the random seed 0 ($np.random.seed(0)$).

5. Pricing of an ATM European call and using the Heston Model and Monte-Carlo simulation and $\rho = -0.3$. We get Call price = 2.88 and put price = 2.8.

6. Pricing of an ATM European call and using the Heston Model and Monte-Carlo simulation and $\rho = -0.7$. We get Call price = 2.13 and put price = 3.42.

7. We compute delta and gamma using numerical approximations. Given a function f, we assume that

- $f'(x) = (f(x + h) - f(x - h))/(2h)$: this will be used for computing Delta
- $f''(x) = (f(x + h) + f(x - h) - 2f(x))/(h^2)$: this formula will account for computation Gamma.

When applying those approximations with $h = \epsilon \times S_0$ and $\epsilon = 0.01$ we get:

- For $\rho = -0.3$:
    - Call Delta = 0.53 and Call Gamma = -0.2
    - Put Delta = -0.46 and Put Gamma = 0.15
- For $\rho = -0.7$:
    - Call Delta = 0.47 and Call Gamma = -0.12
    - Put Delta =-0.50 and Put Gamma = 0.17

Team Member B: Jump Modeler

$\mu = -0.5;\ \delta = 0.22$

8. Using the Merton Model, pricing an ATM European call and an ATM European put with jump intensity parameter equal to 0.75 gave us:

- European Call Price: **8.27**
- European Put Price: **7.41**

9. Using the Merton Model, price an ATM European call and an ATM European put with jump intensity parameter equal to 0.25.

- European Call Price: **6.73**
- European Put Price: **5.82**

10. Calculating the delta and gamma for each of the options in Questions 8 and 9 yielded:

i. Greeks for Merton Model (Jump Intensity = 0.75)

- Call Delta: 0.6529, Gamma: 0.0208
- Put Delta: -0.3481, Gamma: 0.0208

ii. Greeks for Merton Model (Jump Intensity = 0.25)

- Call Delta: 0.5895, Gamma: 0.0264
- Put Delta: -0.4092, Gamma: 0.0264

Team Member C: Model Validator

Question 11.

I. Theoretical analysis

Put-call parity is a foundational identity in European option pricing, based on the principle of no arbitrage. It defines a relationship between a European call and a European put with the same strike price and time to maturity, and a position in the underlying asset financed at the risk-free rate.

General formula:

$$C - P = S_0 - K_e^{-rT}$$

Where:

C: price of the European call option

P: price of the European put option

S_0: Current price of the underlying asset

K: strike price

r: continuously compounded risk-free interest rate

T: time to maturity in years

This relationship assumes:

a) No dividends.

b) Efficient markets.

c) European-style options (exercisable only at maturity).

Although the formula is derived under the Black-Scholes framework, it is expected to hold approximately in more complex models like Heston (stochastic volatility) and Merton (jump-diffusion). However, practical deviations can arise due to the stochastic nature of the models and numerical simulation errors.

II. Numerical validation (Python Monte Carlo Simulation)

Model parameters:

$$\boldsymbol{S_0} = 80, \boldsymbol{K} = 80, \boldsymbol{r} = 5.5\%, \boldsymbol{T} = 0.25 \text{ years}$$

Theoretical Put-Call Parity Value:

$$S_0 - K_e^{-rT} = 80 - 80 * e^{-0.055*0.25} \approx 1.09$$

Simulated results:

Q	Model	Parameter	C	P	C - P	$S_0 - K_e^{-rT} \approx 1.09$	Parity Satisfied
5	Heston	p = -0.30	2.89	2.80	0.09	1.09	No
6	Heston	p = -0.70	2.08	3.88	-1.30	1.09	No
8	Heston	λ = 0.75	5.06	9.45	-4.39	1.09	No
9	Heston	λ = 0.25	5.82	6.54	-0.72	1.09	No

III. Conclusion

While put-call parity is theoretically expected to hold in all arbitrage-free markets, none of the models or parameterizations evaluated satisfied the condition exactly. This outcome is likely due to several practical and model-specific factors:

a) Monte Carlo simulation noise due to a finite number of paths

b) Model-specific non-lognormal asset price distributions

c) Volatility skew, jump risk, and time discretization effects

These findings highlight the importance of model features and numerical accuracy in advanced option pricing frameworks.

Question 12.

I. Theoretical context

Option prices vary with strike price, and this sensitivity is best understood through moneyness, defined as:

$$\text{Moneyness} = \frac{S_0}{K}$$

Where:

S_0 is the current stock price

K is the option's strike price

This question explores how the Heston model (stochastic volatility) and the Merton model (jump diffusion) price European call and put options across a range of strikes, chosen to reflect a spectrum of moneyness from out-of-the-money (OTM), at-the-money (ATM), to in-the-money (ITM).

II. Simulation Design

Parameters used for both models:

- Stock price: S_0=80 (spot price)
- Risk-free rate: r=5.5%
- Volatility (for Merton base): σ=35%
- Time to maturity: T=0.25 (3 months)

Heston Model Parameters:

- Initial variance v_0=0.032
- Mean-reversion speed κ=1.85
- Long-run variance θ=0.045,
- Correlation ρ=−0.30

Merton Model Parameters:

- Jump intensity λ=0.75
- Jump mean μ=−0.5
- jump volatility δ=0.22

Strike Prices chosen based on moneyness:

Moneyness	**Strike (*K*)**
0.85	94.12
0.90	88.89
0.95	84.21
1.00	80.00
1.05	76.19
1.10	72.73
1.15	69.57

Simulations were performed using 5,000 Monte Carlo paths and 100 time steps for each strike and model.

III. Simulated Results

Strike	Moneyness	Call (Heston)	Put (Heston)	Call(Merton)	Put (Merton)
94.12	0.85	0.09	14.00	1.41	19.42
88.89	0.90	0.43	9.18	2.41	15.43
84.21	0.95	1.28	5.53	3.72	12.09
80.00	1.00	2.93	2.86	5.20	9.45
76.19	1.05	5.14	1.40	7.01	7.21
72.73	1.10	7.83	0.68	8.88	5.36
69.57	1.15	10.23	0.33	10.99	4.04

IV. Observations

a) Call prices increase as strike decreases (i.e., deeper ITM), while put prices decrease, which aligns with theoretical expectations.

b) The Merton model consistently produces higher prices for both calls and puts due to the inclusion of jump risk.

c) The Heston model's pricing reflects volatility clustering and skew, especially noticeable near ATM levels.

d) Both models reflect asymmetries not captured by Black-Scholes, reinforcing the relevance of advanced stochastic processes in real-world pricing.

V. Conclusion

This exercise illustrates how option pricing changes with strike when modeled under more realistic dynamics. The Heston model captures volatility behavior over time, while the Merton model incorporates discontinuous jumps in the asset price. The difference in results highlights the importance of selecting a model that aligns with the risk characteristics of the underlying asset and the type of derivative being priced.

Step 2

All team members:

Q13. American Call Option under the Heston Model

Objective:

To evaluate the value of an American-style call option using the Heston stochastic volatility model and compare it to the European call option price obtained in Question 5.

Theoretical Framework:

1. In the Heston model, volatility follows a mean-reverting stochastic process, allowing for volatility clustering and the volatility smile.

2. The price of an American option must account for the possibility of early exercise, which European options ignore.

3. Although American calls on non-dividend-paying assets are often equal in value to their European counterparts under constant volatility (Black-Scholes), this may not hold under Heston due to the stochastic nature of volatility.

4. We apply the Longstaff-Schwartz (Least Squares Monte Carlo – LSM) method to model early exercise features efficiently under a Monte Carlo framework.

Outcome:

1. The American call price exceeds the European price from Q5 (≈ 2.89), indicating a small early exercise premium.

2. The Delta and Gamma are computed using finite difference approximation by bumping the underlying asset value.

Q14. American Put Option under the Merton Model

Objective:

To price an American-style put option using the Merton jump-diffusion model and compare it with the European put option obtained in Question 8.

Theoretical Framework:

1. The Merton model enhances Black-Scholes by incorporating Poisson-distributed jumps in asset prices, simulating real-world market shocks.

2. American puts can benefit significantly from early exercise, especially when deep in-the-money and under positive interest rates.

3. The jump component can accelerate deep ITM scenarios, making the American feature more valuable.

4. As with Q13, we apply the Longstaff-Schwartz method, now adjusted for non-continuous jump paths.

Outcome:

1. The American put price is slightly higher than the European put from Q8 (≈ 9.45), confirming the value of the early exercise opportunity.

2. Greeks are again estimated using bump-and-revalue techniques, although Gamma is often less stable under discontinuous price paths.

Q15. American Call Option under the Merton Model

Objective:

To evaluate an American-style call option under the Merton model and compare its value to the European call price from Question 8.

Theoretical Framework:

1. The call option's value under Merton reflects both diffusion and jump risks.

2. Although early exercise is rarely optimal for calls, jumps can create rare but significant scenarios where early exercise may provide a premium.

3. The impact of jumps on the upper tail of asset returns can shift the exercise boundary forward in time, especially when the market expects upward jumps.

Outcome:

1. The American call under Merton is slightly more valuable than the European call from Q8 (≈ 5.06), showing that jumps slightly increase early exercise scenarios.

2. Delta and Gamma reflect the jump-adjusted sensitivities, with results aligned to economic intuition: positive Delta, modest Gamma.

Final Observations

Question	Model Type	European Price	American Price	Premium
Q13	Heston Call	≈ 2.89	≈ 3.49	≈ +0.60
Q14	Merton Put	≈ 9.45	≈ 9.51	≈ +0.06
Q15	Merton Call	≈ 5.06	≈ 5.33	≈ +0.27

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