DEPARTMENT OF MECHANICAL AND ENERGY ENGINEERING
机械与能源工程系

CLEAR
Control & Learning for Robotics and Autonomy Lab

Deep RL for MPC of Quadruped Locomotion

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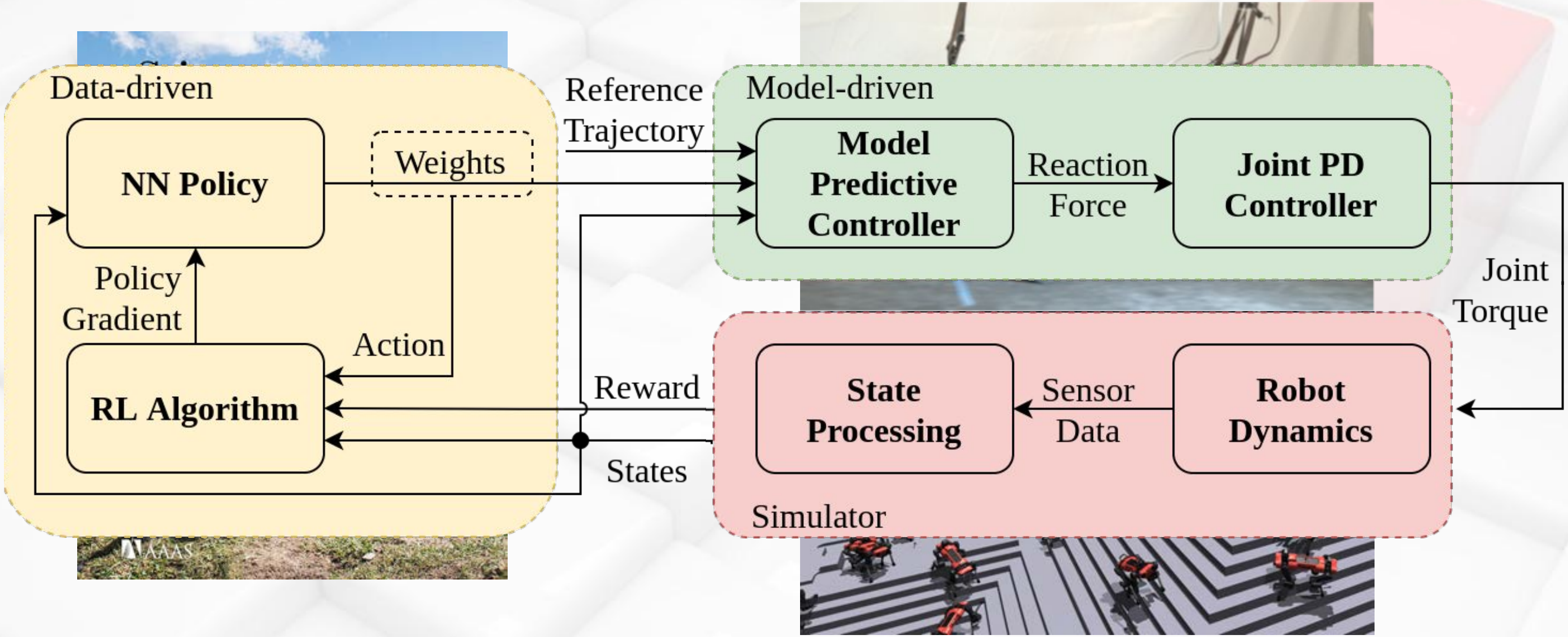
Motivation and Demonstration

Thesis Motivation

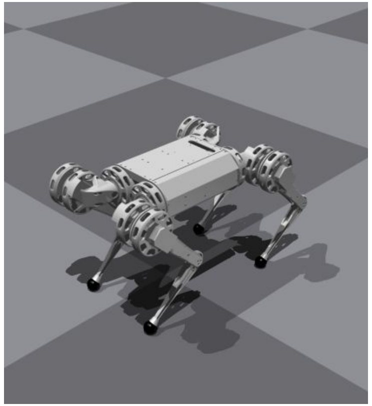


★ A Model-Data Hybrid-Driven Hierarchical Control Method for Quadruped

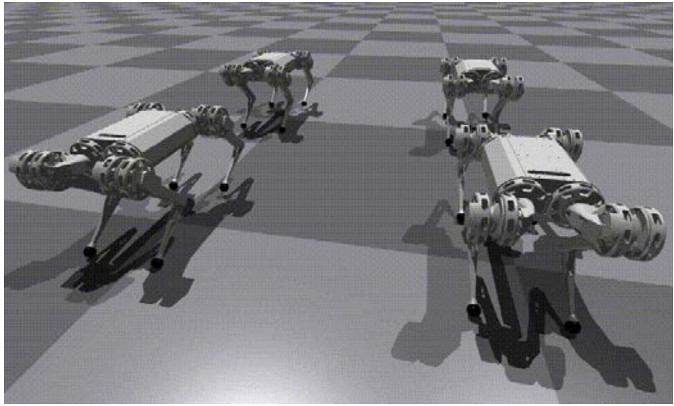
A Quadruped Control & Learning FWK

Unitree Aliengo

Unitree A1

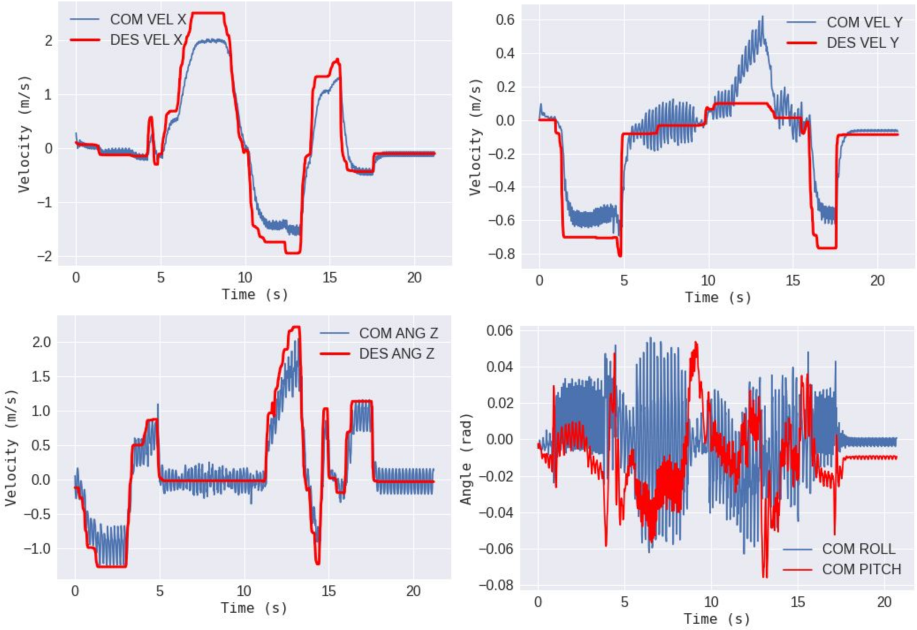


Mini Cheetah



Parallel Simulation Demo

MPC Controller Performance



RL Training Results - Part 1

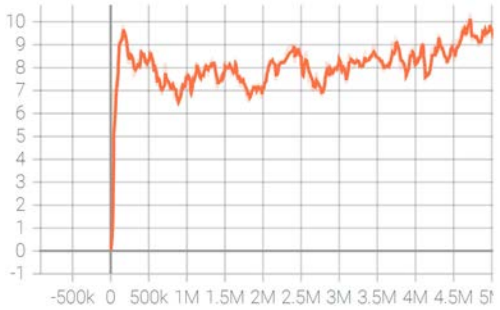
❖ Computing Device

RL Training: NVIDIA GTX 1060

MPC Controller: Intel Core i7-7850H

RAM: 16 G

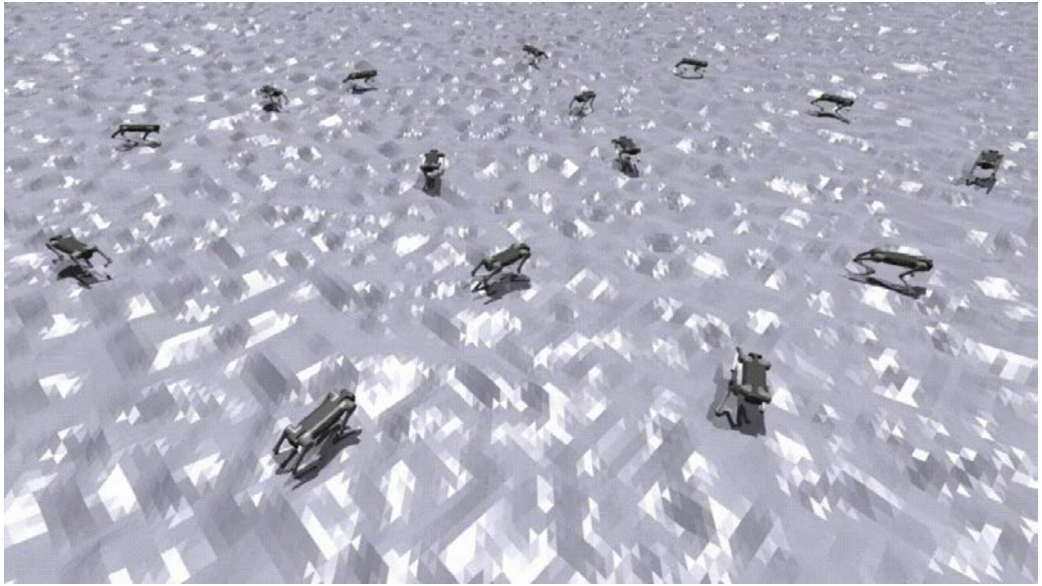
Envs	Batch Size	GPU Occupancy
1	24	35%
4	96	28%
8	192	20%
16	384	**15%**
32	768	8%
64	1536	6%
512	12288	1%



Reward

Entropy Loss

RL Training Results - Part 2

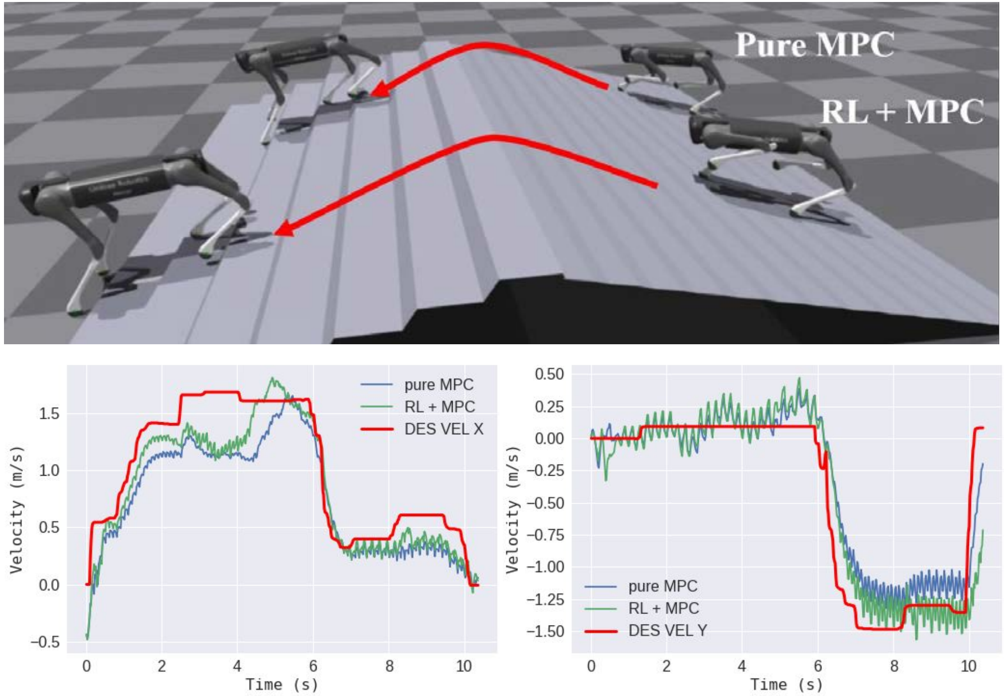


Weight Policy Training

Linear velocity:	[-2, 2]
Angular velocity:	[-1.5, 1.5]
Simulation:	500 Hz
MPC:	50 Hz

Actors:	16
Algorithm:	PPO
Policy:	3 layer MLP
Action clip:	[-1, 1]

Performance Comparison - Part 1



Linear velocity: [-2, 2]

Angular velocity: [-2.5, 2.5]

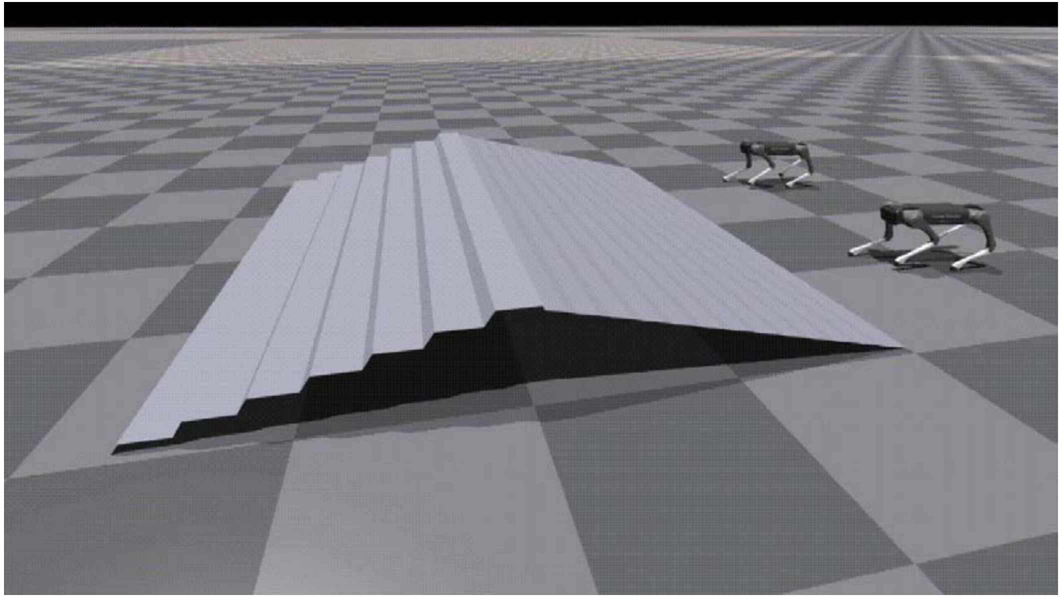
Simulation: 500 Hz

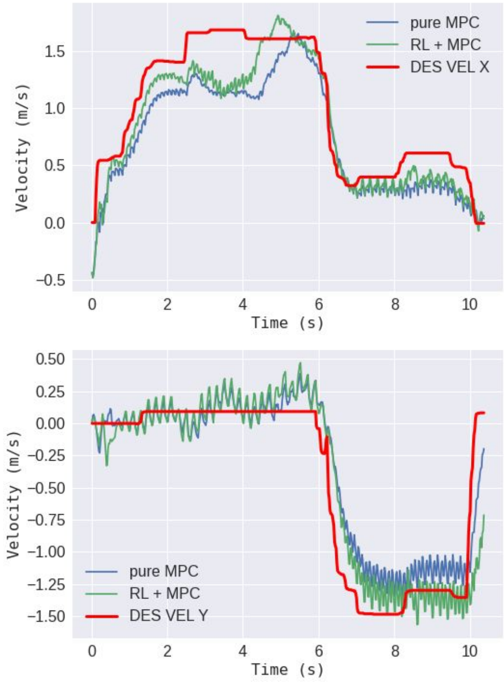
MPC: 50 Hz

Model inference time: ~0.001 s

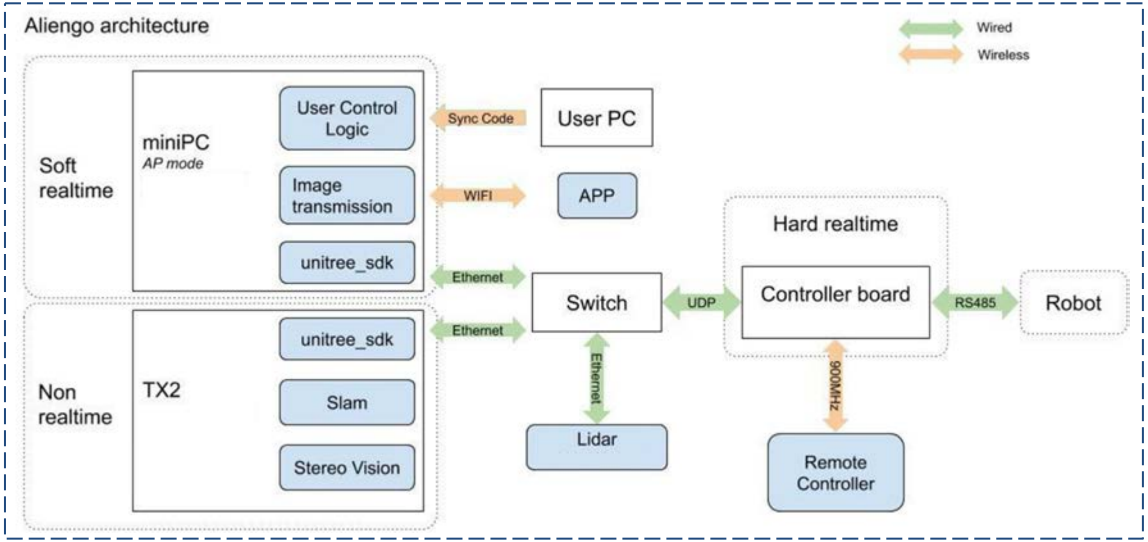
Policy update time: ~0.0015 s

Performance Comparison - Part 2

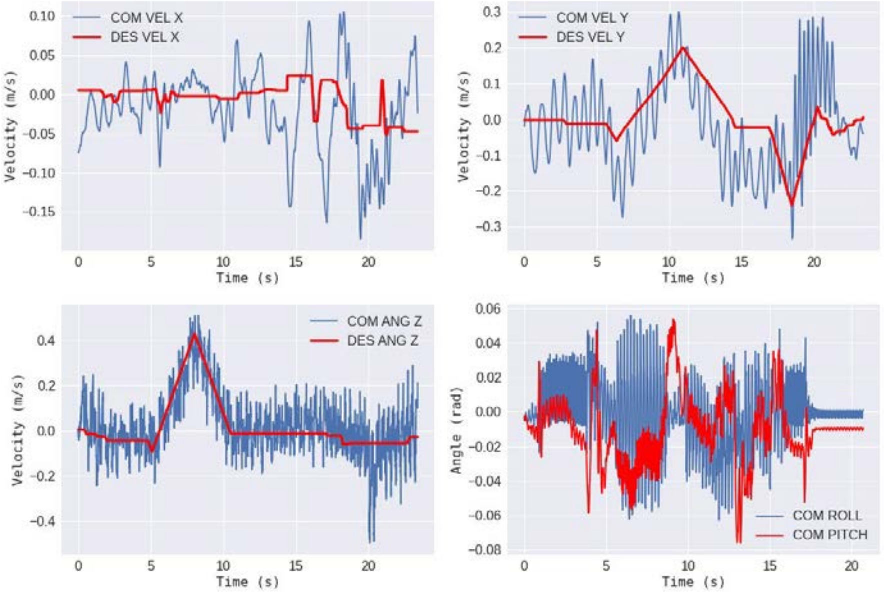




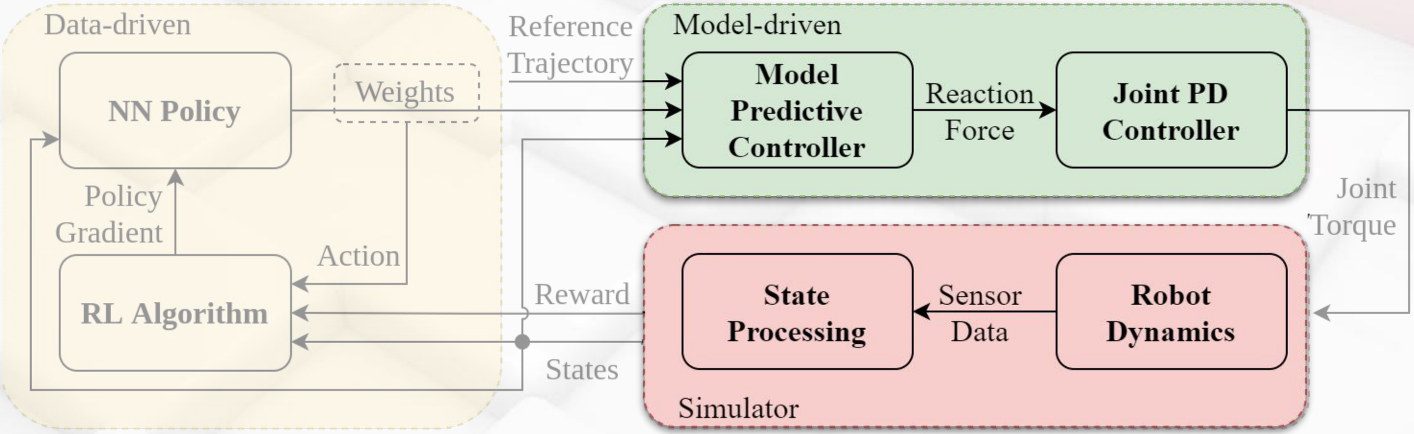
Sim2Real Transfer - Part 1



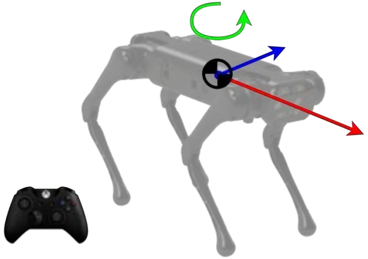
Sim2Real Transfer - Part 2

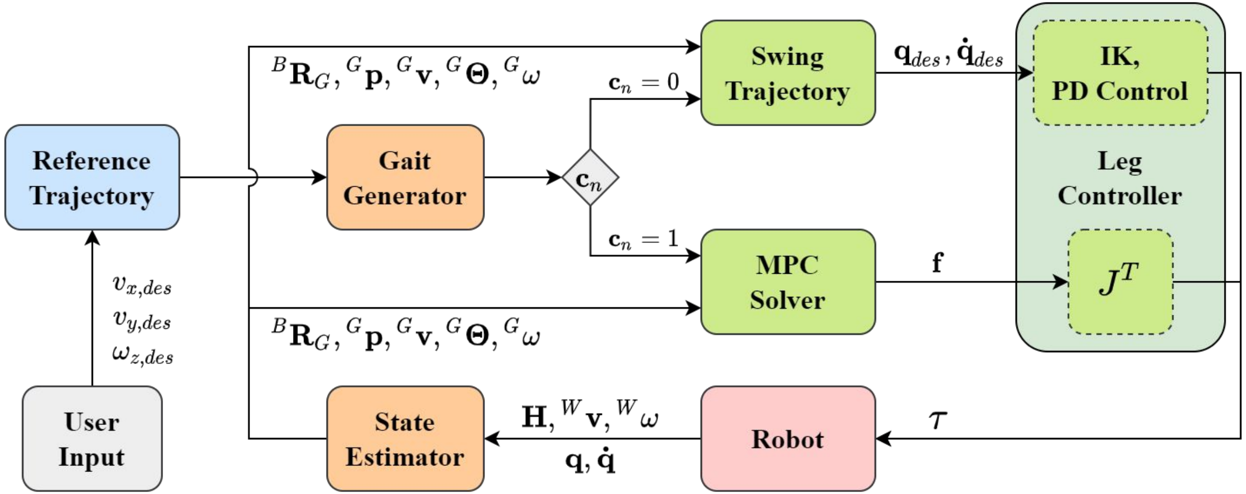


Model Predictive Control Framework



Controller Overview





- ❖ Operator Input:
  - ➢ velocity in xy plane and yaw turn rate.

Di Carlo, Jared, et al. "Dynamic locomotion in the mit cheetah 3 through convex model-predictive control." *2018 IEEE/RSJ international conference on intelligent robots and systems (IROS)*. IEEE, 2018.

Rigid-body Dynamics

❖ Rigid body dynamics:

$$\ddot{\mathbf{p}} = \frac{\sum_{i=1}^{n} \mathbf{f}_i}{m} - \mathbf{g}$$

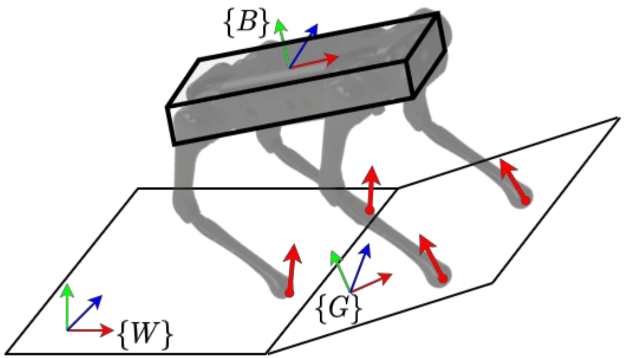
$$\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{I}\boldsymbol{\omega}) = \sum_{i=1}^{n} \mathbf{r}_i \times \mathbf{f}_i$$

$$\dot{\mathbf{R}} = [\boldsymbol{\omega}]_\times \mathbf{R}$$

where

$$\mathbf{R} = \mathbf{R}_z(\psi)\mathbf{R}_y(\theta)\mathbf{R}_x(\phi)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}(\mathbf{I}\boldsymbol{\omega}) = \mathbf{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times (\mathbf{I}\boldsymbol{\omega}) \approx \mathbf{I}\dot{\boldsymbol{\omega}}$$



❖ Approximations:

1. Small value of roll and pitch
2. Robot is not pointed vertically ($\cos(\theta) \neq 0$)
3. The $\omega \times (\mathbf{I}\omega)$ term is small for bodies with small angular velocities

State Space Model

- Calculate state derivatives:

$$\frac{\mathrm{d}}{\mathrm{dt}}\hat{\mathbf{\Theta}} = R_z(\psi)\hat{\mathbf{\Theta}}$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\hat{\mathbf{p}} = \dot{\hat{\mathbf{p}}}$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\hat{\omega} = \hat{\mathbf{I}}^{-1}\sum_{i=1}^{n}\mathbf{r}_i \times \mathbf{f}_i = \hat{\mathbf{I}}^{-1}([\mathbf{r}_1]_\times \mathbf{f}_1 + \cdots + [\mathbf{r}_n]_\times \mathbf{f}_n)$$

$$\frac{\mathrm{d}}{\mathrm{dt}}\dot{\hat{\mathbf{p}}} = \frac{\sum_{i=1}^{n}\mathbf{f}_i}{m} - \mathbf{g} = \frac{\mathbf{f}_1 + \cdots + \mathbf{f}_n}{m} - \mathbf{g}$$

where $n = 4$

- Add additional gravity state:

$$\frac{\mathrm{d}}{\mathrm{dt}}\begin{bmatrix}\hat{\mathbf{\Theta}}\\ \hat{\mathbf{p}}\\ \hat{\omega}\\ \dot{\hat{\mathbf{p}}}\\ \mathbf{g}\end{bmatrix} = \begin{bmatrix}\mathbf{0}_3 & \mathbf{0}_3 & \mathbf{R}_z(\psi) & \mathbf{0}_3 & \mathbf{0}_{3\times1}\\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{I}_3 & \mathbf{0}_{3\times1}\\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_{3\times1}\\ \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & \mathbf{0}_3 & [0\ 0\ 1]^T\\ \mathbf{0}_{1\times3} & \mathbf{0}_{1\times3} & \mathbf{0}_{1\times3} & \mathbf{0}_{1\times3} & 0\end{bmatrix}\begin{bmatrix}\hat{\mathbf{\Theta}}\\ \hat{\mathbf{p}}\\ \hat{\omega}\\ \dot{\hat{\mathbf{p}}}\\ \mathbf{g}\end{bmatrix} + \begin{bmatrix}\mathbf{0}_3 & \cdots & \mathbf{0}_3\\ \mathbf{0}_3 & \cdots & \mathbf{0}_3\\ \hat{\mathbf{I}}^{-1}[\mathbf{r}_1]_\times & \cdots & \hat{\mathbf{I}}^{-1}[\mathbf{r}_4]_\times\\ \mathbf{I}_3/m & \cdots & \mathbf{I}_3/m\\ \mathbf{0}_{1\times3} & \cdots & \mathbf{0}_{1\times3}\end{bmatrix}\begin{bmatrix}\mathbf{f}_1\\ \vdots\\ \mathbf{f}_4\end{bmatrix}$$

$$\dot{\mathbf{x}}(t) = \mathbf{A}_c(\psi)\mathbf{x}(t) + \mathbf{B}_c(\mathbf{r}_1, \ldots, \mathbf{r}_n, \psi)\mathbf{u}(t)$$

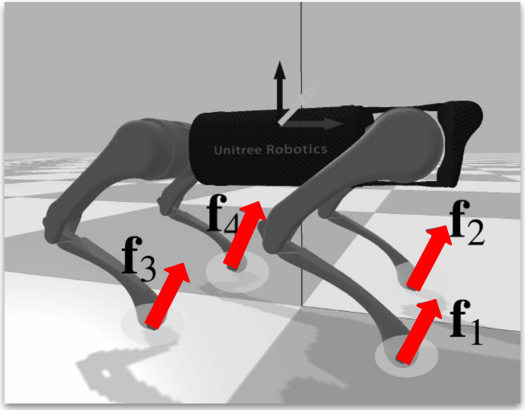
where $\mathbf{A}_c \in \mathbb{R}^{13\times13}$ and $\mathbf{B}_c \in \mathbb{R}^{13\times3n}$.

Stance Leg Control

- Stance Leg Force Control:

$$\boldsymbol{\tau}_i = \mathbf{J}_i^\top \mathbf{R}_i^\top \mathbf{f}_i$$

f is the vector of forces calculated from the **MPC** controller in the world coordinate frame.



- Zero Order Hold Discretization

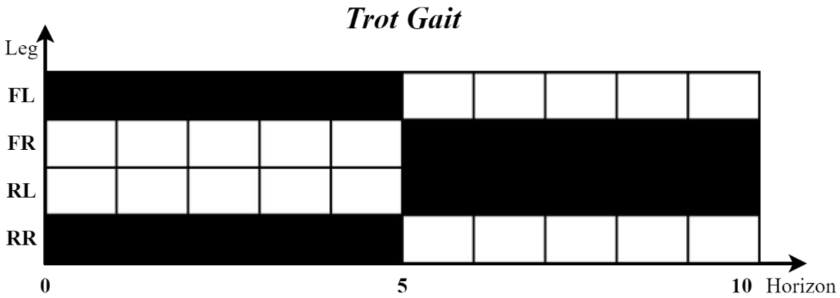
$$\begin{aligned}
\mathbf{x}_{k+1} &= \mathbf{A}_d \mathbf{x}_k + \mathbf{B}_d \mathbf{u}_k \\
\mathbf{A}_d &= e^{\mathbf{A}T} \\
\mathbf{B}_d &= \int_0^T e^{\mathbf{A}\tau}\, d\tau \mathbf{B}
\end{aligned}$$

MPC Formulation

❖ Convex Model Predictive Control Problem

$$\min_{\mathbf{x},\mathbf{u}} \quad \sum_{i=0}^{k-1} ||\mathbf{x}_{i+1} - \mathbf{x}_{i+1,\text{ref}}||_{\mathbf{Q}_i} + ||\mathbf{u}_i||_{\mathbf{R}_i}$$

$$\begin{aligned} \text{subject to} \quad \mathbf{x}_{i+1} &= \mathbf{A}_i\mathbf{x}_i + \mathbf{B}_i\mathbf{u}_i, i = 0 \ldots k-1 \\ \underline{\mathbf{c}}_i \leq \mathbf{C}_i\mathbf{u}_i &\leq \overline{\mathbf{c}}_i, i = 0 \ldots k-1 \\ \mathbf{D}_i\mathbf{u}_i &= 0, i = 0 \ldots k-1 \end{aligned}$$

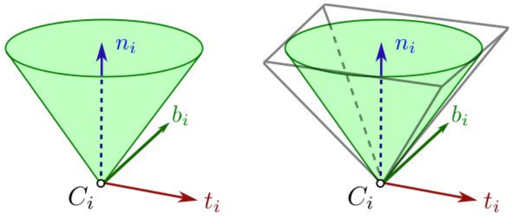


❖ Constraints

➢ Equality constraints: set all forces from feet off the ground to zero, enforcing the desired gait.

➢ Inequality constraints: $f_{\min} \leq f_z \leq f_{\max}$

- max z-force $-\mu f_z \leq \pm f_x \leq \mu f_z$
- friction cone $-\mu f_z \leq \pm f_y \leq \mu f_z$



Di Carlo, Jared, et al. "Dynamic locomotion in the mit cheetah 3 through convex model-predictive control." *2018 IEEE/RSJ international conference on intelligent robots and systems (IROS)*. IEEE, 2018.

QP Formulation

❖ Batch Formulation

$$\mathbf{x_k} = \mathbf{A}^k\mathbf{x}_0 + \sum_{i=0}^{k-1} \mathbf{A}^{k-1-i}\mathbf{B}\mathbf{u}_i$$

$$\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_k \end{bmatrix} = \begin{bmatrix} \mathbf{A}^1 \\ \mathbf{A}^2 \\ \vdots \\ \mathbf{A}^k \end{bmatrix} \mathbf{x}_0 + \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{B} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{AB} & \mathbf{B} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{A}^{k-1}\mathbf{B} & \mathbf{A}^{k-1}\mathbf{B} & \mathbf{0} & \cdots & \mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{u}_0 \\ \mathbf{u}_1 \\ \vdots \\ \mathbf{u}_{k-1} \end{bmatrix}$$

Therefore,

$$\mathrm{X} = \mathrm{A}_{\mathrm{qp}}\mathrm{x}_0 + \mathrm{B}_{\mathrm{qp}}\mathrm{U}$$

where $\mathrm{X} \in \mathbb{R}^{13k}$ and $\mathrm{U} \in \mathbb{R}^{3nk}$

❖ Rewrite the objective function

$$J(\mathrm{U}) = ||\mathrm{A}_{\mathrm{qp}}\mathrm{x}_0 + \mathrm{B}_{\mathrm{qp}}\mathrm{U} - \mathrm{x}_{\mathrm{ref}}||_{\mathrm{L}} + ||\mathrm{U}||_{\mathrm{K}}$$

❖ Convert to convex QP

$$\begin{array}{ll} \min_{\mathrm{U}} & \frac{1}{2}\mathrm{U}^\top \mathbf{H}\mathrm{U} + \mathrm{U}^\top \mathbf{g} \\ \text{s. t.} & \underline{\mathbf{c}} \leq \mathbf{C}\mathrm{U} \leq \overline{\mathbf{c}} \end{array}$$

where

$$\mathbf{H} = 2(\mathrm{B}_{\mathrm{qp}}^\top \mathrm{L}\mathrm{B}_{\mathrm{qp}} + \mathrm{K})$$

$$\mathbf{g} = 2\mathrm{B}_{\mathrm{qp}}^\top \mathrm{L}(\mathrm{A}_{\mathrm{qp}}\mathrm{x}_0 - \mathbf{y})$$

MPC Solver Implementation

QP Solver	Language	Binding	Run Time	Total Time	Choice
CVXOPT	Python	N/A	100 ms	N/A	❌
OSQP	Python	N/A	10 ms	15 ms	❌
OSQP-Eigen	C++	pybind11	1.33 ms	28 ms	❌
qpOASES	C	pybind11	< 1 ms	< 2 ms	✔

Refer to MIT's Implmnt (CVXOPT, OSQP, OSQP-Eigen)

Refer to Yuxiang's Implmnt → qpOASES

Di Carlo, Jared, et al. "Dynamic locomotion in the mit cheetah 3 through convex model-predictive control." *2018 IROS*. IEEE, 2018.
Yang, Yuxiang, et al. "Fast and efficient locomotion via learned gait transitions." Conference on Robot Learning. PMLR, 2022.

Swing Leg Control

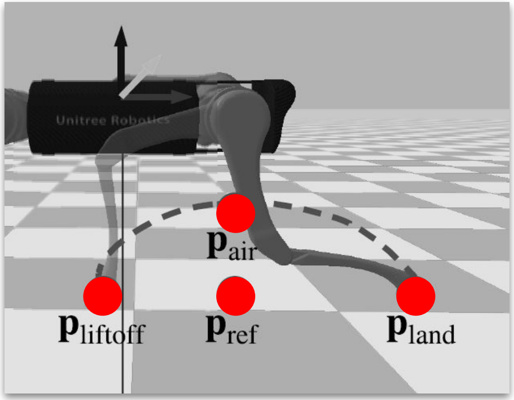
❖ PD Feedback Control:

$$\boldsymbol{\tau}_i = \mathbf{J}_i^T \left[\mathbf{K}_p \left({}^H\mathbf{p}_{i,ref} - {}^H\mathbf{p}_i\right) + \mathbf{K}_d \left({}^H\mathbf{v}_{i,ref} - {}^H\mathbf{v}_i\right)\right]$$

It plans a trajectory using a **cubic Bezier spline**, then uses position feedback control for tracking.

❖ Foot Placement – Raibert heuristic

$$p_{\text{land}} = p_{\text{ref}} + v_{\text{CoM}} T_{\text{stance}} / 2$$

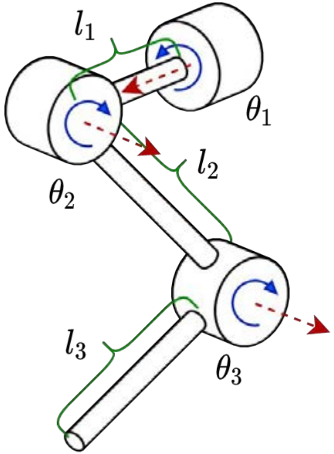


Single Leg Kinematics

- Homogeneous Transformation Matrix

$$\mathbf{H} = Rot_{x,\theta_1} Trans_{y,l_1} Rot_{y,\theta_2} Trans_{z,l_2} Rot_{y,\theta_3} Trans_{z,l_3}$$

$$= \begin{bmatrix} c_{23} & 0 & s_{23} & l_3 s_{23} + l_2 s_2 \\ -s_{23}s_1 & -c_1 & c_{23}s_1 & l_1 c_1 + l_3 s_1 c_{23} + l_2 c_2 s_1 \\ s_{23}c_1 & -s_1 & -c_{23}c_1 & l_1 s_1 - l_3 c_1 c_{23} - l_2 c_2 c_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



- Forward Kinematics

$$\mathbf{p} = \begin{bmatrix} l_3 s_{23} + l_2 s_2 \\ l_1 c_1 + l_3 s_1 c_{23} + l_2 c_2 s_1 \\ l_1 s_1 - l_3 c_1 c_{23} - l_2 c_2 c_1 \end{bmatrix}$$

- Velocity Jacobian Matrix

$$\mathbf{v} = \dot{\mathbf{p}} = J\dot{\boldsymbol{\theta}}$$

$$J = \begin{bmatrix} 0 & l_3 c_{23} + l_2 c_2 & l_3 c_{23} \\ -l_1 s_1 + l_3 c_1 c_{23} + l_2 c_2 c_1 & -l_3 s_1 s_{23} - l_2 s_2 s_1 & -l_3 s_1 s_{23} \\ l_1 c_1 + l_3 s_1 c_{23} l_2 c_2 s_1 & l_3 c_1 s_{23} + l_2 s_2 c_1 & l_3 c_1 s_{23} \end{bmatrix}$$

Ground Slope Estimation

- Use measurements of each footstep location:

$$\boldsymbol{p}_i = (p_i^x, p_i^y, p_i^z)$$

- To approximate the local slope of the walking surface:

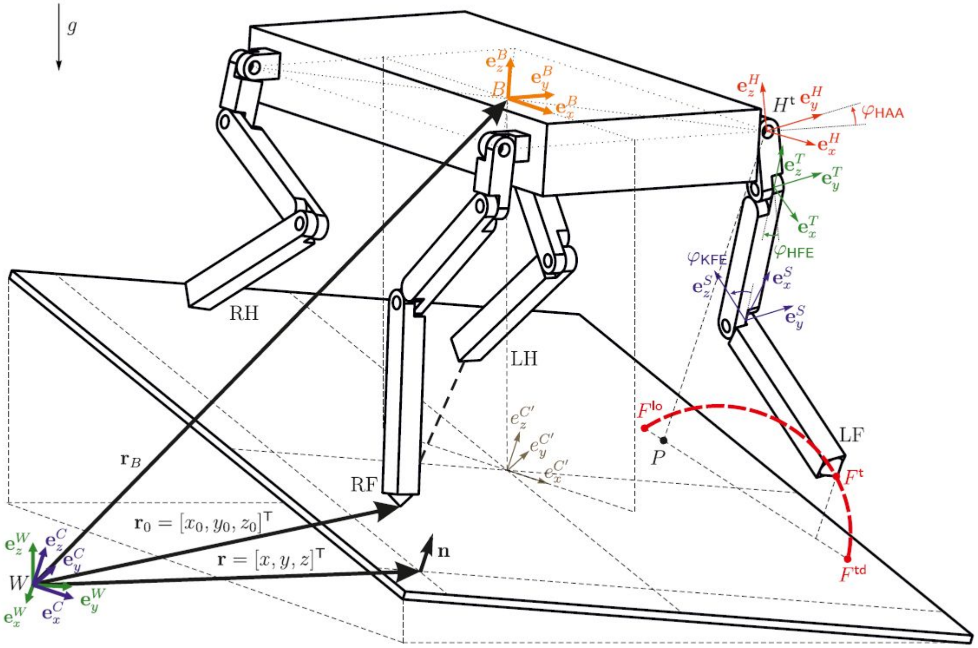
$$z(x, y) = a_0 + a_1 x + a_2 y$$

- Coefficients $\boldsymbol{a}$ are obtained through least squares:

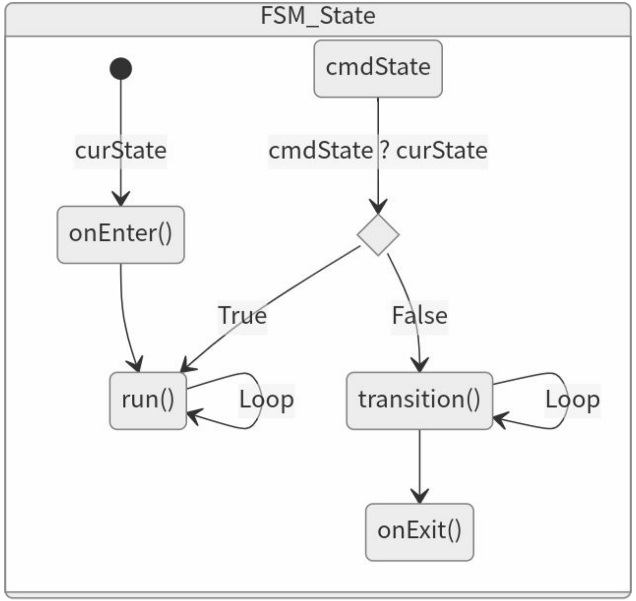
$$\boldsymbol{a} = \left(\boldsymbol{W}^T \boldsymbol{W}\right)^{\dagger} \boldsymbol{W}^T \boldsymbol{p}^z$$

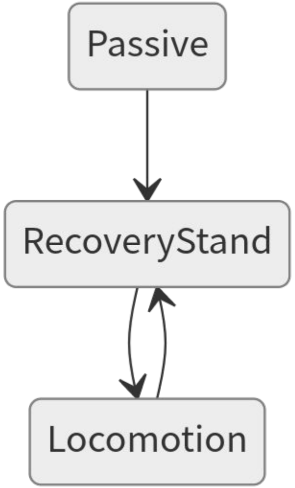
$$\boldsymbol{W} = \begin{bmatrix} \mathbf{1} & \boldsymbol{p}^x & \boldsymbol{p}^y \end{bmatrix}_{4\times 3}$$

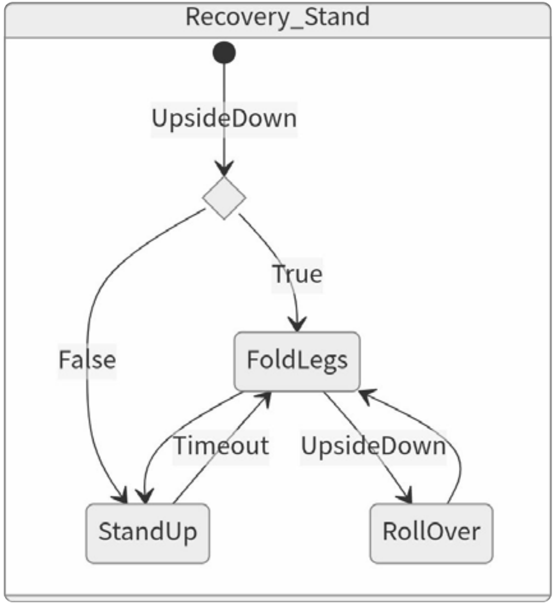
where $\boldsymbol{p}^x = (p_1^x, p_2^x, p_3^x, p_4^x)$



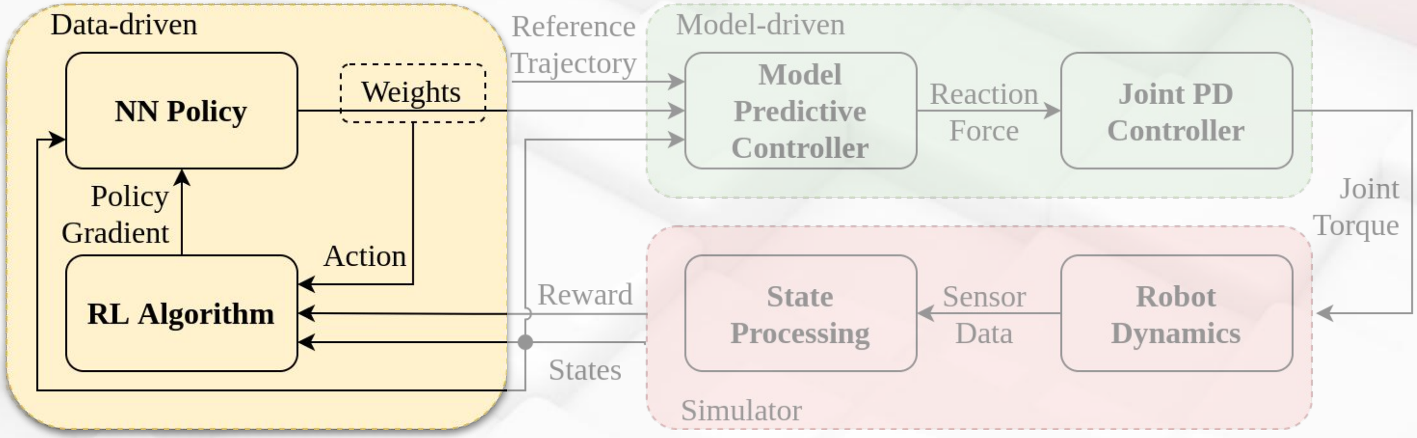
Finite State Machine







Deep RL Training Framework



RL Framework

Simulator: NVIDIA Isaac Gym

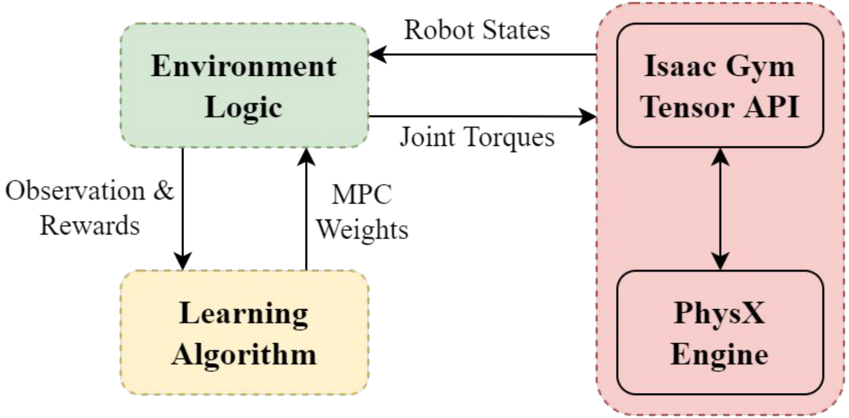
RL Algorithm: PPO (A-C Style)

Policy Network: 3-Layer MLP

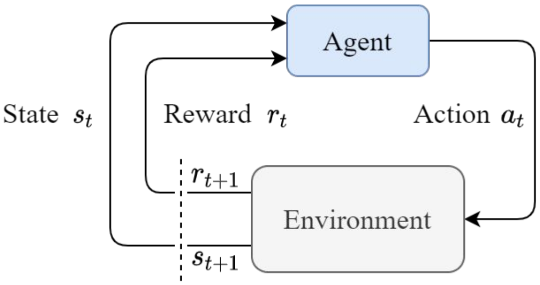
Device: NVIDIA GTX 1060

Parallelism:

- ➢ $B = n_{\text{robots}} n_{\text{steps}}$ (trade off between efficiency and performance)
- ➢ Reset based on a time out breaks the infinite horizon assumption of PPO



RL Algorithm



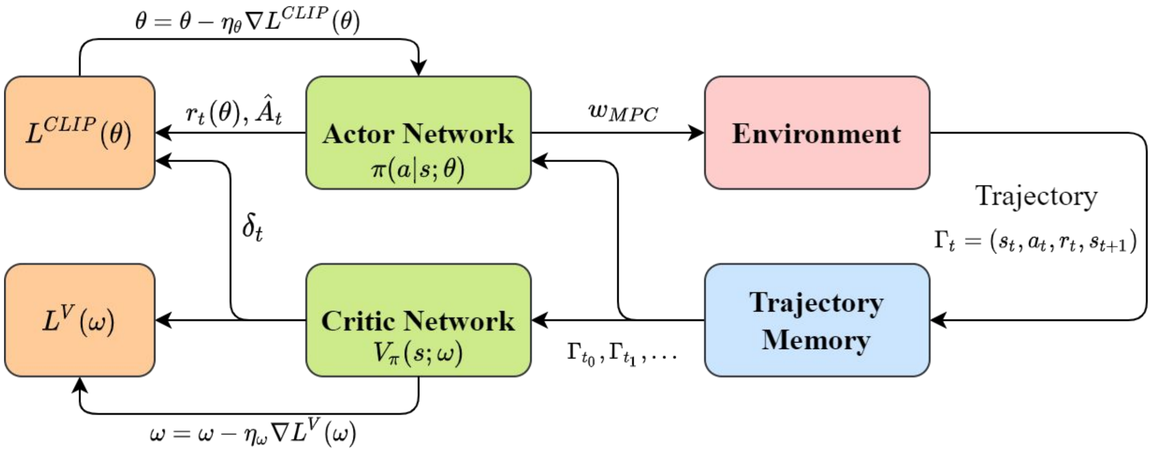
$$\pi(a|s) = \mathbf{P}(a_t = a | s_t = s)$$

$$R_t = \Sigma_{k=0}^{\infty} \gamma^k r_{t+k}, \ \gamma \in (0, 1]$$

$$V_\pi(s) = \mathbf{E}\left[R_t | s_t = s\right]$$

❖ Proximal Policy Optimization

$$L^{CLIP}(\theta) = \hat{\mathbf{E}}\left[\min\left(r_t(\theta)\hat{A}_t, \text{clip}\left(r_t(\theta), 1-\epsilon, 1+\epsilon\right)\hat{A}_t\right)\right]$$

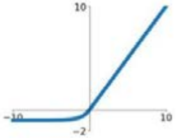


Policy & Observation & Action

- MLP (Multilayer Perceptron)
  - Input Dim: 48
  - Output Dim: 12
  - Hidden Unit: [256, 128, 64]
  - Activation:

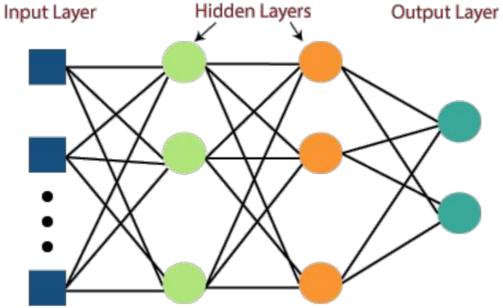
ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Observation Space	Dimensions
COM position	3
COM velocity	3
COM Euler angle	3
COM angular velocity	3
DOF position	12
DOF velocity	12
Actions	12
Total	**48**

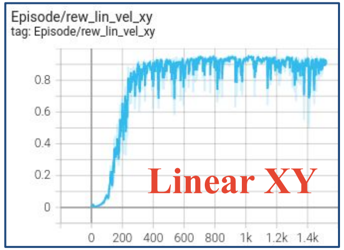
Action Space	Dimensions
Position weights	3
Velocity weights	3
Euler Angle weights	3
Angular Velocity weights	3
Total	**12**

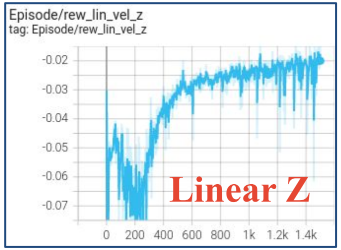


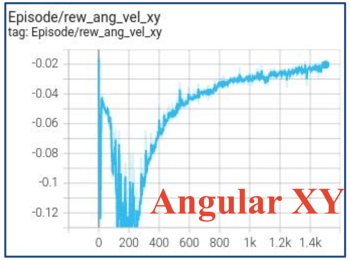
Observation and Reward Design

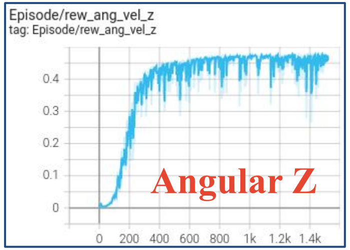
Observation space		Degrees of freedom
Base velocity	positional	3
	angular	3
Body-relative gravity		3
Target X, Y, yaw velocities		3
DOF states	position	12
	velocity	12
Actions		12
Total number of observations		48

Reward	Definition	Weight
Linear velocity tracking	$\phi(\mathbf{v}^*_{b,xy} - \mathbf{v}_{b,xy})$	$1dt$
Angular velocity tracking	$\phi(\boldsymbol{\omega}^*_{b,z} - \boldsymbol{\omega}_{b,z})$	$0.5dt$
Linear velocity penalty	$-\mathbf{v}^2_{b,z}$	$4dt$
Angular velocity penalty	$-\|\|\boldsymbol{\omega}_{b,xy}\|\|^2$	$0.05dt$
Joint motion	$-\|\|\ddot{\mathbf{q}}_j\|\|^2 - \|\|\dot{\mathbf{q}}_j\|\|^2$	$0.001dt$
Joint torques	$-\|\|\boldsymbol{\tau}_j\|\|^2$	$0.00002dt$
Action rate	$-\|\|\dot{\mathbf{q}}^*_j\|\|^2$	$0.25dt$
Collisions	$-n_{collision}$	$0.001dt$









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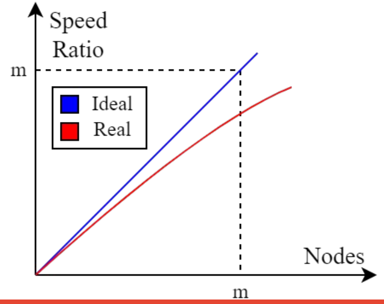
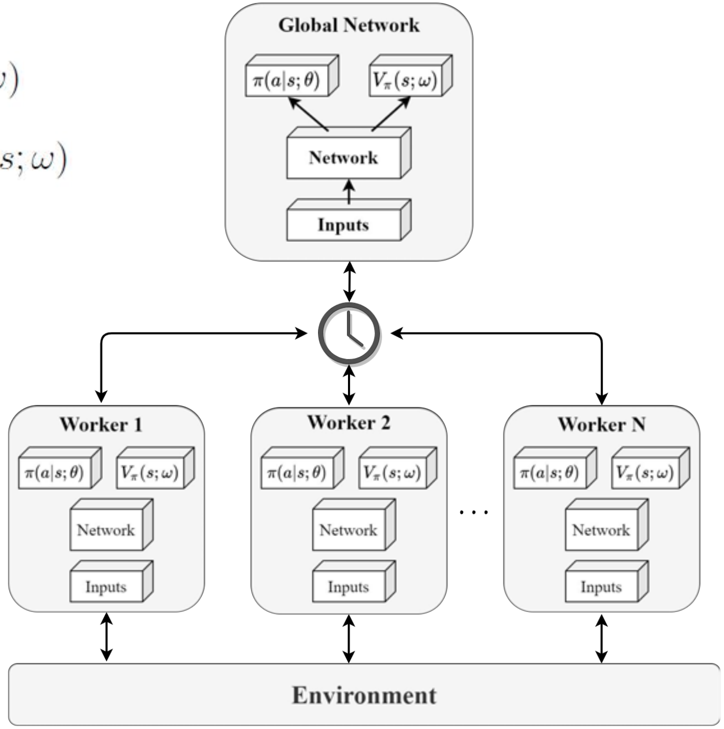
Makoviychuk, Viktor, et al. "Isaac gym: High performance gpu-based physics simulation for robot learning."

Parallel RL Training - Part 1

- Global network maintain $\pi(a|s;\theta)$ and $V_\pi(s;\omega)$
- Each worker have a copy of $\pi(a|s;\theta)$ and $V_\pi(s;\omega)$
- Each worker interact the environment ***n*** steps to gain experience and calc gradients
- Global network update $\pi(a|s;\theta)$ and $V_\pi(s;\omega)$ after receiving **all** gradients from workers

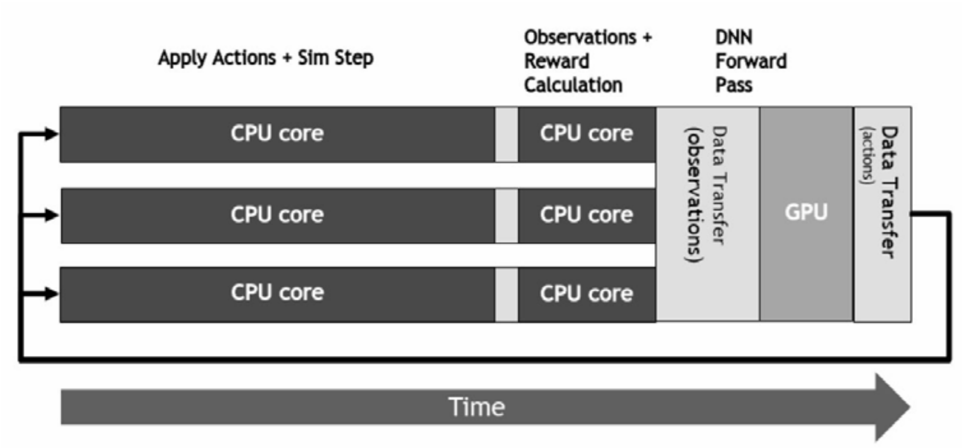




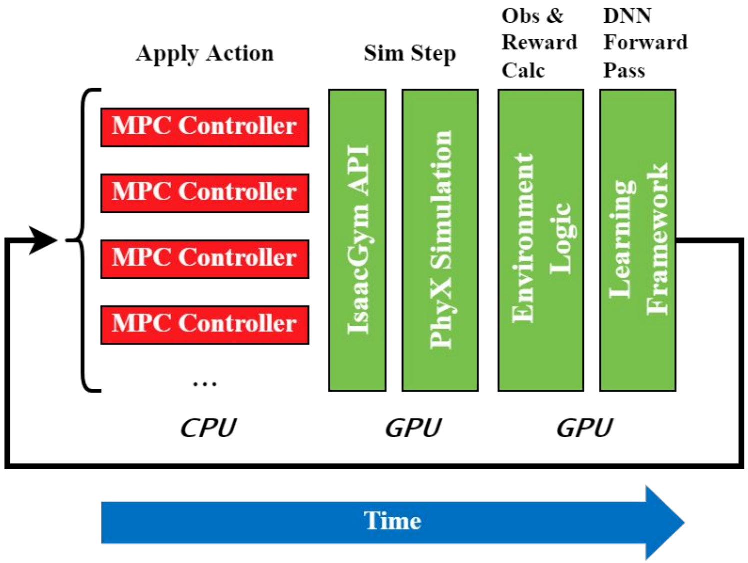
Mnih V, Badia A P, Mirza M, et al. Asynchronous methods for deep reinforcement learning[C]//International conference on machine learning. PMLR, 2016: 1928-1937.

Parallel RL Training - Part 2

- Traditional RL Experience Collection



★ Isaac Gym + MPC Experience Collection



Makoviychuk, Viktor, et al. "Isaac gym: High performance gpu-based physics simulation for robot learning."

Conclusions and Future Work

★ Proposed and implemented a hierarchical control architecture for quadruped

- ◆ Periodic gait such as walking and trotting can be done with simple tuning
- ◆ Sim2Real transfer on real Aliengo robot

➔ Controller Performance :

- ◆ 3 m/s Vx
- ◆ 2 m/s Vy
- ◆ 5 rad/s ω
- ◆ 0.04 rad max orientation deviation
- ◆ 1 ms policy inference

➔ Cons :

- ◆ Insufficient network training steps
- ◆ Only open loop gaits are using
- ◆ No external sensors like camera

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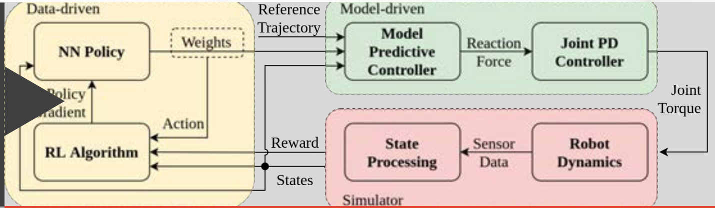
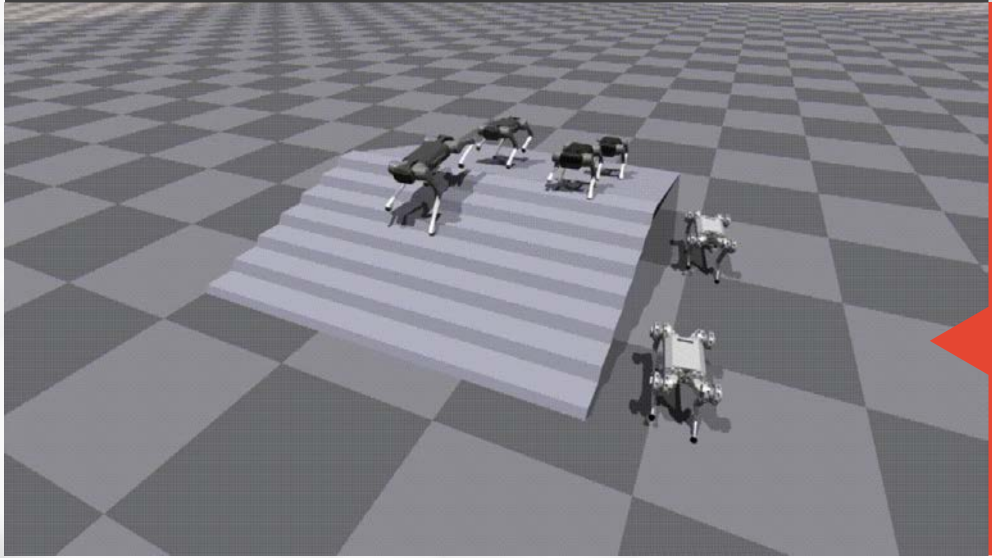
Thanks for Listening

Yulun Zhuang

Quadruped Ctrl and Learn FWK based on IsaacGym

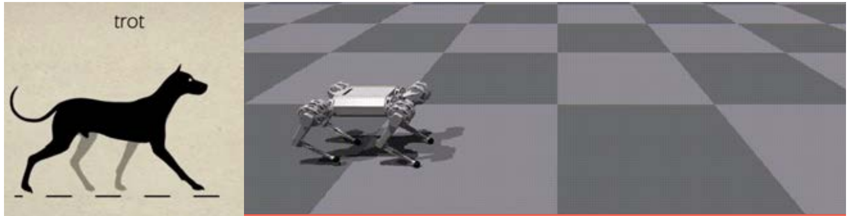




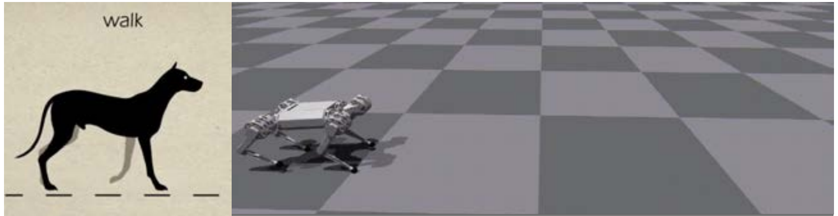
Parallel MPC Control Demo

Simulation:	1000 Hz
Control:	500 Hz
MPC:	50 Hz

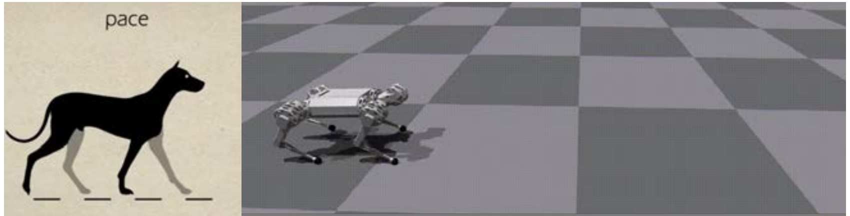
Locomotion Gaits



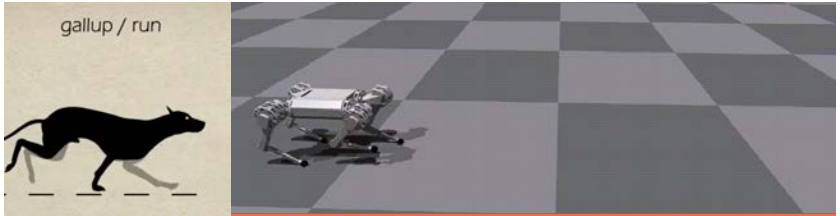
Trot



Walk



Pace



Gallop