GROUP WORK PROJECT # _1__
GROUP NUMBER: __10310____

MScFE 622: Stochastic Modeling

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Statement of integrity: By typing the names of all group members in the text boxes below, you confirm that the assignment submitted is original work produced by the group (excluding any non-contributing members identified with an "X" above).	
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Note: You may be required to provide proof of your outreach to non-contributing members upon request.

N/A

Step 1: Calibration Heston 1993 stochastic volatility model using Lewis (2001) approach and Carr-Madan (1999) approach to double-check results.

To achieve this task, we use call and put option prices having as underlying assets the stock of SM Energy company. MSE is used as the objective function for matching models to market price.

We assume that:

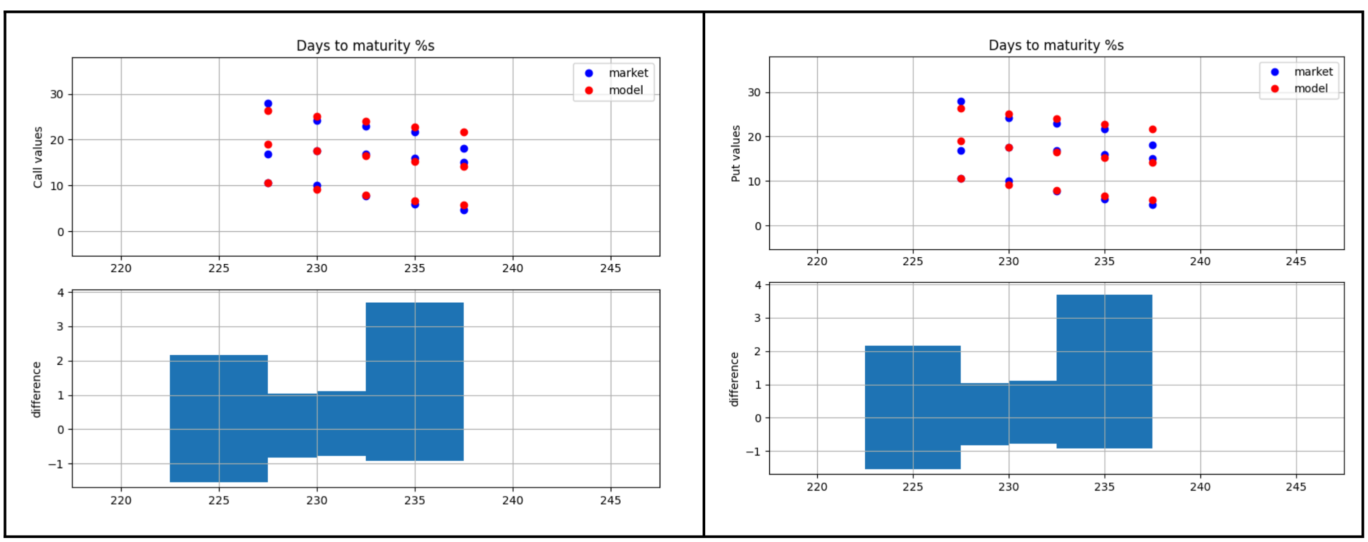
- the stock is actually traded at $S_0 = 232.90$ USD
- The interest rate is constant and equals to $r = 1.50\%$
- There are 250 trading days in a year.

Since the client needs a short maturity derivative, calibration will focus on short maturities (T = 15 days). Furthermore, we use both call and put option prices by deriving put prices from the call-put parity. The calibration process consist in minimizing the cost function $MSE = \frac{1}{n}\sum(model_{price} - market_{price})^2$. It is achieved in two steps: a first step consisting in finding a region (brute force) where to focus to refine the optimization in a second step.

a. **Calibrating using Lewis (2001) approach.**

The model call price is obtained by computing the semi-closed form with Lewis (2001) integral quadrature. The call-put parity is then used to obtain model put price . Here we use 15 days to maturity puts and calls. At the optimum we get a $MSE = 17.31$ This are the parameters resulting from calibration of the Heston (1993) model: $\kappa_\nu = 4.317; \theta_\nu = 0.144; \sigma_\nu = 0.000; \rho = -0.011; \nu_0 = 0.107$

The volatility of volatility is different to 0 from the sixth decimal.



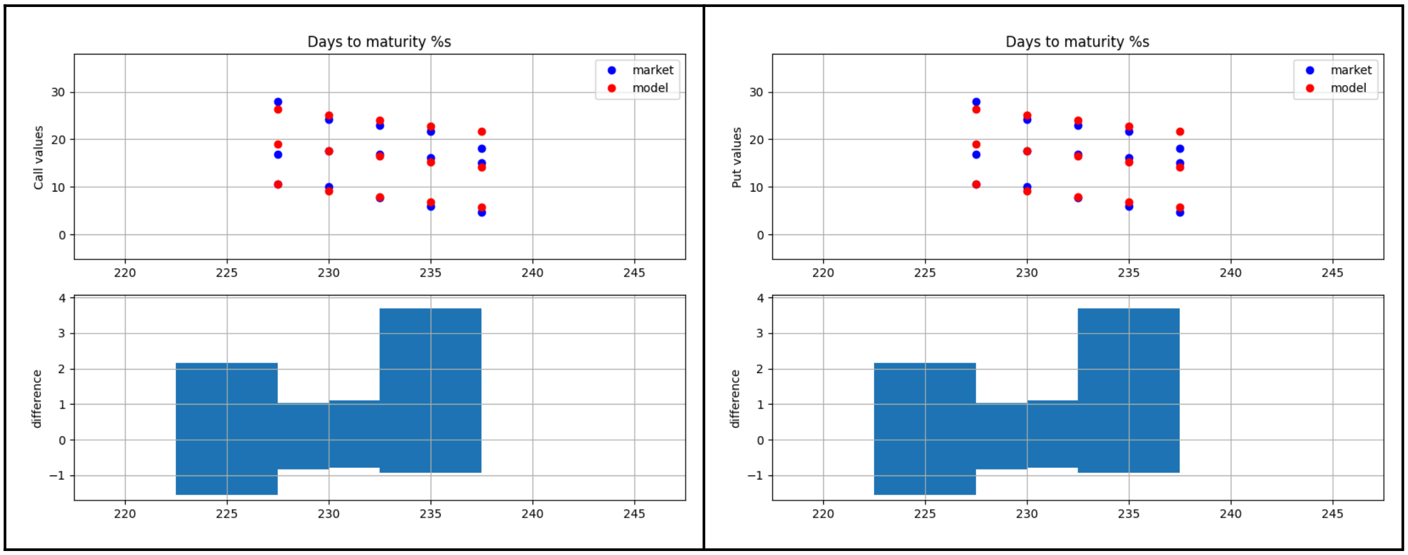
The model prices fit the market prices well at ATM. Differences increase when moving OTM and ITM.

b. Calibrating using Carr-Madan (1999) approach.

Call price from the Heston (1993) model is computed using the semi-closed form from Carr-Madan (1999) and Fast Fourier Transform discretization. The calibration is then repeated as in the previous question. We notice that computation is faster and provides a $MSE = 15.05$. Model parameters obtained are the same as the one with the Lewis approach (the four first decimals).

- $\kappa_\nu = 4.317; \theta_\nu = 0.144; \sigma_\nu = 0.000; \rho = -0.011; \nu_0 = 0.107$

Graphically, the model prices match market prices. ATM derivatives are better fitted than the others.



Given that the Carr-Madan approach achieves a fewer MSE and is the fastest, we can rely on its results for the pricing. In fact, parameters may have differences given more decimals.

c. Pricing an Asian call option on SM assets for the client.

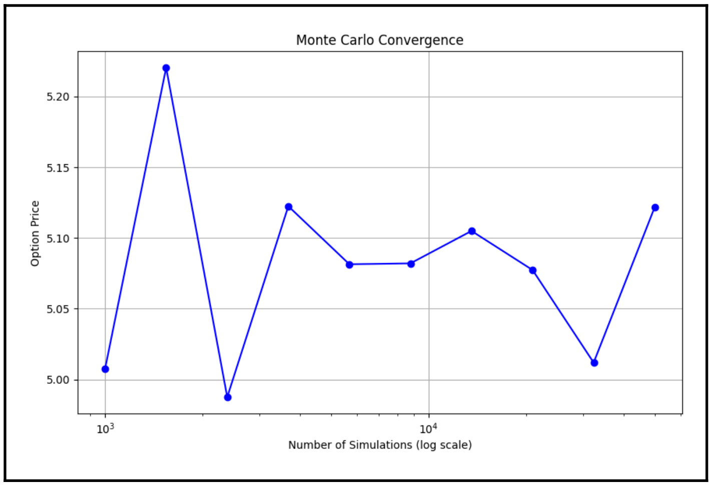
The client wants an ATM Asian call with 20 days-maturity. Given the results of the calibration process, we can use Monte-Carlo simulations to generate multiple trajectories the stock may follow so that on average we can estimate the value of the stock at maturity. The payoff of an Asian call option is

$payoff = max(\frac{1}{T+1}\sum_{t=0}^{T} S_t - K,\ 0)$. By using Monte-Carlo method, the different trajectories give a set of potential payoff from which we derive the mean payoff and finally the option price.

Actually we obtain:

- a fair price equals to $\$5.11$ USD
- With a 4% charge, the client price is $\$5.31$

This price is obtained with a 10 000 Monte-Carlo simulations that leads to an accurate level as shown on the figure below.



Step 2: Calibration of Bates (1996) model using Lewis (2001) approach and Carr-Madan (1999) approach to double-check results.

Since the client is interested in longer maturity instruments, we calibrate the model to T = 60 days to maturity. Bates (1996) model extends Heston (1993) model by adding a jumping term. Bates requires more parameters and is expected to capture information Heston (1993) was unable to reproduce. As in step 1, we calibrate the Bates model both on call and put market prices by computing model price with Lewis (2001) and Carr-Madan (1999) approaches.

The calibration process is done in three stages:

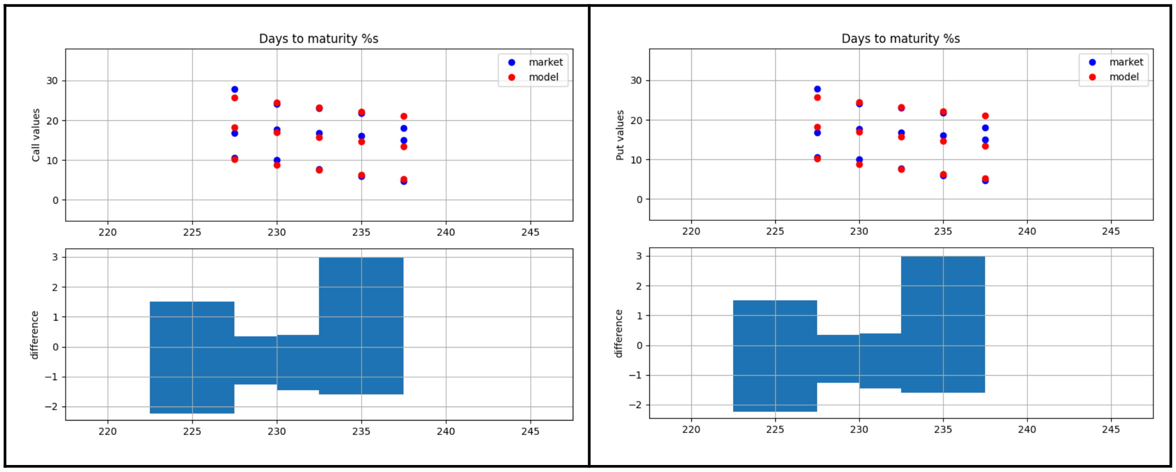
- Calibrating Heston stochastic volatility model parameters as in step 1
- Given the Heston parameters, calibrate the jump parameters in two stages (brute force, and refinement)
- Using obtained parameters as the beginning point, do a full calibration of both Heston and jump parameters at the time.

a. Calibration using Lewis (2001)

The full calibration leads to a $MSE = 16.16$. This are the optimal parameters:

- **Heston:** $\kappa_\nu = 4.374$; $\theta_\nu = 0.140$; $\sigma_\nu = 0.000$; $\rho = -0.012$; $\nu_0 = 0.094$
- **Merton:** $\lambda = 0.000$; $\mu = -0.518$; $\delta = 0.000$

Since the jump intensity is null, there is only an adjustment on the drift under risk-neutral probability that changes the Heston (1993) based model. We can see that the stochastic volatility parameters do not deviate too much from what we obtain under the Heston model (except the long rate volatility).

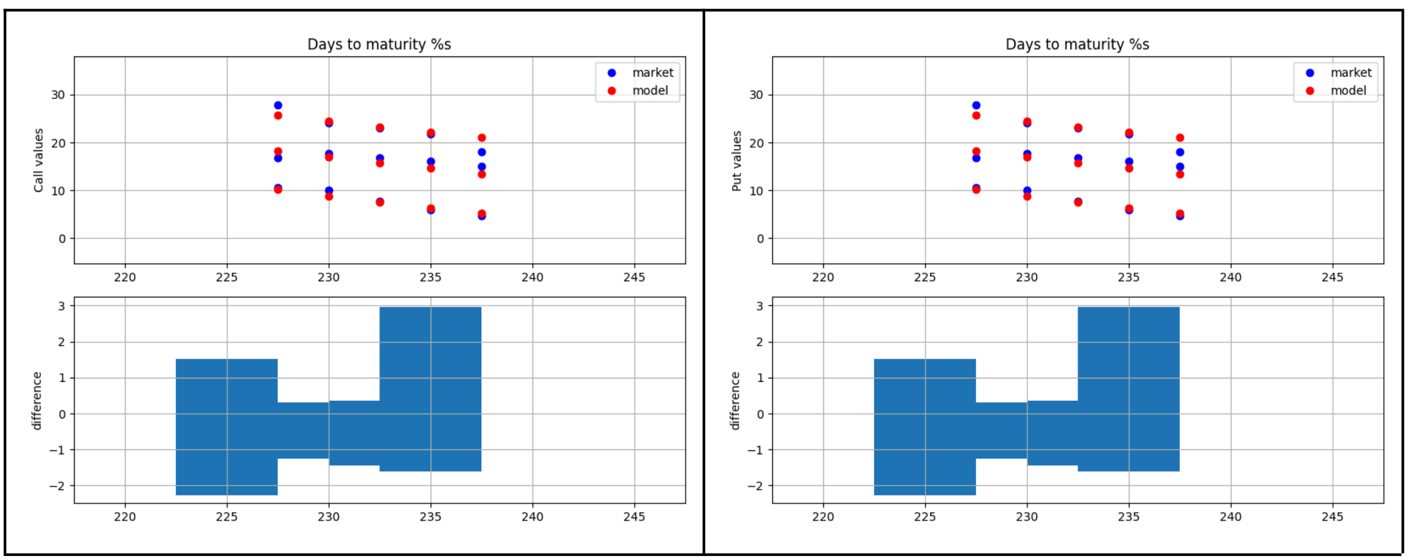


b. Calibrating using Carr-Madan (1999)

The full calibration leads to a $MSE = 16.16$. This are the optimal parameters:

- **Heston:** $\kappa_\nu = 4.373$; $\theta_\nu = 0.139$; $\sigma_\nu = 0.000$; $\rho =- 0.012$; $\nu_0 = 0.094$
- **Merton:** $\lambda = 0.000$; $\mu =- 0.519$; $\delta = 0.000$

These parameters are closed to those obtained using the Lewis approach. However the calibration is timeless, which makes this approach better.



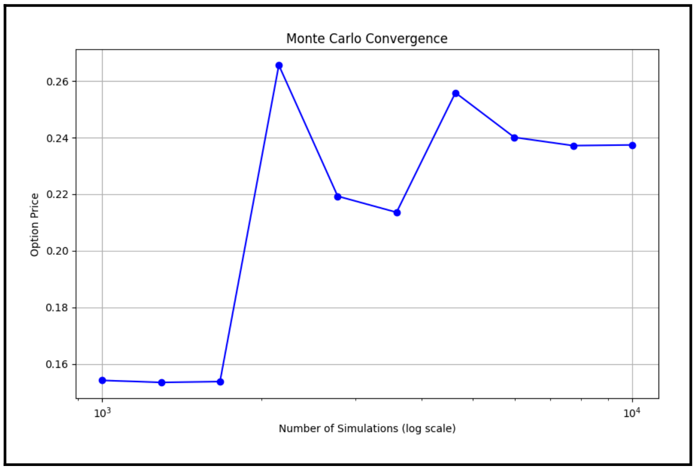
c. Pricing an Asian Put option under Bates (1996)

The client wants 95% moneyness Asian put with 70 days-maturity. Given the results of the calibration process using Carr-Madan (1999) approach, we use Monte-Carlo simulations to generate multiple trajectories the stock may follow so that on average we can estimate the payoff and derive the option price by actualising.

We obtain:

- a fair price equals to $\$0.25$ USD
- With a 4% charge, the client price is $\$0.26$

This price is obtained with a 10 000 Monte-Carlo simulations that leads to an accurate level as shown on the figure below.



Step 3: CIR (1985) model calibration

a. CIR (1985) model calibration to euribor term structure

Calibration process

To calibrate the CIR model to Euribor data, we begin by constructing the term structure of zero-coupon rates from the available Euribor fixings (1W, 1M, 3M, 6M, 12M). Assuming 360 days in a year,

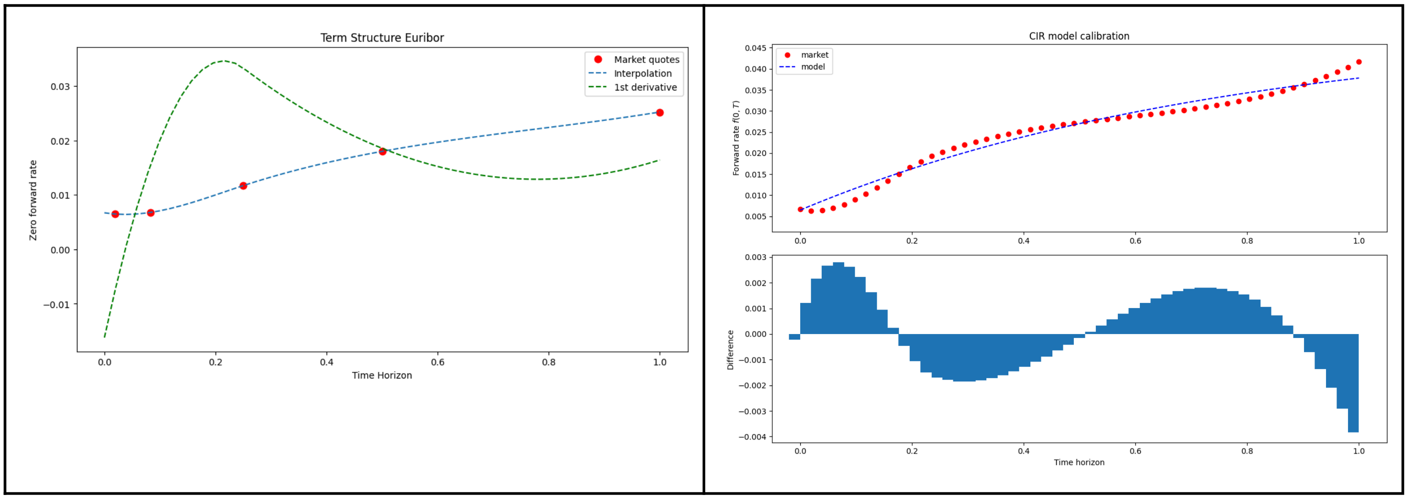
- we compute discount factors as: $P(T) = \frac{1}{1+r_{Euribor} \cdot T}$ where T is the maturity in years and $r_{Euribor}$ is the quoted Euribor rate.
- The continuously compounded zero rates are then obtained by: $z(T) = -\frac{1}{T} \cdot ln(P(T))$.
- We apply cubic spline interpolation to the computed zero rates in order to estimate weekly zero rates for maturities up to one year (52 weeks).

Using the interpolated term structure, we calibrate the CIR model by minimizing the mean squared error between model-implied zero-coupon bond prices and those derived from the interpolated market zero rates. The CIR model is governed by the stochastic differential equation: $dr_t = \kappa(\theta - r_t)dt + \sigma\sqrt{r_t}dW_t$.

Calibration results

The calibration results yield to this parameters: $\kappa = 1.282; \theta = 0.100; \sigma = 0.007$. with the MSE between market and model values:0.000003.

We present the interpolated Euribor term structure and the CIR model calibration results in the figure below.



Discussion

The calibrated model provides a good fit to the interpolated zero-coupon curve. The relatively low volatility parameter $\sigma = 0.007$ suggests modest rate fluctuations, while the mean reversion coefficient $\kappa = 1.282$ indicates high speed of reversion toward the long-term mean $\theta = 0.100$. Overall, the CIR model captures the general shape of the Euribor term structure, although some deviations may occur beyond the 6-month horizon due to the single-factor nature of the model.

b. Simulation of euribor 12-month rates using the CIR model calibrated

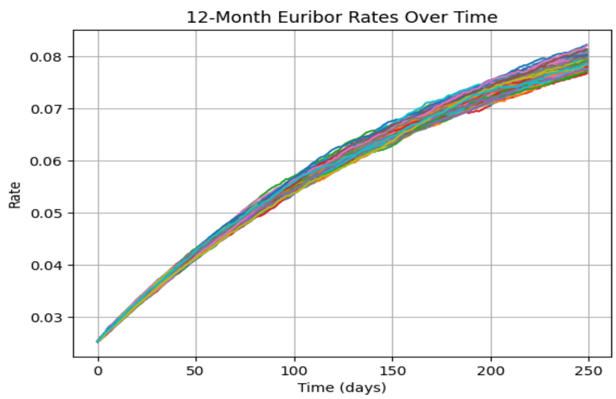
Using the CIR parameters obtained in the previous step ($\kappa = 1.282$; $\theta = 0.100$; $\sigma = 0.007$), we simulate future short-term interest rate paths under the CIR stochastic differential equation.

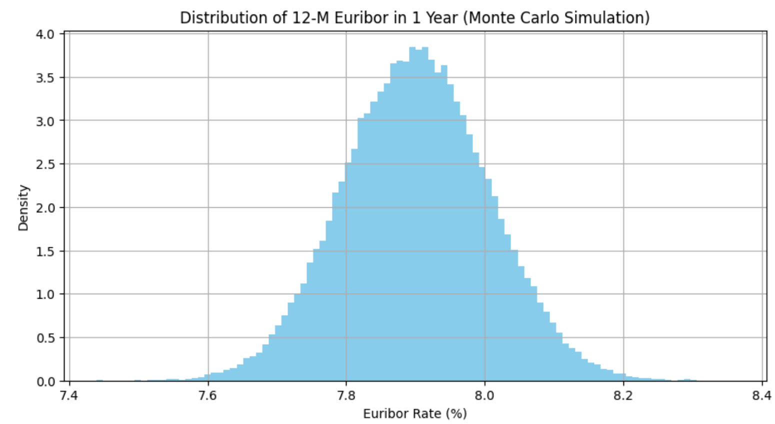
- Starting value: the current 12-month Euribor rate (2.524%)
- Number of Monte-Carlo simulations: 100 000
- Time horizon: 12 months
- Time step: 1 day ($d_t = \frac{1}{250}$)

Each path follows the discretized form of the CIR process (using Euler-Maruyama method)..

Results

Despite the square-root diffusion in the CIR model, the relatively low volatility ($\sigma = 0.007$) results in a narrow spread of simulated outcomes. The distribution of simulated 12-month Euribor rates after one year appears approximately symmetric.





i. At a 95% confidence level, the simulated 12-month Euribor rate in 1-year is expected to fall between 7.697% and 8.107%.

ii. The expected 12-month Euribor rate in 1 year is 7.901%. This value is significantly higher than the current 12-month Euribor (2.524%), indicating upward pressure from the high mean-reversion level ($\theta = 0.100$).

iii. In pricing interest-rate-sensitive instruments such as Asian put and call options, a higher expected Euribor implies an increase in forward rates. This tends to reduce the value of fixed-income instruments or derivatives benefiting from lower rates and increasing the cost of hedging against rising rates.

Therefore, expectations of a significant rate increase driven by the CIR model's parameters suggest that product pricing in one year may be less favorable for clients locking in current lower rates.

Bibliography

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