Properties of the Back-vector Modification for Microfacet BRDF Models

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1 Microfacet BRDF Models

Using the notations from figure 1, a microfacet BRDF [WMLT07, Hei14] takes the form

$$f(\mathbf{v},\mathbf{l}) = \frac{D(\mathbf{h})G_2(\mathbf{v},\mathbf{l})}{4\langle\mathbf{v},\mathbf{n}\rangle\langle\mathbf{l},\mathbf{n}\rangle} \tag{1}$$

where D is the distribution of microfacets, normalized to fulfill $\int_\Omega D(\mathbf{h})\langle\mathbf{n},\mathbf{h}\rangle\, d\mathbf{h} = 1$, and G_2 is the shadowing-masking function. The shadowing-masking function combines the visibility masking function $G_1(\mathbf{v},\mathbf{l})$ for view direction and $G_1(\mathbf{l},\mathbf{v})$ for light direction:

$$G_2(\mathbf{v},\mathbf{l}) = G_2\left(G_1(\mathbf{v},\mathbf{l}), G_1(\mathbf{l},\mathbf{v})\right) \tag{2}$$

Mathematically, the BRDF is plausible if G_2 is symmetric w.r.t. to exchanging $\mathbf{v}$ and $\mathbf{l}$, fulfills $G_2(\mathbf{v},\mathbf{l}) \leq G_1(\mathbf{v},\mathbf{l})$, and if G_1 further establishes the correct projected area of visible micro-surfaces [Hei14]

$$\langle\mathbf{v},\mathbf{n}\rangle = \int_{\{\mathbf{h}\in\Omega:\langle\mathbf{v},\mathbf{h}\rangle\geq 0\}} D(\mathbf{h})G_1(\mathbf{v},\mathrm{reflect}(\mathbf{v},\mathbf{h}))\langle\mathbf{v},\mathbf{h}\rangle\, d\mathbf{h}. \tag{3}$$

Choices for G_1 with this property are the Smith masking function

$$G_1(\mathbf{v},\mathbf{l}) = G_{1,\mathrm{Smith}}(\mathbf{v}) = \frac{\langle\mathbf{v},\mathbf{n}\rangle}{\int_{\{\mathbf{h}\in\Omega:\langle\mathbf{v},\mathbf{h}\rangle\geq 0\}} D(\mathbf{h})\langle\mathbf{v},\mathbf{h}\rangle\, d\mathbf{h}}, \tag{4}$$

where analytic expressions are available for Beckmann and GGX distributions, and the v-cavity masking function

$$G_1(\mathbf{v},\mathbf{l}) = G_{1,\mathrm{vc}}(\langle\mathbf{v},\mathbf{n}\rangle, \langle\mathbf{n},\mathbf{h}\rangle, \langle\mathbf{v},\mathbf{h}\rangle) = \min\left\{\frac{2\langle\mathbf{v},\mathbf{n}\rangle\langle\mathbf{n},\mathbf{h}\rangle}{\langle\mathbf{v},\mathbf{h}\rangle}, 1\right\}, \tag{5}$$

which is generally applicable for distributions with symmetry around the normal.

$\mathbf{v}$	normalized vector to viewer
$\mathbf{l}$	normalized vector to light
$\mathbf{n}$	surface normal
$\mathbf{t_1}, \mathbf{t_2}$	tangent and bitangent (forming an orthonormal coordinate frame with the normal)
$\mathbf{h}$	half vector, $\mathbf{h} = \mathbf{h}(\mathbf{v},\mathbf{l}) = \frac{\mathbf{v}+\mathbf{l}}{\Vert\mathbf{v}+\mathbf{l}\Vert}$
$\langle\mathbf{x},\mathbf{y}\rangle$	dot product of $\mathbf{x}$ and $\mathbf{y}$
$\mathrm{reflect}(\mathbf{x},\mathbf{m})$	reflection of vector $\mathbf{x}$ on surface with normal $\mathbf{m}$, i.e. $\mathrm{reflect}(\mathbf{x},\mathbf{m}) = -\mathbf{x} + 2\langle\mathbf{x},\mathbf{m}\rangle\mathbf{m}$
Ω	hemisphere of directions around the normal

Figure 1: Notations used.

2 Back-vector Modification

If instead of half-vector $\mathbf{h}$ the back-vector $\mathbf{b}$, defined as

$$\mathbf{b}(\mathbf{v},\mathbf{l}) = \mathbf{h}(\mathbf{v}',\mathbf{l}) = \frac{\mathbf{v}'+\mathbf{l}}{\|\mathbf{v}'+\mathbf{l}\|}, \text{ with } \mathbf{v}' = \text{reflect}(\mathbf{v},\mathbf{n}), \tag{6}$$

is used in equation (1), the resulting BRDF is retro-reflective [BPD+14]. Note that instead of explicitly defining a BRDF using the back-vector, we can simply use $\mathbf{v}'$ instead of $\mathbf{v}$, i.e. we define $f_{\text{retro}}(\mathbf{v},\mathbf{l}) := f(\mathbf{v}',\mathbf{l})$ such that the half-vector becomes the back-vector. In the following we show that this BRDF is plausible, i.e. symmetry and energy conservation are guaranteed.

3 Plausibility of the Back-vector Modification

In analogy to $\mathbf{v}'$ we let $\mathbf{l}' := \text{reflect}(\mathbf{l},\mathbf{n})$. First we observe

$$\begin{aligned} \langle \mathbf{v}',\mathbf{n}\rangle &= \langle \mathbf{v},\mathbf{n}\rangle \text{ and} \\ \langle \mathbf{l},\mathbf{n}\rangle &= \langle \mathbf{l}',\mathbf{n}\rangle . \end{aligned} \tag{7}$$

For the back-vector we show $\mathbf{h}(\mathbf{v}',\mathbf{l}) = \text{reflect}\left(\mathbf{h}(\mathbf{l}',\mathbf{v}),\mathbf{n}\right)$, as

$$\begin{aligned} \mathbf{v}'+\mathbf{l} &= (-\mathbf{v}+2\langle \mathbf{v},\mathbf{n}\rangle \mathbf{n}) + \mathbf{l} \\ &= (-\mathbf{v}+2\langle \mathbf{v},\mathbf{n}\rangle \mathbf{n}) + (-\mathbf{l}'+2\langle \mathbf{l}',\mathbf{n}\rangle \mathbf{n}) \\ &= -(\mathbf{l}'+\mathbf{v}) + 2\langle (\mathbf{l}'+\mathbf{v}),\mathbf{n}\rangle \mathbf{n} = \text{reflect}(\mathbf{l}'+\mathbf{v},\mathbf{n}) \end{aligned} \tag{8}$$

and therefore $\|\mathbf{v}'+\mathbf{l}\| = \|\mathbf{l}'+\mathbf{v}\|$ as well. From that we can conclude

$$\begin{aligned} \langle \mathbf{n},\mathbf{h}(\mathbf{v}',\mathbf{l})\rangle &= \langle \mathbf{n},\mathbf{h}(\mathbf{l}',\mathbf{v})\rangle \\ \langle \mathbf{t_1},\mathbf{h}(\mathbf{v}',\mathbf{l})\rangle &= -\langle \mathbf{t_1},\mathbf{h}(\mathbf{l}',\mathbf{v})\rangle \\ \langle \mathbf{t_2},\mathbf{h}(\mathbf{v}',\mathbf{l})\rangle &= -\langle \mathbf{t_2},\mathbf{h}(\mathbf{l}',\mathbf{v})\rangle . \end{aligned} \tag{9}$$

As anisotropic Phong, Beckmann, and GGX distributions are reflection-symmetric around the normal, i.e. they take the form $D(\mathbf{h}) = D\left(\langle \mathbf{n},\mathbf{h}\rangle, |\langle \mathbf{t_1},\mathbf{h}\rangle|, |\langle \mathbf{t_2},\mathbf{h}\rangle|\right)$, we can thus infer that using them with the back-vector obeys symmetry. Further, the reflection symmetry of D along with equation (7) gives

$$\begin{aligned} G_{1,\text{Smith}}(\mathbf{v}) &= \frac{\langle \mathbf{v},\mathbf{n}\rangle}{\int_{\{\mathbf{h}\in\Omega:\langle \mathbf{v},\mathbf{h}\rangle\geq 0\}} D(\mathbf{h})\langle \mathbf{v},\mathbf{h}\rangle\, d\mathbf{h}} \\ &= \frac{\langle \mathbf{v}',\mathbf{n}\rangle}{\int_{\{\mathbf{h}\in\Omega:\langle \mathbf{v}',\mathbf{h}\rangle\geq 0\}} D(\mathbf{h})\langle \mathbf{v}',\mathbf{h}\rangle\, d\mathbf{h}} = G_{1,\text{Smith}}(\mathbf{v}') \end{aligned} \tag{10}$$

for Smith-masking. Next we show

$$\langle \mathbf{v}',\mathbf{l}\rangle = -\langle \mathbf{v},\mathbf{l}\rangle + 2\langle \mathbf{v},\mathbf{n}\rangle\langle \mathbf{l},\mathbf{n}\rangle = \langle \mathbf{v},-\mathbf{l}+2\langle \mathbf{l},\mathbf{n}\rangle \mathbf{n}\rangle = \langle \mathbf{v},\mathbf{l}'\rangle \tag{11}$$

and therefore

$$\langle \mathbf{v}',\mathbf{h}(\mathbf{v}',\mathbf{l})\rangle = \frac{1+\langle \mathbf{l},\mathbf{v}'\rangle}{\|\mathbf{v}'+\mathbf{l}\|} = \frac{1+\langle \mathbf{l}',\mathbf{v}\rangle}{\|\mathbf{l}'+\mathbf{v}\|} = \langle \mathbf{l}',\mathbf{h}(\mathbf{l}',\mathbf{v})\rangle . \tag{12}$$

Using this along with equations (9) and (7) we get

$$\begin{aligned} G_1(\mathbf{v}',\mathbf{l}) &= G_{1,\text{vc}}\left(\langle \mathbf{v}',\mathbf{n}\rangle, \langle \mathbf{n},\mathbf{h}(\mathbf{v}',\mathbf{l})\rangle, \langle \mathbf{v}',\mathbf{h}(\mathbf{v}',\mathbf{l})\rangle\right) \\ &= G_{1,\text{vc}}\left(\langle \mathbf{v},\mathbf{n}\rangle, \langle \mathbf{n},\mathbf{h}(\mathbf{l}',\mathbf{v})\rangle, \langle \mathbf{l}',\mathbf{h}(\mathbf{l}',\mathbf{v})\rangle\right) \\ &= G_{1,\text{vc}}\left(\langle \mathbf{v},\mathbf{n}\rangle, \langle \mathbf{n},\mathbf{h}(\mathbf{v},\mathbf{l}')\rangle, \langle \mathbf{l}',\mathbf{h}(\mathbf{v},\mathbf{l}')\rangle\right) \\ &= G_1(\mathbf{v},\mathbf{l}') \end{aligned} \tag{13}$$

and similarly $G_1(\mathbf{l}, \mathbf{v}') = G_1(\mathbf{l}', \mathbf{v})$ for the v-cavity masking function. In summary, we can conclude the symmetry of the BRDF

$$\begin{aligned} f_{\text{retro}}(\mathbf{v}, \mathbf{l}) = f(\mathbf{v}', \mathbf{l}) &= \frac{D(\mathbf{h}(\mathbf{v}', \mathbf{l}))G_2(G_1(\mathbf{v}', \mathbf{l}), G_1(\mathbf{l}, \mathbf{v}'))}{4 \langle \mathbf{v}', \mathbf{n} \rangle \langle \mathbf{l}, \mathbf{n} \rangle} \\ &= \frac{D(\mathbf{h}(\mathbf{l}', \mathbf{v}))G_2(G_1(\mathbf{v}, \mathbf{l}'), G_1(\mathbf{l}', \mathbf{v}))}{4 \langle \mathbf{v}, \mathbf{n} \rangle \langle \mathbf{l}', \mathbf{n} \rangle} = f(\mathbf{l}', \mathbf{v}) = f_{\text{retro}}(\mathbf{l}, \mathbf{v}). \end{aligned} \tag{14}$$

For the albedo we simply have

$$\rho_{\text{retro}}(\mathbf{v}) = \int_\Omega f_{\text{retro}}(\mathbf{v}, \mathbf{l}) \langle \mathbf{l}, \mathbf{n} \rangle \, d\mathbf{l} = \int_\Omega f(\mathbf{v}', \mathbf{l}) \langle \mathbf{l}, \mathbf{n} \rangle \, d\mathbf{l} = \rho(\mathbf{v}') \tag{15}$$

i.e. we get the albedo of the regular microfacet BRDF for the reflected direction. Which, using equation (3) and $\frac{d\mathbf{h}(\mathbf{v}', \mathbf{l})}{d\mathbf{l}} = \frac{1}{4\langle \mathbf{l}, \mathbf{h}(\mathbf{v}', \mathbf{l}) \rangle} = \frac{1}{4\langle \mathbf{v}', \mathbf{h}(\mathbf{v}', \mathbf{l}) \rangle}$ [WMLT07], can be shown to fulfill energy conservation

$$\begin{aligned} \rho(\mathbf{v}') &= \int_\Omega \frac{D(\mathbf{h}(\mathbf{v}', \mathbf{l}))G_2(\mathbf{v}', \mathbf{l})}{4 \langle \mathbf{v}', \mathbf{n} \rangle \langle \mathbf{l}, \mathbf{n} \rangle} \langle \mathbf{l}, \mathbf{n} \rangle \, d\mathbf{l} \\ &\leq \int_{\{\mathbf{h} \in \Omega : \langle \mathbf{v}', \mathbf{h} \rangle \geq 0\}} \frac{D(\mathbf{h})G_1(\mathbf{v}', \text{reflect}(\mathbf{v}', \mathbf{h}))}{\langle \mathbf{v}', \mathbf{n} \rangle} \langle \mathbf{v}', \mathbf{h} \rangle \, d\mathbf{h} = 1. \end{aligned} \tag{16}$$

4 Implementation Notes

Given an implementation of a regular microfacet BRDF, extending it to retro-reflection is straightforward:

- Evaluation merely needs to replace $\mathbf{v}$ with $\mathbf{v}'$ upfront.
- Similarly, importance sampling of $\mathbf{l}$ given $\mathbf{v}$ can be realized by replacing $\mathbf{v}$ with $\mathbf{v}'$ upfront and then importance sampling the regular microfacet BRDF. This may include low variance sampling using the domain of visible microfacets [Hd14].
- As the albedos of standard BRDF and retro-reflective BRDF are essentially identical, compensating for energy loss in the sense of [KSK01] can be realized using the same data tables.

References

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