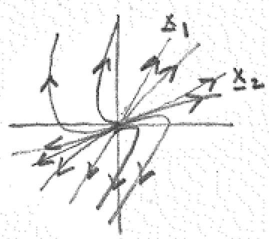
3. Types of critical points for 2d systems

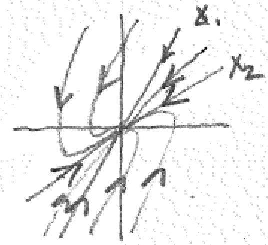
We can use the eigenvalues and eigenvectors to anticipate what solutions around critical points ($\underline{y}' = \underline{0}$, steady states) look like. This will be really useful for controls!

If the eigenvalues are real and distinct:



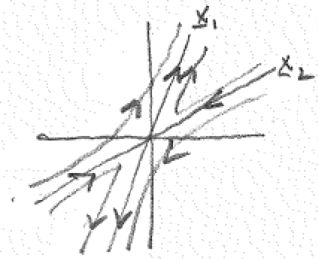
$\lambda_1 > \lambda_2 > 0$

improper node
(unstable)



$\lambda_1 < \lambda_2 < 0$

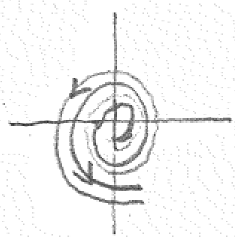
improper node
(stable)



$\lambda_1 > 0$, $\lambda_2 < 0$

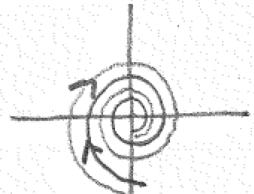
saddle

If the eigenvalues are complex $\lambda_{1,2} = \alpha \pm \omega i$:



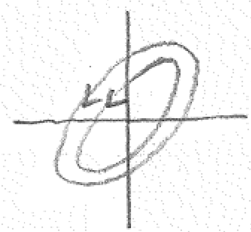
$\alpha > 0$

spiral
(unstable)



$\alpha < 0$

spiral
(stable)

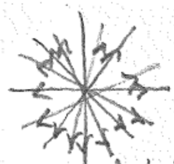


$\alpha = 0$

center

The direction depends on the matrix and can be checked for some point (e.g. $(1,0)$). If $\underline{\underline{A}} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $b-c>0 \rightarrow$ clockwise
$b-c<0 \rightarrow$ counterclockwise

If the eigenvalues are <u>real</u> and <u>repeated</u> ($\lambda_1 = \lambda_2$):



proper node / star

$\underline{\underline{A}}$ is multiple of $\underline{\underline{I}}$

degenerate node

Direction depends on $\underline{\underline{A}}$ as for complex

Unstable vs. stable
$\lambda>0$ $\quad$ $\lambda<0$