PR details

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1 ReHLine with quadratic coefficient

Below we will provide update formulas to extend ReHLine algorithm to the following set of problems:

$$\min_{\mathbf{w} \in R^d} \frac{1}{2} \mathbf{w}^T \mathbf{G} \mathbf{w} - \mu^T \mathbf{w} + \sum_{i=1}^{n} \sum_{l=1}^{L} \text{ReLU}(u_{li} \mathbf{x}_i^T \mathbf{w} + v_{li}) +$$

$$+ \sum_{i=1}^{n} \sum_{h=1}^{H} \text{ReHU}_{\tau_{hi}}(s_{hi} \mathbf{x}_i^T \mathbf{w} + t_{hi}) \text{ s.t. } \mathbf{A}\mathbf{w} + \mathbf{b} \geq \mathbf{0}$$

1. CD update λ_{li}

$$\begin{cases} \lambda_{li}^{new} = \mathcal{P}_{[0,1]} \left(\lambda_{li} + \frac{u_{li} \mathbf{x}_i^T \mathbf{w}^{old} + v_{li}}{u_{li}^2 \mathbf{x}_i^T \mathbf{G}^{-1} \mathbf{x}_i} \right) \\ \mathbf{w}^{new} = \mathbf{w}^{old} - (\lambda_{li}^{new} - \lambda_{li}^{old}) u_{li} \mathbf{G}^{-1} \mathbf{x}_i \end{cases}$$

2. CD update γ_{hi}

$$\begin{cases} \gamma_{hi}^{new} = \mathcal{P}_{[0,\tau_{hi}]} \left(\gamma_{hi} + \frac{s_{hi} \mathbf{x}_i^T \mathbf{w}^{old} - \gamma_{hi}^{old} + t_{hi}}{s_{hi}^2 \mathbf{x}_i^T \mathbf{G}^{-1} \mathbf{x}_i + 1} \right) \\ \mathbf{w}^{new} = \mathbf{w}^{old} - (\gamma_{hi}^{new} - \gamma_{hi}^{old}) s_{hi} \mathbf{G}^{-1} \mathbf{x}_i \end{cases}$$

3. CD update ξ_k

$$\begin{cases} \xi_k^{new} = \max \left(0, \xi_k^{old} - \frac{\mathbf{a}_k^T \mathbf{w}^{old} + b_k}{\mathbf{a}_k^T \mathbf{G}^{-1} \mathbf{a}_k} \right) \\ \mathbf{w}^{new} = \mathbf{w}^{old} - (\xi_k^{new} - \xi_k^{old}) \mathbf{G}^{-1} \mathbf{a}_k \end{cases}$$

4. Primal-dual relationship

$$\mathbf{w} = \mathbf{G}^{-1} \left(\mu + \mathbf{A}^T \xi - \bar{\mathbf{U}}_{(3)} \text{vec}(\mathbf{\Lambda}) - \bar{\mathbf{S}}_{(3)} \text{vec}(\mathbf{\Gamma}) \right)$$

Note that nothing much changes to the original algorithm other than tucked in $\mathbf{G}^{-1}$ terms and presence of μ term in the primal-dual relationship.