Some definitions

- Center-of-momentum gamma

$$\gamma_C = \frac{E}{E_C} = \frac{m_1\gamma_1 + m_2\gamma_2}{m_1\gamma_1^* + m_2\gamma_2^*}$$

- Center-of-momentum reduced mass

$$\mu_R^* = \frac{m_1\gamma_1^* m_2\gamma_2^*}{m_1\gamma_1^* + m_2\gamma_2^*},$$

- Relative velocity in center-of-momentum frame (not Lorentz invariant)

$$v_{\text{rel}}^* = |\mathbf{v}_1^* - \mathbf{v}_2^*| = \frac{p_1^*}{\mu_R^*},$$

- Lorentz-invariant relative velocity computed in the center-of-momentum frame

$$v_{\text{rel,invar}}^* = \frac{|\mathbf{v}_1^* - \mathbf{v}_2^*|}{1 - \mathbf{v}_1^* \cdot \mathbf{v}_2^*/c^2}$$

$$= v_{\text{rel}}^* / \left(1 + \frac{(p_1^*)^2}{m_1\gamma_1^* m_2\gamma_2^* c^2}\right)$$

Superscript * referrers to quantities computed in the center-of-momentum frame

Simplification of last two expressions in Eq. 9 of Perez 2012

$$\frac{\gamma_C p_1^*}{m_1\gamma_1 + m_2\gamma_2} = \frac{p_1^*}{m_1\gamma_1^* + m_2\gamma_2^*} = \frac{p_1^*}{\mu_R^*}\frac{m_1\gamma_1^* m_2\gamma_2^*}{\left(m_1\gamma_1^* + m_2\gamma_2^*\right)^2},$$

$$\left(\frac{m_1\gamma_1^* m_2\gamma_2^*}{\left(p_1^*\right)^2}c^2 + 1\right)^2 = \frac{\left(m_1\gamma_1^* m_2\gamma_2^*\right)^2}{\left(p_1^*\right)^4}c^4\left(1 + \frac{\left(p_1^*\right)^2}{m_1\gamma_1^* m_2\gamma_2^* c^2}\right)^2$$

$$\Rightarrow \frac{\gamma_C p_1^*}{m_1\gamma_1 + m_2\gamma_2} \times \left(\frac{m_1\gamma_1^* m_2\gamma_2^*}{\left(p_1^*\right)^2}c^2 + 1\right)^2 = \frac{p_1^*}{\mu_R^*}\frac{m_1\gamma_1^* m_2\gamma_2^* \left(\mu_R^*\right)^2}{\left(p_1^*\right)^4}c^4\left(1 + \frac{\left(p_1^*\right)^2}{m_1\gamma_1^* m_2\gamma_2^* c^2}\right)^2$$

$$= v_{\mathrm{rel}}^* \frac{m_1\gamma_1^* m_2\gamma_2^* c^4}{\left(\mu_R^* v_{\mathrm{rel}}^* v_{\mathrm{rel,invar}}^*\right)^2}$$

Eq. 9 of Perez is same as expression used for s12 in Coulomb collision method

Eq. 9 of Perez

$$s_{12} = \frac{(q_1 q_2)^2 \ln \Lambda}{4\pi\epsilon_0^2 c^4 m_1\gamma_1 m_2\gamma_2} \frac{\gamma_C p_1^*}{m_1\gamma_1 + m_2\gamma_2} \times \left(\frac{m_1\gamma_1^* m_2\gamma_2^*}{(p_1^*)^2} c^2 + 1 \right)^2 n_{12}\Delta t$$

$$= \frac{(q_1 q_2)^2 \ln \Lambda}{4\pi\epsilon_0^2 c^4 m_1\gamma_1 m_2\gamma_2} v_{\mathrm{rel}}^* \frac{m_1\gamma_1^* m_2\gamma_2^* c^4}{\left(\mu_R^* v_{\mathrm{rel}}^* v_{\mathrm{rel,invar}}^*\right)^2} n_{12}\Delta t$$

$$= \pi \left(\frac{q_1 q_2}{2\pi\epsilon_0 \mu_R^* v_{\mathrm{rel}}^* v_{\mathrm{rel,invar}}^*} \right)^2 \ln \Lambda v_{\mathrm{rel}}^* \frac{\gamma_1^*\gamma_2^*}{\gamma_1\gamma_2} n_{12}\Delta t$$

$$= \pi b_0^2 \ln \Lambda v_{\mathrm{rel}}^* \frac{\gamma_1^*\gamma_2^*}{\gamma_1\gamma_2} n_{12}\Delta t$$

s12 in WarpX